QCOMP103-B: Hello World

L'algorithme de Deutsch, démonstration de la supériorité quantique

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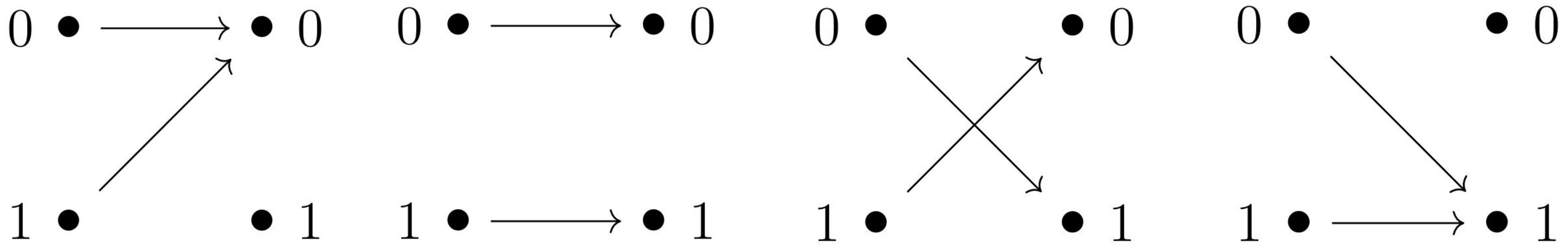
Plan du cours

- Le probleme
- Premier essai
- Second essai
- The best of both worlds

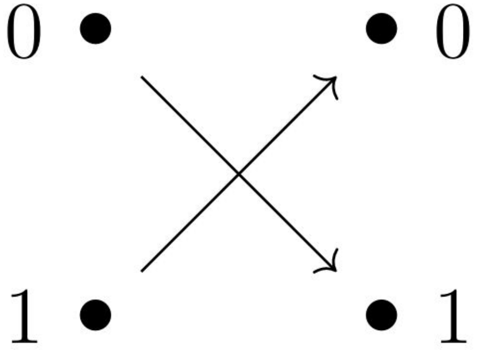
> Computer science is no more about computers than astronomy is about telescopes
>
> *E.W. Dijkstra*

