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1. $f(n) = 10n\log n + 500n + n^2 + 123$

$f \in O(g(n))$ $g(n) - ?$

By definition of Big-O notation for at least one choice of a constant $c > 0$ exist such number $n_0$, that $0 <= f(x) <= c \cdot g(x)$ (*) holds for all $n > n_0$

Let $g(n)$ be $n^2$ (because in $f(n)$ it is the biggest term and $\log n < n$)

$$10n\log n + 500n + n^2 + 123 <= c \cdot n^2 \quad / : n^2$$

$$\frac{10\log n}{n} + \frac{500}{n} + 1 + \frac{123}{n^2} <= c$$

then for $n_0 = 1$

$n = 1$ $\quad 0 + 500 + 1 + 123 = 624$

for all $n > 1$ $\quad \frac{10\log n + 500}{n} + 1 + \frac{123}{n^2} <= \frac{10\log n + 500}{n-1} + 1 + \frac{123}{(n-1)^2}$

$$\frac{10\log n \, n + 500n - 10\log n - 500 - 10\log n \cdot n - 500n}{n(n-1)} + \frac{123n^2 - 123 \cdot 2n + 123 - 123n^2}{n^2(n+1)^2} <= 0$$

$$\frac{10\log n + 500}{n(n-1)} + \frac{123 \cdot 2n - 123}{n^2(n+1)^2} >= 0$$

That means that for $c = 624$ and $n_0 = 1$ inequality* holds for every $n > n_0$.

2. $n^{\frac{9}{2}} + 7n^4\log n + n^2$.

$n^{\frac{9}{2}} = O(n^{\frac{9}{2}})$

$n^2 = O(n^2)$

$7n^4\log n <= c \cdot n^4\log n$ for $c = 7$ and any $n$ (by definition of Big-O notation)

$7n^4\log n = O(n^4\log n)$

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Among functions $n^{\frac{9}{2}}$, $n^2$, $n^4\log n$ max function is $n^{\frac{9}{2}}$, because

$$n^4\log n <= n^{\frac{9}{2}} \cdot c$$

So, $n^{\frac{9}{2}}+7n^4\log n+n^2=O(n^{\frac{9}{2}})$

3. $6^{n+1}+6(n+1)!+24n^{42}$

$$6^{n+1}=6\cdot 6^n$$

$6\cdot 6^n <= c\cdot 6^n$, for $c=6$ and any $n$.

$24\cdot n^{42} <= c\cdot n^{42}$ for $c=24$ and any $n$.

$6(n+1)! <= c\cdot(n+1)!$ for $c=6$ and any $n$.

So, we have $6^{n+1}=O(6^n)$, $6(n+1)!=O(n+1)!$, $24n^{42}=O(n^{42})$.

By theorem of functions whose asymptotic behaviors are known
if $f_1(n)=O(g_1(n))$ and $f_2(n)=O(g_2(n))$, then $f_1(n)+f_2(n)=O(\max(g_1(n),g_2(n)))$

Among functions $6^n$, $(n+1)!$, and $n^{42}$ max function is $(n+1)!$

Finally, we have $6^{n+1}+6(n+1)!+24n^{42}=O(n+1)!$