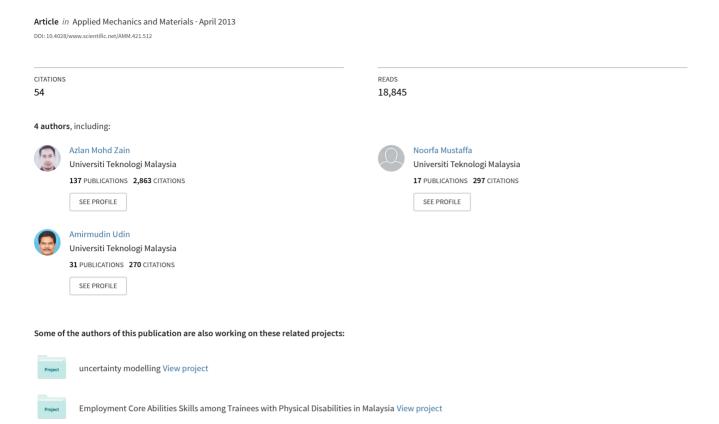
Firefly Algorithm for Optimization Problem



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Abstract. This paper reviews the applications of Firefly Algorithm (FA) in various domain of optimization problem. Optimization is a process of determining the best solution to make something as functional and effective as possible by minimizing or maximizing the parameters involved in the problems. Several categories of optimization problem such as discrete, chaotic, multi-objective and many more are addressed by inspiring the behavior of fireflies as mentioned in the literatures. Literatures found that FA was mostly applied by researchers to solve the optimization problems in Computer Science and Engineering domain. Some of them are enhanced or hybridized with other techniques to discover better performance. In addition, literatures found that most of the cases that used FA technique have outperformed compare to other metaheuristic algorithms.

Introduction

Optimization problem is a computational problem in which the object is to find the best of all possible solutions. In other word, optimization problem is to find a solution in the feasible region which has the minimum (or maximum) value of the objective function. In an optimization problem, the types of mathematical relationships between the objective and constraints and the decision variables determine how hard it is to solve, the solution methods or algorithms that can be used for optimization to discover the solution that truly optimal.

Optimization always present in day life since ancient times. In order to deal with the restrictions, individual has to produce solution that serve the source limits and lack knowledge of some problems. Most of the solutions based on nature observations in the beginning. Gradually, knowledge will be developed based on this nature observations and past experiences. In day life, all solutions found will be tested and may be replaced by a better solution to ensure high performance of the problem.

Single objective optimization is often used by researchers in solving real world problem. Sometimes, a better way will be achieved by defining multiple objectives in solving a problem. Multi-objective optimization is a process of solving a problem by simultaneously optimizing two or more objectives subjected to constraints [1]. *Optimal solution is solution that is not dominated by other solution in the search space* [1] where the optimal solution is called Pareto optimal.

Metaheuristic Algorithm

In general, metaheuristic is a general algorithmic framework for addressing intractable problems. Metaheuristic is an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space while learning strategies are used to structure information to find efficiently near-optimal solutions ([2].

In computer science, metaheuristic designates a computational method that optimizes problem by iteratively trying to improve the solution of a problem. There are few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions. However, metaheuristics do not guarantee an optimal solution is ever found.

[3] concluded that metaheuristic approaches are very suitable for solving non-deterministic polynomial-hard (NP-hard) optimization and complicated search problems because they obtained better quality solutions compare to heuristic approaches especially on hybrid techniques. Furthermore, the approaches are more efficient to solve the problems.

This is proved by [4], metaheuristics are designed to deal with complex optimization problems because of the other optimization techniques are not very efficient to solve the problems. Hence, these approaches were recognized as one of the most practical approaches for solving complex problems.

Fundamental Properties of Metaheuristics

Metaheuristics designed to search solution for an optimization problem. There are some fundamentals properties of metaheuristics as stated by [5]:

- 1) Metaheuristics are strategies that act as guidance for searching process.
- 2) The goal of metahueristics is to explore the search space efficiently to find optimal or near-optimal solutions.
- 3) The techniques used to solve problems are from simple local search procedures to complex learning process.
 - 4) Metaheuristic algorithms are approximation solutions and usually non-deterministic.