$$f : \{0,\ 1\} \longrightarrow \{0,\ 1\}$$

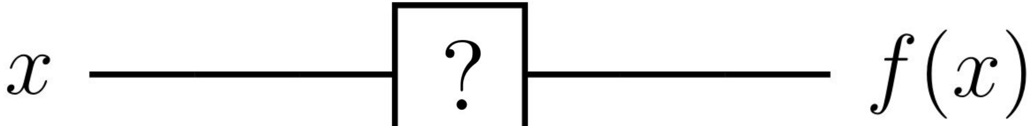


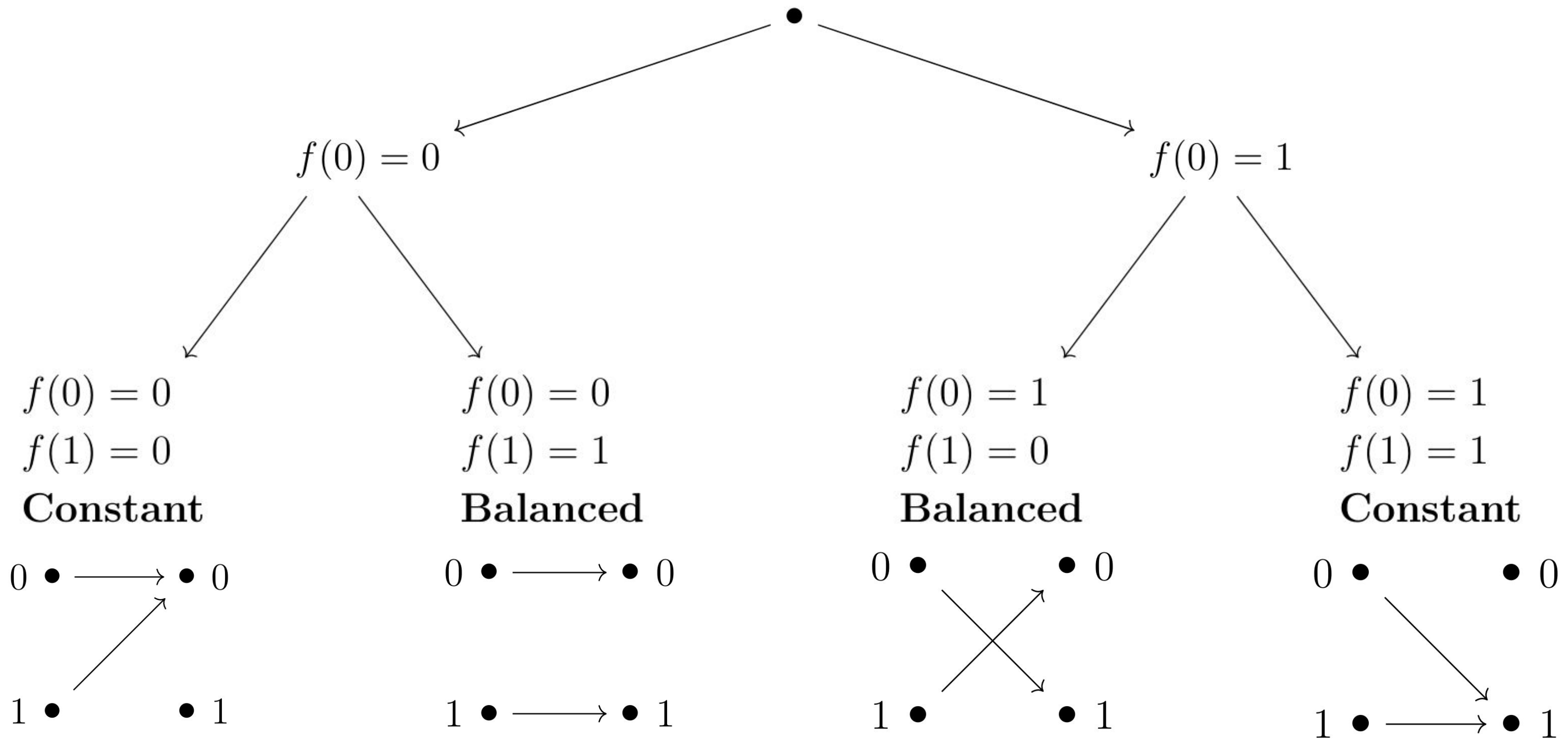
$$f(0) \neq f(1) \qquad\qquad f(0) = f(1)$$

Balanced **Constant**

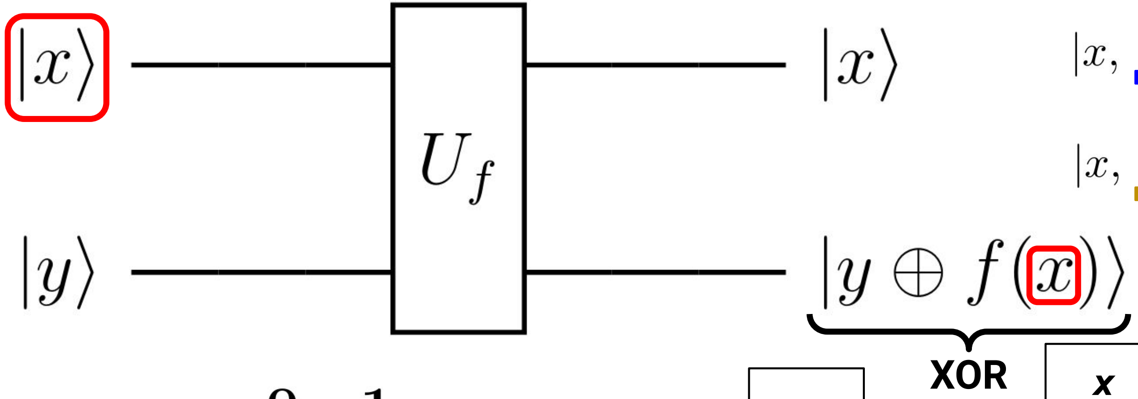


$$\begin{array}{cc} & \begin{array}{cc} \mathbf{0} & \mathbf{1} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$$





