A Three-dimension Stacking Model with Modified Genetic Algorithm

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2022 IEEE International Symposium on Product Compliance Engineering - Asia (ISPCE-ASIA) | 979-8-3503-3248-3/22/$31.00 ©2022 IEEE | DOI: 10.1109/ISPCE-ASIA57917.2022.9971062

***Abstract*—The three-dimensional stacking problem (3D-SP) is a challenging task in cold chain warehouse. Different from common three-dimensional packing problem, 3D-SP problem is more operable and can be stacked from all directions of the pallet. Based on this characteristic, we construct our model by considering the utilization rate of pallet space and the stability criterion of goods together as objective function. Further, four constraints are designed, which are placement direction, pallet space and no overlapping. According to the characteristics of the problem, a new improved genetic algorithm is proposed. In specific, the order of goods placement is regarded as individual, and with the consideration of order feature, we designed a more reasonable crossover and mutation operator. Compared with traditional greedy and genetic algorithm, our algorithm outperforms them and proved to be effective on 3D-SP problem.**

***Index Terms*—stacking problems, multiple targets, genetic algorithms, NP problems**

I. Introduction

The Bin Packing Problem (BPP) appears as the mainproblem in a large number of industrial applications. It is such a Non-deterministic Polynomial-hard (NP-hard) combinatorial grouping problem that often occurs in reallife, including computer science, engineering, logistics, and manufacturing. As an advanced variety of BPP, the 3D stacking problem, which studies how to pack items with a minimum number of pallets, has been studied at home and abroad since the 1980s, hence there are also many predecessors' experiences for study [1]. Theoretically, the 3D stacking problem is a NP-hard Problem [2] as well.

In the actual scene, the 3D stacking problem of cold storage goods has various constraints and unknowns [3]. In the actual situation, the order quantity is large and the specifications are multifarious, so the real-time order pre-processing cannot be achieved; different from containers, pallets have no vertical walls to block the placement of goods [4], so the risk of goods collapsing is higher; and even for regular goods with complete packaging, there is still a risk of deformation of the lower goods due to the excessive pressure exerted by the upper goods; the contact surface does not provide enough support for the goods [5], causing the goods to slide or roll. This requires high center of gravity distribution of stacking results [6].

The 3D-stacking model has more combination possibilities and randomness than the 2D model [7], so it has the probability of curse of dimension [8]. When establishing the 3D stacking model, the model should be properly simplified without being too divorced from reality, and various assumptions should be established. For example, it is assumed that there is a pretreatment process from the known size to the stacked goods [8], and the rigid rectangular goods with equal density everywhere are loaded orthogonally on the limited rectangle provided by the flat pallet [10], and there is few that certain goods can only be loaded in a specific direction. While satisfying the efficient utilization of tray space, the weight method is used to add the center of gravity constraint to the objective equation, which makes the center of gravity distribution of stacking results more stable.

In our work, we took the stacking order of goods as an individual, and adjusted the stacking order and placement mode to make the goods more compact and the center of gravity more reasonable [11]. In order to explore the feasibility of the genetic algorithm, this paper compares the stacking order after the iteration of the genetic algorithm with the stacking order obtained by the greedy algorithm [12] for each time of stacking the goods with the current maximum bottom area first, and the stacking order placed randomly, in terms of the fitness function, the number of required pallets, and the gravity center ratio. In order to explore the universality of the genetic algorithm, we do the following:

The utilization rate of pallet space and the stability criterion of goods are combined to build a mathematical model;

Four constraints, including placement direction, tray space and non overlapping, are designed;

An improved genetic algorithm is proposed according to the characteristics of the problem.

II. Three-dimensional stacking problem

A. Assumption

In the ideal assumption, the three-dimensional stacking model can be simplified as that rigid rectangular goods with equal density can be loaded orthogonally on the limited rectangle provided by the flat pallet, and there is no case that certain goods can only be loaded in a specific direction.

1) Length sort: Assuming that no matter what the specification of goods is, $L_{ij} \geq B_{ij} \geq H_{ij}$.

