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2) To apply Master Theorem $T(n)$ must be considered following conditions: $\sqrt{k} \geq 1$ and $k^2 > 1 \Rightarrow k > 1$.

$c \cdot \sqrt[3]{n}$ must be asymptotically positie $\Rightarrow$ plus we have $c^6 > k^5$; so $c > 1$.

Let consider third case:

$c \cdot n^{\frac{1}{3}} = \Omega\left(n^{\log_{k^2}\sqrt{k} + e}\right)$: by Ω-notation $f(n) = \Omega(g(n))$, if exist $p > 0$ and n_0, that inequality $g(n) \cdot p <= f(n)$ holds for all $n > n_0$.

$$n^{\log_{k^2}\sqrt{k} + e} \cdot p <= c \cdot n^{\frac{1}{3}} \quad (*)$$

$$n^{\frac{1}{4}} \cdot n^{e} \cdot p <= c \cdot n^{\frac{1}{3}}$$

$$n^{e} \cdot p <= c \, n^{\frac{1}{12}}$$

for example $(p = c)$

for $e = \frac{1}{12}$ and $p <= c$ for any n inequality $(*)$ holds

And $\sqrt{k} \cdot c \cdot \left(\frac{n}{k^2}\right)^{\frac{1}{3}} \leq m \cdot c \cdot n^{\frac{1}{3}}$ for some constant $m < 1$.

$$k^{-\frac{1}{6}} \cdot c \cdot n^{\frac{1}{3}} \leq m \cdot c \cdot n^{\frac{1}{3}}$$

$$\frac{1}{\sqrt[6]{k}} \leq m$$

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We can take $m = \frac{1}{\sqrt[6]{k}}$ because $\frac{1}{\sqrt[6]{k}}$ - const and $\frac{1}{\sqrt[6]{k}} < 1$ $(k > 1)$.

Finally, $T(n) = \Theta\left(c \cdot n^{\frac{1}{3}}\right) = \Theta\left(n^{\frac{1}{3}}\right)$ for $k > 1$.

1) $T(n) = \sqrt{k} \cdot T\left(\frac{n}{k^2}\right) + c \cdot \sqrt[3]{n}$ $\quad T(1) = 0$

$$T\left(\frac{n}{k^2}\right) = \sqrt{k} \cdot T\left(\frac{n}{(k^2)^2}\right) + c \cdot \sqrt[3]{\frac{n}{k^2}} = \dots = \left(\sqrt{k}\right)^p T\left(\frac{n}{k^{2p}}\right) + \sum_{i=0}^{p-1} (\sqrt{k})^i \sqrt[3]{\frac{n}{k^{2i}}}$$

If we set $p = \log_{k^2} n = \frac{1}{2} \log_k n$

$$T(n) = (\sqrt{k})^{\frac{1}{2}\log_k n} \underbrace{T\left(\frac{n}{n}\right)}_{T(1)=0} + c \sum_{i=0}^{\frac{1}{2}\log_k n - 1} (\sqrt{k})^i \sqrt[3]{\frac{n}{k^{2i}}} = c \sum_{i=0}^{\frac{1}{2}\log_k n - 1} \left((\sqrt{k})^i \sqrt[3]{\frac{n}{k^{2i}}} \right)$$