QINI-BASED UPLIFT REGRESSION

By Mouloud Belbahri,* Alejandro Murua, Olivier Gandouet[‡] and Vahid Partovi Nia§

Université de Montréal*¶, TD Insurance‡ and École Polytechnique de Montréal§

Uplift models provide a solution to the problem of isolating the marketing effect of a campaign. For customer churn reduction, uplift models are used to identify the customers who are likely to respond positively to a retention activity only if targeted, and to avoid wasting resources on customers that are very likely to switch to another company. We introduce a Qini-based uplift regression model to analyze a large insurance company's retention marketing campaign. Our approach is based on logistic regression models. We show that a Qini-optimized uplift model acts as a regularizing factor for uplift, much as a penalized likelihood model does for regression. This results in interpretable parsimonious models with few relevant explanatory variables. Our results show that performing Qini-based parameters estimation significantly improves the uplift models performance.

1. Introduction. This work proposes a methodology that identifies characteristics associated with a home insurance policy that can be used to infer the link between marketing intervention and policy renewal rate. Using the resulting statistical model, the goal is to predict which customers the company should focus on, in order to deploy future retention campaigns.

A subscription-based company loses its customers when they stop doing business with their service. Also known as customer attrition, customer churn can be a drag on the business growth. It is less expensive to retain existing customers than to acquire new customers, so businesses put effort into marketing strategies to reduce customer attrition. Customer loyalty, on the other hand, is usually more profitable because the company have already earned the trust and loyalty of existing customers. Businesses mostly have a defined strategy for mitigating customer churn. Organizations are able to determine their success rate in customer loyalty and identify improvement strategies using available data and learning about churn.

With the increasing amount of data available, a company tries to find the causal effects of customer churn. The term *causal*, as in causal study, refers

 $[\]P Corresponding \ author: \ murua@dms.umontreal.ca$

 $MSC\ 2010\ subject\ classifications:$ casual inference, Kendall's correlation, lasso, logistic regression, marketing, penalization

to a study that tries to discover a cause-effect relationship. The statement Acauses B means that changing the value of A will change the distribution of B. When A causes B, A and B will be associated but the reverse is not, in general, true, since association does not necessarily imply causation. There exists two frameworks for discussing causation [Pearl, 2009]. We will consider the statistical framework for causal inference formally introduced by Rubin [1974], which uses the notation of counterfactual random variables. This framework is also associated with the potential outcome framework Nevman, 1923, also known as the Rubin causal model [Holland, 1986]. Suppose a company decides to deploy a marketing campaign, and that customers are randomly divided into two groups. The first group is targeted with a marketing initiative (treatment group), and the second group serves as control (or baseline). A potential outcome is the theoretical response each customer would have manifested, had it been assigned to a particular group. Under randomization, association and causation coincide and these outcomes are independent of the assignment other customers receive. In practice, potential outcomes for an individual cannot be observed. Each customer is only assigned to either treatment or control, making direct observations in the other condition (called the counterfactual condition) and the observed individual treatment effects, impossible [Holland, 1986].

In marketing, the responses of customers in the treatment and control groups are observed. This makes it possible to calculate and compare the response rate of the two groups. A campaign is considered successful if it succeeds in increasing the response rate of the treated group relative to the response rate of the control group. The difference in response rate is the increase due to the campaign. To further increase the returns of future direct marketing campaigns, a predictive response model can be developed. Response models [Smith and Swinyard, 1982, Hanssens et al., 2003, Coussement et al., 2015] of client behavior are used to predict the probability that a client responds to a marketing campaign (e.g. renews subscription). Marketing campaigns using response models concentrate on clients with high probability of positive response. However, this strategy does not necessarily cause the renewal. In other words, the customers could renew their subscription without marketing effort. Therefore, it is important to extract the cause of the renewal, and isolate the effect of marketing.

Data from one of the leader north-American insurers is at our disposal to evaluate the performance of the methodology introduced in this work. This company is interested in designing retention strategies to minimize its policyholders' attrition rate. For that purpose, during three months, an experimental loyalty campaign was implemented, from which policies coming

up for renewal were randomly allocated into one of the following two groups: a treatment group, and a control group. Policyholders under the treatment group received an outbound courtesy call made by one of the company's licensed insurance advisors, with the objective to reinforce the customers confidence in the company, to review their coverage and address any questions they might have about their policy. No retention efforts were applied to the control group. The goal of the study is develop models that will be used to identify which clients are likely to benefit from a call at renewal, that is, clients that are likely to renew their policy *only* if they are called by an advisor during their renewal period. Also, clients that are not targeted will most likely

- renew their policy on their own,
- cancel their policy whether they receive a call or not,
- cancel their policy *only* if they receive a call.

Table 1 shows the marketing campaign retention results. The observed difference in retention rates between the treated group and the control group is small, but there is some evidence of a slightly negative impact of the outbound call. Even if the difference is slightly negative, it may be the case that the campaign had positive retention effects on some subgroup of customers, but they were offset by negative effects on other subgroups. This can be explained by the fact that some customers are already dissatisfied with their insurance policies and have already decided to change them before receiving the call. This call can also trigger a behavior that encourages customers to look for better rates.

 $\begin{tabular}{ll} Table 1 \\ Renewal\ rate\ by\ group\ for\ n=20,997\ home\ insurance\ policies. \end{tabular}$

	Control	Called	Overall
Renewed policies	2,253	18,018	20,271
Cancelled policies	72	654	726
Renewal rate	96.90%	96.50%	96.54%

In a randomized experiment, researchers often focus on the estimation of average treatment effects; the effect of the marketing initiative on a particular client is determined from this estimate. However, there might be a proportion of the customers that responds favorably to the marketing campaign, and another proportion that does not. A decision based on an average treatment effect at the individual level would require an adjustment because of the heterogeneity in responses that can be originated by many factors.

The so-called uplift model [Radcliffe and Surry, 1999, Hand and Yu, 2001, Lo, 2002] provides a solution to the problem of isolating the marketing effect.

Instead of modeling the class probabilities, uplift attempts to model the difference between conditional class probabilities in the treatment (e.g., a marketing campaign) and control groups. Uplift modeling aims at identifying groups on which a predetermined action will have the most positive effect.

Assessing model performance is complex for uplift modeling, as the actual value of the response, that is, the *true* uplift, is unknown at the individual subject level. To overcome this limitation, one can assess model performance by comparing groups of observations. This is done through the Qini coefficient [Radcliffe, 2007], which plays a similar role as the Gini coefficient [Gini, 1997] in Economics. The Qini coefficient is a single statistics drawn form the Qini curve. This latter object is a generalization of the Lorenz curve [Lorenz, 1905] traditionally used in direct marketing for response models.

As in all regression-based modeling, an issue in uplift modeling is the ease of interpretation of the results. The model becomes harder to interpret when the number of potential explanatory variables, that is the *dimension* of the explanatory variables increases. When the variable dimension is small, knowledge-based approaches to select the optimal set of variables can be effectively applied. When the number of potentially important variables is too large, it becomes too time-consuming to apply a manual variable selection process. In this case one may consider using automatic subset selection tools. Variable selection is an important step. It reduces the dimension of the model, avoids overfitting, and improves model stability and accuracy [Guyon and Elisseeff, 2003]. Well-known variable selection techniques such as forward, backward, stepwise [Montgomery et al., 2012], stagewise [Hastie et al., 2007], lasso [Tibshirani, 1996], and LARS [Efron et al., 2004b], among others, are not designed for uplift models. One might need to adapt them to perform variable selection in this context.

We propose a new way to perform model selection in uplift regression models. Our methodology is based on the maximization of a modified version of the Qini coefficient, the adjusted Qini, that we introduced in Section 2.1. Because model selection corresponds to variable selection, the task is haunting and intractable if done in a straightforward manner when the number of variables to consider is large, e.g. $p \approx 100$, like in the case of the insurance data. To realistically search for a good model, we conceived a searching method based on an efficient exploration of the regression coefficients space combined with a lasso penalization of the log-likelihood. There is no explicit analytical expression for the adjusted Qini surface (nor for the Qini curve), so unveiling it is not easy. Our idea is to gradually uncover the adjusted Qini surface in a manner inspired by surface response designs. The goal is to find the global maximum or a reasonable local maximum of the adjusted

Qini by exploring the surface near optimal values of the coefficients. These coefficient values are given by maximizing the lasso penalized log-likelihood. The exploration is done using Latin hypercube sampling structures [McKay et al., 2000] centered in a sequence of penalized estimates of the coefficients.

The rest of the paper is organized as follows. We first present the current uplift models in Section 2 and Section 3 introduces the notation and details of Qini-based uplift regression. Sections 4 and 5 present the computational results of the proposed methodology on synthetic and real datasets. Final remarks and conclusion are given in Section 6.

2. Uplift modeling. Let Y be the 0-1 binary response variable, T the 0-1 treatment indicator variable and X_1, \ldots, X_p the explanatory variables (predictors). The binary variable T indicates if a unit is exposed to treatment (T=1) or control (T=0). Suppose that n independent units are observed $\{(y_i, \mathbf{x}_i, t_i)\}_{i=1}^n$, where $\mathbf{x}_i = (x_{i1}, \ldots, x_{ip})$ are realisations of the predictors random variables. For $i = 1, \ldots, n$, an uplift model estimates

(1)
$$u(\mathbf{x}_i) = \Pr(Y_i = 1 \mid \mathbf{x}_i, T_i = 1) - \Pr(Y_i = 1 \mid \mathbf{x}_i, T_i = 0),$$

where the notation $Pr(Y_i = y_i \mid \mathbf{x}_i, T_i = t_i)$ stands for the corresponding conditional probability. Uplift modeling was formally introduced in Radcliffe and Surry [1999] under the appellation of differential response modeling where a thorough motivation and several practical cases promoted uplift modeling in comparison with common regression or basic tree-based methods that were used to predict the probability of success for the treatment group. They showed that conventional models, which were referred to as response models, did not target the people who were the most positively influenced by the treatment. In [Radcliffe and Surry, 1999] and [Hansotia and Rukstales, 2002, the methods introduced are tree-based algorithms similar to CART [Breiman et al., 1984], but using modified split criteria that suited the uplift purpose. The method proposed by Hansotia and Rukstales [2002] uses the uplift's absolute difference $\Delta = |u_l - u_r|$, where u_l, u_r are the observed uplifts in the left and right child nodes, respectively. It is also possible to use the difference in node sizes as some sort of penalty term to adjust the differences in uplift [Radcliffe and Surry, 2011]. Other split criteria proposed in the literature are based on the χ^2 statistic [Su et al., 2009, Radcliffe and Surry, 2011, which is usually a function of Δ^2 . All these splitting criteria rely on maximizing heterogeneity in treatment effects (Δ).

