Hotel Cancellation Strategies Under Online Advanced Booking

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Abstract - We segment heterogeneous customers that are different with each other in arriving probability into two types and study possible pricing strategies together with two types of booking cancellation policies, deadline cancellation and refund cancellation. We find that if hotel is risk neutral, the two types of cancellation contracts are the same as each other. However, for a risk-averse hotel, booking contract with refund cancellation policy is better than that with deadline cancellation policy whereas for a risk-seeking hotel, booking contract with deadline cancellation policy is better.

Keywords - Online Booking Contracts; Pricing Strategy; Cancellation.

I. INTRODUCTION

A. Motivation and Research Problem

Nowadays, traveling has become one of people's favorite recreation ways. According to a report of World Tourism Organization (UNWTO), International tourist arrivals reached 1,323 million in 2017, which grows by some 84 million, or 7%, over 2016. Tourism has grown above average, at around 4% per year, for eight straight years. Accompanied by the increase of travel market, it is found that the internet has particularly played a more and more important role in tourism industry. People have been used to in advance book tickets and rooms through online travel agencies (OTAs) for their journey plan today. Therefore, the online reservation system (ORS) is widely used in hospitality industries and makes it possible for econsumers to reserve hotel rooms at anywhere and anytime. For example, the Chinese Tourism Annual Report 2016 stated that, in 2015, the online tourist trading in China is about US\$ 70.7 billion, grew by 49.6%.

However, as every coin has two sides, the adoption of the ORS enables a hotel to receive more bookings but meanwhile makes it more convenience for a customer, who has in advance booked a hotel room, to cancel his booking due to schedule changes. As a result, it is particularly important for a hotel to design cancellation policy, especially when the ORS is widely used (see, for example, [1]). When booking a hotel room, a customer who takes the cancellation into account might concern two things, time and money, specifically, what the cancellation deadline of his booking is and how much the cancellation refund would be, if he has prepaid for his booked room and finally decides to cancel the booking.

Therefore, which type of cancellation policy should be adopted in the hotel's prepayment booking contract, and given a specific cancellation regulation, what the hotel's pricing strategy should be need to be learned. Though, several studies have focus on the cancellation policies of hotel online booking problems, the researchers mainly give some empirical details of hotel booking cancellation (see, [1], [2], [3] and [4] for examples). So far, there is no research work has analytically learned the above questions that we focus on.

In this paper, we consider two most commonly used hotel room prepayment booking contracts that differ in cancellation policies: deadline cancellation policy and refund cancellation policy. Under prepayment booking contract with deadline cancellation policy, a customer can freely cancel his booking room before a given cancellation deadline and get full refund. Usually, this cancellation deadline is before the customer's check-in time, for example, a customer books a room for the night on June 1, the cancellation deadline might be 18:00 pm on May 31. After the deadline, no cancellation with refund would be accepted. Under the prepayment booking contract with refund cancellation policy, a customer can get some refund from the hotel when he cancels his booking at any moment before his check-in time.

We use two models to capture a monopolistic hotel's decision problems under the above two types of cancellation strategies respectively. We classify customers into two types with different arriving probabilities based on their travel behavior and habit. The first type of customer has a higher arriving probability, for example, the travelers who goes on trip for business and/or the well-planed travelers who often travels for long distance. The second type of customer has lower arriving probability, for example, the travelers with casual plans. Beyond arriving probabilities, we also consider that the customers are heterogeneous and differ in their own willingness to pay for a same hotel room.

Our study shows how should the hotel choose cancellation strategies and set its price by comparing the hotel's profits under different types of cancellation policies. It is found that, in a single period decision problem, the two types of cancellation contract are indeed equivalent when the monopolistic hotel is risk neutral and a customer's booking cancellation rate is uniformly distributed in the time interval between its booking and expected checkin time. However, for a risk-averse hotel, booking contract with refund cancellation policy is better than that with deadline cancellation policy whereas for a risk-seeking hotel, booking contract with deadline cancellation policy is better.