Many different metaheuristics are in existence to solve optimization problems. Somehow, there are some new variants are continually being proposed. In last decade, the inspiration of swarm intelligence (SI) to optimization techniques has become more popular among researchers to solve the optimization problems. Normally, swarm intelligence is made up of a population of agents that interact with each other in their environment [6]. The inspirations of SI come from nature, especially the biological systems where the algorithms imitate the behaviour of swarms of social insects including ant colonies, bird flocking, animal herding, bacterial growth and fish schooling [7].

Regarding to [8], the use of SI approach is because of their robustness, flexibility, decentralized and self-organized characteristics. Furthermore, SI is able to operate themselves independently without guidance from coordinator or external controller [9]. The most significant algorithms that have many contributions to the field are Genetic Algorithm (GA), Firefly Algorithm (FA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Levy Flight, Artificial Bee Colony Algorithm (ABC), Hunting Search (HuS), Simulated Annealing (SA) and many more.

The main purpose of this paper is to review the applications of FA in optimization problems. Firefly Algorithm inspires the flashing behaviour of fireflies in solving problems. The behaviour and methodology of FA will be discussed in the next section.

Firefly Algorithm

Fireflies are winged beetles or insects that produce light and blinking at night. The light has no infrared or an ultraviolet frequency which is chemically produced from the lower abdomen is called bioluminescence. They use the flash light especially to attract mates or prey. The flash light also used as a protective warning mechanism to remind the fireflies about the potential predators.

Firefly algorithm formulated by Yang [10] is a metaheuristic algorithm that is inspired by the flashing behavior of fireflies and the phenomenon of bioluminescent communication. [10] formulated the Firefly Algorithm with the following assumptions:

- 1) A firefly will be attracted to each other regardless of their sex because they are unisexual.
- 2) Attractiveness is proportional to their brightness whereas the less bright firefly will be attracted to the brighter firefly. However, the attractiveness decreased when the distance of the two fireflies increased.
 - 3) If the brightness of both fireflies is the same, the fireflies will move randomly.

The generations of new solutions are by random walk and attraction of the fireflies [11, 12, 13]. The brightness of the fireflies should be associated with the objective function of the related problem. Their attractiveness makes them capable to subdivide themselves into smaller groups and each subgroup swarm around the local models. Thus, FA is suitable for optimization problems as stated by [14, 15].

From the previous literatures, many researchers have stated that FA developed by Yang in 2008 is a very powerful technique to solve constrained optimization problems and NP-hard problems [16]. [17] stated that FA has widely been applied to solve continuous mathematical functions but has been rarely reported. For applied mathematics, the algorithm must be just a simple math and logic [18]. The behaviour of FA is simple and therefore it is suitable to solve the continuous mathematical functions.

[16] said that FA is very efficient and can outperform other conventional algorithms based on statistical performances measured using standard stochastic test functions. The algorithm works based on global communications among the fireflies. Hence, it can find global and local optimal simultaneously. [15] said that FA use mainly real random numbers. Different fireflies work independently and it is suitable for parallel implementation. [24] said that FA is one of the technique that recently used by researchers to solve optimization problems in dynamic environment.

The attractiveness of the firefly

The attractiveness which is its brightness, I of firefly i on the firefly j is based on the degree of the brightness of the firefly i and the distance r_{ij} between the firefly i and the firefly j [19] as in Eq.1.

$$I(r) = \frac{I_s}{r^2} \tag{1}$$

Suppose there are n fireflies; and x_i corresponds to the solution for firefly i. The brightness of the firefly i, is associated with the objective function $f(x_i)$. The brightness I of a firefly is chosen to reveal its recent position of its fitness value or objective function f(x) as in Eq.2.

$$I_i = f(x_i) \tag{2}$$

The less bright (attractive) firefly is attracted and moved to the brighter one; and each firefly has a certain attractiveness value β . However, the attractiveness value β is relative based on the distance between fireflies. The attractiveness function of the firefly is established by Eq. 3.

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{3}$$

where β_0 is the firefly attractiveness value at r = 0 and γ is the media light absorption coefficient.