2.1. Adjusted Qini. Evaluating uplift models requires the construction of the Qini curve and the computation of the Qini coefficient [Radcliffe,

2007]. The motivation to consider the Qini curve comes from the fact that a good model should be able to select individuals with highest uplift first. More explicitly, for a given model, let $\hat{u}_{(1)} \geq \hat{u}_{(2)} \geq ... \geq \hat{u}_{(n)}$ be the sorted predicted uplifts. Let $\phi \in [0,1]$ be a given proportion and let $N_{\phi} = \{i: \hat{u}_i \geq \hat{u}_{(\lceil \phi n \rceil)}\} \subset \{1,\ldots,n\}$ be the subset of individuals with the $\phi n \times 100\%$ highest predicted uplifts \hat{u}_i (here $\lceil s \rceil$ denotes the smallest integer larger or equal to $s \in \mathbb{R}$). Because N_{ϕ} is a function of the predicted uplifts, N_{ϕ} is a function of the fitted model. For a parametric model such as (7), N_{ϕ} is a function of the model's parameters estimates, and should be denoted $N_{\phi}(\hat{\boldsymbol{\theta}})$. To simplify the notation, we prefer to omit this specification.

As a function of the fraction of population targeted ϕ , the incremental uplift is defined as

$$h(\phi) = \sum_{i \in N_{\phi}} y_i t_i - \sum_{i \in N_{\phi}} y_i (1 - t_i) \left\{ \sum_{i \in N_{\phi}} t_i / \sum_{i \in N_{\phi}} (1 - t_i) \right\},\,$$

where $\sum_{i \in N_{\phi}} (1 - t_i) \neq 0$, with h(0) = 0. The incremental uplift has been normalized by the number of subjects treated in N_{ϕ} . The relative incremental uplift $g(\phi)$ is given by $g(\phi) = h(\phi) / \sum_{i=1}^{n} t_i$. Note that g(0) = 0 and g(1) is the overall sample observed uplift

$$g(1) = \left(\sum_{i=1}^{n} y_i t_i / \sum_{i=1}^{n} t_i\right) - \left(\sum_{i=1}^{n} y_i (1 - t_i) / \sum_{i=1}^{n} (1 - t_i)\right).$$

The Qini curve is constructed by plotting $g(\phi)$ as a function of $\phi \in [0,1]$. This is illustrated in Figure 1. The curve can be interpreted as follows. The x-axis represents the fraction of targeted individuals and the y-axis shows the incremental number of positive responses relative to the total number of targeted individuals. The straight line between the points (0,0) and (1,g(1)) in Figure 1 represents a benchmark to compare the performance of the model to a strategy that would randomly target subjects. In other words, when the strategy is to treat individuals randomly, if a proportion ϕ of the population is treated, we expect to observe an uplift equal to ϕ times the global uplift. The Qini coefficient q is a single index of model performance. It is defined as the area between the Qini curve and the straight line

(2)
$$q = \int_0^1 Q(\phi) \, d\phi = \int_0^1 \{g(\phi) - \phi \, g(1)\} \, d\phi,$$

where $Q(\phi) = g(\phi) - \phi$ g(1). This area can be numerically approximated using a Riemann method such as the trapezoid rule formula: the domain of

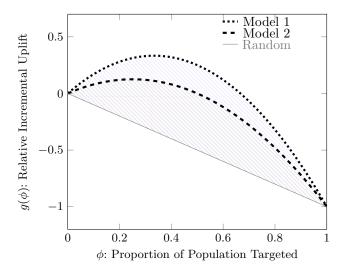


FIG 1. Example of Qini curves corresponding to two different uplift models compared to a random targeting strategy.

 $\phi \in [0,1]$ is partitioned into J panels, or J+1 grid points $0 = \phi_1 < \phi_2 < \dots < \phi_{J+1} = 1$, to approximate the Qini coefficient q (2) by its empirical estimation

(3)
$$\hat{q} = \frac{1}{2} \sum_{j=1}^{J} (\phi_{j+1} - \phi_j) \{ Q(\phi_{j+1}) + Q(\phi_j) \}.$$

In general, when comparing several models, the preferred model is the one with the maximum Qini coefficient [Radcliffe, 2007].

Another visualization associated with uplift model validation is based on the observed uplifts in each of the J bins used to compute the Qini coefficient: a good model should induce a decreasing disposition of the observed uplifts in these bins. Figure 2 illustrates good and bad uplift models as barplots of observed uplifts associated with each of the J bins. A decreasing disposition of the uplift values in the J bins is an important property of an uplift model. To measure the degree to which a model does this correctly, we suggest the use of the Kendall rank correlation coefficient [Kendall, 1938]. The goal is to find a model that maximizes the correlation between the predicted uplift and the observed uplift. The Kendall's uplift rank correlation is defined as

(4)
$$\rho = \frac{2}{J(J-1)} \sum_{i < j} \operatorname{sign}(\bar{u}_i - \bar{u}_j) \operatorname{sign}(\bar{u}_i - \bar{u}_j),$$

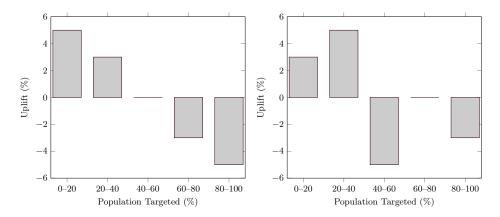


FIG 2. Theoretical predicted uplift barplots with 5 panels corresponding to two different models. A good model should order the observed uplift from highest to lowest. The Kendall's uplift rank correlation is $\rho = 1$ for the left barplot and $\rho = 0.4$ for the right barplot.

where \bar{u}_k is the average predicted uplift in bin $k, k \in 1, ..., J$, and \bar{u}_k is the observed uplift in the same bin.

From a business point of view, this statistic and the associated barplot are easier to interpret than the Qini and the Qini curve. However, we do not advise the use of ρ alone for model selection. If two models have the same \hat{q} , the preferred one should be the one with the highest ρ . But, when two models \hat{q} differ, it is not clear that the preferred one should be the one with the highest ρ . In Figure 3, we show an example of two models where the one in the left has a perfect Kendall's uplift rank correlation ($\rho = 1$) but with a Qini coefficient much smaller than the model on the right panel. In this scenario, the model with $\rho = 0.8$ is the best.

We propose an appropriate combination of (3) and (4): the *adjusted Qini* coefficient which is given by

(5)
$$\hat{q}_{\mathrm{adj}} = \rho \, \max\{0, \hat{q}\}.$$

The adjusted Qini coefficient represents a trade-off between maximizing the area under the Qini curve and grouping the individuals in decreasing uplift bins.

A note on the estimation of the Qini curve. The number of bins J may be seen as a hyper-parameter. Its choice will certainly affect the computation of the adjusted Qini coefficient. In practice, the sample is divided into quantiles (J=5) or deciles (J=10). In order to have a hint on what adequate values for J are, suppose that the relative incremental uplift function $g(\phi)$ is twice-differentiable, with bounded second derivative. Consider the trapezoid rule

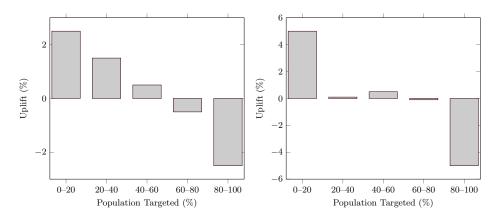


FIG 3. Theoretical predicted uplift barplots with 5 panels corresponding to two different models. The left panel model has a much smaller value of \hat{q} than the one on the right panel. However, $\rho = 1$ for the left panel and $\rho = 0.8$ for the right panel.

approximation to the integral q based on J bins. Let us assume that the bin sizes are proportional to 1/J. It is well-known that under these assumptions the error of the approximation is order $\mathcal{O}(1/J^2)$. Since $g(\cdot)$ is unknown, one need to estimate it with data. Let \hat{g}_j be the estimate of $g(\phi_j)$, $j=1,\ldots,J$. Suppose that \hat{g}_j is obtained as a mean of n/J random variables observed in the j-th bin. We suppose that these random variables are independent and identically distributed with mean $g(\phi_j)$ and a certain finite variance. The weak law of large numbers says that \hat{g}_j converges to $g(\phi_j)$, and the error in this approximation is of order $\mathcal{O}(J/\sqrt{n})$. It turns out that we need J to minimize $\kappa_1/J^2 + \kappa_2 J/\sqrt{n}$, where κ_1, κ_2 are constants. The solution is $J = \mathcal{O}(n^{1/6})$. So, for example, if $n \approx 1000$, then the optimal $J \approx 3$. Hence, the usual values of J = 5 and J = 10 seem reasonable to estimate the Qini [Radcliffe, 2007].