2) Known order: Assuming that the goods can know the order content in advance, the goods required for the order can be preprocessed, and stacking order can be controlled to achieve the maximum utilization of tray space.

3) Regular rectangles: Assuming that all goods are regular rectangles, which can be assembled into a whole through orthogonal arrangement.

4) Rigid body: Assuming that all rectangular goods are rigid bodies and will not deform when squeezed, this simplification can avoid the changes in specifications that are difficult to quantify when the goods are squeezed and deformed, and can also reduce the complexity of calculation caused by uneven density distribution of goods with different gravity centers.

5) The center of gravity: Assuming that the center of gravity of the goods will collapse in the air, regardless of the fact that multiple goods will lift other goods without supporting the center of gravity.

B. Parameter

According to the consideration of modeling simplification and actual situation, the parameters shown in I.

Among this,

$$C^* = \lceil [\frac{\sum_{i=1}^{n} l_{ij} * b_{ij} * h_{ij}}{1200mm^*1000mm^*1300mm}] \rceil \tag{1}$$

$$G^* = \frac{\sum_{i=1}^{n} (X_{i2}/2) * w_i}{\sum_{i=1}^{n} w_i} \tag{2}$$

The six placement modes represented by s_{ik} are shown in the 1.

C. Objective Function

Then the objective function is:

$$\text{Min}\, z = \alpha \frac{C}{C^*} + (1-\alpha) \frac{G}{G^*} \tag{3}$$

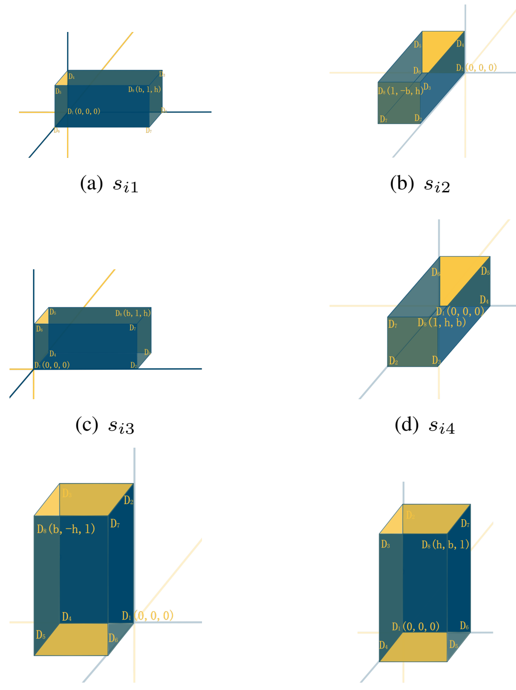
There α is the weight of , and $\alpha \in (0,1)$ of the ratio C / C^* between the actual pallet quantity and the ideal pallet minimum quantity in the objective function. the $(1-\alpha)$ and C / C^* are the same.

D. Constraint Condition

1) Placement direction constraint: When $L : B \geq 3 : 1$, L cannot be placed on the Z-axis; When$B : H \geq 3 : 1$, B cannot be placed on the Z-axis. That is, when the goods are in strip or stick shape, the goods cannot be placed vertically; when the goods are in plate shape, the goods cannot be placed vertically. When the goods are in plate shape, the goods cannot be placed upright, which can avoid the collapse to a certain extent. As shown in 2.

2) Pallet space constraint:

$$(0,0,0) \leq (x_{d_{io}}, y_{d_{io}}, z_{d_{io}}) \leq (100, 120, 130) \tag{4}$$



(a) s_{i1} (b) s_{i2}

(c) s_{i3} (d) s_{i4}

(e) s_{i5} (f) s_{i6}

Fig. 1: The six placement modes

(a) Rules for placing strip goods (b) Rules for placing plate goods

Fig. 2: Rules for special shaped goods

3) No overlapping constraint:

$$\begin{aligned} & a = \max(x_{d_{io}}) \leq \min(x_{d_{po}}) \mid \max(x_{d_{po}}) \leq \min(x_{d_{io}}) \\ & b = \max(y_{d_{io}}) \leq \min(y_{d_{po}}) \mid \max(y_{d_{po}}) \leq \min(y_{d_{io}}) \\ & c = \max(z_{d_{io}}) \leq \min(z_{d_{po}}) \mid \max(z_{d_{po}}) \leq \min(z_{d_{io}}) \\ & t = a + b + c > 0 \quad d_i \in P_u \quad d_p \in P_e \end{aligned} \tag{5}$$