I – Le probleme (cf. polycopié page 25)



$$|x,\ 0\rangle \longrightarrow |x,\ 0 \oplus f(x)\rangle = |x,\ f(x)\rangle$$

$$|x,\ 1\rangle \longrightarrow |x,\ 1 \oplus f(x)\rangle = \left|x,\ \overline{f(x)}\right\rangle$$

$$f = \begin{array}{cc} & \begin{array}{cc} \mathbf{0} & \mathbf{1} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$$

$$\overline{f} = \begin{array}{cc} & \begin{array}{cc} \mathbf{0} & \mathbf{1} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \end{array} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$U_f = \begin{array}{ccc} & & \begin{array}{cccc} x & x & x & x \\ \mathbf{00} & \mathbf{01} & \mathbf{10} & \mathbf{11} \end{array} \\ \begin{array}{c} \overline{f(x)} \\ f(x) \\ f(x) \\ \overline{f(x)} \end{array} & \begin{array}{c} \mathbf{00} \\ \mathbf{01} \\ \mathbf{10} \\ \mathbf{11} \end{array} & \begin{pmatrix} 0 & \boxed{1} & 0 & 0 \\ \boxed{1} & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & \boxed{1} \end{pmatrix} \end{array}$$

I – Le probleme (cf. polycopié page 25)

Kahoot

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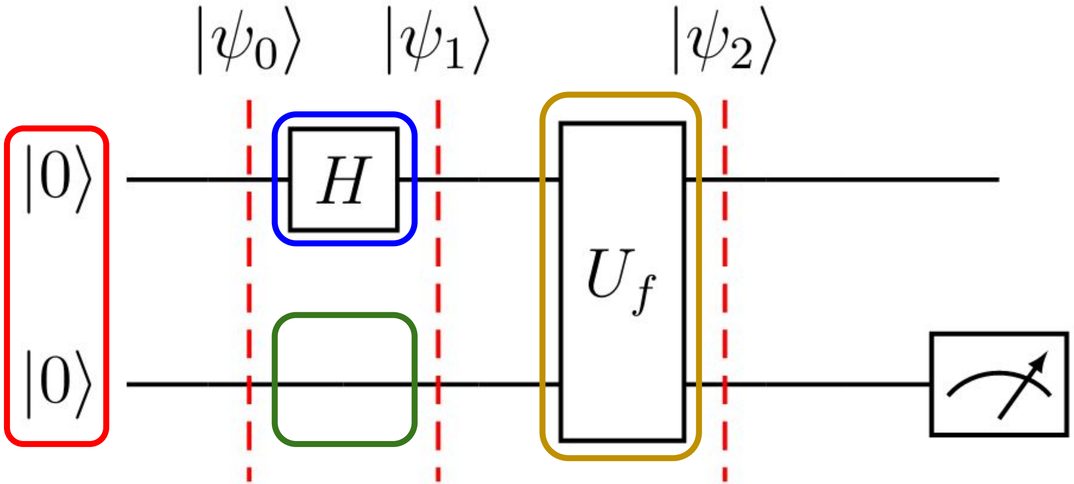
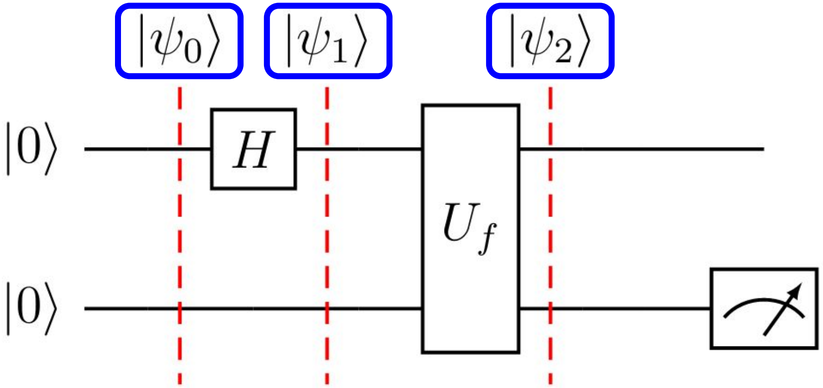
4. Tenez vous prêt à jouer

$$|x\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad H * |0\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$U_f(H \otimes I)(|0,\ 0\rangle)$$