2.2. Brief overview of previous work on uplift modeling. The intuitive approach to uplift modeling is to build two separated classification models. Hansotia and Rukstales [2001] used the two-model approach which consists in direct subtraction of models for the treated and untreated groups. The asset of this technique is its simplicity. However, in many cases this approach performs poorly [Radcliffe and Surry, 2011]. Both models focus on predicting the class probabilities instead of making the best effort to predict the uplift, i.e., the difference between two probabilities. General discussions following differential response modeling and the two-model approach appeared in Hansotia and Rukstales [2002] where the technique known as incremental

value modeling was introduced. This uses the difference in response rates in the two groups (treatment and control) as the split criterion of a regression tree. Also, Lo [2002] introduced the true lift modeling using a single standard logistic regression model which explicitly added interaction terms between each explanatory variable and the treatment indicator. The interaction terms measure the additional effect of each explanatory variable because of treatment. The model yields an indirect estimation of the causal effect by subtracting the corresponding prediction probabilities, which are obtained by respectively setting the treatment indicator variable to treated and control in the fitted model. The disadvantage with this solution is that it is not optimized with respect to the goodness-of-fit measures designed for uplift. Instead, the parameters are estimated with respect to the likelihood. Our results show that estimating the regression parameters by maximizing the adjusted Qini significantly improves the uplift models performance.

Most current approaches that directly model the uplift causal effect are adaptations of classification and regression trees [Breiman et al., 1984]. Rzepakowski and Jaroszewicz [2010] propose a tree-based method based on generalizing classical tree-building split criteria and pruning methods. The approach is based on the idea of comparing the distributions of outcomes in treatment and control groups, using a divergence statistic, such as the Kullback-Leibler divergence or a modified Euclidean distance [Rzepakowski and Jaroszewicz, 2012, Guelman et al., 2012, Rzepakowski and Jaroszewicz, 2010. Another non-parametric method is discussed in [Alemi et al., 2009, Su et al., 2012]. Therein the uplift is estimated from the nearest neighbors containing at least one treated and one control observation. This method quickly becomes computationally expensive when dealing with large datasets, because the entire dataset has to be stored in order to predict the uplift for new observations. For a more detailed overview of the uplift modeling literature, the reader is referred to the works of Kane et al. [2014], Gutierrez and Gérardy [2017] and Devriendt et al. [2018].

From a complexity point of view, parametric models are simpler than non-parametric ones such as regression trees, because for parametric models, the number of parameters is kept small and fixed. Although, for many analysts prediction is the main target, from a business point of view, model interpretation is very important. Knowing which variables and how these variables discriminate between groups of clients is one of the main goal of uplift modeling for marketing. For these reasons, in this work we focus on parametric models. We develop our methodology for the logistic regression since interpretation of the odds ratios is well-known. However, our estimation procedure can be easily generalized to other parametric models.

3. Qini-based logistic regression for uplift. Logistic regression is a well-known parametric model for binary response variables. Given a p-dimensional predictor vector \mathbf{x}_i , $i \in \{1, ..., n\}$, logistic intercept $\theta_o \in \mathbb{R}$, and logistic regression coefficients $\boldsymbol{\beta} \in \mathbb{R}^p$, the model is

$$p_i = p_i(\theta_o, \boldsymbol{\beta}) = \Pr(Y_i = 1 \mid \mathbf{x}_i, \theta_o, \boldsymbol{\beta}) = (1 + \exp\{-(\theta_o + \mathbf{x}_i^{\top} \boldsymbol{\beta})\})^{-1}$$

or, equivalently, $\log \operatorname{it}(p_i) = \theta_o + \mathbf{x}_i^{\top} \boldsymbol{\beta}$, where $\operatorname{logit}(p_i) = \log\{p_i/(1-p_i)\}$. Throughout the paper, the superscript $^{\top}$ stands for the transpose of a column vector or matrix. In the uplift context, one need to add explicit interaction terms between each explanatory variable and the treatment indicator. Let γ denote the treatment effect, $\boldsymbol{\beta}$, the vector of main effects, $\boldsymbol{\delta}$, the vector of interactions effects, and θ_o , the intercept. The model is

$$p_i(\theta_o, \boldsymbol{\theta}) = \Pr(Y_i = 1 \mid \mathbf{x}_i, t_i, \theta_o, \boldsymbol{\theta}) = (1 + \exp\{-(\theta_o + \gamma t_i + \mathbf{x}_i^{\top} [\boldsymbol{\beta} + t_i \boldsymbol{\delta}])\})^{-1}$$

where $\theta = (\beta, \gamma, \delta)$, denotes all model parameters except for the intercept θ_o . The likelihood function associated with the uplift model is

(7)
$$\mathcal{L}(\theta_o, \boldsymbol{\theta}) = \prod_{i=1}^n p_i(\theta_o, \boldsymbol{\theta})^{y_i} \{1 - p_i(\theta_o, \boldsymbol{\theta})\}^{(1-y_i)},$$

where $\{y_i: i=1,\ldots,n\}$ are the observed response variables. The maximum likelihood estimates of $(\theta_o, \boldsymbol{\theta})$ will be denoted by $(\hat{\theta}_o, \hat{\boldsymbol{\theta}})$, with $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\beta}}, \hat{\gamma}, \hat{\boldsymbol{\delta}})$. The predicted uplift associated with the covariates vector \mathbf{x}_{n+1} of a future individual is estimated by

$$\hat{u}(\mathbf{x}_{n+1}) = \left(1 + \exp\{-(\hat{\theta}_o + \hat{\gamma} + \mathbf{x}_{n+1}^{\intercal}[\hat{\boldsymbol{\beta}} + \hat{\boldsymbol{\delta}}])\}\right)^{-1} - \left(1 + \exp\{-(\hat{\theta}_o + \mathbf{x}_{n+1}^{\intercal}\hat{\boldsymbol{\beta}})\}\right)^{-1}.$$

We propose to select a regression model that maximizes the adjusted Qini coefficient. To realistically search for a good model, we conceived a searching method based on Latin hypercube sampling of the regression coefficients space combined with a lasso penalization of the log-likelihood. The procedure is explained in the following sections.

3.1. Estimation of the Qini maximizer. Because the adjusted Qini function is not straightforward to optimize with respect to the parameters, one needs to explore the parameters space in order to find the maximum of the adjusted Qini.

Latin hypercube sampling (LHS) is a statistical method for quasi-random sampling based on a multivariate probability law inspired by the Monte Carlo method [McKay et al., 2000]. The method performs the sampling by

ensuring that each sample is positioned in a space Ω of dimension p as the only sample in each hyperplane of dimension p-1 aligned with the coordinates that define its position. Each sample is therefore positioned according to the position of previously positioned samples to ensure that they do not have any common coordinates in the Ω space. When sampling a function of p variables, the range of each variable is divided into M equally probable intervals. M sample points are then placed to satisfy the Latin hypercube requirements; this forces the number of divisions, M, to be equal for each variable. Also this sampling scheme does not require more samples for more dimensions (variables); this independence is one of the main advantages of this sampling scheme. We use LHS to find the coefficient parameters that maximize the adjusted Qini. The procedure to search for the Qini maximizer is explained next. It is based on the lasso penalized likelihood and several LHS structures.

3.1.1. Penalized log-likelihood. In the context of linear regression, the effectiveness of penalization has been amply supported practically and theoretically in several studies. In order to decrease the mean squared error of least squares estimates, ridge regression [Hoerl and Kennard, 1970] has been proposed as a trade-off between bias and variance. This technique adds an L_2 -norm penalization term to the least squares loss. The lasso (least absolute shrinkage and selection operator) penalization technique [Tibshirani, 1996] uses an L_1 -norm penalization which sets some of the regression coefficients to zero (sparse selection) while shrinking the rest. The elastic net penalization technique [Zou and Hastie, 2005] linearly combines the L_1 and L_2 -norms to provide better prediction in the presence of collinearity. Other penalization techniques such as scad [Fan and Li, 2001] and bridge regression [Frank and Friedman, 1993], offer interesting theoretical properties, including consistency.

Here, we focus on sparse estimation of the coefficients. That is, the selection of a small subset of features to predict the response. This is often achieved with a L_1 -norm penalization. Given $\lambda \in \mathbb{R}^+$, in the context of linear regression, the lasso penalization [Tibshirani, 1996] finds the estimate of the coefficients $\hat{\boldsymbol{\beta}}(\lambda)$ that maximizes the penalized log-likelihood, say $\ell(\beta) + \lambda \sum_{j=1}^p |\beta_j|$. Setting the penalization constant $\lambda = 0$ returns the least squares estimates which performs no shrinking and no selection. For $\lambda > 0$, the regression coefficients $\hat{\boldsymbol{\beta}}(\lambda)$ are shrunk towards zero, and some of them are set to zero (sparse selection). Friedman et al. [2007] proposed a fast pathwise coordinate descent method to find $\hat{\boldsymbol{\beta}}(\lambda)$, using the current estimates as warm starts. In practice, the value of λ is unknown. Cross-validation is often

used to search for a good value of the penalization constant. The least angle regression (or LARS algorithm) efficiently computes a path of values of $\beta(\lambda)$ over a sequence of values of $\lambda = \lambda_1 < \cdots < \lambda_j < \cdots < \lambda_{\min(n,p)}$, for which the parameter dimension changes [Efron et al., 2004a]. The entire sequence of steps in the LARS algorithm with p < n variables requires $\mathcal{O}(p^3 + np^2)$ computations, which is the cost of a single least squares fit on p variables. Extensions to generalized linear models with nonlinear loss functions require some form of approximation. In particular, for the logistic regression case, which is our model of interest, Friedman et al. [2010] extend the pathwise coordinate descent algorithm [Friedman et al., 2007] by first, approximating the log-likelihood (quadratic Taylor expansion about current estimates), and then using coordinate descent to solve the penalized weighted least-squares problem. The algorithm computes the path of solutions for a decreasing sequence of values for $\lambda = \lambda_{\min(n,p)} > \cdots > \lambda_j > \cdots > \lambda_1$, starting at the smallest value for which the entire vector $\hat{\boldsymbol{\beta}} = 0$. The algorithm works on large datasets, and is publicly available through the R package qlmnet [Friedman et al., 2009], which we use in this work. In what follows, we will refer to the sequence of regularizing constant values given by qlmnet as the logistic-lasso sequence.