B. Related Literature

The information technology has greatly changed hotel customers' behavior as well as hospitality industry in the last decades. Due to the increasing popularity of the Internet[5], travelers have tended to move their information search and travel arrangements activities online. By perfectly meeting customers' information needs about products and services prior to purchase, the hotel industry

is benefiting from information technology, especially from online reservation, in satisfying customer expectations, in improving service, in increasing revenue, and in decreasing costs [6]. At the beginning of online marketing era, the growth of online travel booking was mainly attributed to three aspects: price transparency, the perception of lower prices, and the economies of bundling[7].

Since online marketing has become essential to the tourism and hospitality industry, many academic researchers have studied the behavior of online travelers to understand their requirements and desires. Reference [8] studied which characteristics of websites and hotels are attractive to travelers. Reference [9] believed that the personal variables such as familiarity with e-commerce, prior satisfaction with ecommerce, and receptivity to innovation might precede the formation of online satisfaction and trust. Reference [10] studied how the presentation of information such as page position and the presence of images will possibly affect online travelers' decisionmaking process. Reference [11] reached to the conclusion that tourism innovativeness is able to drive the adoption of tourism products or new technologies in the tourism domain. Reference [12] used an online travel context to test how source, content style, and peripheral credibility cues in online postings influence consumers' attitudes and purchase intentions. On the basis of 638 completed questionnaires, Reference [13] used a principal component factor analysis to derive three dimensions of motivation as information quality, sensitivity content, and time why travelers make hotel room reservation online.

Market segmentation is considered to be having a significant influence on the pricing and revenue management both in practice and in academic research. Especially the rapid development of the B2C e-commerce has raise enough attention of the researchers. Taking hotel rooms as an example, Reference [14] built a pricing model with a new market segmentation based on the lead time and suggested that an appropriate policy of market segmentation in using of online reservation systems is benefit for the service suppliers as well as the consumers. Reference [15] first segmented the consumer market into four segments, and then by comparing the optimal expected revenue associated with different segment combinations, they identify the optimal customer segment combination that generates the highest expected revenue for the firm.

In addition, the market segmentation based on price discrimination can be realized by different characters of the consumers, like expenditure-based market[16], activities-based segmentation[17], benefit-based approach[18], and culture [19].

Pricing strategy have always been the emphasis not only in hospitality but also in revenue management of any industry. Reference [20] show that a firm can increase its expected profit by offering a pricing arrangement that is contingent on whether the seller is able to obtain a higher price within a specified period. Reference [21] used metasearch price data to analyze the best price clauses of online travel agents. Reference [22] argued hotels had to commit to raise prices over time to discourage consumers postponing the purchase in anticipation of last-minute

discounts. Reference [23] tried to identify the characters that may possibly influence hotels' pricing strategy of being dynamic or uniform.

As far as we know, our paper is the first that tries to obtain the optimal pricing policy for combined contracts based on segmentation respect to both the customers' arriving probabilities and willingness to pay through the online reservation system under different cancellation policies.

II. METHODOLOGY

In this section, we first discuss customers' booking behavior and then use the customers' utility functions to derive related demands. Based on the demand functions, we therefore construct the hotel's profit functions for two types of cancellation policies.

We consider a hotel that is selling one type of room (e.g., the Poppies Samui in Thailand) and focus on the hotel's online channel decision problem. Note that he decision rules of one-room-type hotel somehow could also be used by most of the hotels that operate only one main room type (e.g., the budget hotels mainly focusing on the standard room). Customers book rooms of this hotel through the ORS individually and each customer needs one room. The customers that book hotel rooms through other ways except for the ORS are neglected (similar assumption could be see in [24], [25] for example).

In our model, we consider the customers are heterogeneous. They differ with each other in their willing-to-pay prices p_r and arriving probabilities α , where p_r is uniformly distributed between 0 and 1 and α is classified into two types, $\alpha = \alpha_H$ (high arriving probability, e.g., the business visitors) and α_L (low arriving probability, e.g., the leisure travelers) with $0 < \alpha_L < \alpha_H < 1$. Let 1-k and k be the proportions of customers with arriving probabilities α_H and α_L , respectively. The time interval between a customer's booking time and check-in time is denoted as T_A and we assume $0 < T_A < 1$. For simplicity, the potential market size is normalized as 1.

