BA 64018-003 Assignment 3 - Transportation Model

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Question

Heart Start produces automated external defibrillators (AEDs) in each of three different plants (A, B and C). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping

```
table <- matrix(c(20,14,25,400,100,
                  12,15,14,300,125,
                  10,12,15,500,150,
                  80,90,70,"-","-"),nrow = 4, byrow = T)

colnames(table) <- c("Ware. 1", "Ware. 2", "Ware. 3", "P.Cost", "P.Capacity")
rownames(table) <- c("Plant A", "Plant B", "Plant C", "M.Demand")

table <- as.table(table)


library(kableExtra)

table%>%
  kable()%>%
  kable_classic()%>%
  column_spec(2, border_left = T)%>%
  column_spec(6, border_left = T)%>%
  row_spec(3, extra_css = "border-bottom:dotted;")
```

	Ware. 1	Ware. 2	Ware. 3	P.Cost	P.Capacity
Plant A	20	14	25	400	100
Plant B	12	15	14	300	125
Plant C	10	12	15	500	150
M.Demand	80	90	70	•	•

1. Formulate and solve this transportation problem using R
2. Formulate the dual of this transportation problem
3. Make an economic interpretation of the dual

Solution

Q.1 In formulating the above problem, we need to find the total production cost for each of the plant to the warehouses. Note that the total production cost is the addition of the amount used in producing each of the goods and the cost of supplying them, that is, transport cost, to each of the warehouse. Hence, the table below is total plant cost of production, supply capacity (Production capacity) and monthly demand;

```
table <- matrix(c(420,414,425,100,
                  312,315,314,125,
                  510,512,515,150,
                  80,90,70,"240/375"), nrow = 4, byrow = TRUE)

colnames(table) <- c("Ware. 1", "Ware. 2", "Ware. 3", "P.Capacity")
rownames(table) <- c("Plant A", "Plant B", "Plant C", "M.Demand")

table <- as.table(table)

table%>%
  kable()%>%
  kable_classic()%>%
  column_spec(2, border_left = T)%>%
  column_spec(5, border_left = T)%>%
  row_spec(3, extra_css = "border-bottom:dotted;")
```

	Ware. 1	Ware. 2	Ware. 3	P.Capacity
Plant A	420	414	425	100
Plant B	312	315	314	125
Plant C	510	512	515	150
M.Demand	80	90	70	240/375

Drawing the pictorial transportation diagram of the table above,

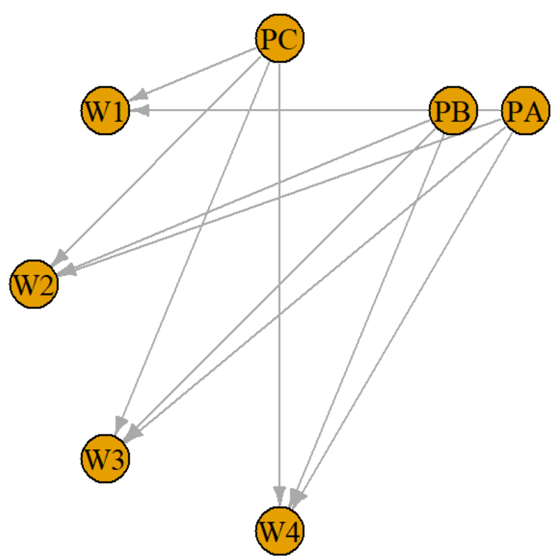
```
library(igraph)
```

```
##
## Attaching package: 'igraph'
```

```
## The following objects are masked from 'package:stats':
##
##     decompose, spectrum
```

```
## The following object is masked from 'package:base':
##
##     union
```

```
plants <- c("PA","PB","PC")
supply <- c(100,125,150)

warehouses <- c("W1","W2","W3","W4")
demand <- c(80,90,70,135)

total_vertices <- length(plants)+length(warehouses)
g <- graph.empty(n=total_vertices,directed = TRUE)

V(g)$name <- c(plants, warehouses)


for(i in 1:length(plants)) {
  for(j in 1:length(warehouses)) {
    weight <- min(supply[i], demand[j])
    if(weight > 0) {
      g <- add_edges(g, edges = c(plants[i], warehouses[j]), weight = weight)
    }
  }
}


layout <- layout_in_circle(g, order = c(1,2,3,4,5,6,7,8))

plot(g, layout=layout, vertex.label.color = "black", vertex.size = 20, edge.arrow.size = 0.5)
```