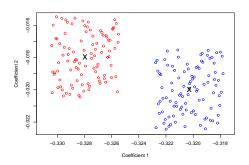
3.1.2. Qini-optimized uplift regression. Recall the uplift model likelihood given in (7). The vector of parameters $\boldsymbol{\theta} = (\boldsymbol{\beta}, \gamma, \boldsymbol{\delta})$ is a p' = (2p+1)-dimensional vector. Because of the considerations mentioned in the previous sections, in order to select an appropriate sparse model for uplift, we adapt the lasso algorithm to explore a relatively small set of reasonable models, so as to avoid an exhaustive model search. The penalized uplift model log-likelihood is given by

(8)
$$\ell(\theta_o, \boldsymbol{\theta} \mid \lambda) = \sum_{i=1}^{n} (y_i \log\{p_i(\theta_o, \boldsymbol{\theta})\} + (1 - y_i) \log\{1 - p_i(\theta_o, \boldsymbol{\theta})\}) + \lambda \|\boldsymbol{\theta}\|_1,$$

where $p_i(\theta_o, \boldsymbol{\theta})$ is as in (6), and $\|\cdot\|_1$ stands for the L_1 -norm. For any given λ , the parameters that maximize the penalized log-likelihood (8) are denoted by

(9)
$$(\hat{\theta}_o(\lambda), \hat{\boldsymbol{\theta}}(\lambda)) = \underset{\boldsymbol{\theta}_o, \boldsymbol{\theta}}{\operatorname{argmax}} \ \ell(\boldsymbol{\theta}_o, \boldsymbol{\theta} \mid \lambda).$$

Applying the pathwise coordinate descent algorithm to the uplift model, we get a sequence of critical penalization values $\lambda_1 < \cdots < \lambda_{\min\{n,p'\}}$ and corresponding model parameters $\{(\hat{\theta}_o(\lambda_j), \hat{\boldsymbol{\theta}}(\lambda_j))\}_{j=1}^{\min\{n,p'\}}$ associated with different model dimensions $m \in \{1, \dots, p'\}$.



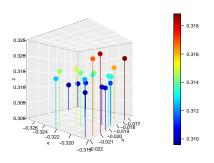


FIG 4. Example of Latin hypercube sampling around two different estimates $\hat{\boldsymbol{\theta}}(\hat{\lambda})$ for two coefficients. The idea is to sample points (left panel) and compute \hat{q}_{adj} at these points in order to maximize directly the adjusted Qini coefficient (right panel).

- 3.1.3. The LHS search. For each λ_j , $j=1,...,\min\{n,p'\}$, we generate a LHS comprising L points $\{\hat{\theta}(\hat{\lambda}_j)_l\}_{l=1}^L$ in the neighborhood of $\hat{\theta}(\hat{\lambda}_j)$, and evaluate the adjusted Qini on each of these points. The optimal coefficients are estimated as those coefficients among the $(\min\{n,p'\}\times L)$ LHS points that maximize the adjusted Qini. Figure 4 illustrates the procedure.
- 3.1.4. A simpler estimate of the Qini-based uplift regression parameters. We also consider a simpler two-stage procedure to find a good uplift model. This one is based only on the penalized log-likelihood and does not require the posterior LHS-based search for the optimal coefficients. Let $\hat{q}_{\rm adj}(\lambda)$ be the adjusted Qini coefficient associated with the model with parameters $(\hat{\theta}_o(\lambda), \hat{\boldsymbol{\theta}}(\lambda))$. The first stage of the procedure solves

(10)
$$\hat{\lambda} = \operatorname{argmax} \left(\hat{q}_{\operatorname{adj}}(\lambda_j) : j = 1, \dots, \min\{n, p'\} \right),$$

where as before, the sequence $\lambda_1 < \cdots < \lambda_{\min\{n,p'\}}$ is the logistic-lasso sequence. On the second-stage, a reduced model that only include those explanatory variables associated with non-zero entries of the estimated parameter $\hat{\boldsymbol{\theta}}(\hat{\lambda})$ is fitted without penalization, that is, with λ set to zero. The parameters are estimated with maximum likelihood. This yields the selected model. In our simulations, this model performs well. It also serves to show that the value of the penalization parameter $\hat{\lambda}$ that maximizes the Qini or adjusted Qini, is not necessarily the same as the one that maximizes the penalized log-likelihood.

4. Simulations. We conduct a simulation study to examine the performance of Qini-based uplift regression. More specifically, we compare the

different proposed parameters estimation methods by varying both the complexity of the data, and the number of predictors in the model. In order to create realistic scenarios, we based our artificial data generation on the home insurance policy data described in the introduction. We take advantage of the opportunity to have real data in order to generate realistic scenarios. We proceed as follows. First, we fit a non-parametric model on a random sample $\mathcal D$ of the home insurance policy data. Based on the resulting model, we can extract the probabilities

$$p_1(\mathbf{x}) = \Pr(Y = 1 \mid \mathbf{x}, T = 1), \text{ and } p_0(\mathbf{x}) = \Pr(Y = 1 \mid \mathbf{x}, T = 0),$$

for any given value \mathbf{x} . Then, we use these probabilities to generate synthetic data. We start by creating a bootstrap sample \mathcal{S} of size $n_{\mathcal{S}}$ from \mathcal{D} . For each observation $\mathbf{x}_i \in \mathcal{S}$, we generate a random vector $\tilde{y}_i = (\tilde{y}_{i0}, \tilde{y}_{i1})$, where \tilde{y}_{i0} is the binary outcome of a Bernoulli trial with success probability $p_0(\mathbf{x}_i)$, and \tilde{y}_{i1} is the binary outcome of a Bernoulli trial with success probability $p_1(\mathbf{x}_i)$, $i = 1, \ldots, n_{\mathcal{S}}$. The augmented synthetic dataset $\{(\mathbf{x}_i, t_i, \tilde{y}_i)\}_{i=1}^{n_{\mathcal{S}}}$, which we are going to denote again by \mathcal{S} , is the data of interest in the simulation. For each simulated dataset, we implement the following models:

- (a) a multivariate logistic regression without penalization as in (7). This is the baseline model, and we will refer to it as *Baseline*.
- (b) our Qini-based uplift regression model that uses several LHS structures to search for the optimal parameters (see Section 3.1.2). We denote this model by Q+LHS.
- (c) our Qini-based uplift regression model that uses the simpler estimate of the regression parameters as explained in Section 3.1.4. We denote this model by Q+lasso.

Note that our Q+LHS method is a derivative free optimization procedure. Another derivative free optimization method is the well-known Nelder-Mead method [Nelder and Mead, 1965]. In order to obtain benchmarks for the LHS search, we implement the following Nelder-Mead Qini-based uplift regression models:

- (d) Base+NM, which initializes the Nelder-Mead algorithm with the maximum likelihood estimates (the Baseline model solution) and which searches for coefficients that maximize the adjusted Qini coefficient.
- (e) Q+NM, which initializes the Nelder-Mead algorithm with coefficients from the lasso-sequence (the first-stage of Q+lasso) and which searches for coefficients that maximize the adjusted Qini coefficient.

Data generation. As discussed in Section 2.2, several tree-based methods have been suggested in the uplift literature. Here, we use the uplift random

forest [Guelman et al., 2012] as the data generating process. We chose this method due to its simplicity, and because it is readily available in ${\bf R}$ through the package uplift [Guelman, 2014]. Algorithm 1 describes the associated methodology.

Algorithm 1 Uplift Random Forest [Guelman et al., 2012]

- 1: $B \leftarrow$ number of bootstrap samples
- 2: for b = 1 to B do
- 3: Draw a bootstrap sample of size n_S with replacement from the data
- 4: Fit an uplift decision tree T_b to the bootstrap data
- 5: Output the ensemble of uplift trees T_b ; $b = \{1, 2, ..., B\}$ and the predicted probabilities $\Pr(Y = 1 \mid \mathbf{x}, T = 1)$ and $\Pr(Y = 1 \mid \mathbf{x}, T = 0)$ obtained by averaging the predictions of the individual trees in the ensemble

In our simulations, we vary two parameters: the depth of the trees used to fit the uplift random forests, and the number of variables k considered when fitting the uplift logistic models. Algorithm 2 details the procedure.

We define 21 scenarios by varying two parameters: (i) the depth of the uplift random forest trees used to generate the synthetic data is either 1, 2 or 3, and (ii) the number of total covariates k considered to build the forest model, $k \in \{10, 20, 30, 50, 75, 90, 97\}$. For scenarios 1-7, the depth is 1, and we vary k; for scenarios 8-14, the depth is 2; and for scenarios 15-21, the depth is 3. Each scenario was replicated 100 times.

Algorithm 2 Simulations

- 1: $M \leftarrow \text{number of simulations}$
- 2: for m = 1 to M do
- 3: Draw a stratified sample \mathcal{D}_m of size $n_{\mathcal{S}}$ without replacement from the data
- 4: Fit an uplift random forest of a given tree depth to the sampled data \mathcal{D}_m
- 5: Generate a complete (i.e., including the binary responses) synthetic data S_m with data D_m
- 6: Fit an uplift random forest of the same given tree depth to the synthetic data S_m using all p predictors
- 7: **for** each k **do**
- 8: Sample $k \leq p$ random predictors for modeling
- 9: Fit the different uplift logistic models with k predictors on S_m
- 10: Output the average and standard errors of the metrics for each method

The sample means of \hat{q} (2), and $\hat{q}_{\rm adj}$ (5) and their corresponding standard errors are reported in Tables 2, and 3, respectively. Since the conclusions are similar for the three tree depths, we report only the results associated with depth 3, that is, for the most complex model. For each comparison group, we also report the corresponding performance of an uplift random forest (RF)

fitted to the synthetic data using all available predictors (i.e., k = 97).

Table 2 Qini coefficient (\hat{q}) averaged over 100 simulations. Standard-errors are reported in parenthesis. The RF model (with k=97 and depth= 3) performance is 1.60 (0.039). n=5000 observations.