4) No dangling constraint: In particular, when a, b and c are taken as true, t = 3 indicates that the goods to be stacked and the goods to be inspected are in a common point, t = 2 is collinear, and t = 1 is coplanar. As shown in 3.

No dangling constraint by a pseudo code is 1.

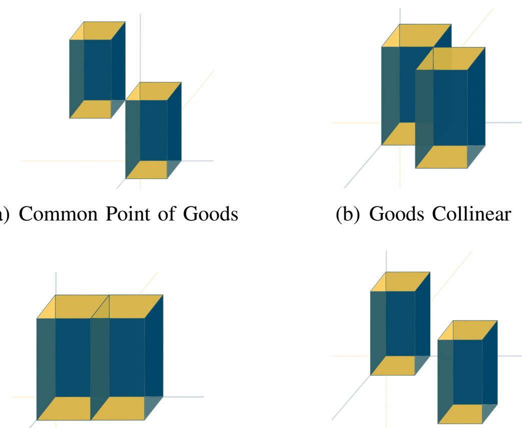
III. The Improved Genetic Algorithm to Solve Stacking Problem

A. Chromosome

The placement mode of this batch of goods, i.e. the placement direction and placement position of each goods, will be described by whether the length, width and height of the object coincide with or are parallel to the X, Y and Z axes, and the placement position will be represented by the X, Y and Z

TABLE I: Parameters table

Parameter	
i = 1, 2, 3, ..., n	The i-th goods in n goods in a batch of orders
j = 1, 2, 3	X, Y and Z-axis directions in spatial orientation
l_{ij}, b_{ij}, h_{ij}	The i-th cargo in the direction of its inherent length, width and height
w_i	The weight of the i-th cargo
s_{ik}, k = 1, 2, ..., 6	Six bottom areas of the i-th cargo, corresponding to six placement methods
d_{io}, o = 1, 2, ..., 8	Eight vertices of the i-th cargo
$P_e, P_u, P_e \cup P_u = P$	Placed, to be placed pallet space, and whole pallet space
C	The number of ideal pallets
C^*	The minimum number of ideal pallets
X	The height of the nth cargo in the Y-axis direction in the tray space
G	Synthetic centroid
G^*	Lowest synthetic centroid



(a) Common Point of Goods (b) Goods Collinear

(c) Goods Coplanar (d) no common ground between the goods

Fig. 3: Spatial relationship between goods

Algorithm 1 No dangling constraint

Input: Cargo X, Y axis coordinates
Output: Cargo Z coordinate

1: **if** d_{i0}satisfies the non overlapping constraint and does not satisfy $\max\left(x_{d_{i0}}\right) \leq \min\left(x_{d_{p0}}\right) \left|\max\left(x_{d_{i0}}\right) \leq \min\left(x_{d_{p0}}\right)\right| \max\left(y_{d_{i0}}\right) \leq \min\left(y_{d_{p0}}\right) \mid \max\left(y_{d_{i0}}\right) \leq \min\left(y_{d_{p0}}\right)$; **then**

2: **if** the cargo centroid c_i is not satisfied $x_{C_{i0}} \leq \min\left(x_{d_{p0}}\right) \left|\max\left(x_{D_{po}}\right) \leq x_{C_{i0}}\right| y_{C_{i0}} \leq \min\left(y_{d_{p0}}\right) \mid \max\left(y_{d_{p0}}\right) \leq y_{C_{i0}}$; **then**

3: Goods can be placed, and Zd1 is the maximum value of the p-th goods on the Z-axis;

4: **else**

5: goods collapse and cannot be placed;

6: The goods are the goods at the bottom of the pallet, which can be placed, and z_{d_1} = 0, $d_i \in P_u \quad d_p \in P_e$

axis coordinates of the eight vertices; The fitness of the goods placed this time, that is, after the goods are placed in this order, the objective function value is calculated, and the variation of the objective function is taken as the fitness of the individual.