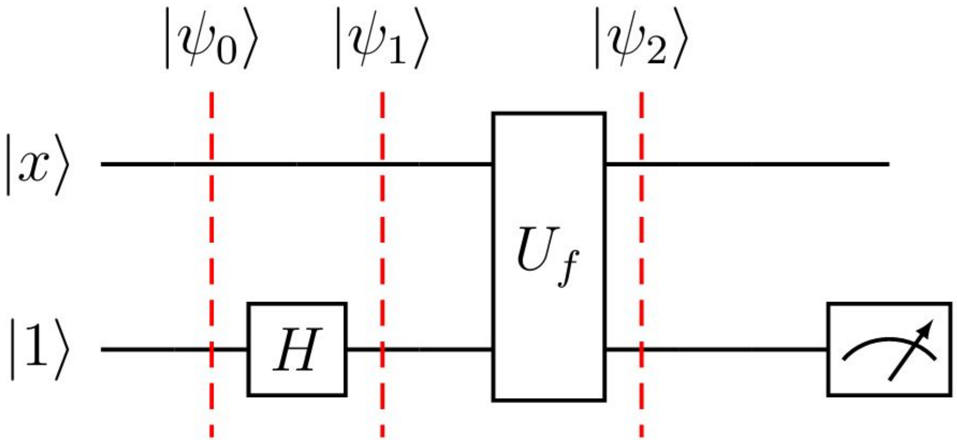
$$\begin{array}{c} \mathbf{00} \\ \mathbf{01} \\ \mathbf{10} \\ \mathbf{11} \end{array} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



$$|\psi_0\rangle = |0\rangle \otimes |0\rangle = |0,\ 0\rangle$$

$$|\psi_1\rangle = \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right] \otimes |0\rangle = \frac{|0,\ 0\rangle + |1,\ 0\rangle}{\sqrt{2}}$$

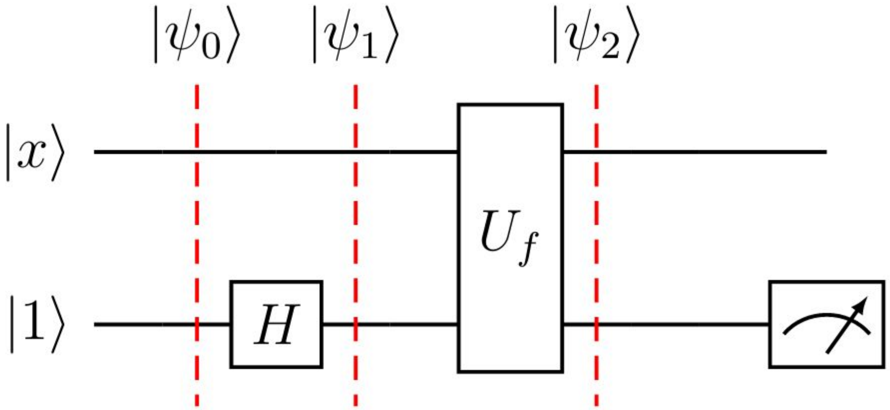
$$|\psi_2\rangle = \frac{|0,\ f(0)\rangle + |1,\ f(1)\rangle}{\sqrt{2}}$$



$$\frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle$$

$$U_f \; * \; (I \otimes H) \; * \; |x,\ 1\rangle$$

III - Deuxieme essai (cf. polycopié page 28)



$$|\psi_0\rangle = |x,\ 1\rangle$$

$$|\psi_1\rangle = |x\rangle \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] = \frac{|x,\ 0\rangle - |x,\ 1\rangle}{\sqrt{2}}$$

$$|\psi_2\rangle = |x\rangle \otimes \left[\frac{|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle}{\sqrt{2}}\right] = |x\rangle \otimes \left[\frac{|f(x)\rangle - \overline{|f(x)\rangle}}{\sqrt{2}}\right]$$

III - Deuxieme essai (cf. polycopié page 28)

$$|\psi_2\rangle = |x\rangle \otimes \left[\frac{|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle}{\sqrt{2}}\right] = |x\rangle \otimes \left[\frac{|f(x)\rangle - \overline{|f(x)\rangle}}{\sqrt{2}}\right]$$

$$|\psi_2\rangle = \begin{cases} |x\rangle \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], \text{ if } f(x) = 0 \\ |x\rangle \otimes \left[\frac{|1\rangle - |0\rangle}{\sqrt{2}}\right], \text{ if } f(x) = 1 \end{cases}$$

$$a - b = (-1)(b - a)$$

$$|\psi_2\rangle = (-1)^{f(x)} |x\rangle \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] = (-1)^{f(x)} \frac{|x,\ 0\rangle - |x,\ 1\rangle}{\sqrt{2}}$$