\overline{k}	Baseline	Q+lasso	Q+LHS	Base+NM	Q+NM
10	0.51 (0.023)	0.56 (0.020)	0.76 (0.021)	0.64 (0.019)	0.68 (0.021)
20	0.74 (0.020)	$0.83 \ (0.022)$	1.03 (0.024)	$0.93 \ (0.025)$	0.95 (0.020)
30	$0.94 \ (0.023)$	$1.01 \ (0.023)$	$1.20 \ (0.023)$	1.04 (0.024)	1.13(0.026)
50	1.09(0.039)	1.19 (0.025)	$1.48 \ (0.026)$	$1.23 \ (0.031)$	1.35 (0.029)
75	0.97 (0.073)	1.37(0.036)	1.48 (0.033)	1.35 (0.059)	1.49 (0.056)
90	1.00 (0.084)	$1.36 \ (0.045)$	$1.54 \ (0.036)$	$1.40 \ (0.071)$	1.47 (0.064)
97	$0.83 \ (0.083)$	$1.36 \ (0.055)$	1.59 (0.037)	$1.31\ (0.088)$	$1.41 \ (0.079)$

In Table 2, we compare the performance of the models according to the Qini coefficient \hat{q} . We observe that performing variable selection driven by the adjusted Qini coefficient (Q+lasso) significantly improves the performance of the baseline model. As expected, the models using a LHS-driven optimization perform better. The performance of Q+lasso is similar to the Base+NM performance, and is slightly lower than the others. Using the lasso-sequence in order to initialize the posterior searches (Q+LHS) or Q+NM improves the performance of the final models. However, for the Q+NM solution, the standard error of the Qini coefficient increases with k. It is almost twice the standard errors from Q+LHS for $k \geq 75$. Using all predictors (k=97) enables the Q+LHS models to achieve the same performance as the RF model.

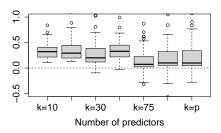
Table 3 Adjusted Qini coefficient (\hat{q}_{adj}) averaged over 100 simulations. Standard-errors are reported in parenthesis. The RF model (with k=97 and depth= 3) performance is 1.40 (0.048). n=5000 observations.

\overline{k}	Baseline	Q+lasso	Q+LHS	Base+NM	Q+NM
10	0.36 (0.027)	0.39 (0.026)	0.72 (0.024)	0.54 (0.025)	0.61 (0.027)
20	$0.58 \ (0.025)$	$0.68 \ (0.027)$	1.02 (0.025)	$0.83 \ (0.025)$	$0.91 \ (0.027)$
30	0.83 (0.026)	0.92(0.029)	$1.20 \ (0.023)$	1.03 (0.029)	1.13 (0.029)
50	0.97(0.041)	1.12(0.026)	$1.48 \ (0.026)$	$1.23 \ (0.033)$	1.35 (0.028)
75	$0.90 \ (0.069)$	$1.32 \ (0.039)$	$1.48 \ (0.033)$	$1.35 \ (0.061)$	$1.49 \ (0.058)$
90	0.94 (0.081)	1.28 (0.049)	1.54 (0.036)	1.37(0.071)	1.46 (0.069)
97	0.79 (0.085)	$1.30 \ (0.063)$	1.59 (0.037)	$1.23 \ (0.089)$	1.32 (0.081)

In Table 3, we compare the main statistic of interest, that is, the adjusted Qini coefficient. These results corroborate the findings from the previous

table. Guiding the variable selection by this statistic leads to significant improvements from the results of the baseline model. Similarly, estimating the parameters with a derivative-free maximization of the adjusted Qini coefficient improves the performance of the models in comparison to maximum likelihood estimation. The best results are obtained with models that make use of the lasso-sequence in order to explore the space of the parameters (Q+LHS) and Q+NM. As in Table 2, the Q+LHS models give the best results. Moreover, when using all available predictors, the Q+LHS models outperform both the RF and the Q+NM models.

The difference in performance between Q+LHS and Q+lasso is significant. The left panel of Figure 5 display boxplots of the differences in performance between these two models in each simulation. It is clear that Q+LHS perform much better than Q+lasso most of the time. The relative performance of Q+lasso improves slightly when the number of predictors approaches the total number of predictors available (p=97). The same pattern is observed in the difference of performance between Q+LHS and Base+NM (see right panel of Figure 5). This confirms the importance of the use of the lasso-sequence in order to estimate the model's parameters.



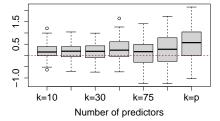


FIG 5. Comparison between Q+LHS and Q+lasso (left panel) and Q+LHS and Base+NM (right panel). Boxplots of the differences in terms of \hat{q}_{adj} as a function of the number of predictors used in the models over the 100 simulations with n=5000 observations. The black lines represent the differences medians.

Choosing an appropriate sparse model. Next, we compare the models selected by the Qini-based uplift regression and the classical lasso approach where the penalization constant is chosen by cross-validation on the log-likelihood. Consider the values of the logistic-lasso sequence $\lambda_1 < \cdots < \lambda_{\min\{n,p'\}}$ sorted according to the results of Q+lasso. That is, consider the permutation $(\pi_1, \pi_2, \dots, \pi_{\min\{n,p'\}})$ of $(1, 2, \dots, \min\{n, p'\})$ so that $\lambda_{\min\{n,p'\}} \leq \cdots \leq \lambda_{\pi_1}$, where the relation $\lambda_{\pi_i} \leq \lambda_{\pi_j}$ means that $\hat{q}_{adj}(\lambda_{\pi_i}) \leq \hat{q}_{adj}(\lambda_{\pi_j})$. We

look at the value of $\tilde{\lambda} \in \{\lambda_1, \ldots, \lambda_{\min\{n,p'\}}\}$ that is chosen by cross-validation of the log-likelihood, and report its ranking based on the sorted Q+lasso sequence $\lambda_{\pi_{\min\{n,p'\}}} \preceq \cdots \preceq \lambda_{\pi_1}$. Comparing the two models is equivalent to check when lasso finds the "best" λ , that is, when λ_{π_1} is equal to $\tilde{\lambda}$. We repeated the simulation 100 times, each time using n=5000 observations randomly selected from the full data set. The barplot in Figure 6 shows that only 7% of the time $\tilde{\lambda}$ also maximizes \hat{q}_{adj} . Observe that 4% of the time $\tilde{\lambda}$ is positioned 10 in the ranking, and 12% of the time, it is positioned 39. These results clearly show that choosing the penalization constant by cross-validation of the log-likelihood does not solve the problem of maximizing the adjusted Qini coefficient, and therefore, is not necessarily appropriate for uplift models.

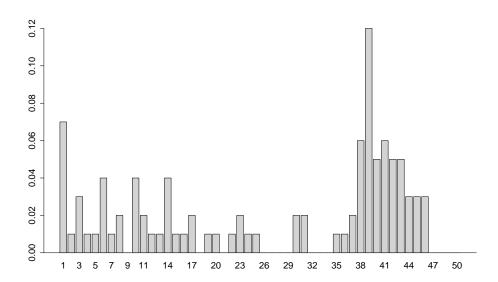


Fig 6. Barplot of the distribution of the Q+lasso rankings associated with $\tilde{\lambda}$.

5. Insurance data analysis. Recall the insurance data introduced in Section 1. The insurance company is interested in designing retention strategies to minimize its policyholders' attrition rate. An experimental loyalty campaign was implemented, from which policies coming up for renewal were randomly allocated into one of the following two groups: treatment group, and control group. The goal of this section is to analyze the marketing campaign results so as to identify both the set of persuadable clients, and the set

of clients that should not be disturbed. Table 4 describes some of the p=97 available explanatory variables in the dataset, in addition to the treatment (Called or Control) and outcome (Renewed or Cancelled the policy) variables.

Table 4 Descriptive statistics of some available variables for n=20,997 home insurance policies. With randomization, the difference of means between treatment and control groups is significantly not different from 0 for all available predictors. For privacy concerns, we hide some values with *.

$nide\ some\ values\ witn\ au.$					
	Control	Called	Diff Mean	Diff SD	Domain
Sample size	18,672	2,325			
Credit Score	756.93	756.00	-0.00	1.46	\mathbb{R}^+
	44.97	756.92 45.26	0.30	0.25	\mathbb{R}^+
Age (Years)	44.97	45.20	0.50	0.25	11/2
Genger Male	0.59	0.60	0.01	0.01	(0.1)
Marital Status	0.59	0.00	0.01	0.01	$\{0, 1\}$
	0.00	0.00	0.00	0.00	(0.1)
Divorced	0.02	0.02	0.00	0.00	$\{0,1\}$
Married	0.69	0.69	0.00	0.01	$\{0,1\}$
Single	0.23	0.23	0.00	0.01	$\{0,1\}$
Seniority (Years)	9.57	9.73	0.16	0.16	\mathbb{R}^+
Policy Premimm (\$)	di	at.			- 1
New Premium	*	*	-7.44	16.14	\mathbb{R}^+
Old Premium	*	*	-5.18	15.12	\mathbb{R}^+
Territory					
Rural	0.06	0.06	0.00	0.01	$\{0, 1\}$
Products					
Auto and Home	0.86	0.85	-0.01	0.01	$\{0, 1\}$
Auto Policies Count	1.05	1.04	-0.01	0.01	\mathbb{N}
Mortgage Count	0.66	0.67	0.01	0.01	\mathbb{N}
Residences Count	1.07	1.08	0.01	0.01	\mathbb{N}
Endorsement Count	1.98	2.00	0.02	0.03	\mathbb{N}
Neighbourhood's Retention	0.87	0.87	0.00	0.00	[0, 1]
Type of Dwelling					
Family House	0.69	0.70	0.00	0.01	$\{0, 1\}$
Duplex	0.03	0.03	-0.00	0.00	$\{0,1\}$
Apartment	0.19	0.19	-0.00	0.01	$\{0,1\}$
Year of Construction	1982.82	1983.52	0.71	0.58	N
Extra Options					
Option 1	0.20	0.20	0.00	0.01	$\{0, 1\}$
Option 2	0.73	0.73	0.00	0.01	$\{0,1\}$
Option 3	0.71	0.72	0.01	0.01	$\{0,1\}$
- r · · · ·	···-		0.0-	0.0-	(~,-)

Parameters estimation. We fit the Qini-based uplift regression Q+LHS to the data using the methodology described in the previous sections. For comparison purposes, we also considered the model Q+lasso. Although, we

are interested in interpretable parametric models, we also fit an uplift random forest (RF) as a benchmark for our comparison.

