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Functional Programming

Algebraic datatypes and collections

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Credit where credit is due

These slides are based on original slides by Michael R. Hansen at DTU. Thank you!

The original slides have been used for a functional programming course at DTU

Last week

We covered a substantial part of F#

- Higher-order functions
- Function composition
- Piping commands
- More lists

Folds (loops)

```
fold f acc [x1; x2; …; xn]
```

returns

```
f (… (f (f acc x1) x2) … xn-1) xn
```

```
let acc = fold f init xs
```

vs.

```
acc = init;
for(i = 0; i < xs.Length; i++)
{
    acc = f (acc, xs[i]);
}
```

Folds (loops)

```
let rec fold f acc =
    function
    | []        -> acc
    | x :: xs -> fold f (f acc x) xs

fold : ('b -> 'a -> 'b) -> 'b -> 'a list -> 'b
```

Folds (loops)

```
let rec fold f acc =
    function
    | []       -> acc
    | x :: xs -> fold f (f acc x) xs
```

```
fold : (‘b -> ‘a –> ‘b) -> ‘b -> ‘a list -> ‘b
```

```
let rec fold f acc xs =
    match xs with
    | []       -> acc
    | x :: xs -> fold f (f acc x) xs
```

Folds (loops)

```
let rec fold f acc =
    function
    | []          -> acc
    | x :: xs -> fold f (f acc x) xs
```

```
fold : (‘b -> ‘a —> ‘b) -> ‘b -> ‘a list -> ‘b
```

```
fold (fun acc x -> acc + x) 0 [1; 2; 3; 4] =
```

fold (+) 0 [1; 2; 3; 4] =
(+) ((+) ((+) (((+) 0 1) 2)) 3) 4 =
(((0 + 1) + 2) + 3) + 4 =
10

Folds (loops)

```
let rec fold f acc =
    function
    | []        -> acc
    | x :: xs -> fold f (f acc x) xs
```

```
fold : ('b -> 'a -> 'b) -> 'b -> 'a list -> 'b
```

```
fold (fun acc x -> acc * x] 1 [1; 2; 3; 4] =
```

```
fold (*) 1 [1; 2; 3; 4] =
```

(*) ((*) ((*) (((*) 1 1) 2)) 3) 4 =
(((1 * 1) * 2) * 3) * 4 =
24

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
```

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
 let ??? lst =
     lst |>
     List.tail |>
     fold (fun (y, acc) z -> (z, y <= z && acc))
          (List.head lst, true) |>
     snd
```

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
```

```
let ??? ls
    lst |>
    List.t
    fold (
         (List.head lst, true) |>
    snd
```

List.pairwise $[x_1; \ldots; x_n]$ =
$[(x_1, x_2); (x_2, x_3); \ldots; (x_{n-1}, x_n)]$

```
let ??? lst =
   lst |>
   List.pairwise |>
   List.forall (fun (x, y) -> x <= y)
```

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
let ??? lst =
    lst |>
    List.tail |>
    fold (fun (y, acc) z -> (z, y <= z && acc))
         (List.head lst, true) |>
    snd
let ??? lst =
   lst
   List.pairwise |>
   List.forall (fun (x, y) -> x <= y)
```

Function composition

Functions can be composed
In mathematics we write

$$(f \circ g)(x) = f(g(x))$$

In F# we write

```
g >> f
```

Function composition

For example

Assume $f(x) = x * 3$ and $g(y) = y + y$

$$(f \circ g)(z) = (z + z) * 3$$

In F# we can write

```
let h = (fun y -> y+y) >>
        (fun x -> x*3)
```

```
h 0  = (0 + 0) * 3 = 0
h 4  = (4 + 4) * 3 = 24
h 20 = (20 + 20) * 3 = 120
```

Function composition

Functions can be composed
In mathematics we write

$$(f \circ g)(x) = f(g(x))$$

In F# we write

```
g >> f
```

Function composition

Functions can be composed
In mathematics we write

$$(f \circ g)(x) = f(g(x))$$

In F# we write

```
g >> f        or        f << g
```

Function composition

Functions can be composed
In mathematics we write

$$(f \circ g)(x) = f(g(x))$$

In F# we write

`g >> f` or `f << g`

The arrows point towards the outermost function

Function composition

Functions can be composed
In mathematics we write

$$(f \circ g)(x) = f(g(x))$$

In F# we write

```
g >> f        or        f << g
```

i.e. the function that will run last

Function composition

Functions can be composed
In mathematics we write

$$(f \circ g)(x) = f(g(x))$$

In F# we write

g >> f   or   f << g

```
let (>>) g f = fun x -> f (g x)
```

```
let (<<) f g = fun x -> f (g x)
```

Operators |> and <|

The operator |> means "send the value as argument to the function on the right"

`x |> f` is equivalent to `f x`

The operator <| means "send the value as argument to the function on the left"

`f <| x` is equivalent to `f x`

Operators |> and <|

The operator |> means "send the value as argument to the function on the right"

This just seems like wasted effort… `x |> f` is harder to read than `f x`

`f <| x` is equivalent to `f x`

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
```

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
(fun x -> x+x) 4 |> (*) 3 =
```

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
(fun x -> x+x) 4 |> (*) 3 =
4 + 4 |> (*) 3
```

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
(fun x -> x+x) 4 |> (*) 3 =
4 + 4 |> (*) 3 =
8 |> (*) 3
```

