Step-Up Barrier-Protected S&P500 ELN

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Group 2

Agenda

Step-Up Barrier-Protected S&P500 ELN

1. Economic motivation (Kelly)
2. Product design (Simon)
3. Pricing (Sam)
4. Hedging strategy (Vincent)

Economic Motivation

Opportunities: Post COVID-19 pandemic

- ➢ Large fiscal stimulus
- ➢ Vaccine rollout
- ➢ Economic recovery

Concerns: High uncertainty, volatile stock market

- ➢ Rise in US treasury yield
- ➢ Resurgence of COVID-19 cases
- ➢ Capital gains tax hike
- ➢ Geopolitical tensions

Rising S&P 500 Index

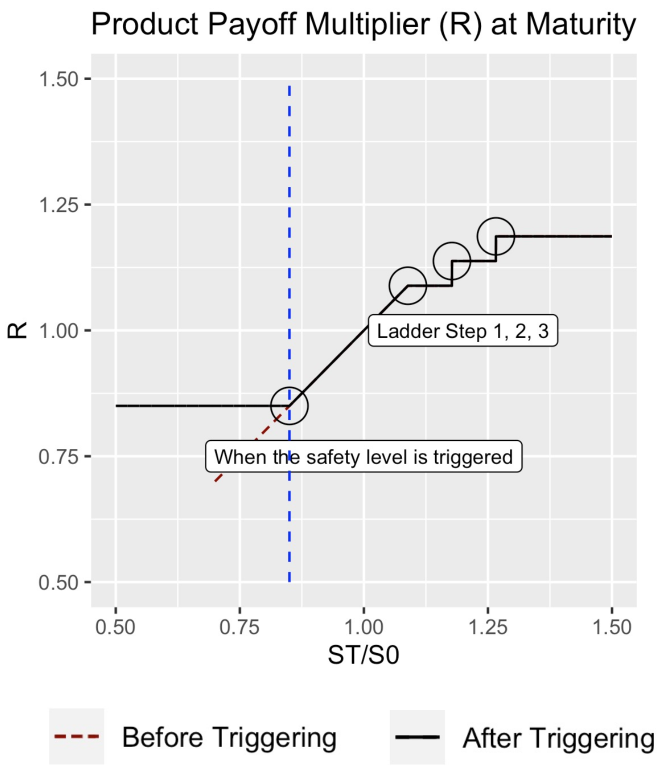
High S&P 500 PE Ratio

Source: Macrotrend

Innovative structured product:
- Downside protection
- Gain in bullish market

Target investors:
- Conservative investors who have
- Bullish views on U.S. equity market

Product Overview



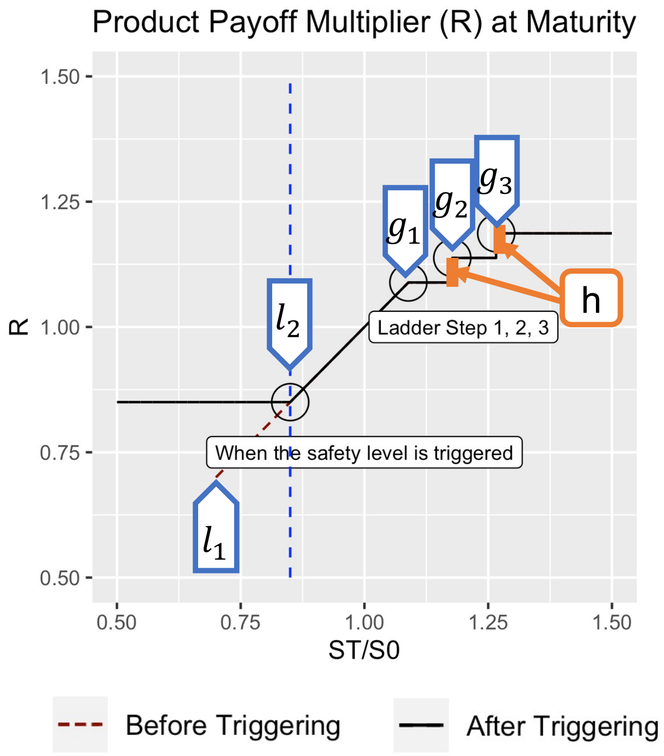
Basic information

Product type	Equity Linked Fund (ELF)
Underlying	S&P 500 Index
Dividend	No
Tenor	6 months

