

Spreading Processes Network Analysis

we want to use networks now :-)

How does something spread in a network?
rumors, ideas, disease, weather, temperature
All models origin from epidemiology ❤

Network effects

we add knowledge on top of our network!

SI-model

S - susceptible to infection I - infected

β - infection rate/probability no healing

Effects:

1. The more connected / clustered,
the faster the spread

a. connected to degree distribution

SIS-model

Same as above, but including healing

μ - recovery rate

now the max # of infected is less than 100%
Endemic state

Endemic state

$G(n, p) = \text{if } \lambda > \frac{1}{n+1} \text{ then endemic state}$

$$\lambda = \frac{\beta}{\mu}$$

Scale free can always be endemic no matter λ

SIR-model

Same as SIS, but once you're I, you cannot
become S, but become R Recovered or Removed
R-nodes will disconnect the network

Regular Cells SIR

Cancer cells SI

Triggering Mechanism

Contagion: $S \rightarrow I$

single- vs. multiple exposure

4 Contagion Models

1. single exposure

2. Threshold

β is $p(S \rightarrow I)$, and $(1-(1-\beta)^n)$ is $p(S \rightarrow I)$ after n exposures

3. Cascade

β is now min. fraction of neighbors

you go $S \rightarrow I$ if $\frac{I}{N}$ of neighbors $> \beta$

4. Percolation

no β , instead $K = \# \leftarrow$ number of neighbors
to be I for you to become I

Reachability	Thresh	Casc	Perco
HUBS	Easy	Hard	Easy
Peripheral	Hard	Easy	Impossible usually ~

Link Prediction

Classical Link Prediction

Next edge? missing edges?

1. Observe Links
2. Hypotheses for link creation process
3. Operationalize it

Preferential Attachment \leftarrow worst method

we 'prefer' high degree nodes

\hookrightarrow the more links you have, the more likely you get new

\hookrightarrow potential links are scored by product of nodes' degrees

Bias \rightarrow high degree nodes

Common Neighbors \leftarrow Triangles tend to close

we prefer other nodes for which we have common neighbors

\hookrightarrow potential links are scored by count of common neighbors

\hookrightarrow possibly $\rightarrow \%$ of set(neighbors) that are common

Adamic - Adar \leftarrow common neighbors correction

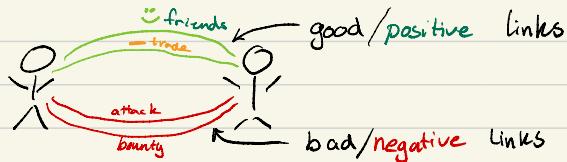
Common neighbors are weighted \rightarrow hubs penalized discounted

Since hubs cannot necessarily introduce all neighbors \hookrightarrow

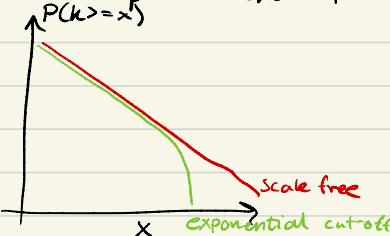
\hookrightarrow weight of neighbor is $\frac{1}{\log(d)}$ degree

\hookrightarrow variations: $\frac{1}{d}$ etc.

Signed Networks



We can plot friendship and enemy degree dist



Funny:

limit on # friends,
no limit on # enemies!

Link predictions
in signed networks

Social Balance Theory undirected

1. Friend of my Friend \rightarrow is a friend! ↗ balance
 2. Enemy of my Friend \rightarrow is my enemy.
- ~? enemy of my enemy is my enemy? ↘ unbalance
- %? friend of my friend is my enemy? ↘

This theory is used to determine sign of link

Social Status Theory directed \rightarrow endorsement

sending connections creates hierarchy \Rightarrow

F.ex ratings/trust networks



Multilayer Link Prediction

which two nodes? AND how? which layer?

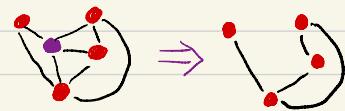
Layer Independence
Multilayer score

The Mesoscale

Dividing our network structure into smaller substructures

Ego Network

Only consider nodes that have incident edges to "chosen" node, and all edges between them.

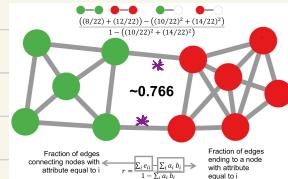


Edge Creation Tendencies

Homophily edges connect similar nodes
Heterophily edges connect different nodes

Quantifying homophily assortativity

Measuring the probability of connecting 2 nodes that are similar:



Granovetter

Strength of weak ties*

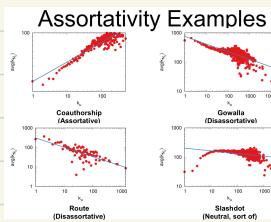
Weak ties are actually more beneficial

knowledge sharing!