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
(fun x -> x+x) 4 |> (*) 3 =
4 + 4 |> (*) 3 =
8 |> (*) 3 =
(*) 3 8
```

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
(fun x -> x+x) 4 |> (*) 3 =
4 + 4 |> (*) 3 =
8 |> (*) 3 =
(*) 3 8 =
3 * 8
```

Remember that `(*) 3`
is the same as `fun x -> 3 * x`

Operators |> and <|

Both operators can be composed
(but we usually use |> for that)

```
4 |> (fun x -> x+x) |> (*) 3 =
(fun x -> x+x) 4 |> (*) 3 =
4 + 4 |> (*) 3 =
8 |> (*) 3 =
(*) 3 8 =
3 * 8 =
24
```

Remember that `(*) 3`
is the same as `fun x -> 3 * x`

Operators |> and <|

While

`x |> f`

is not easier to read than

`f x`,

`x |> f |> g |> h`

is easier to read than

`h (g (f x))`

(especially when the functions are large or partially applied with many arguments) and naturally shows how x is transformed by the functions

Piping and HoFs

We can do great things with piping, function composition and higher-order functions

Recall our equation solver

```
let solve (a, b, c) =
  let sqrtD =
    let d = b * b - 4.0 * a * c
    if d < 0.0 || a = 0.0 then
      raise SolveSDP
    else
       sqrt d
  ((-b + sqrtD) / (2.0 * a),
   (-b - sqrtD) / (2.0 * a))
```

Which solves the formula $ax^2 + bx + c = 0$

Piping and HoFs

Create a function that given a list of second degree polynomials (as defined before), return the smallest non-negative root smaller than 100 (assuming one exists).

Piping and HoFs

Create a function that given a list of second degre), return th aller than 100 (assuming one exists).

Start with the list of polynomials

```
sdps
```

Current result type: `(float * float * float) list`

Piping and HoFs

Create a function that given a list of second degre
),
return th
aller than 100 (assuming one exists).

Solve the polynomials

```
sdps |>
List.map solve
```

Current result type: `(float * float) list`

Piping and HoFs

Create a function that given a list of second degr[...]),
return [...]aller
than 100 (assuming one exists).

Flatten the list of solutions - rather than a list of pairs of roots return a list of roots

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) []
```

Current result type: `float list`

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Piping and HoFs

Create a function that given a list of second
deg),
return aller
than 100 (assuming one exists).

Remove all roots smaller than zero

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0)
```

Current result type: `float list`

Piping and HoFs

Create a function that given a list of second
degr),
return aller
than 100 (assuming one exists).

Remove all roots smaller than zero

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0)
```

Same as `(fun x -> 0.0 < x)`

Current result type: `float list`

Piping and HoFs

Create a function that given a list of second
deg),
return aller
than 100 (assuming one exists).

Return the smallest root that is smaller than 100.0

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

Current result type: float

Piping and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

is much easier to read than

```
List.fold min 100.0
  (List.filter ((<) 0.0)
    (List.fold (fun acc (a, b) -> a::b::acc) []
                (List.map solve sdps)))
```

Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

… and is identical to

```
(List.map solve >>
 List.fold (fun acc (a, b) -> a::b::acc) [] >>
 List.filter ((<) 0.0) >>
 List.fold min 100.0) sdps
```

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Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```


```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

```
[(3., 2., 1); (6., 5., 4); (9., 8., 7)] |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

```
[(0.14, -0.8); (0.28, -1.12); (0.32, -1.2)] |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

```
[0.14; -0.8; 0.28; -1.12; 0.32; -1.2] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

```
[0.14; 0.28; 0.32] |>
List.fold min 100.0
```

Piping, composition, and HoFs

```
sdps |>
List.map solve |>
List.fold (fun acc (a, b) -> a::b::acc) [] |>
List.filter ((<) 0.0) |>
List.fold min 100.0
```

```
0.14
```

This week

- Inductively defined datatypes
- Expression trees
- Collections (sets and maps)
- Data representation
- Possibly live coding

Questions?

Inductively defined types

- … or algebraic datatypes
- … or disjoint unions

Allow us to concisely say how
members of types are created

Inductively defined types

```
type month =
| January | February | March | April
| May | June | July | August
| September | October | November
| December


type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday
```

Inductively defined types

```
type month =
| January | February | March | April
| May
| Sep
| Dec
```

Observation 1

We use 'type', not 'let'

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday
```

Inductively defined types

```
type month =
| January | February | March | April
| May
| Sep
| Dec
```

Observation 2

Behave similarly to enum-types in Java

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday
```

Programming

```
type month =
 | January | February | March | April
 | May | June | July | August
 | September | October | November
 | December

 let numberOfDays =
 function
  | January | March | May | July
  | August | October | December -> 31
  | February -> 28
  | _ -> 30
```

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Programming

```
type month =
  | December
```

We can pattern match on algebraic types

```
let numberOfDays =
function
| January | March | May | July
| August | October | December -> 31
| February -> 28
| _ -> 30
```

Programming

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday

let nextWeekday =
  function
  | Monday -> Tuesday
  | Tuesday -> Wednesday
  | Wednesday -> Thursday
  | Thursday -> Friday
  | _ -> Monday
```

Arguments

Algebraic types can take arguments

```
type shape =
| Circ of float             (* radius *)
| Rect of float * float (* sides *)

let area =
  function
  | Circ r -> System.Math.PI * r * r
  | Rect (w, h) -> w * h
```