Product highlights:

- Step-up return
- Large protection in case of market crash
- Effect of dividend payment is considered

Payoff Overview



Payoff at t = T:

X = RF, where R is determined by:

Value of R	Condition	Explanation
$\frac{S_T}{S_0}$	$\frac{S_T}{S_0} < g_1$	Floating gain/loss
g_1	$g_1 \le \frac{S_T}{S_0} < g_2$	Fixed gain, 1st step
$g_1 + h$	$g_2 \le \frac{S_T}{S_0} < g_3$	Fixed gain, 2nd step
$g_1 + 2h$	$\frac{S_T}{S_0} \le g_3$	Fixed gain, 3rd step
Additional protection		
l_2	$inf_{0 \le t \le T}\{S_t/S_0\} < l_1$ And $S_T/S_0 < l_2$	Fixed loss if barrier is triggered

Variable & Parameter Settings

Variables	Explanation
X	Payoff (Claim) at t = T
R	Product price multiplier = 1 + product return
S_t/S_0	Index price multiplier = 1 + index return
F	Face value:= \$100
P_0	Price:= \$100

Parameter	Value
g_1	$1 + \mu_{S\&P500,\,6mo} \approx 1.08$
g_2	$1 + 2 \times \mu_{S\&P500,\,6mo} \approx 1.16$
g_3	$1 + 3 \times \mu_{S\&P500,\,6mo} \approx 1.24$
l_1	0.70
l_2	0.85
h	To be determined later

Note: $l_1 < l_2 < 1 < g_1 < g_2 < g_3; h > 0$

Economic motivation | Product design | Pricing | Hedging strategy

Replicating portfolio

Case 1: $S_T < S_0$, *barrier option never triggers*	
Loss from Index	$S_T - S_0$

Case 2: $S_T < S_0$, *barrier option triggers*	
Loss from Index	$S_T - S_0$
Protection from Barrier	$\max(l_2 S_0 - S_T, 0)$
Total	$S_T - S_0 + \max(l_2 S_0 - S_T, 0)$

Replicating portfolio

Case 3: $S_T > S_0, S_T < g_1 S_0$	
Gain from Index	$S_T - S_0$

Case 4: $S_T > S_0, g_1 S_0 \leq S_T < g_2 S_0$	
Gain from Index	$S_T - S_0$
Ceiling by short call	$-\max(S_T - g_1 S_0, 0)$
Total	$S_T - S_0 - \max(S_T - g_1 S_0, 0)$

Economic motivation Product design **Pricing** Hedging strategy

Replicating portfolio

Case 5: $S_T > S_0, g_2 S_0 \le S_T < g_3 S_0$	
Gain base on Case 4	$S_T - S_0 - \max(S_T - g_1 S_0, 0)$
Gain from digital option	hS_0
Total	$S_T - S_0 - \max(S_T - g_1 S_0, 0) + hS_0$

Case 6: $S_T > S_0, S_T \ge g_3 S_0$	
Gain base on Case 5	$S_T - S_0 - \max(S_T - g_1 S_0, 0) + hS_0$
Gain from digital option	hS_0
Total	$S_T - S_0 - \max(S_T - g_1 S_0, 0)$ $+ 2hS_0$

Economic motivation | Product design | Pricing | Hedging strategy

Pricing Formula

Basic BS Model

Price Dynamics: $dS_t = rS_t dt + \sigma S dB_t$

BS Model with Dividend

Price Dynamics: $dS_t = (r - q)S_t dt + \sigma S_t dB_t$

Assumptions:

- <u>Underlying pays dividend continuously</u>
- Underlying asset return is following a lognormal random walk
- Interest rate and volatility are known and are constant through time
- European style exercise only
- Frictionless trading
- Trading is carried on continuously
- Short-selling allowed

