Computational Numerical Statistics
Assignment 10

Migla Miskinyte Reis

09.12.20

Exercise 1. (0.1 points) **Points:** ______

\Let X ~N(μ, σ^2), with σ^2 that is known and p.d.f

$$\text{f(x;}\mu\text{,}\sigma^2\text{)} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \text{ where} \quad \text{x} \in \text{R} \quad \text{and } \mu \in \text{R}$$

1.1 Show that this family of distributions belongs to the exponential family. This is a Gaussian normal distribution and it is known that it can fit a class of exponential families. For two paramether exponential family we can write a function as:

$$\text{f(x;}\theta\text{)} = \exp\left[\frac{x\theta - b(\theta)}{a(\psi)} + c(x,\psi)\right], \text{ where } \psi \text{ is dispersion parameter and } \theta \text{ is a location parameter, b is a cumulant function.}$$

Thus we can re-write our normal distribution:

$$\text{f(x;}\mu\text{,}\sigma^2\text{)} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] =$$

by taking a log:

$$\log f(x;\mu,\sigma^2) = -\tfrac{1}{2}\log(2\pi\sigma^2) - \tfrac{(x-\mu)^2}{2\sigma^2}$$

and the exponential:

$$= \exp\left[-\frac{1}{2}\log(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$= \exp\left[-\frac{1}{2}\log(2\pi\sigma^2) - \frac{(x^2 - 2\mu + \mu^2)}{2\sigma^2}\right]$$

$$= \exp\left[(x\mu - \frac{\mu^2}{2})/\sigma^2 + \frac{-1}{2}(\frac{x^2}{\sigma^2} + log(2\pi\sigma^2))\right]$$

Here, we can clearly see that in our equation $x\theta$ is $x\mu$, $b(\theta)$ is $\frac{\mu^2}{2}$, $a(\psi)$ is σ^2 and the rest of the equation is $c(x, \psi)$

1.2 Is your formulation in 1.1 the canonical form? If not, write it down and identify the canonical link.

In our case we can already see our formulation is the canonical form and that μ parameter is θ, because $x\theta$ $= x\mu$.

1.3 Use the canonical form to show that $\mathbb{E}(X) = \mu$ and V(X) $= \sigma^2$

From the canonical function of exponential distribution we have already shown that $x\theta = x\mu$, $b(\theta) = \frac{\mu^2}{2}$, $a(\psi) = \sigma^2$. From the properties of exponential family, we know that we can derive moments from canonical function:

$$E\left[X\right] = b'(\theta) =(\frac{\theta^2}{2})' = (\frac{\mu^2}{2})' = \mu$$

And:

$$Var\left[X\right] = \psi b''(\theta) =\sigma^2(b)'' = \sigma^2 1 = \sigma^2$$