The option type

Options are used to encode partial functions

```
type 'a option =
| None         (* No result *)
| Some of 'a (* result *)
```

You can think of options as terms that can be set to null (but nice, and type-safe, and does not cause as many bugs)

Head and tails of a list

The functions List.head and List.tail will throw an exception if you try to get the head of an empty list (`List.head []`)

```
let safeHead =
   function
   | []     -> None
   | x :: _ -> Some x
```

```
let safeTail =
   function
   | []      -> None
   | _ :: xs -> Some xs
```

The option type

Some useful functions

```
Option.get : ‘a option -> ‘a

Option.map : (‘a -> ‘b) -> ‘a option -> ‘b option

Option.defaultValue :
        ‘a -> ‘a option -> ‘a
```

The option type

Some useful functions

```
Option.get : ‘a option -> ‘a

Option                                                   ion

Option.defaultValue :
        ‘a -> ‘a option -> ‘a
```

Option.get None throws an exception, similarly to taking the head of an empty list

Examples

```
let safeHead =
   function
   | []       -> None
   | x :: _   -> Some x
```

```
let safeTail =
   function
   | []        -> None
   | _ :: xs   -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
Option.get (safeHead [1;2;3]) = 1
```

which is the same as

```
[1;2;3] |> safeHead |> Option.get = 1
```

Examples

```
let safeHead =
  function
  | []       -> None
  | x :: _   -> Some x
```

```
let safeTail =
  function
  | []        -> None
  | _ :: xs   -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
Option.get (safeHead [])
```

which is the same as

```
[] |> safeHead |> Option.get
```

Examples

```
let safeHead =
   function
   | []       -> None
   | x :: _   -> Some x
```

```
let safeTail =
   function
   | []        -> None
   | _ :: xs   -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
Option.get (safeTail [1;2;3]) = [2; 3]
```

which is the same as

```
[1;2;3] |> safeHead |> Option.get = [2; 3]
```

Examples

```
let safeHead =
  function
  | []     -> None
  | x :: _ -> Some x
```

```
let safeTail =
  function
  | []      -> None
  | _ :: xs -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
Option.map (fun x -> x + 10) (safeHead [1;2;3]) = Some 11
```

which is the same as

```
[1;2;3] |> safeHead |> Option.map (fun x -> x + 10) = Some 11
```

Examples

```
let safeHead =
  function
  | []     -> None
  | x :: _ -> Some x
```

```
let safeTail =
  function
  | []      -> None
  | _ :: xs -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
Option.map (fun x -> x + 10) (safeHead []) = None
```

which is the same as

```
[] |> safeHead |> Option.map (fun x -> x + 10) = None
```

Examples

```
let safeHead =
   function
   | []       -> None
   | x :: _   -> Some x
```

```
let safeTail =
   function
   | []        -> None
   | _ :: xs   -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
defaultValue 0 (safeHead [1;2;3]) = 1
```

which is the same as

```
[1;2;3] |> safeHead |> defaultValue 0 = 1
```

Examples

```
let safeHead =
   function
   | []       -> None
   | x :: _   -> Some x
```

```
let safeTail =
   function
   | []        -> None
   | _ :: xs   -> Some xs
```

```
Option.get : 'a option -> 'a
Option.map : ('a -> 'b) -> 'a option -> 'b option
Option.defaultValue :'a -> 'a option -> 'a
```

```
defaultValue 0 (safeHead []) = 0
```

