Feedforward Control

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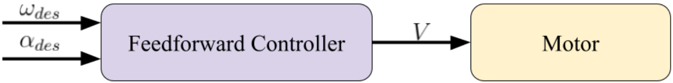
Introduction

In this lecture, we will discuss feedforward control in the context of the *FIRST Tech Challenge* (FTC).

For the purposes of this lecture, we will discuss a simple, yet practical example: controlling the velocity of a motor.



Motor model

Let's take a quick look at a simple model for a motor. More information about the derivation of such a model can be found in this slideshow.

$$
\begin{aligned}
\dot{\omega} &= -\frac{k_\tau k_V}{RJ}\omega + \frac{k_\tau}{RJ}V \\
&= A\omega + BV
\end{aligned}
$$

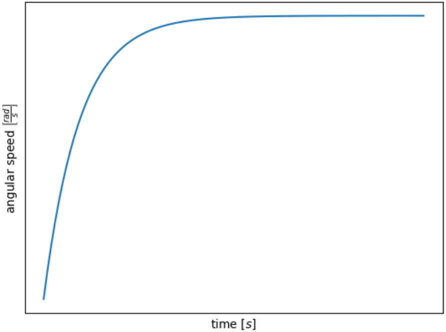
where $\dot{\omega}$ is the motor's angular acceleration, ω is the motor's speed, V is the input voltage, k_τ is the motor's torque constant, k_V is the motor's back-EMF constant, R is the motor's internal resistance and J is the inertia of the motor's shaft. These constants are often found on the datasheet of a motor, or can be determined experimentally. We write, the collection of constants as A and B for simplicity

Note: this is a linear system with state variable ω and input V.

Note: we will treat V as an actual input voltage to the motor, however in the FTC SDK this value is between -1 and 1, where 1 corresponds to the maximum battery voltage available.

Motor model

Let's take a look at what happens to a motor when we apply a constant input voltage.



We see that the motor speeds up rather quickly then the speed decays into a **steady-state**.

Feedfoward - model

So, what makes this model useful? Well, we can use it to predict what input voltage V will produce a certain [angular] speed ω and acceleration *omėga* for the motor.

Let's keep $\dot{\omega}$ as some constant angular acceleration α and rearrange the model and solve for V:

$$\begin{aligned} V &= -\frac{A}{B}\omega + \frac{1}{B}\alpha \\ &= k_\omega \omega + k_\alpha \alpha \end{aligned}$$

Now, we can predict what input voltage we may need to produce a desired velocity ω_{des} and acceleration α_{des}. In practice, the constants k_ω and k_α are determined through manual tuning.

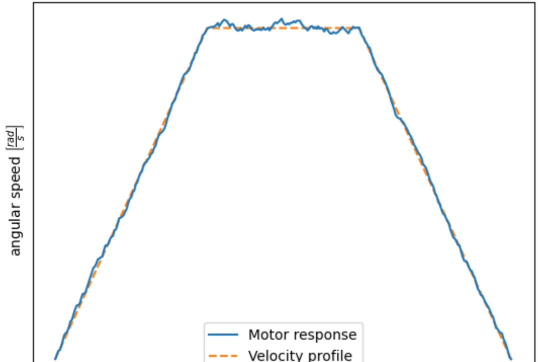
Note: in typical FTC documentation/learning material, ω is represented as v and α is represented as a

Feedforward - tracking

To test the feedforward control on our motor, we can build a simple trapezoidal velocity profile. For k timesteps, we will increase our speed from 0 to some ω_{max} by α each timestep:

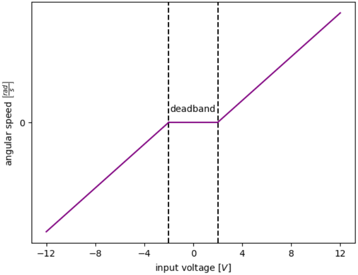
$$\omega_{k+1} = \omega_k + \alpha \Delta t$$

Then we will keep a constant speed for another k timesteps, before we decrease back to zero for the final k timesteps. We can feed in the value of ω_k and $\{-\alpha, 0, \alpha\}$ into our feedforward expression to generate an input voltage to our motor.



Feedforward - incorporating friction

One thing we did not consider is the physical resistance the motor may encounter when attempting to move. Usually, this resistance stems from friction. The figure below showcases this issue. At small input voltages, the motor is still unable to move. We will call this the **deadband**.



To overcome this, we introduce an additional feedforward term to overcome static friction. Since friction acts in the opposite direction of our movement, we will always apply the friction feedforward in the direction of our desired movement. Our full feedforward expression becomes:

$$V = k_\omega \omega_{des} + k_\alpha \alpha_{des} + k_s \text{sign}(\omega_{des})$$