Similarly, when placing, the vertex position is still used as the basis for recording the placement position and placement direction, and six placement directions are judged by the placement position. The X and Y axis coordinates of the center of gravity also depend on the coordinates of the four vertices of the bottom area to determine whether it is dangling [16].

The length, width, height and quality of the goods are the input variables of the goods, and the detailed information of the order table needs to be imported. The placement position is the random placement of goods at one vertex. There may be multiple placement modes through constraint selection. The positions of the other seven vertices can be calculated, and then the individual fitness can be calculated.

B. Crossover and Mutation

The placing order is not certain, in this case, the common single point crossing or double point crossing may cause duplication or default of order goods.In order to avoid such a situation, this paper chooses to cross within the individual, that is, any two sections of the cargo placing order are exchanged in order to obtain a new placing order, as shown in 4.

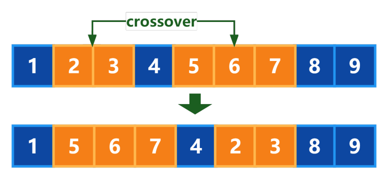


Fig. 4: Goods are placed in cross order.

In addition, since the iteration object is the placement order, the common variation method for the number sequence is not suitable for the problems with aftereffect such as the three-dimensional stacking problem, and the rash variation will also cause the repetition or default of the order goods. In order to avoid such a situation, this paper still carries out variation within the individual, that is, randomly select any section of the goods placement order, insert any position in the extracted placement order, and obtain a new placement order, as shown in 5.

Although it is not a typical crossover and mutation operation, through such changes, a new cargo placement order can still be generated, and the placement mode of the cargo can be changed through the change of the placement order. The

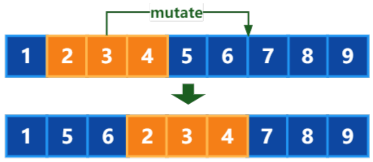


Fig. 5: Goods are placed in inserted order.

new placement mode is obtained through calculation. After the new placement mode is determined, the target function value is calculated, and the lowest objective function value, i.e. the maximum fitness, is selected to change the individual fitness [17].

C. Fitness Function and Selection

The fitness function is a variant of the objective function mentioned above:

$$\text{fitness } = 1 - \left[\alpha \frac{C}{C^*} + (1-\alpha) \frac{G}{G^*}\right] \tag{6}$$

The second is the actual synthetic centroid height, which needs to be determined after the placement position of each cargo is determined, and then the actual synthetic centroid height can be calculated. The above two actual values are divided by the number of ideal pallets and the height of the ideal synthetic center of mass. Both are divided by the ideal minimum value to obtain the actual maximum value. If it is converted into a fitness function, the greater the fitness, that is, the more the individual adapts to the environment, the easier it is to inherit.

In the initial feasible solutions, there is a satisfactory solution calculated by the greedy algorithm, so its fitness can be taken as a selection standard. If the individual fitness after the exchange mutation fails to meet the selection standard, it will not participate in the population updating [18]. Only the individual who meets the selection standard can form a new generation of population with other individuals, and iterating the population repeatedly to achieve higher fitness.

In the selection operator, this paper adopts the roulette selection method which repeats K times [19]. The selection methods commonly used in genetic algorithm include random traversal sampling method and tournament selection method. Roulette selection method is also known as proportional selection method. As the name implies, in the selection process, the probability of each gene individual being selected is positively correlated with its fitness. Let the probability that each individual can be selected to be inherited into the next generation of new population be$P\left(x_i\right)$, and the environmental fitness of each individual be$f\left(x_i\right)$, then:

$$P\left(x_i\right) = \frac{f\left(x_i\right)}{\sum_{j=1}^{n} f\left(x_j\right)} \tag{7}$$

And record the accumulation probability of each gene individual. Let the accumulation probability be q_i, then

$$q_i = \sum_{j=1}^{i} P\left(x_j\right) \tag{8}$$

And a random number is arbitrarily taken between 0 and 1 through the random function. If the random number is less than the accumulation probability, the new individual after genetic operation is selected.