which is the same as

```
[] |> safeHead |> defaultValue 0 = 0
```

Recursive

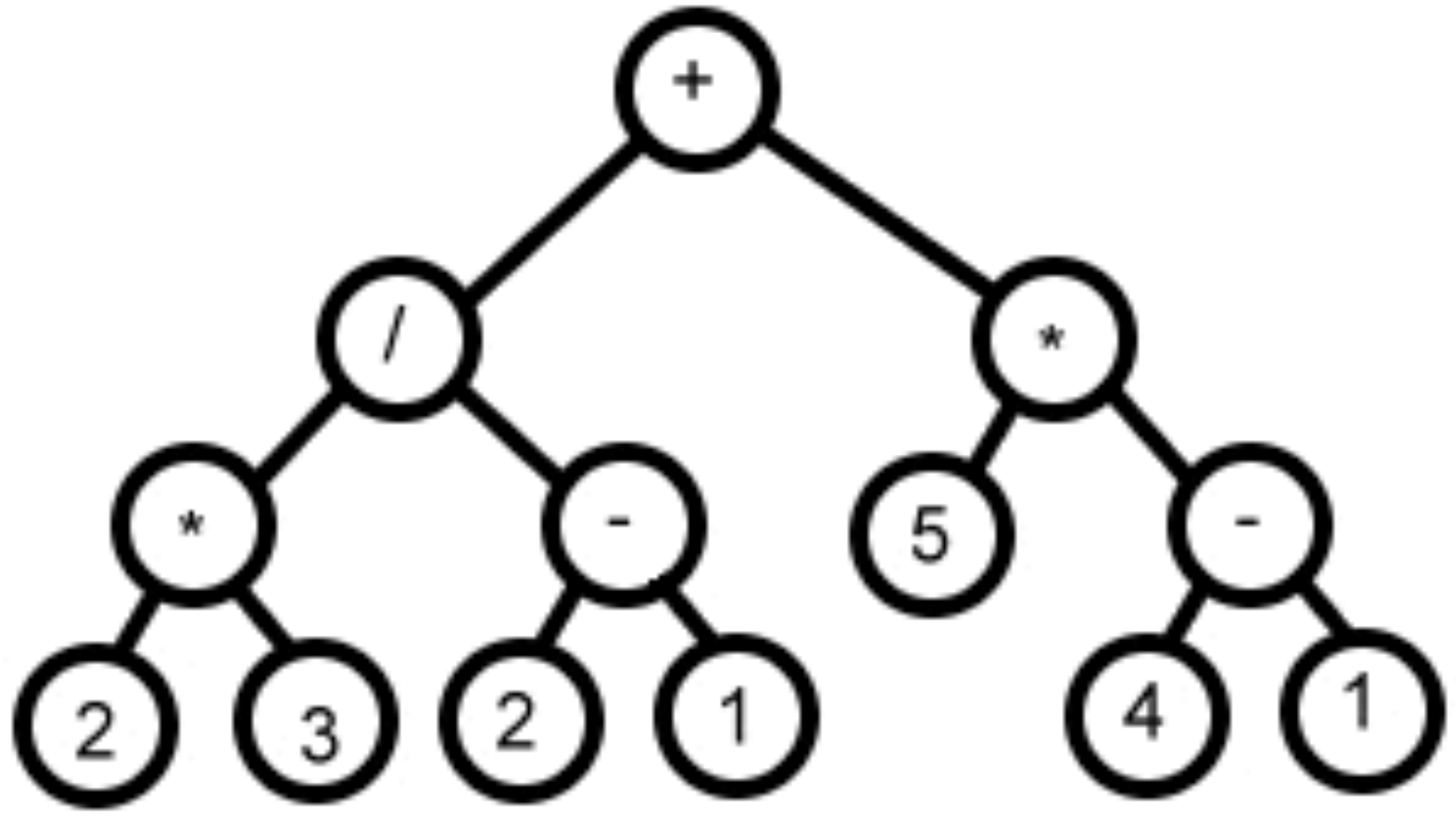
Algebraic types can be recursive and polymorphic

```
type 'a myList =
| Nil                      (* empty list *)
| App of ('a * 'a myList) (* cons *)

let rec length =
  function
  | Nil -> 0
  | App (_, lst) -> 1 + length lst
```

Expression trees

- Expression trees are heavily used by compilers
- Nodes contain operators
- Leaves contain values



Expression tree for 2*3/(2-1)+5*(4-1)