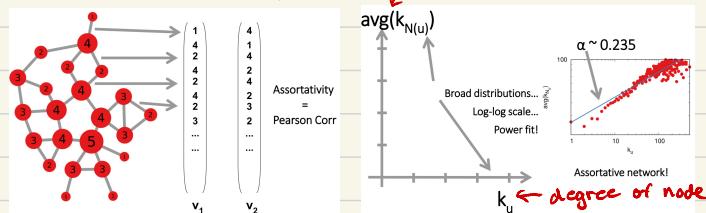
Echo Chamber

New edges with similar attributes,
Remove edges with different attributes.

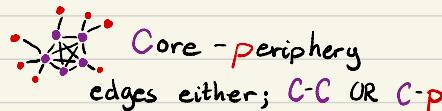
Degree assortativity



Nodes are generally connected with nodes of a similar degree.



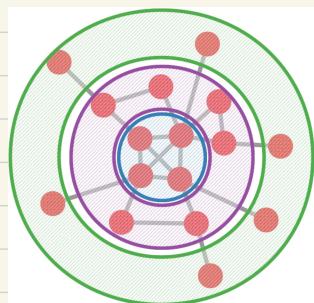
Core-Periphery



Periphery is low-degree nodes, Core is high-degree densely connected

Pure C-p is too strict and rare, Instead:

Continuous model



Core

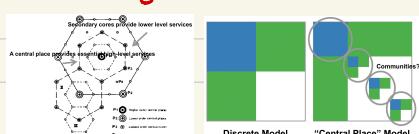
Outer Core

Periphery

Community Discovery

must be different since CP only views 1 community... But each community has a Core-Periphery model within itself..

Hotelling's law: Service-Rich center; Copenhagen city center mobility links; People outside travel in



Community Discovery

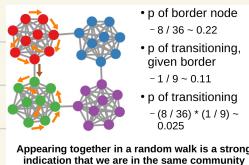
Definition

A community is densely connected within itself, and sparsely connected to the outside...

Random walk approach

In a random walk you have:

- ↳ way higher probability of choosing nodes that are part of same community
- ↳ walking an edge indicates with high probability being part of same community



Label Propagation

Every node in own community

Look at all neighbors

adopt majority community, ties broken randomly

→ Modularity to evaluate quality of partition
BUT! *

1. Build null-expectation (Random Graph)

2. Observe & subtract null-expectation

Disassortative

$$\sum_{vu} \left[A_{vu} - \frac{k_v k_u}{2 \cdot |E|} \delta(C_v, C_u) \right]$$

Normalized

connected? 1 or 0

No community

community

$\frac{1}{2|E|}$

degrees

#edges in same community?

* Not always... M tends to prefer communities of a given size... Does not like small ones $\approx \sim \sqrt{|E|}$

↳ merges 2 cliques when small

Also might give high M for disconnected clique

Temporal communities

What if we have a time dimension?

↳ communities can grow, shrink, merge, born, die

Hierarchical Community

"Not all levels will have valid communities for our initial assumption"

When we consider communities nested within a higher grain community.

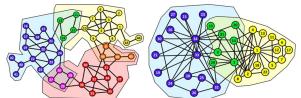
↳ higher grain community might have lower internal density, but can still offer meaning

Bottom-up: start with network. Do community discovery. Communities are now nodes in new network. Repeat!

Top-down: everything in one \rightarrow smaller

High edge-betweenness nodes are connecting communities by def!

Overlapping Communities



Nodes belong to several communities

↳ we can split node in 2 (1 for each community)

CONGA: Using Pair edge betweenness:

Merge nodes with min. score + sum scores
repeat until $|E|=1$

↳ we can cluster edges (edges considered as a group in bipartite networks)

then: weighted projection on edges, community discovery, and move back to graph!

Bipartite Network

CD on projection \Rightarrow information loss,
unclear mapping

Solution:

Bi-clique Percolation

TA Guidance

Helpful Measures

many connections?
triangles close?

Measure

Density
Clustering coeff
Diameter
Path length dist

Networkx

density(G)

(average-)clustering(G)
diameter (G)

Purpose

What do these measures say about our particular network?

Use gephi → experiment w. layout types
→ plot with positions

General Focus areas: explain method
explain application
explain results

Remember Limitations:

Randomness?

Our assumptions?