If the random number is greater than the accumulation probability, the original individual before genetic operation is selected, and the cycle is k times until all individuals in a generation of population are traversed. Since the accumulation probability is superimposed according to the individual fitness, the stronger the individual fitness, the more it accounts for in the accumulation probability, the greater the probability that the random number is less than the accumulation probability corresponding to the individual, and the greater the probability that the individual can inherit to the next generation population. However, due to the randomness of random numbers, it is still possible to select the original individual as the genetic object. Although this selection method will lose some better solutions, it can alleviate the premature convergence of the model to a certain extent, so that the population falls into the local optimal solution [20]. The algorithm logic is shown in 6.

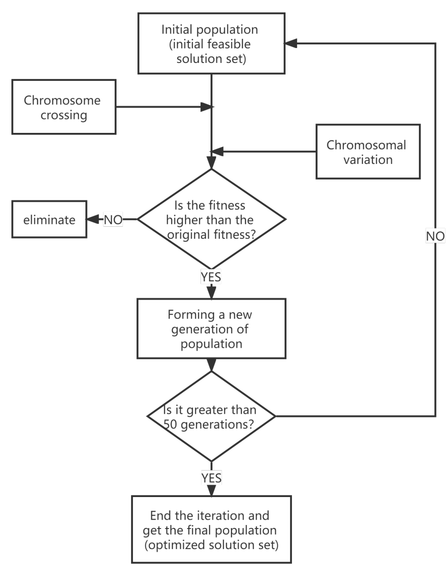


Fig. 6: Logic flow chart of genetic algorithm.

We can see from the Fig.6 that after continuous iteration, the genetic algorithm crosses and mutates from the initial feasible solution. Through selection based on fitness, the optimized solution set is finally obtained and the satisfactory solution is achieved.

IV. Algorithm Simulation and Result Analysis

The ordering list of the most fitness placement order in the latest population after iteration is output in the form of an excel table that records the placement order, placement direction, number of trays and the most important fitness. We compare the fitness of random order, greedy algorithm performs order and genetic algorithm performs order under different batch quantities and size of goods. In order to make it more concrete, we use charts to show the comparison results, as shown in 7, 8 and 9.