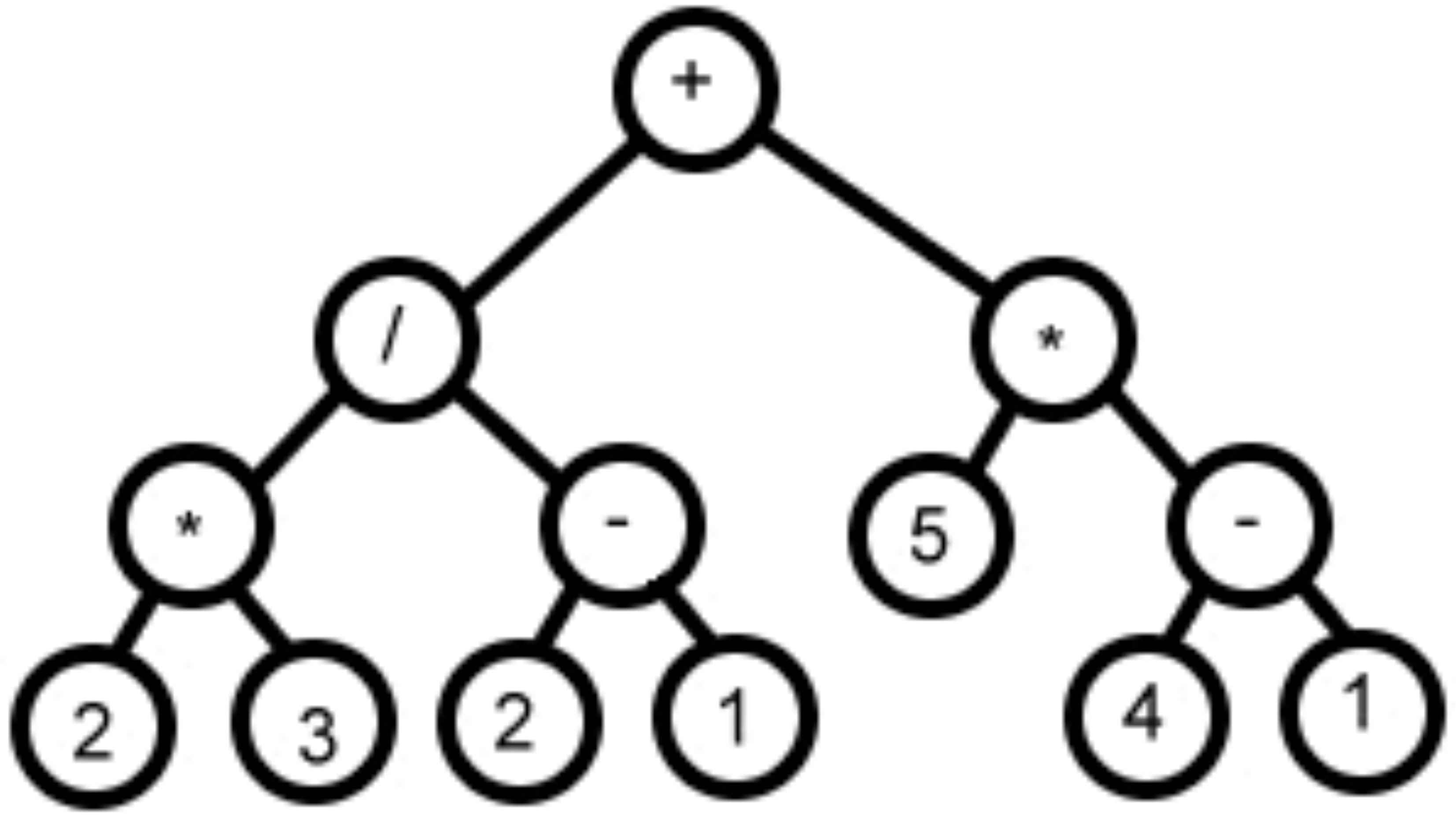
Expression trees

Expression trees are quite easy to code

```
type term =
| Const of int
| Add of term * term
| Sub of term * term
| Mul of term * term
| Div of term * term
```

Expression trees

```
type term =
  | Const of int
  | Add of term * term
  | Sub of term * term
  | Mul of term * term
  | Div of term * term
```



Expression tree for 2*3/(2-1)+5*(4-1)

```
Add
   (Div (Mul (Const 2, Const 3),
             Sub (Const 2, Const 1)),
    Mul (Const 5,
             Sub (Const 4, Const 1))
```

Exression trees

… and also nice to recurse over

```
let rec show : term -> string =
    function
    | Const f -> sprintf "%d" f
    | Add (t1, t2) ->
      "(" + show t1 + " + " + show t2 + ")"
    | Sub (t1, t2) ->
      "(" + show t1 + " - " + show t2 + ")"
    | Mul (t1, t2) ->
      "(" + show t1 + " * " + show t2 + ")"
    | Div (t1, t2) ->
      "(" + show t1 + " / " + show t2 + ")"
```

Expression Trees

Recall the expression tree from before:

```
let t : term =
   Add
      (Div (Mul (Const 2, Const 3),
            Sub (Const 2, Const 1)),
       Mul (Const 5,
            Sub (Const 4, Const 1)))
```

```
> show t;;
val it : string =
   "(((2 * 3) / (2 - 1)) + (5 * (4 - 1)))";;
```

Collections

F# has support for all collections from the .NET framework

- Lists
- Sets
- Maps
- Hash tables
- …

Sets

- Represents mathematical sets
- Intersection, union, …
- Not mutable
- Unordered
- Can only store values of comparison type
- Created inline by `set [a1; a2; …; an]` where all duplicates are removed

Sets (some functions)

```
Set.empty       : Set<'a>
```

```
Set.empty = set []
```

Sets (some functions)

```
Set.empty       : Set<'a>
Set.singleton : 'a -> Set<'a>
```

```
Set.singleton 42 = set [42]
```

Sets (some functions)

```
Set.empty     : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union     : Set<'a> -> Set<'a> -> Set<'a>
```

```
Set.union (Set.singleton 42)
          (Set.singleton 123) = set [42; 123]
```

Sets (some functions)

```
Set.empty      : Set<'a>
Set.singleton  : 'a -> Set<'a>
Set.union      : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect  : Set<'a> -> Set<'a> -> Set<'a>
```

```
Set.intersect (set [1;2;3]) (set [3;4;5]) = set [3]
```

Sets (some functions)

```
Set.empty        : Set<'a>
Set.singleton    : 'a -> Set<'a>
Set.union        : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect    : Set<'a> -> Set<'a> -> Set<'a>

Set.map          : ('a -> 'b) -> Set<'a> -> Set<'b>
```

```
Set.map (fun x -> x + 2) (set [3;4;5]) = set [5;6;7]
```

Sets (some functions)

```
Set.empty       : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union       : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect : Set<'a> -> Set<'a> -> Set<'a>

Set.map         : ('a -> 'b) -> Set<'a> -> Set<'b>
```

```
Set.map (fun x -> x + 2) (set [3;4;5]) = set [5;6;7]
Set.map ((+) 2) (set [3;4;5]) = set [5;6;7]
```

Sets (some functions)

```
Set.empty       : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union       : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect : Set<'a> -> Set<'a> -> Set<'a>

Set.map         : ('a -> 'b) -> Set<'a> -> Set<'b>
```

```
Set.map (fun x -> x + 2) (set [3;4;5]) = set [5;6;7]

Set.map ((+) 2) (set [3;4;5]) = set [5;6;7]

Set.map (fun x -> x % 2 = 0) (set [3;4;5]) =
    set [false; true]
```

Sets (some functions)

```
Set.empty        : Set<'a>
Set.singleton    : 'a -> Set<'a>
Set.union        : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect    : Set<'a> -> Set<'a> -> Set<'a>

Set.map          : ('a -> 'b) -> Set<'a> -> Set<'b>
Set.filter       : ('a -> bool) -> Set<'a> -> Set<'a>
```

```
Set.filter (fun x->x%2=0) (set[3;4;5]) = set[4]
```