In order to choose the optimal value from the logistic-lasso sequence of penalization constant values $\{\lambda_1,\ldots,\lambda_{\min\{n,p'\}}\}$, we use a 5-fold cross-validation on the adjusted Qini statistics. We compare the resulting models with the one yielded by applying the classical lasso approach, that is, with the model associated with the value of the penalization constant that maximizes the cross-validated log-likelihood. We will refer to this latter model as MLE+lasso. The two-stage approach was used in all the cases. The first stage estimates the best λ in the logistic-lasso sequence by cross-validation. The second stage fits the non penalized logistic regression model with the subset of selected variables.

For the Q+LHS model, for each λ_j , we perform a LHS search to directly maximize the adjusted Qini coefficient. In this case, applying the LHS search leads to the selection of the model associated with the penalization constant $\approx 3 \times 10^{-5}$, while for the MLE+lasso logistic regression, it is $\approx 10^{-3}$. The number of selected variables are, respectively, 163 and 53 out of a total of 195 main and interaction effect terms. If we use the simple search method described in Section 3.1.4, which was denoted by Q+lasso in the previous section, the optimal value of the penalization constant is $\approx 4 \times 10^{-5}$. In this case, the number of selected variables is 156.

In order to have a fair comparison, we followed a process similar to the one applied to the Q+LHS model to fit the uplift RF model. The accuracy of a random forest can be sensitive to several training hyper-parameters: number of trees (from 10 to 200, with increments of 10 trees), maximum depth on each tree (from 1 to 10), minimum number of observations per node (either 100, 200 or 500), and split criterion, either Euclidean distance or Kullback-Leibler divergence; see Guelman et al. [2012] for more details on the split criteria. The optimal RF hyper-parameters were those that maximized the adjusted Qini coefficient with a 5-fold cross-validation over the grid given by the possible values of the hyper-parameters. Hence, the chosen RF was composed of 100 trees of maximum depth 3, with a minimum of 200 observations per node, with trees splitted according to the Kullback-Leibler criterion. The final RF was fitted using all available data.

Figures 7 and 8 show the performance of the models in terms of the Qini curve and the uplift barplot (Kendall's rank correlation), respectively. As expected, the Qini-based uplift regression models outperform the classic lasso approach, that is, the MLE+lasso model, both in terms of overall adjusted Qini coefficient (see Table 5) and in terms of sorting the individuals in decreasing order of uplift (see Figure 8). Moreover, the performance of the

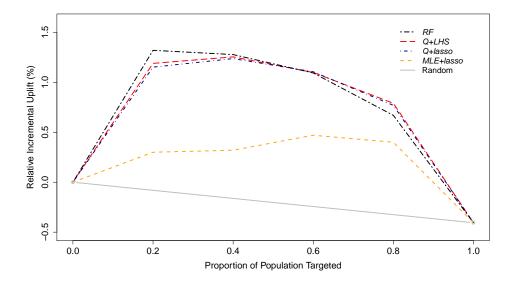


Fig 7. Performance of the final models based on the Qini curves.

Qini-based uplift regression is slightly lower, but comparable to the one of the RF model ($\hat{q}_{adi} = 1.07$), even though the RF is more complex. Indeed, with 100 trees of depth 3, the RF model can both model non-linearity and interactions between covariates, which makes interpretation of the final RFhard. However, the in-sample performance is similar to our models. This is interesting because it shows that it is possible to get powerful models without loosing interpretation when estimating the parameters with the adjusted Qini function. Based on the final models, we can identify both (i) the group of clients at the top 20% of predicted uplifts, that is, the clients to pursue in the marketing, and (ii) the group of clients at the bottom 20% of predicted uplifts, that is, the clients not to disturb with any marketing. The group of clients at the top 20% of predicted uplifts provides very strong return on investment cases when applied to retention activities. For example, by only targeting the persuadable customers in an outbound marketing campaign, the contact costs and hence the return per unit spend can be dramatically improved [Radcliffe and Surry, 2011]. We observe from Table 5 that the RF model finds the highest top 20% uplift group which presents an uplift of 6.41%, while the Q+LHS model finds the lowest bottom 20% uplift group which shows an uplift of approximately -6%. Note that the overall uplift is approximately -0.5%. The performance of the Q+lasso model is slightly

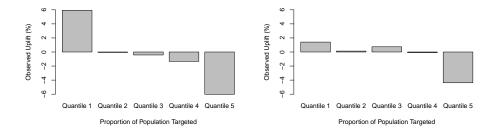


FIG 8. Performance of the Q+LHS and MLE+lasso models based on uplift Kendall's correlations. The left barplot corresponds to Q+LHS ($\rho = 1$) and the right barplot to to the MLE+lasso model ($\rho = 0.8$).

lower than the one of the Q+LHS model.

Table 5 Uplift comparison of the top and bottom 20% uplift groups estimated by the models: RF, Q+LHS, Q+lasso, and MLE+lasso.

Method	$\hat{q}_{ m adj}$	Top 20% Uplift	Bottom 20% Uplift
RF	1.07	6.41%	-5.36%
Q+LHS	1.03	5.94%	-6.02%
Q+lasso	1.02	5.76%	-5.93%
$MLE{+}lasso$	0.39	1.41%	-4.38%

Model interpretation. Because the Q+LHS model is a logistic model, we can interpret the results through its coefficients. The usual approach is that of the odds ratios. For a specific variable, the odds ratio is computed by fixing the other covariates at fixed values, such as their mean, which is what we have done here. Since the company is not interested in all the variables included in the model, we will analyze a subset with relevant interpretation for the business. In addition, for confidentiality reasons, we do not show the analysis of variables related to the insurance premium. The following variables are chosen by our model: client's credit score, age, gender and marital status (single or not); client's products: whether it is a single line (home) or a multi-line (automobile and home) account; client's number of automobile policies, mortgages and residences; and whether the client's has extra options (additional endorsements) in his/her account. For a model with p variables, the odds ratio $OR_{X_i}(t)$ for a specific variable X_j is given

by

$$\frac{\Pr(Y = 1 \mid X_j = x_j + 1, T = t) / \Pr(Y = 0 \mid X_j = x_j + 1, T = t)}{\Pr(Y = 1 \mid X_j = x_j, T = t) / \Pr(Y = 0 \mid X_j = x_j, T = t)}$$

$$= \frac{\exp(\hat{\beta}_j(x_j + 1) + \hat{\delta}_j t \ (x_j + 1))}{\exp(\hat{\beta}_j x_j + \hat{\delta}_j t \ x_j)} = \exp(\hat{\beta}_j) \exp(\hat{\delta}_j t) ,$$

where T is the treatment indicator, and where for a binary variable, such as $extra\ options$, x_j is set to 0 in the above expression. When the company does not call a client (T=0), the odds ratio is $\mathrm{OR}_{X_j}(0) = \exp(\hat{\beta}_j)$ and when the company calls a client (T=1), the odds ratio is $\mathrm{OR}_{X_j}(1) = \exp(\hat{\beta}_j) \exp(\hat{\delta}_j) = \mathrm{OR}_{X_j}(0) \exp(\hat{\delta}_j)$. Table 6 gives the estimated odds ratios $\mathrm{OR}_{X_j}(0)$ and $\mathrm{OR}_{X_j}(1)$ with 95% confidence intervals. We can see, for example, that when the company does not call a client which has $extra\ options$ in his/her policy, his/her odds ratio of renewing the policy is 0.35 while when the company calls that same client, the odds ratio becomes 1.28.

Table 6

Odds ratios and 95% confidence intervals estimated by the Qini-based uplift regression model (Q+LHS) for some of the selected variables, * represents significant coefficients.

	$\exp(\hat{\beta}_j)$	CI (95%)	$\exp(\hat{\beta}_j + \hat{\delta}_j)$	CI (95%)
Credit Score	0.998	(0.994; 1.003)	1.001	(0.999; 1.001)
Age (Years)	0.995	(0.969; 1.023)	0.998	(0.989; 1.006)
Genger				
Male	1.297	(0.778; 2.163)	0.962	(0.814; 1.135)
Marital Status				
Single	2.759	(0.956; 7.963)	0.697	(0.452; 1.077)
Products				
Auto and Home	1.619	(0.586; 4.472)	*1.418	(1.017; 1.977)
Auto Policies Count	2.106	(0.653; 6.790)	*1.996	(1.368; 2.918)
Mortgage Count	1.381	(0.789; 2.418)	*1.366	(1.137; 1.642)
Residences Count	0.505	(0.172; 1.489)	*1.788	(1.122; 2.848)
Extra Options	*0.350	(0.141; 0.874)	1.276	(0.949; 1.715)

Next, we use the Q+LHS model predictions to describe in more detail the two extreme groups found by the model (top 20% and bottom 20% predicted uplifts). This furnishes the insurance company with typical profiles of clients that are persuadables (top 20%), and clients that should not be targeted (bottom 20%). Table 7 shows descriptive statistics of some selected predictors for both groups. A MANOVA comprising only these two groups for the selected variables, followed by ANOVA tables involving individual selected variables separately, show that all mean differences were statistically significant (p-value < 0.0005). Looking at the average profiles of persuadable

Table 7 Profiles of the persuadables and do not disturb groups predicted by the Qini-based uplift regression model (Q+LHS) for some of the selected variables. Note that all group means are significantly different from 0 (p-value < 0.0005).