The movement towards attractive firefly

[19] described the movement of a firefly i at position x_i moving to a brighter firefly j at position x_j by eEq.4.

$$x_i(t+1) = x_i(t) + \beta_0 e^{-\gamma r^2} (x_i - x_i) + \alpha \varepsilon_i \tag{4}$$

where $\beta_0 e^{-\gamma r^2} (x_i - x_j)$ is due to the attraction of the firefly x_j and $\alpha \varepsilon_i$ a randomization parameter; so if $\beta_0 = 0$ then it turns out to be a simple random movement.

The algorithm compares the attractiveness of the new firefly position with old one. If the new position produces higher attractiveness value, the firefly is moved to the new position; otherwise the firefly will remain in the current position. The termination criterion of the FA is based on an arbitrary predefined number of iterations or predefined fitness value.

The brightest firefly moves randomly based on Eq.5.

$$x_i(t+1) = x_i(t) + \alpha \varepsilon_i \tag{5}$$

Application of Firefly Algorithm

From the previous literatures, many researchers have stated that FA developed by Yang in 2008 is a very powerful technique to solve constrained optimization problems and NP-hard problems [16].

[20] introduced a hybrid algorithm between FA and genetic operators to solve mono alphabetic substitution cipher. Results showed that the algorithm works best for large input cipher text length while smaller input will run the program for multiple times to get the best result.

- [21] used a hybrid algorithm which is Harmony-Seeking Firefly Algorithm (HSFA) that hybrid FA with Harmony Search (HS) to obtain nearly-optimal sensor replacement trajectories for robotic fleet in a limited time. Results showed that a solution can be achieved in a short duration for vehicle routing problem by using HSFA compare to NN.
- [22] introduced FA in Radial Basis Function (RBF) Network and results are compared to Gradient Descent (GD), PSO and ABC algorithms. Results showed that FA has better performance of area under curve (AUC) in corresponding receive operating characteristic (ROC).
- [23] focused on phase equilibrium calculations and phase stability analysis of reactive and non-reactive systems. The three mentioned techniques were tested and results showed that FA was the most reliable technique to solve phase equilibrium and phase stability problems while CMA-ES can find global minimum reliably and accurately with smaller iterations.

Based on previous researches, some analysis has been done and graphs were produced to show the widely used of FA for optimization problems.

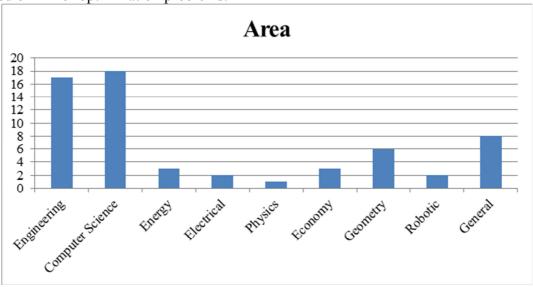


Fig 1: Firefly Algorithm Application in Optimization Problem based on Area

Fig 1 shows the distribution of FA application in optimization problem based on area from 2008 to 2013. There are 8 areas of optimization problem that used FA in their case study. Based on the figure, there are two major area which are Engineering and Computer Science domain that widely used FA in determining solutions for the problem. The number of case study in Engineering domain that used FA is 17 while in Computer Science domain is 18 case studies.

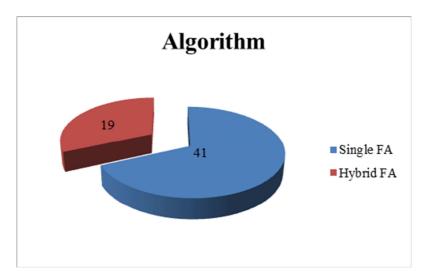


Fig 2: Comparison of Single and Hybrid Firefly Algorithm Applications

As depicted in Fig 2, the number of single FA used to solve optimization problems in double than hybrid FA. It showed that, there is only a few researchers had hybrid FA with other algorithm to solve optimization problems. Hence, the hybrid algorithms perform better than the single FA. Applications of hybrid FA seem to be better than single FA because the other techniques improving the FA in terms of processing time and performance. Sometimes, the single FA acquires a lot of time to get the optimal parameter values. Therefore, FA should be hybridized with other techniques to reduce the processing time.

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