QCOMPPW100: Quantum computing kickoff practical work

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Based on:
MITx: 8.370.1x Quantum Information Science I, Part I

Practical work Outline: During This practical work we will simulate a quantum computer

Contents

1 Introduction

The goal of this practcal work is to make you program a simulation of a quantum computer.You will have to proagram it in Python3.The only librairie you have the right to use is numpy and math. You will have to program the function: quantumComputer(nbQbits: int, quantumGates: list). nbQbits is the number of Qbits in the circuit of the circuit. quantumGates is the list of quantum gates of circuit.

1.1 Forbiden function

You does not have the rigth to use the use the function numpy.kron().

1.2 Given file

You can download the given file here: LINK Content of the only given file quantumComputer.py: Verbatim You will have to program in this file. TypeOfQuantumGate is an numration for each type of gate our quantum computer handle.
QuantumGate is the class of the quantum gate. TypeOfGate is the type of the gate (a value of TypeOfQuantumGate). fQbit is the position in the circuit of the first input Qbit of the gate. sQbit is the position in the circuit of the second input Qbit of the gate (if the gate has two input). You can add functions to quantum gate.

1.3 reference

You can find an implementation of this practical work at : LINK. This implementation Qiskit and you does not have the rigth to do so. Yet this can be usefull to test your code and disambiguate what you are ask to do. Your practical work has to behave the same way. The reference raise exception for some type of input. Your quantum computer will never be tested for those kind of input.

1.4 Submission

For the submission you will have to download the repo: LINK.You will have to submit you work before TIME.

2 Generate state vector

For validating this section, your quantum computer will have to handle an **empty** list of quantum gate as input.This function output a vector of $2^n bQbit$ values, with the value first value being equal to one and the other values must be equal to zeroes. The function must pass this test(no assert must be thrown):

```
    arr = np.array([1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j])
comparison = np.array_equal(arr, quantumComputer(3,[]))
assert(comparison)
```

Tips: This code does not throw any error assert

```
assert(1. == 1. + 0.j)
```

Tips: checks the function numpy.zeros ;)

3 Kronecker product

To validate this section you will have to program the function kroneckerProduct. This function takes in input to matrix m1 and m2. This function ouput the Kronecker product of m1 and m2. If you do not remember who does work the kronecker product you can check: LINK The function must pass this test:

```
m1 = [1,2]
m2 = [[4,5],[6,7]]
assert(np.array_equal(kroneckerProduct(m1,m2) , [[ 4,  5,  8, 10],[ 6,  7, 12, 14]]))
```

4 Handle one gate having one input

The goal of this section is to make your quantum computer handle one gate with one input.

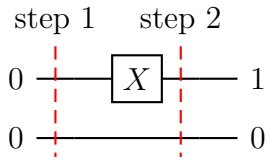
4.1 Iterative kronecker product

To do so, you will have to program the function computeMatrix. This function takes in input an integers : nbQbit, a 2x2 matrix, and an integer fQbit. This function output: $I_2^{\otimes(nbQbit-fQbit-1)} \otimes matrix \otimes I_2^{\otimes fQbit}$. The function must pass this test:

```
m = [[0,1],
     [1,0]]
arr = [[0., 1., 0., 0.],
 [1., 0., 0., 0.],
 [0., 0., 0., 1.],
 [0., 0., 1., 0.]]
assert(np.array_equal(createMatFromBaseMatrix(m,2,0), arr))
```

4.2 Handle one not gate

Now you have all the key in hand to handle one not gate. To do so, if there is a NOT gate the circuit you will have to compute its corresponding matrix according to the number of qubits in the circuit and its position in the circuit. Then you will just have to multiply the transposition of your state vector with this matrix.
Here the evolution of the state vector during the computation:



At step 1 the state vector is equal to: $\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}$
At step 2 the state vector is equal to:

$$\begin{bmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix} \times \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^t = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

Your quantum computer must pass this test:

```
arr = quantumComputer(2,[QuantumGate(TypeOfQuantumGate.NOT,0)])
assert(np.array_equal(arr,[0., 1., 0., 0.]))
```

4.3 Handle one Hadamard gate

Now you will have to handle one Hadamard gate.To compute the corresponding matrix of an Hadamard gate you just have to act like for the NOT gate, except that you use the Hamadamard matrix as a 2x2 matrix:

$$\frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Your quantum computer must pass this test:

```
arr =quantumComputer(1,[QuantumGate(TypeOfQuantumGate.HADAMARD,0)])
assert(np.isclose(arr,[1/math.sqrt(2), 1/math.sqrt(2) ]).all())
```

5 Handle many gates

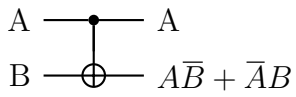
For validating this section, your quantum computer must handle many gate. To do so first you have to compute the corresponding matrix of every gate and then multiply them in the reverse order to get the matrix corresponding to you circuit. If as input you have such list list of quantum gate : [gate A, gate B ,gate c], you have to compute mat C X mat B X mat A. Your quantum computer must pass this test:

```
arr =quantumComputer(1,[QuantumGate(TypeOfQuantumGate.NOT,0),
QuantumGate(TypeOfQuantumGate.HADAMARD,0)])
assert(np.isclose(arr,[1/math.sqrt(2), -1/math.sqrt(2) ]).all())
```

6 Handle CNOT gate

6.1 Compute CNOT matrix

Program the function computeCNOT. This function takes in input an integers: nbQbit, an integers fQbit and integer sQbit. Consedering fQubit as the control qubit. This function ouputs the corresponding CNOT matrix. To compute this matrix there is an elegant way you can find here:



And has this truth table:

Input	Output
00	00
01	01
10	11
11	10

So in our quantum computing world for 2 qubit, we have a susch "truth table":

Input	Output
$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$
$\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$
$\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$
$\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$	$\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

So we have a such corresponding matrix:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

Here a pseudo-code algorithm that compute the corresponding matrix:

```
    computeCNOT(nQubits, fQubit,sQubit):
    create  a (2**nbQubits) * (2**nbQubits) matrix m, filled with 0.
    for i in 0, 2**nbQubits :
        s = tobinarystring(i)
        if(i[fQubit] == 1):
            s[sQubit] = !s[sQubit]
            m[i,s] =1
        else:
        m[i,i] =1
    return m
```

Your function compute CNOT must pass this test:

```
    arr = [[1., 0., 0., 0.],
 [0., 0., 0., 1.],
 [0., 0., 1., 0.],
 [0., 1., 0., 0.]]
assert(np.array_equal(computeCNOT(2,0,1) , arr))
```

6.2 Back to our quantum computer

Now you have all you need to handle CNOT gate in our quantum computer. Just to do it! Your quantum computer must pass this test now:

```
arr =quantumComputer(2,[QuantumGate(TypeOfQuantumGate.NOT,1),
    QuantumGate(TypeOfQuantumGate.CNOT,1,0)])
assert(np.array_equal(arr , [0,0,0,1]))
```

7 Compute Probability

Program the function computeProbability. This function takes in inputs an array of complex number: tab. This function output an array of the same dimension. Every coefficient of the output is computed this way:

$$output[i] = \frac{|tab[i]|^2}{\Sigma_{k=0}^{tab.length()}|tab[k]|^2}$$

This function must pass this test:

```
arr =quantumComputer(1,[QuantumGate(TypeOfQuantumGate.NOT,0),
    QuantumGate(TypeOfQuantumGate.HADAMARD,0)])
assert(np.isclose(computeProbability(arr),[0.5,0.5]).all())
```

This function compute the probabilty of each combination to comes out when you mesure every qubits.