	- '-	,
	Persuadables	Do Not Disturb
Number of observations	4199	4200
Observed Uplift	5.94%	-6.02%
Predicted Uplift (\pm S.E.)	$4.94\%~(\pm 0.10\%)$	$-5.45\%~(\pm0.07\%)$
Credit Score	$771 (\pm 60)$	$736 \ (\pm 77)$
Age (Years)	$46.1 \ (\pm 11.6)$	$41.4 (\pm 11.8)$
Genger		
Male	$50\% \ (\pm 50\%)$	$61\% \ (\pm 49\%)$
Marital Status		
Single	$13\% \ (\pm 33\%)$	$43\% \ (\pm 49\%)$
Products		
Auto and Home	$83\% \ (\pm 37\%)$	$66\% \ (\pm 47\%)$
Auto Policies Count	$1.04\ (\pm0.64)$	$0.76 \ (\pm 0.63)$
Mortgage Count	$0.63\ (\pm0.63)$	$0.51\ (\pm 0.55)$
Residences Count	$1.15 \ (\pm 0.41)$	$1.02 \ (\pm 0.19)$
Extra Options	$87\% \ (\pm 33\%)$	41% (±49%)

and do not disturb clients, we can say that a persuadable client has a higher credit score and is slightly older than a client that should not be targeted. A persuadable client is less likely to be single and more likely to hold both company insurance products (i.e., home and auto policies). Also, this type of client holds more auto policies in his/her account, more mortgages, more residences in his/her name and is more likely to have extra coverage options. Thus, it seems that a persuadable client is a customer with many products to insure. The correlation matrices associated with these two groups are displayed in image format in Figure 9. There are some obvious patterns that distinguish the two groups. For example, credit score is slightly correlated with *client age* for persuadable clients, but not for do-not-disturb clients. Client age is negatively correlated with marital status for do-not-disturb clients, but only slightly correlated for persuadables. Indeed, there are several differences in the marital status correlations in both groups. Also, the number of mortgages and residences are more correlated for persuadables than do-not-disturb, and the number of mortgages and whether or not a client has extra options are more correlated for do-not-disturb than persuadables.

The differences between these two groups can also be observed through the odds ratios. For any specific variable X_j which takes average values $x_i^{(p)}$

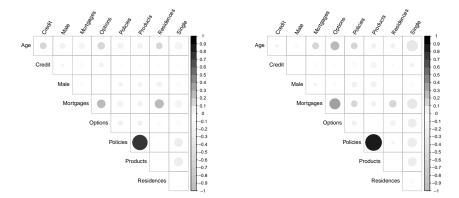


FIG 9. Correlations of selected variables of interest for Persuadable (left panel) and Donot-disturb (right panel) clients.

(for persuadable clients), and $x_j^{(d)}$ (for do-not-disturb clients), consider the odds ratio $OR_{X_j}^{(group)}(t)$ between persuadable and do-not-disturb clients

$$\frac{\Pr(Y=1 \mid X_{j} = x_{j}^{(p)}, T=t) / \Pr(Y=0 \mid X_{j} = x_{j}^{(p)}, T=t)}{\Pr(Y=1 \mid X_{j} = x_{j}^{(d)}, T=t) / \Pr(Y=0 \mid X_{j} = x_{j}^{(d)}, T=t)}$$

$$= \left(\exp(\hat{\beta}_{j}) \exp(\hat{\delta}_{j}t)\right)^{x_{j}^{(p)} - x_{j}^{(d)}} = \left(\operatorname{OR}_{X_{j}}(t)\right)^{x_{j}^{(p)} - x_{j}^{(d)}},$$
(11)

where T is the treatment indicator. Table 8 shows these odds ratios for the two values of $T \in \{0,1\}$. For example, if we only consider extra options, when the insurance company calls a client (i.e., T=1), the odds ratio between a persuadable client (Extra Options= 87%) and a do-not-disturb client (Extra Options= 41%) is about 1.12 with a 95% confidence interval of [0.98; 1.28]. On the other hand, when the company does not call a customer (i.e., T=0), the odds ratio becomes 0.62 with a 95% confidence interval of [0.41; 0.94]. These results are quite logical in the sense that the odds of renewing the insurance policy are higher for the persuadable clients if the company calls, while the same odds are higher for the do-not-disturb clients if the company does not call.

Overall, we observe that when calling a client, the odds of renewing the insurance policy of persuadable clients are almost twice (1.81) the odds of donot-disturb clients with a 95% confidence interval of [1.43; 2.29]. Conversely, when the company does not call a client, the odds of renewing the insurance

Table 8 Odds ratios $\mathrm{OR}_{X_j}^{(\mathrm{group})}(t)$ of the persuadable compared to the do not disturb clients (Eq. 11) and 95% confidence intervals estimated by the Qini-based uplift regression model (Q+LHS) for some of the selected variables. The $\Delta=x_j^{(\mathrm{p})}-x_j^{(\mathrm{d})}$ column represents the difference of group means from Table 7.

	Δ	Control	CI (95%)	Called	CI (95%)
Overall	-	0.52	(0.26; 1.03)	1.81	(1.43; 2.29)
Credit Score	35	0.95	(0.82; 1.09)	1.02	(0.98; 1.07)
Age (Years)	4.7	0.98	(0.86; 1.11)	0.99	(0.95; 1.03)
Genger					
Male	-11%	0.97	(0.92; 1.03)	1.00	(0.99; 1.02)
Marital Status					
Single	-30%	0.73	(0.54; 1.01)	1.11	(0.98; 1.27)
Products					
Auto and Home	17%	1.09	(0.91; 1.29)	1.06	(1.00; 1.12)
Auto Policies Count	0.28	1.23	(0.89; 1.71)	1.21	(1.09; 1.35)
Mortgage Count	0.12	1.04	(0.97; 1.11)	1.04	(1.02; 1.06)
Residences Count	0.13	0.92	(0.80; 1.05)	1.08	(1.02; 1.15)
Extra Options	46%	0.62	(0.41; 0.94)	1.12	(0.98; 1.28)

policy of persuadable clients are half (0.52) the odds of do-not-disturb clients with a 95% confidence interval of $[0.26;\ 1.03]$. Hence, based on our model, by calling identified persuadable clients and not calling identified do-not-disturb clients in future marketing campaigns should result in increased retention rates for the company.

Uplift prediction. The main objective in analyzing the insurance data is to estimate the parameters of the parametric uplift model which maximizes the Qini. Based on these estimates, we were able to provide useful insights to the company. In order to prevent overfitting, we made use of 5-fold cross validation in the fitting process. However, since uplift models can also be used for predicting future clients behaviour, it is important to evaluate outof-sample performance. In Table 5, we showed the in-sample performance. Since we do not have a test sample, in order to evaluate the out-of-sample performance, we proceeded in the following way. We ran 30 experiments. For each experiment, we reserved 25% randomly drawn observations for out-ofsample performance (test-set). We used the remaining observations to fit the models. These observations were further randomly divided into training-set, which comprised 2/3 of the remaining observations, and validation-set. We compare two ways of fitting the models. First, we only use the training set to fit the models, and compute the test-set performance through the adjusted Qini coefficient; the validation-set was not use in this process. Second, as

before, we use the training data to fit the models, but the model parameters and/or coefficients are chosen so as to find the best fit for the validation-set (cross-validation). For each experiment, the training-set size was 10,394 observations, the validation-set size was 5,354 observations, and the test-set size was 5,249 observations. To fit the RFs models, we searched for the hyper-parameters that maximize the adjusted Qini coefficient following the same procedure that was applied in the first part of the insurance data analysis.

The results of the only training-set way of fitting the models are displayed in Table 9. We see that the RF model shows the highest performance in the training-set. However, there are strong signs of overfitting. The Q+LHS models clearly outperform the other methods.

Table 9
Out-of-sample performance when models are trained using the training observations only.
The adjusted Qini coefficients are averaged over 30 experiments. Standard-errors are shown in parenthesis.

Method	training-set	test-set
RF	1.195 (0.020)	0.048 (0.014)
Q+LHS	$0.993 \ (0.029)$	$0.703\ (0.060)$
Q+lasso	$0.896 \ (0.025)$	0.093 (0.021)
MLE+lasso	0.481 (0.033)	0.033(0.012)

In order to mitigate the overfitting seen in the experiments where the models were fit using only the training observations, we now choose the model that maximizes the adjusted Qini on the validation set. Then, we score the observations from the test set to measure performance from a predictive point of view. The average results are presented in Table 10.

Table 10
Out-of-sample performance when models are trained with cross-validation (i.e., using both training and validation sets). The adjusted Qini coefficients are averaged over 30 experiments. Standard-errors are shown in parenthesis.

Method	training-set	validation-set	test-set
RF	$0.896 \ (0.031)$	0.152 (0.037)	0.071 (0.018)
Q+LHS	0.885 (0.032)	$0.859\ (0.051)$	$0.556\ (0.024)$
Q+lasso	0.618 (0.041)	0.450 (0.030)	0.127 (0.017)
MLE+lasso	0.303 (0.037)	0.057 (0.028)	0.049(0.009)

Based on these experiments, we see that the Q+LHS model gives the best results in terms of prediction. We are not surprised by the performance of the RF model because we had experimented with these RF models in the past, and we have not able to get better predictive performance in other marketing campaign initiatives.

6. Conclusion. Our goal was to analyze the data of a marketing campaign conducted by an insurance company to retain customers at the end of their contract. A random group of policyholders received an outbound courtesy call made by one of the company's licensed insurance advisors, with the objective to reinforce the customers confidence in the company, to review their coverage and address any questions they might have about their renewal. In the database at our disposal, an independent group of clients was observed and serves as control. In order to evaluate the causal effect of the courtesy call on the renewal or cancellation of the insurance policy of its clients, an uplift model needed to be applied.

We have developed a methodology for estimating parameters of a logistic regression in the context of uplift models. This is based on a new statistic specially conceived to evaluate uplift models. The statistic, the adjusted Qini, is based on the Qini coefficient. It takes into account the correlation between the observed uplift and the predicted uplift by a model. Maximizing the adjusted Qini to choose an adequate model for uplift acts as a regularizing factor to select parsimonious models, much as lasso does for regression models.