Sets (some functions)

```
Set.empty       : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union       : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect : Set<'a> -> Set<'a> -> Set<'a>

Set.map         : ('a -> 'b) -> Set<'a> -> Set<'b>
Set.filter      : ('a -> bool) -> Set<'a> -> Set<'a>
```

```
Set.filter (fun x->x%2=0) (set[3;4;5]) = set[4]

Set.filter (fun x->x%2=1) (set[3;4;5]) = set[3;5]
```

Sets (some functions)

```
Set.empty       : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union       : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect : Set<'a> -> Set<'a> -> Set<'a>

Set.map         : ('a -> 'b) -> Set<'a> -> Set<'b>
Set.filter      : ('a -> bool) -> Set<'a> -> Set<'a>
Set.fold        : ('b -> 'a -> 'b) -> 'b ->
                  Set<'a> -> 'b
```

```
Set.fold (fun x y -> x + y) 0 (set[3;4;5]) = 12
```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Sets (some functions)


```
Set.empty       : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union       : Set<'a> -> Set<'a> -> Set<'a>
Set.intersect : Set<'a> -> Set<'a> -> Set<'a>

Set.map         : ('a -> 'b) -> Set<'a> -> Set<'b>
Set.filter      : ('a -> bool) -> Set<'a> -> Set<'a>
Set.fold        : ('b -> 'a -> 'b) -> 'b ->
                  Set<'a> -> 'b
```

```
Set.fold (fun x y -> x + y) 0 (set[3;4;5]) = 12

Set.fold (+) 0 (set[3;4;5]) = 12
```

Sets (some functions)

```
Set.empty
```

Result: `set []`

Sets (some functions)

```
Set.empty |>
Set.add "hello"
```

Result: `set ["hello"]`

Sets (some functions)

```
Set.empty |>
Set.add "hello" |>
Set.add "there"
```

Result: `set ["hello"; "there"]`

Sets (some functions)

```
Set.empty |>
Set.add "hello" |>
Set.add "there" |>
Set.union (Set.ofList ["there";"you";"are"])
```

Result: `set ["hello"; "there"; "you"; "are"]`

Sets (some functions)

```
Set.empty |>
Set.add "hello" |>
Set.add "there" |>
Set.union (Set.ofList ["there";"you";"are"]) |>
Set.intersect (Set.ofList ["hello";"there";"you"])
```

Result: `set ["hello"; "there"; "you"]`

Sets (some functions)

```
Set.empty |>
Set.add "hello" |>
Set.add "there" |>
Set.union (Set.ofList ["there";"you";"are"]) |>
Set.intersect (Set.ofList ["hello";"there";"you"]) |>
Set.map String.length
```

Result: `set [5; 3]`

Sets (some functions)

```
Set.empty |>
Set.add "hello" |>
Set.add "there" |>
Set.union (Set.ofList ["there";"you";"are"]) |>
Set.intersect (Set.ofList ["hello";"there";"you"]) |>
Set.map String.length |>
Set.filter (fun x -> x % 2 = 1)
```

Result: `set [5; 3]`

Sets (some functions)

```
Set.empty |>
Set.add "hello" |>
Set.add "there" |>
Set.union (Set.ofList ["there";"you";"are"]) |>
Set.intersect (Set.ofList ["hello";"there";"you"]) |>
Set.map String.length |>
Set.filter (fun x -> x % 2 = 1) |>
Set.fold (+) 0
```

Result: 8

Maps

- Represents mathematical maps
- add, lookup, …
- Not mutable
- Unordered
- Can only have keys of comparison type

Maps (some functions)

```
Map.empty    : Map<'a,'b>
```

```
Map.empty = map []
```

Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
```

```
Map.add "k" 4 Map.empty = map [("k", 4)]
```

Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList   : ('a*'b) list -> Map<'a,'b>
```

```
Map.ofList [("k",4); ("l",5)] =
    map [("k", 4); ("l",5)])
```

Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList   : ('a*'b) list -> Map<'a,'b>
Map.find     : 'a -> Map<'a,'b> -> 'b
```

```
Map.find "k" (Map.add "k" 4 Map.empty) = 4
```

which is the same as

```
Map.empty |> Map.add "k" 4 |> Map.find "k" = 4
```

Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList   : ('a*'b) list -> Map<'a,'b>
Map.find     : 'a -> Map<'a,'b> -> 'b
```

```
Map.find "l" (Map.add "k" 4 Map.empty)
```

Both are identical and throw exceptions

```
Map.empty |> Map.add "k" 4 |> Map.find "l"
```

Maps (some functions)

```
Map.empty   : Map<'a,'b>
Map.add     : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList  : ('a*'b) list -> Map<'a,'b>
Map.find    : 'a -> Map<'a,'b> -> 'b
Map.tryFind : 'a -> Map<'a,'b> -> 'b option
```

```
Map.tryFind "k" (Map.add "k" 4 Map.empty) = Some 4
```

which is the same as

```
Map.empty |> Map.add "k" 4 |> Map.tryFind "k" =
    Some 4
```

Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList   : ('a*'b) list -> Map<'a,'b>
Map.find     : 'a -> Map<'a,'b> -> 'b
Map.tryFind  : 'a -> Map<'a,'b> -> 'b option

Map.map      : ('a->'b->'c) -> Map<'a,'b> -> Map<'a,'c>
```

```
Map.map (fun k v->k+string v) (Map.ofList [("k",4);("l",5)]) =
   map [("k","k4");("l","l5")]
```

which is the same as