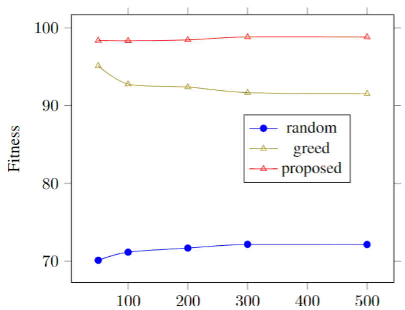


Fig. 7: Fitness of different algorithm results

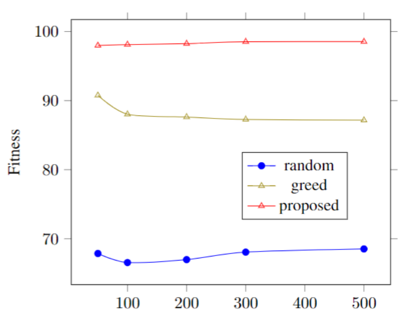


Fig. 8: Batch quantity of goods of different algorithm results

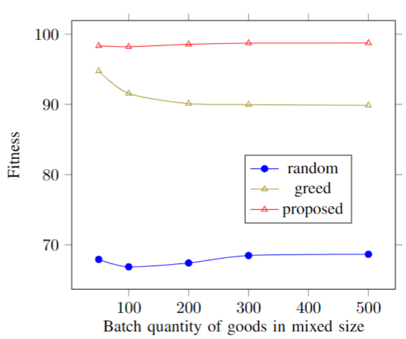


Fig. 9: Number of pallets of different algorithm results

First, we can see from the above figure that the fitness of the rearranged cargo sequence based on the greedy algorithm after the stacking is greatly improved compared with the completely random sequence, and it is very close to the ideal optimal solution. However, the genetic algorithm is closer to the optimal solution based on the greedy algorithm. Since the ideal synthetic centroid height actually violates the non overlap constraint, it cannot be executed under the actual stacking, Therefore, in the actual situation, the optimal solution can not reach 100%, so it can be considered that the result obtained by genetic algorithm is a relatively satisfactory solution.However, the batch quantity of goods has little impact on the fitness. Only in the application of greedy algorithm, there is a big drop from 50 to 100, which may indicate that greedy algorithm is more suitable for the scenario of goods stacking in smaller batches.

TABLE II: Number of pallets

order	random			greed			gentic		
size	Small	Large	Mixed	Small	Large	Mixed	Small	Large	Mixed
50	21	34	24	10	16	13	6	12	8
100	51	65	63	21	33	24	11	22	13
200	105	137	131	43	68	49	23	46	27
300	184	235	213	69	107	74	36	67	42
500	297	361	342	147	158	135	59	124	78

TABLE III: Center of gravity ratio

order	random			greed			gentic		
size	Small	Large	Mixed	Small	Large	Mixed	Small	Large	Mixed
50	5.113	4.427	4.735	3.155	2.772	2.935	2.858	2.146	2.763
100	5.602	4.826	5.126	3.207	2.863	3.101	2.674	2.100	2.446
200	5.651	4.893	5.147	3.389	3.086	3.435	2.357	2.003	2.248
300	5.704	4.967	5.158	3.592	3.317	3.746	2.233	2.000	2.132
500	5.775	5.100	5.162	4.010	3.578	4.257	2.162	1.97	2.078

Then, by comparing the results of three different size of goods, it can be found that the greedy order performs worse in the case of large size of goods and better in the case of small size of goods. In any case, the gene sequence is quite stable regardless of the quantity or size of goods, and is better than the greedy sequence.

At the same time, it can be concluded that number of pallets and center of gravity ratio of different situations, as shown in II and III.

In Table II, we can find that the trend of the three orders can be regarded as linear single increase, that is, for different batch quantity of goods, the utilization rate of tray space basically maintains a stable level, and will not change dramatically with the batch size. On the whole, the performance of the three orders in small size is better than that in large.

In Table III, the difference is that the change of the center of gravity ratio with respect to batches is more obvious. With the increase, the random order rises significantly and then remains unchanged. The application of genetic algorithm drops significantly and then remains unchanged. The greedy algorithm shows a single increase trend. It can be understood that the application of genetic algorithm plays a significant role in the stability of large batches of goods, while the greedy algorithm increases the instability of goods after the batch quantity increases, which is prone to collapse.Contrary to Table II, the three orders here are better than those in the situation of large size.

In addition to the comparison between different algorithms, we also constantly experiment and compare by adjusting the weight parameters to explore a better objective function. Through experiments under the application of different weights, we found that different weight coefficients are also of great significance to the quality of the algorithm. With the increase of weight coefficients α, the fitness result drops rapidly after reaching the peak. It can be seen that the larger the weight coefficient is, the better. A weight coefficient of about 0.1 is more suitable for the algorithm, that is, 90% considers the tray space utilization rate, and 10% considers the impact of the center of gravity on the results.

To sum up, the genetic algorithm can correct and optimize the limitations and irrationality of the greedy algorithm by iterating the initial placement sequence including the greedy algorithm sequence to form a combination of large goods and small goods.

V. Conclusion

In this paper, we used several algorithms in the initial population of genetic algorithm, sorting based on the bottom area and weight of the goods, a better initial solution is obtained. Through the analysis of the experimental results, it can be concluded that the application effect of genetic algorithm is better than that of greedy algorithm, and is significantly better than that of random stacking.Therefore, we conclude that the improved three-dimensional stacking algorithm based on genetic algorithm is effective. However, the model in this paper can not deal with the order of goods with random and unknown specifications very well, and there are still some deficiencies in practical value to be remedied.

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