Bartłomiej Nieledź

Odkształcenie sprężyste

$$-\frac{d}{dx}\left(E(x)\frac{du(x)}{dx}\right)=0$$

$$u(2)=0 \qquad E(x)=\begin{cases}3; & x\in[0,1]\\ 5; & x\in(1,2]\end{cases}$$

$$\frac{du(0)}{dx}+u(0)=10 \Rightarrow u'(0)=10-u(0)$$

gdzie u to poszukiwana funkcja $[0,2]\ni x \rightarrow u(x)\in\mathbb{R}$

$$-\frac{d}{dx}\left(E(x)\frac{du(x)}{dx}\right)=-\left(\underbrace{\frac{dE(x)}{dx}}_{=0}\frac{du(x)}{dx}+E(x)\frac{d^2u(x)}{dx^2}\right)=-E(x)\frac{d^2u(x)}{dx^2}$$

$$-E(x)u''=0$$

$$\int_0^2 -E(x)u''\cdot v\,dx=\int_0^2 0\cdot v\,dx \qquad \forall v\in V$$

$$\int_0^2 -E(x)u''v\,dx=0$$

$$\left[-E(x)u'v\right]_0^2-\int_0^2 -E(x)\cdot u'v'\,dx=0$$

$$-5u'(2)v(2)+3u'(0)v(0)+\int_0^2 E(x)\cdot u'v'\,dx=0$$

$u'(0)=10-u(0)$

$u(2)=0 \Rightarrow v(2)=0$

$$3\cdot(10-u(0))\cdot v(0)+\int_0^2 E(x)u'v'\,dx=0$$

$$\underbrace{\int_0^2 E(x)u'v'\,dx-3u(0)v(0)}_{B(u,v)}=\underbrace{-30v(0)}_{L(v)}$$