In the table above, entity where at which the Monthly Demand intersects Production Capacity shows an unbalance situation where the Production Capacity (supply) is more than the Monthly Demand, and therefore, to rectified this situation, we need want to balance the situation by creating a DUMMY Warehouse named as Warehouse D.

```
table <- matrix(c(420,414,425,0,100,
                  312,315,314,0,125,
                  510,512,515,0,150,
                  80,90,70,135,375), nrow = 4, byrow = TRUE)

colnames(table) <- c("Ware. 1", "Ware. 2", "Ware. 3","Ware. D", "P.Capacity")
rownames(table) <- c("Plant A", "Plant B", "Plant C", "M.Demand")

table <- as.table(table)

table%>%
  kable()%>%
  kable_classic()%>%
  column_spec(2, border_left = T)%>%
  column_spec(6, border_left = T)%>%
  column_spec(5,extra_css = "color: red; border-top")
```

	Ware. 1	Ware. 2	Ware. 3	Ware. D	P.Capacity
Plant A	420	414	425	0	100
Plant B	312	315	314	0	125
Plant C	510	512	515	0	150
M.Demand	80	90	70	135	375

Having balance the table, we want to formulate the transportation model for the Heart Start. However, we represented each Plant A - C supply to warehouse take symbol of a, b and c and the Warehouse take 1,2 3, and d for dummy warehouse respectively. Hence, the Objection function for Transport cost equals:

$$Min\ TC = 420a_1 + 414a_2 + 425a_3 + 0a_d + 312b_1 + 315b_2 + 314b_3 + 0b_d + 510c_1 + 512c_2 + 515c_3 + 0c_d$$

Setting the constraints of the variables:

Supply Constraints:

Plant A:

$$a_1 + a_2 + a_3 + a_d \leq 100$$

Plant B:

$$b_1 + b_2 + b_3 + b_d \leq 125$$

Plant C:

$$c_1 + c_2 + c_3 + c_d \leq 150$$

Demand Constraints:

Warehouse 1:

$$a_1 + b_1 + c_1 \geq 80$$

Warehouse 2:

$$a_2 + b_2 + c_2 \geq 90$$

Warehouse 3:

$$a_3 + b_3 + c_3 \geq 70$$

Warehouse D:

$$a_d + b_d + c_d \geq 135$$

Non-negativity of the decision variables:

$$a_i, b_i, c_i$$

where i = 1, 2, 3, d

Solving the formulated transportation model in R.

```
library(lpSolve)

t.costs <- matrix(c(420,414,425,0,
                  312,315,314,0,
                  510,512,515,0), nrow = 3)
```

Set up constraint signs and right-hand sides (supply side)

```
sup.signs <- rep("<=",3)
sup.rhs <- c(100,125,150)
```

Set up constraint signs and right-hand sides (demand side)

```
dmd.signs <- rep(">=",4)
dmd.rhs <- c(80,90,70,135)
```

Run the model

```
lptrans_HS <- lp.transport(t.costs,"min",sup.signs,sup.rhs,dmd.signs,dmd.rhs)
```

To get the values of 12 decision variables

```
lptrans_HS$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   10   90    0    0
## [2,]   55    0   70    0
## [3,]   15    0    0  135
```

```
lptrans_HS$objval
```

```
## [1] 33345
```

```
lptrans <- lp.transport(t.costs, "min",sup.signs,sup.rhs,dmd.signs,dmd.rhs)
```

```
optimal_solution <- lptrans$solution
basis <- lptrans$Basis
```