Pricing Formula

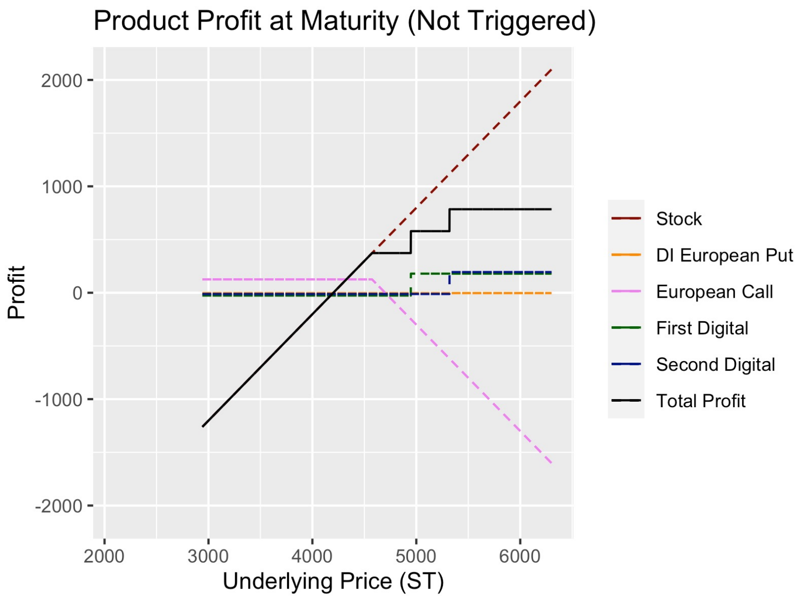
	Position	Function	Notation	Pricing Formula
Long stock	1	S	S	S_0
Long down-and-in European put	1	$F_{DI}(0, S_0; P(S_0, l_2 S_0))$ (Barrier $= l_1 S_0$)	F_{DI}	$\left(\frac{L}{S}\right)^{\frac{2\left(r-q-\frac{\sigma^2}{2}\right)}{\sigma^2}} \times \left[C\left(\frac{L^2}{S_0}, K\right) - C\left(\frac{L^2}{S_0}, L\right) - (L-K)e^{-rT}N\left(d_2(L,S)\right)\right] \times 1\{K > L\} + [P(S_0, L \wedge K) - (L \wedge K - K)e^{-rT}N(-d_2(S_0, L \wedge K))];$ $Where\ K = l_1 \times S_0, L \wedge K = \min(L, K)$
Short European call	-1	$C(S_0, g_1 S_0)$	C	$Se^{-qT}N(d_1(S_0, g_1 S_0)) - Ke^{-rT}N(d_2(S_0, g_1 S_0))$
$(hS_0)\times$Long 1st Binary Call	hS_0	$C_{Digital1}(S_0, g_2 S_0)$	C_{D1}	$e^{-rT}N(d_1(S_0, g_2 S_0))$
$(hS_0)\times$Long 2nd Binary Call	hS_0	$C_{Digital2}(S_0, g_3 S_0)$	C_{D2}	$e^{-rT}N(d_1(S_0, g_3 S_0))$
Total Price	$\Pi_0 = S_0 + F_{DI} - C + hS_0 \times (C_{D1} + C_{D2})$			