Since the Qini is a difficult statistic to compute, maximizing the adjusted Qini directly is not an easy task. Instead, we proposed to use lasso-type likelihood penalization to search the space of appropriate uplift models, so as to only consider relevant variables for uplift. Since the usual lasso is not designed for uplift models, we adapted it, by selecting the value $\hat{\lambda}$ of the lasso penalization constant that maximizes the adjusted Qini. At first, this ensures that the selected variables (i.e., those associated with non-zero regression coefficients) are important variables for estimating uplift. Then, in a second step, we estimate the parameters that maximize the adjusted Qini by searching a Latin hypercube sampling (LHS) surface around the lasso estimates. A variant of this procedure consists of estimating the parameters as those that maximize the likelihood associated with the model selected by $\hat{\lambda}$, using only the selected variables.

Experimental evaluation showed that for the first stage of the Qini optimized uplift regression, choosing the penalization constant from the logistic-lasso sequence by maximizing the adjusted Qini dramatically improves the performance of uplift models. This is the Q+lasso model. In addition, using a LHS search on the second stage leads to a direct maximization of the adjusted Qini coefficient, and to a further boost in the performance of the model. The resulting model is the Q+LHS model. In addition, our empirical studies clearly show that the performance of a Qini-based regression model is much better than the performance of the usual lasso penalized logistic

regression model.

Concerning the particular marketing data available to us from the insurance company, we selected two final models and compared them to the usual lasso regression approach as well as the uplift random forest. The results show that our method clearly surpasses the usual approach in terms of performance. We argue that this is due to the Qini-based methods performing variable selection explicitly build for optimizing uplift. Although, even if overall, the marketing campaign of the insurance company did not appear to be successful, the uplift models with the selection of the right variables identify a group of customers for which the campaign worked very well. Indeed, the results show that a persuadable client is a customer with many products to insure. Also, notice there is a subgroup of clients for whom the call had a negative impact. This can be explained by the fact that some customers are already dissatisfied with their insurance policies and have already decided to change them before receiving the call. This call can also trigger a behavior that encourages customers to look for better rates. For future campaigns, the company can target only those customers for whom the courtesy call will be useful and remove and investigate more the clients for whom the marketing campaign had a negative effect.

Acknowledgements. Mouloud Belbahri and Alejandro Murua were partially funded by The Natural Sciences and Engineering Research Council of Canada grant 2019-05444. Vahid Partovi Nia was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) discovery grant 418034-2012.

References.

Farrokh Alemi, Harold Erdman, Igor Griva, and Charles H Evans. Improved statistical methods are needed to advance personalized medicine. The open translational medicine journal, 1:16, 2009.

Leo Breiman, Jerome Friedman, Charles J Stone, and Richard A Olshen. <u>Classification and regression trees</u>. CRC press, 1984.

Kristof Coussement, Paul Harrigan, and Dries F Benoit. Improving direct mail targeting through customer response modeling. Expert Systems with Applications, 42(22):8403–8412, 2015

Floris Devriendt, Darie Moldovan, and Wouter Verbeke. A literature survey and experimental evaluation of the state-of-the-art in uplift modeling: A stepping stone toward the development of prescriptive analytics. Big data, 6(1):13–41, 2018.

Bradley Efron, Trevor Hastie, Iain Johnstone, Robert Tibshirani, et al. Least angle regression. The Annals of statistics, 32(2):407–499, 2004a.

Bradley Efron, Trevor Hastie, Iain Johnstone, Robert Tibshirani, et al. Least angle regression. The Annals of statistics, 32(2):407–499, 2004b.

Jianqing Fan and Runze Li. Variable selection via nonconcave penalized likelihood and its

- oracle properties. <u>Journal of the American Statistical Association</u>, 96(456):1348–1360, 2001.
- LLdiko E Frank and Jerome H Friedman. A statistical view of some chemometrics regression tools. Technometrics, 35(2):109–135, 1993.
- Jerome Friedman, Trevor Hastie, Holger Höfling, Robert Tibshirani, et al. Pathwise coordinate optimization. The annals of applied statistics, 1(2):302–332, 2007.
- Jerome Friedman, Trevor Hastie, and Rob Tibshirani. glmnet: Lasso and elastic-net regularized generalized linear models. R package version, 1(4), 2009.
- Jerome Friedman, Trevor Hastie, and Rob Tibshirani. Regularization paths for generalized linear models via coordinate descent. Journal of statistical software, 33(1):1, 2010.
- Corrado Gini. Concentration and dependency ratios. <u>Rivista di politica economica</u>, 87: 769–792, 1997.
- Leo Guelman. uplift: Uplift modeling. R package version 0.3, 5, 2014.
- Leo Guelman, Montserrat Guillén, and Ana M Pérez-Marín. Random forests for uplift modeling: an insurance customer retention case. In Modeling and Simulation in Engineering, Economics and Management, pages 123–133. Springer, 2012.
- Pierre Gutierrez and Jean-Yves Gérardy. Causal inference and uplift modelling: A review of the literature. In <u>International Conference on Predictive Applications and APIs</u>, pages 1–13, 2017.
- Isabelle Guyon and André Elisseeff. An introduction to variable and feature selection. Journal of machine learning research, 3(Mar):1157–1182, 2003.
- David J Hand and Keming Yu. Idiot's bayes not so stupid after all. <u>International Statistical</u> Review, 69(3):385–398, 2001.
- Behram Hansotia and Brad Rukstales. Incremental value modeling. <u>Journal of Interactive</u> Marketing, 16(3):35, 2002.
- Behram J Hansotia and Bradley Rukstales. Direct marketing for multichannel retailers: Issues, challenges and solutions. <u>Journal of Database Marketing and Customer Strategy</u> Management, 9(3):259–266, 2001.
- Dominique M Hanssens, Leonard J Parsons, and Randall L Schultz. <u>Market response</u> models: Econometric and time series analysis, volume 12. Springer Science & Business Media, 2003.
- Trevor Hastie, Jonathan Taylor, Robert Tibshirani, Guenther Walther, et al. Forward stagewise regression and the monotone lasso. <u>Electronic Journal of Statistics</u>, 1:1–29, 2007.
- Arthur E Hoerl and Robert W Kennard. Ridge regression: Biased estimation for nonorthogonal problems. <u>Technometrics</u>, 12(1):55–67, 1970.
- Paul W Holland. Statistics and causal inference. <u>Journal of the American Statistical Association</u>, 81(396):945–960, 1986.
- Kathleen Kane, Victor SY Lo, and Jane Zheng. Mining for the truly responsive customers and prospects using true-lift modeling: Comparison of new and existing methods. <u>Journal of Marketing Analytics</u>, 2(4):218–238, 2014.
- Maurice G Kendall. A new measure of rank correlation. Biometrika, 30(1/2):81–93, 1938. Victor SY Lo. The true lift model: a novel data mining approach to response modeling in database marketing. ACM SIGKDD Explorations Newsletter, 4(2):78–86, 2002.
- Max O Lorenz. Methods of measuring the concentration of wealth. <u>Publications of the</u> American Statistical Association, 9(70):209–219, 1905.
- Michael D McKay, Richard J Beckman, and William J Conover. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics, 42(1):55–61, 2000.
- Douglas C Montgomery, Elizabeth A Peck, and G Geoffrey Vining. Introduction to linear

regression analysis, volume 821. John Wiley & Sons, 2012.

John A Nelder and Roger Mead. A simplex method for function minimization. The computer journal, 7(4):308-313, 1965.

Neyman. On the application of probability theory to agricultural experiments. Annals of Agricultural Sciences, 1923.

J Pearl. Causal inference in statistics: An overview. Statistics Surveys, 3:96–146, 2009.

Nicholas J Radcliffe and Patrick D Surry. Real-world uplift modelling with significancebased uplift trees. White Paper TR-2011-1, Stochastic Solutions, 2011.

NJ Radcliffe. Using control groups to target on predicted lift: Building and assessing uplift models. Direct Market J Direct Market Assoc Anal Council, 1:14-21, 2007.

NJ Radcliffe and PD Surry. Differential response analysis: Modeling true response by isolating the effect of a single action. Credit Scoring and Credit Control VI. Edinburgh, Scotland, 1999.

Donald B Rubin. Estimating causal effects of treatments in randomized and nonrandomized studies. Journal of Educational Psychology, 66(5):688, 1974.

Piotr Rzepakowski and Szymon Jaroszewicz. Decision trees for uplift modeling. In 2010 IEEE International Conference on Data Mining, pages 441–450. IEEE, 2010.

Piotr Rzepakowski and Szymon Jaroszewicz. Decision trees for uplift modeling with single and multiple treatments. Knowledge and Information Systems, 32(2):303-327, 2012.

Robert E Smith and William R Swinyard. Information response models: An integrated approach. Journal of Marketing, 46(1):81–93, 1982.

Xiaogang Su, Chih-Ling Tsai, Hansheng Wang, David M Nickerson, and Bogong Li. Subgroup analysis via recursive partitioning. Journal of Machine Learning Research, 10 (Feb):141-158, 2009.

Xiaogang Su, Joseph Kang, Juanjuan Fan, Richard A Levine, and Xin Yan. Facilitating score and causal inference trees for large observational studies. Journal of Machine Learning Research, 13(Oct):2955–2994, 2012.

R. Tibshirani. Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society. Series B (Methodological), pages 267–288, 1996.

Hui Zou and Trevor Hastie. Regularization and variable selection via the elastic net. Journal of the Royal Statistical Society: Series B, 67(2):301–320, 2005.

Département de mathématiques et de statistique. Département de mathématiques et de statistique. Université de Montréal,

CP 6128, Succ. Centre-Ville.

Montréal, Québec H3C 3J7 Canada E-MAIL: belbahrim@dms.umontreal.ca

TD Insurance.

50 Place Crémazie,

Montréal, Québec H2P 1B6 Canada E-MAIL: olivier.gandouet@tdassurance.com

Université de Montréal, CP 6128, Succ. Centre-ville.

Montréal, Québec H3C 3J7 Canada

E-MAIL: murua@dms.umontreal.ca

Advanced Analytics, Research and Development, Department of Mathematics and Industrial Engineering,

ÉCOLE POLYTECHNIQUE DE MONTRÉAL, 2900 Boulevard Édouard Montpetit. Montréal, Québec H3T 1J4 Canada E-MAIL: vahid.partovinia@polymtl.ca