Q.2 In formulating the dual of the transportation problem, supply and demand constraints are represented as

$$s_i$$

and

$$d_j$$

respectively. Hence, the objective function of the dual is:

$$MaxZ \ = 80d_1 + 90d_2 + 70d_3 + 135d_4 - 100s_1 - 125s_2 - 150s_3$$

Constraints:

$$d_i - s_j >= z_i j$$

Thus,

Plant A transportation of goods to the warehouses are:

$$d_1 - s_1 \geq z_1 1 = 420$$

$$d_2 - s_1 \geq z_1 2 = 414$$

$$d_3 - s_1 \geq z_1 3 = 425$$

$$d_4 - s_d \geq z_1 d = 0$$

Plant B transportation of goods to the warehouses are:

$$d_1 - s_2 \geq z_1 1 = 312$$

$$d_2 - s_2 \geq z_1 2 = 315$$

$$d_3 - s_3 \geq z_1 3 = 314$$

$$d_4 - s_d \geq z_1 d = 0$$

Plant C transportation of goods to the warehouses are:

$$d_1 - s_3 \geq z_1 1 = 510$$

$$d_2 - s_3 \geq z_1 2 = 512$$

$$d_3 - s_3 \geq z_1 3 = 515$$

$$d_4 - s_d \geq z_1 d = 0$$

Where all

$$d_i \geq 0$$

for j = 1, 2, 3, 4 and

$$s_j \leq 0$$

Q.3

Economic Interpretation of the Dual.

The economic interpretation of the dual function is based on the economic principle of:

i. **Break Even Rule**. This is referred to as Marginal Revenue (MR) equals Marginal Cost (MC). Going by the dual constraints above,

$$d_i - s_j >= z_i j$$

. This means

$$d_i \geq s_j + z_i j$$

. The demand is the revenue, marginal revenue, gotten by the producer while the total production, marginal cost, is the combination of supply and transportation cost.

At plant A, more goods were supplied to warehouse 2 where

$$d_2 \geq s_1 + z_1 2 = 414$$

, that is,

$$MR \geq MC$$

. On the other hand, less goods were supply to warehouse 1 where

$$d_1 \leq s_1 + z_1 1 = 420$$

, that is,

$$MR \leq MC$$

.

At plant B, the goods supplied were more evenly close to warehouse 2 and 3. More goods were supplied to Warehouse 3 where

$$d_3 \geq s_3 + z_1 3 = 314$$

, and less good to warehouse 1 where

$$d_1 \geq s_2 + z_1 1 = 312$$

.

At plant C, less goods were supplied to warehouse 1 and more goods to warehouse D, that is, dummy warehouse. More goods were supplied to Warehouse D where

$$d_4 \geq s_d + z_1 d = 0$$

.

In conclusion, at plants A, goods were supplied to warehouses 1 and 2, and the production at plant B and C were supplied to 1 & 3 and 1 & 4 respectively. By the quantity of good supplied to each of the warehouses shows the mind of the producer and where his Marginal Revenue exceeds Marginal Cost.

i. The Producer business of using Warehouses for good supply.

In our model, the result shows that if the producer will maximise profit, the warehouse that can be use and the quantiy of good that can be supply were shown. For plant A, the model revealed that for their to be profit maximisation given the production capacity and the monthly demand, it is better to supply goods to only warehouses 1 and 2. For the pther two plants (B & C), the model revealed that profit will be maximised if product are supplied to warehouses 1 & 3 and 1 & 4. However, it is worthy to note that warehouse 4 is a dummy warehouse, that is, if producer will have to maximise profit at plant C, a warehouses that matches the features of the dummy warehouse is needed wherein the unbalance production and demand can be reworked to get a balance situation.