```
[("k",4);("l",5)]|>Map.ofList|>Map.map(fun k v->k+string v) =
map [("k","k4");("l","l5")]
```

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Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList   : ('a*'b) list -> Map<'a,'b>
Map.find     : 'a -> Map<'a,'b> -> 'b
Map.tryFind : 'a -> Map<'a,'b> -> 'b option

Map.map      : ('a->'b->'c) -> Map<'a,'b> -> Map<'a,'c>
Map.filter   : ('a->'b->bool) -> Map<'a,'b> -> Map<'a,'b>
```

```
Map.filter (fun _ v->v%2=0) (Map.ofList [("k",4);("l",5)]) =
   map [("k",4)]
```

Maps (some functions)

```
Map.empty    : Map<'a,'b>
Map.add      : 'a -> 'b -> Map<'a,'b> -> Map<'a,'b>
Map.ofList   : ('a*'b) list -> Map<'a,'b>
Map.find     : 'a -> Map<'a,'b> -> 'b
Map.tryFind  : 'a -> Map<'a,'b> -> 'b option

Map.map      : ('a->'b->'c) -> Map<'a,'b> -> Map<'a,'c>
Map.filter   : ('a->'b->bool) -> Map<'a,'b> -> Map<'a,'b>
Map.fold     : ('c->'a->'b->'c) -> 'c -> Map<'a,'b > -> 'c

Map.fold (fun acc k v->acc+k+string v)
         ""
         (Map.ofList [("k",4);("l",5)]) = "k4l5"
```

Higher-order functions

For lists you can potentially get away without using higher-order functions by using recursion

For sets and maps you still can (translate them to lists, work on the lists and then translate them back) but this is a **really** bad idea.

Practice using higher-order functions :)

Program interpreters

Functional languages are great for working directly on abstract syntax trees

An imperative language

```
type aExp = …
type bExp = …
```

Assuming we have types for arithmetic and boolean expressions

The abstract syntax tree of our language is defined like this

```
type stm =
  | Skip
  | Ass of string * aExp
  | Seq of stm * stm
  | ITE of bExp * stm * stm
  | While of bExp * stm
```

State

In order to keep track of program state we need a mapping from program variables to values

State

In order to keep track of program state we need a mapping from program variables to values

```
type state = Map<string, int>
```

Arithmetic expressions

```
type aExp =
| N of int
| V of string
| Add of (aExp * aExp)
| Mul of (aExp * aExp)
| Sub of (aExp * aExp)
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
type aExp =
 | N of int
 | V of string
 | Add of (aExp * aExp)
 | Mul of (aExp * aExp)
 | Sub of (aExp * aExp)
```

```
let rec evalA arith st =
  match arith with
  | N n -> n
  | V v -> Map.find v st
  | Add(a, b) ->
     let va = evalA a st
     let vb = evalA b st
     va + vb
  …
```

Arithmetic expressions

Arithmetic expressions can be evaluated
in the context of a state

```
evalA : aExp -> state -> int
```

```
let rec evalA arith st =
  match arith with
  | N n -> n
  | V v -> Map.find v st
  | Add(a, b) ->
      let va = evalA a st
      let vb = evalA b st
      va + vb
  …
```

```
let rec evalA arith =
  match arith with
  | N n -> fun st -> n
  | V v -> fun st ->
             Map.find v st
  | Add(a, b) ->
      fun st ->
         let va = evalA a st
         let vb = evalA b st
         va + vb
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
let rec evalA arith =
  match arith with
  | N n -> fun st -> n
  | V v -> fun st ->
             Map.find v st
  | Add(a, b) ->
      fun st ->
         let va = evalA a st
         let vb = evalA b st
         va + vb
```

```
let rec evalA arith =
  match arith with
  | N n -> fun _ -> n
  | V v -> Map.find v
  | Add(a, b) ->
      fun st ->
         (evalA a st) +
         (evalA b st)
  …
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
let rec evalA arith =
  match arith with
  | N n -> fun _ -> n
  | V v -> Map.find v
  | Add(a, b) ->
      fun st ->
        (evalA a st) +
        (evalA b st)
  …
```

```
let rec evalA arith =
  match arith with
  | N n -> fun _ -> n
  | V v -> Map.find v
  | Add(a, b) ->
      fun st ->
        (+)
        (evalA a st)
        (evalA b st)
  …
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
let rec evalA arith =
  match arith with
  | N n -> fun _ -> n
  | V v -> Map.find v
  | Add(a, b) ->
      fun st ->
        (+)
        (evalA a st)
        (evalA b st)
  …
```

```
let binop : ('a -> 'b -> 'c)
            -> (state -> 'a)
            -> (state -> 'b)
            -> state -> 'c =
  fun f x y s -> f (x s) (y s)
```

Arithmetic expressions

```
let rec evalA arith =
  match arith with
  | N n -> fun _ -> n
  | V v -> Map.find v
  | Add(a, b) ->
      fun st ->
        (+)
        (evalA a st)
        (evalA b st)
  …
```

```
let binop : (‘a -> ‘b -> ‘c)
             -> (state -> ‘a)
             -> (state -> ‘b)
             -> state -> ‘c =
   fun f x y s -> f (x s) (y s)

let rec evalA arith =
  match arith with
  | N n       -> fun _ -> n
  | V v       -> Map.find v
  | Add(a, b) -> binop (+)
                       (evalA a)
                       (evalA b)
```