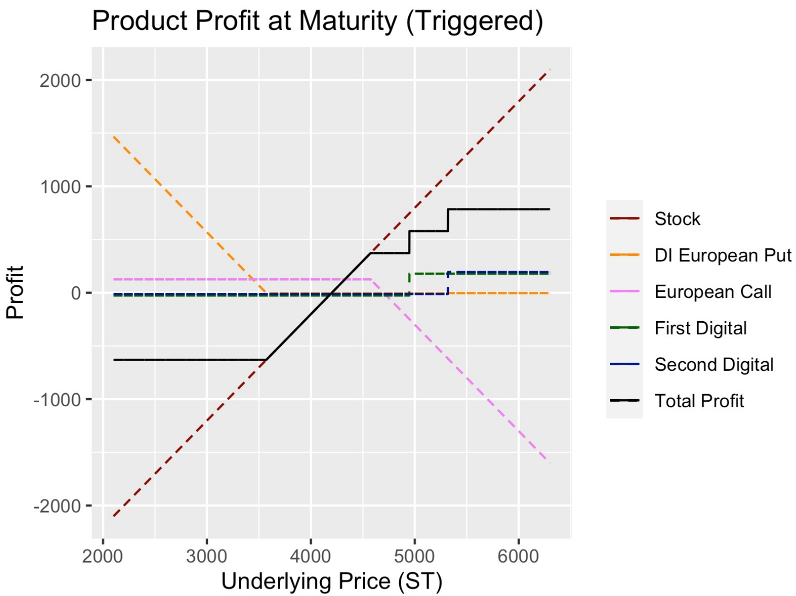
Note: $$d_1(S,K) = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, d_2(S,K) = d_1(S,K) - \sigma\sqrt{T}$$

$$\Pi_0 = 0.98 S_0\ (2\%\ commission) \rightarrow h = \frac{C - F_{DI} - 0.02 S_0}{S_0(C_{D1} + C_{D2})}$$

Replicating portfolio

Breakdown of product's profit into stocks and options (Triggered / Not triggered)





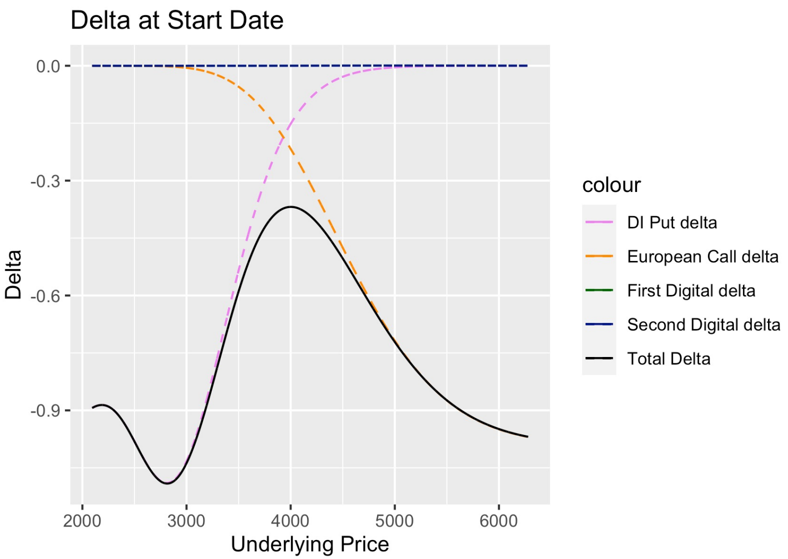
Hedging Strategy

General Approach	
Delta Hedging	- Theoretically applicable - Replicated by down-and-in barrier put option, call option and digital call option - Calculate delta by first order derivative of the pricing equation with respect to the stock price, i.e. $\Delta = \frac{\partial P}{\partial S}$ **Problems:** 1. Frequent re-adjustment 2. Sensitive for barrier option 3. Large delta for binary call if price approaches the strike price

