

Task 2 3.2.1.

①

function search(value)	cost	time
{ for array in arrays	C_1	$\sum_{j=0}^{k-1} b_j$ (or +1 if extra cond.)
{ int l = 0, r = array.length;	C_2	$\sum_{j=0}^{k-1} b_j$
while (l < r)	C_3	$\sum_{j=0}^{k-1} b_j \cdot C_j + 1$
{ int mid = (l + r) / 2;	C_4	$\sum_{j=0}^{k-1} b_j \cdot C_j$
if (array[mid] < value)	C_5	$\sum_{j=0}^{k-1} b_j \cdot C_j$
{ l = mid + 1; }	C_6	$\sum_{j=0}^{k-1} \sum_{m=1}^{q_j} (t_m)$
else if (array[mid] > value)	C_7	$\sum_{j=0}^{k-1} \sum_{m=1}^{q_j} (1 - t_m)$
{ r = mid; }	C_8	$\sum_{j=0}^{k-1} \sum_{m=1}^{q_j} t_m$
else	C_9	0
{ return true; }	C_{10}	1
}		
return false;	C_{11}	1
}		

 $C_j \quad 1 \leq C_j \leq \text{number of shifts of pointers l/r.}$
 $t_m = 0/1$
if() == true

 $t_m = 0/1$
else if() == true

② Worst-case: binary representation of n - all 1.

$$\sum_{j=0}^{k-1} b_j = k$$

We will not find our value and j times will shift our pointers l and r , so $C_j = j$. Also, every time in while-loop we will execute only C_7 and C_8 ($t_m = 0$ for every m)

$$\sum_{j=0}^{k-1} \sum_{m=1}^{q_j} (1 - t_m) = \sum_{j=0}^{k-1} j$$

$$\sum_{j=0}^{k-1} j = \frac{0+k-1}{2} \cdot k = \frac{k^2 - k}{2}$$

$$T(n) = C_1 \cdot k + C_2 \cdot k + C_3 \left(\frac{k^2 - k}{2} + 1 \right) + C_4 \left(\frac{k^2 - k}{2} \right) + C_5 \left(\frac{k^2 - k}{2} \right)$$

$$+ C_7 \cdot \left(\frac{k^2 - k}{2} \right) + C_8 \left(\frac{k^2 - k}{2} \right) + C_{11} = \frac{k^2}{2} (C_3 + C_4 + C_5 + C_7 + C_8) + k(C_1 + C_2) -$$

$$- \frac{k}{2} (C_3 + C_4 + C_5 + C_7 + C_8) + C_3 + C_{11}, \text{ where } [k = \log(n+1)]$$

$$\textcircled{3} \quad T(n) = \frac{k^2}{2} \cdot \text{const} + k \text{ const} + \text{const} = O(\log_2^2(n+1)) = O(\log_2^2 n)$$