Arithmetic expressions

Arithmetic expressions can be evaluated
in the context of a state

```
evalA : aExp -> state -> int
```

```
let binop f x y s = f (x s) (y s)
let rec evalA =
    function
    | N n        -> fun _ -> n
    | V v        -> Map.find v
    | Add(a, b) -> binop (+) (evalA a) (evalA b)
    | Sub(a, b) -> binop (-) (evalA a) (evalA b)
    | Mul(a, b) -> binop (*) (evalA a) (evalA b)
```

Boolean expressions

```
type bExp =
| TT
| FF
| Eq of (aExp * aExp)
| Lt of (aExp * aExp)
| Neg of bExp
| Con of (bExp * bExp)
```

Boolean expressions

Boolean expressions can be evaluated in the context of a state

```
evalB : bExp -> state -> bool
```

```
let rec evalB =
    <Assignment for this week>
```

Evaluating a program

```
type stm =
  | Skip
  | Ass of string * aExp
  | Seq of stm * stm
  | ITE of bExp * stm * stm
  | While of bExp * stm
```

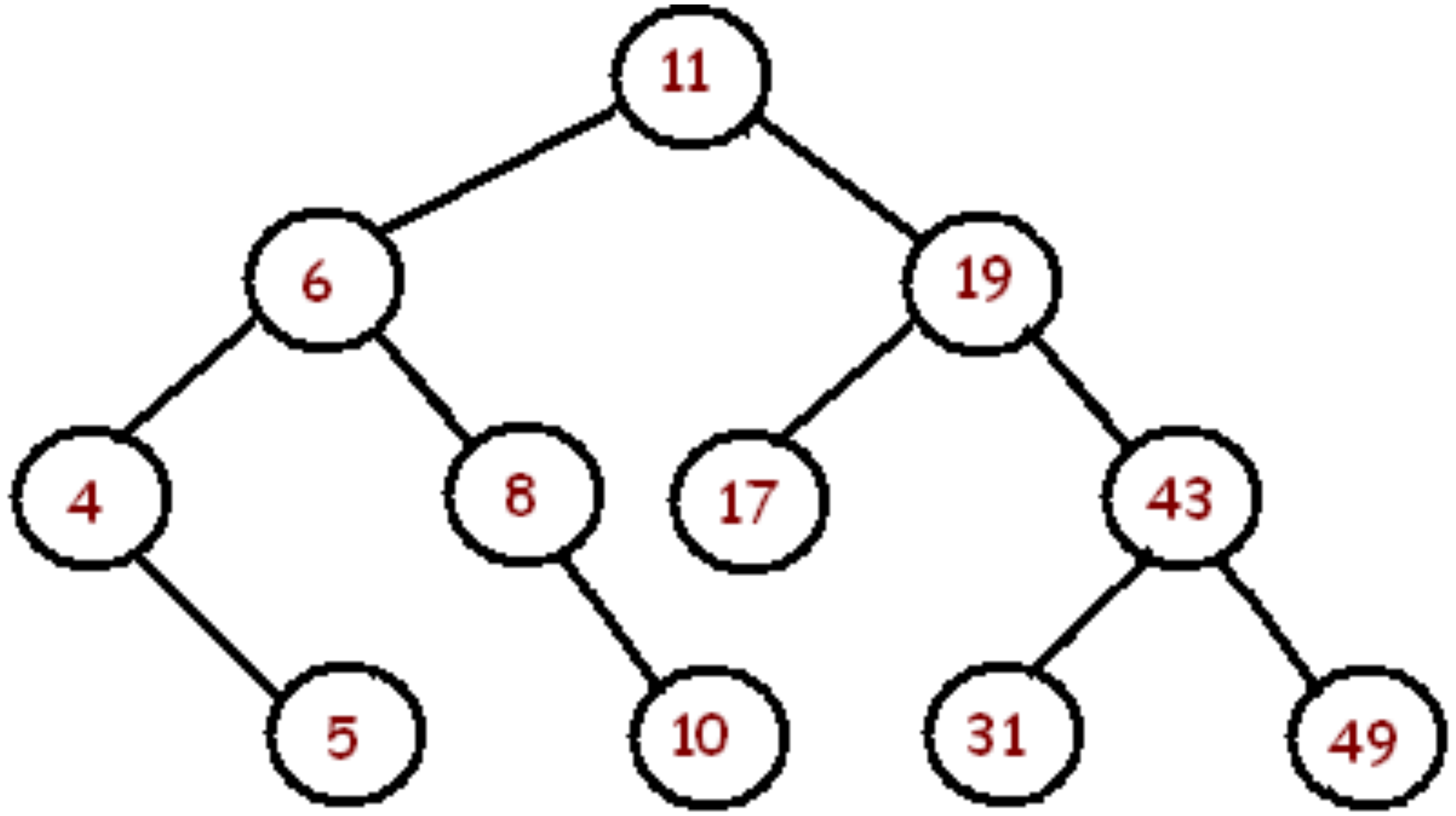
Programs evaluation is done by updating the state

```
evalS : stm -> state -> state
```

```
let rec evalS =
    <Assignment for this week>
```

Binary search trees

- All elements in the left subtree are smaller than or equal to the root
- All elements in the right subtree are greater than the root



Let’s code

Questions?