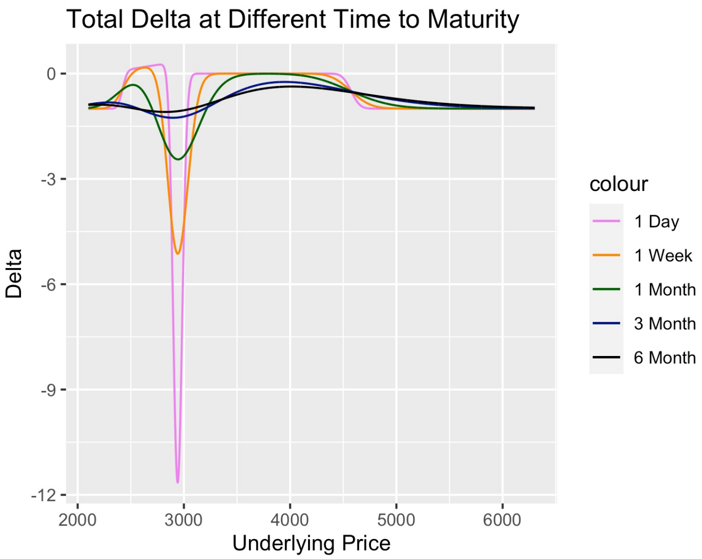
>>>Alternative Approach<<<	
Long stock	Long $(S_0/S_{0,ETF})$ units of SPY
Long DI European put	If signal of market crash appears / really triggered, buy European put $P(S_0, l_2 S_0)$. **Advantage:** no position required at the beginning **Problem:** put may be expensive at that time
Short call	Short corresponding call $C(S_0, g_1 S_0)$
Long binary call	Long bullish spread **Problem:** the hedging position is too large

Hedging Strategy

High sensitivity of Delta (Δ) to underlying price and time to maturity



Sensitivity to underlying price
(t = 0)



Sensitivity to underlying price
& time to maturity

Note: the long-stock position (1) is excluded from total delta here.

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Thank you!

Appendix: Hedging strategy - Delta calculation

	Price (at t=0), single	Delta (at t=0)
Long stock	S_0	1
Long down-and-in European put	$\left(\frac{L}{S}\right)^{\frac{2\left(r-q-\frac{\sigma^2}{2}\right)}{\sigma^2}} \times \left[C\left(\frac{L^2}{S_0},K\right) - C\left(\frac{L^2}{S_0},L\right) - (L-K)e^{-rT}N\big(d_2(L,S)\big)\right] \times 1\{K > L\} +$ $[P(S_0, L \wedge K) - (L \wedge K - K)e^{-rT}N(-d_2(S_0, L \wedge K))];$ $Where\ K = l_1 \times S_0$ $L \wedge K = \min(L,K)$	$= \left(\frac{L}{S}\right)^{\frac{2v}{\sigma^2}}\left(-\left(\frac{L^2}{S^2}\right)e^{-q\tau}N\left(d_1\left(\frac{L^2}{S},K\right)\right) + \left(\frac{L^2}{S^2}\right)e^{-q\tau}N\left(d_1\left(\frac{L^2}{S},L\right)\right)\right.$ $\left. + \frac{(L-K)e^{-r\tau}n\big(d_2(L,S)\big)}{-S\sigma\sqrt{\tau}}\right)$ $+ L^{\frac{2v}{\sigma^2}}\left(-\frac{2v}{\sigma^2}\right)S^{-\frac{2v}{\sigma^2}-1}$ $\times\left(C\left(\frac{L^2}{S},K\right) - C\left(\frac{L^2}{S},L\right) - (L-K)e^{-r\tau}N\big(d_2(L,S)\big)\right)$ $+ e^{-q\tau}N\big(d_1(S,L)\big) + \frac{\Big((L-K)e^{-r\tau}n\big(-d_2(S,L)\big)\Big)}{S\sigma\sqrt{\tau}}$
Short European call	$Se^{-qT}N(d_1(S_0, g_1S_0)) - Ke^{-rT}N(d_2(S_0, g_1S_0))$	$e^{-qT}N(d_1(S_0, g_1K))$
$(hS_0)\times$Long First Binary Call	$e^{-rT}N(d_1(S_0, g_2S_0))$	$\frac{e^{-rT}}{\sigma ST}N(d_2(S_0, g_2S_0))$
$(hS_0)\times$Long Second Binary Call	$e^{-rT}N(d_1(S_0, g_3S_0))$	$\frac{e^{-rT}}{\sigma ST}N(d_2(S_0, g_3S_0))$
Total	Π_0 $= S_0 + F_{DI}(0, S_0; P(S_0, l_2S_0)) - C(S_0, g_1S_0)$ $+ hS_0C_{Digital}(S_0, g_2S_0) + hS_0(S_0, g_3S_0)$	

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