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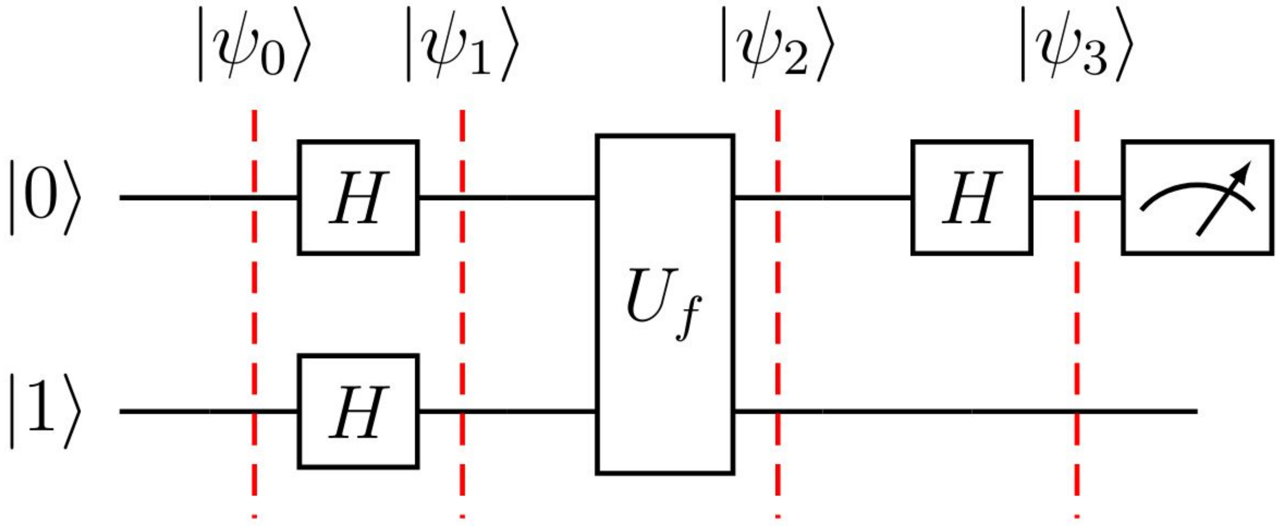
Niveau 2 et 3: Le premier et le deuxième essai



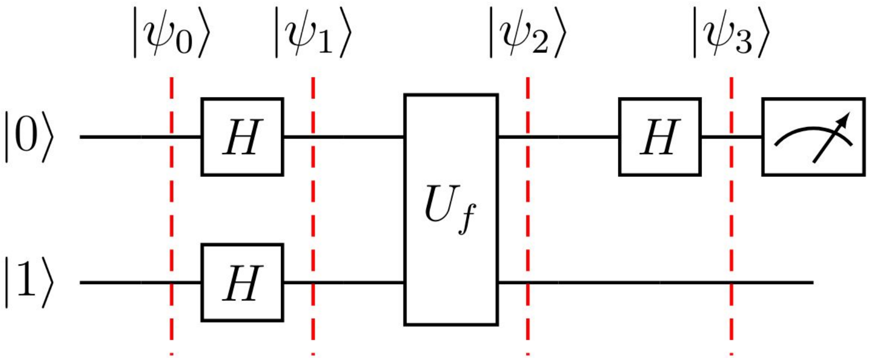
$$\begin{array}{c} \mathbf{00} \\ \mathbf{01} \\ \mathbf{10} \\ \mathbf{11} \end{array} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$(H \otimes I) \ * \ U_f \ * \ (H \otimes H) \ * \ |0,\ 1\rangle$$

IV – The best of both worlds (cf. polycopié page 28)

$$|\psi_0\rangle = |0,\ 1\rangle$$

$$|\psi_1\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$



On se souvient que quand le **qubit du haut** était **fixé** dans l'état ***x*** on avait:

$$(-1)^{f(x)}\,|x\rangle \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

Mais maintenant le **qubit du haut** est en **superposition**, on a donc:

$$|\psi_2\rangle = \left[\frac{(-1)^{f(0)}\,|0\rangle + (-1)^{f(1)}\,|1\rangle}{\sqrt{2}}\right] \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

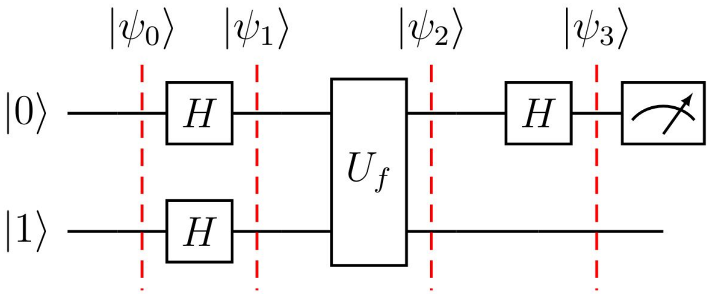
$$|\psi_2\rangle = \left[\frac{\boxed{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}}{\sqrt{2}}\right] \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

Si ***f*** est **"constant"** $f(0) = f(1)$ $\quad +1(|0\rangle + |1\rangle)$ or $-1(|0\rangle + |1\rangle)$

Si ***f*** est **"balanced"** $f(0) \neq f(1)$ $\quad +1(|0\rangle - |1\rangle)$ or $-1(|0\rangle - |1\rangle)$

$$|\psi_2\rangle = \begin{cases} (\pm 1)\left[\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right] \otimes \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{constant}. \\ (\pm 1)\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \otimes \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{balanced}. \end{cases}$$

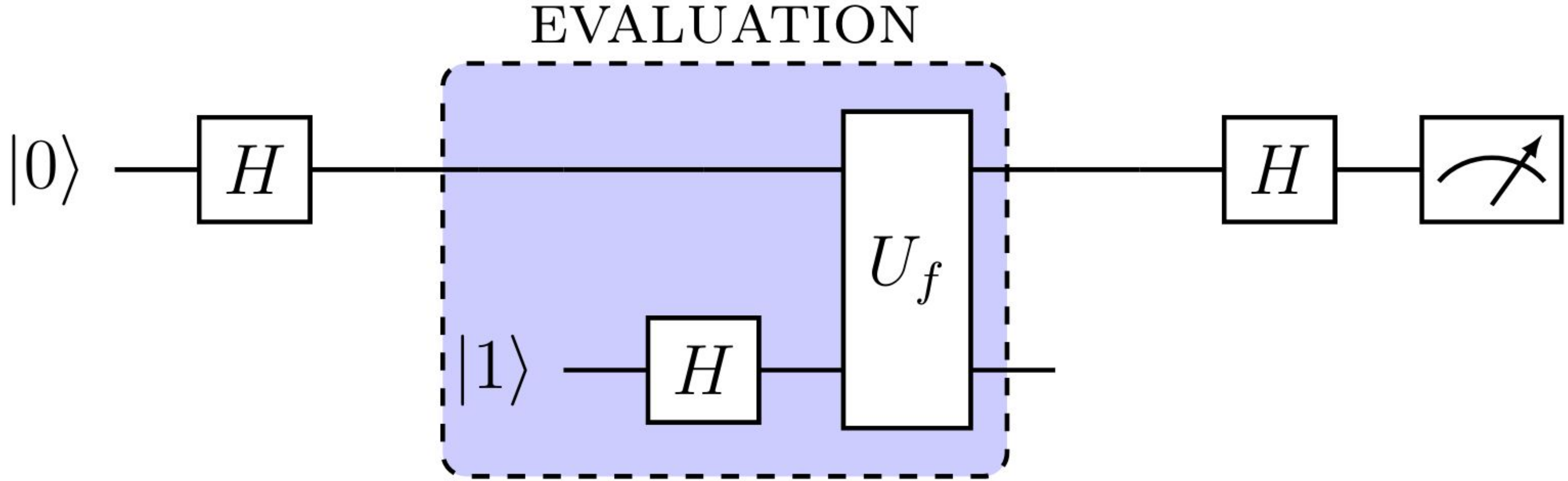
$$|\psi_2\rangle = \begin{cases} (\pm 1)\boxed{\dfrac{|0\rangle + |1\rangle}{\sqrt{2}}} \otimes \left[\dfrac{|0\rangle - |1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{constant}. \\ (\pm 1)\boxed{\dfrac{|0\rangle - |1\rangle}{\sqrt{2}}} \otimes \left[\dfrac{|0\rangle - |1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{balanced}. \end{cases}$$



On applique la **porte d'Hadamard** sur le **qubit du haut** et on obtient:

$$|\psi_3\rangle = \begin{cases} (\pm 1)\boxed{|0\rangle} \otimes \left[\dfrac{|0\rangle - |1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{constant}. \\ (\pm 1)\boxed{|1\rangle} \otimes \left[\dfrac{|0\rangle - |1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{balanced}. \end{cases}$$

$$|\psi_3\rangle = \begin{cases} (\pm 1)\,|0\rangle \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{constant}. \\ (\pm 1)\,|1\rangle \otimes \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], & \text{if } f \text{ is } \textbf{balanced}. \end{cases}$$



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