The hotel faces a fixed daily management cost F and a variable operational cost β with $0 < \beta < 1$ for each room if a customer lives in the room. The hotel needs to decide the price p for each room per day.

For the deadline cancellation policy (Model I), a customer can freely cancel the room he has booked before the cancellation deadline T and get full refund. Customers are able to cancel the reservation and receive a full refund p when the time $t \in [0,T]$ (note that $0 < T < T_A$). So the probability that a customer cancels his booking before cancellation deadline is T/T_A . Note that the customers total arriving probability is $1-\alpha$, and therefore, the probability that the customer cancels his booking and gets the full refund is $\theta = (1-\alpha)*T/T_A$. As a result, the customer's utility is

$$U = (p_r - p)^* \alpha - p^* (1 - \alpha)(1 - T/T_A)$$
.

Obviously, when $U \ge 0$ the customer will choose to book a room from the hotel. By this way, we can obtain the demand of hotel rooms q. Based on the demand we can construct the profit model of hotel as

$$\pi(p,T) = q^*\alpha^*(p-\beta) + p^*(1-T/T_A)(1-\alpha)^*q - F.$$

The hotel makes decisions on its room price and cancellation deadline to maximize the profit.

For the refund cancellation policy (Model II), a customer can get an amount of refund from the hotel when he cancels his booking before his check-in time. Denote the customer's received refund as rp, where r is the ratio of refund with 0 < r < 1. Therefore, the customer's utility model is

$$U = (p_r - p)^* \alpha - p^* (1 - \alpha)(1 - r)$$
.

Therefore, when $U \ge 0$, the customer will choose to book a room from the hotel, and in this way we can also obtain the hotel's received demand of rooms q. As a result, we get the hotel's profit model as follow

$$\pi = q * \alpha * (p - \beta) + p * (1 - r)(1 - \alpha) * q - F.$$

The hotel maximizes its profit through making decisions on room price and the ratio of refund.

III. DECISION ANALYSIS

A. Risk Neutral Hotel

Proposition 1: For risk neutral decision makers, Model I and Model II are equal.

To better understand Proposition 1, we could rewrite the customer's utility function and the hotel's profit function for model I and II. Specifically, the risk neutral customers' utility function is

$$U = (p_r - p)^* \alpha - p^* (1 - \alpha)(1 - t),$$

and the risk neutral hotel's decision model is

$$\pi = q * \alpha * (p - \beta) + p * (1 - t)(1 - \alpha) * q - F$$
,

where t represents different decision variables in the two models. In Model I, t represents the ratio of time interval for cancelling the reservation and the total time interval, i.e., $t=T/T_A$, and in Model II, t happens to be the ratio of refund, i.e., t=r. Note that, given T_A , choosing optimal t is indeed equivalent to choosing optimal T. Therefore, Model I that aims to choose the optimal T is equivalent to Model II that aims to choose the best t. As a result, we only learn model II for illustration.

As mentioned, a customer will book a room when his utility is above zero, i.e.,

$$U = (P_r - P)^* \alpha - p^* (1 - \alpha)(1 - r) \ge 0.$$

For the customer with arriving probability α_{H} , the quantity of rooms demanded is

$$q_H = \{1 - p * [(1 - \alpha_H) * (1 - r) + \alpha_H] / \alpha_H\} * (1 - k)$$

For the customer with arriving probability $\alpha_{\scriptscriptstyle L}$, the quantity of rooms demanded is

$$q_L = \{1 - p * [(1 - \alpha_L) * (1 - r) + \alpha_L] / \alpha_L\} * k$$

The total quantity of rooms demanded is $q = q_H + q_L$ and the actual quantity of customers who really arrive at and live in the hotel is

$$q' = q_H * \alpha_H + q_L * \alpha_L$$
.

Therefore, the decision problem of hotel is $\pi = (q_H * \alpha_H + q_L * \alpha_L) * (P - \beta) + p * (1 - r) * [q_H * (1 - \alpha_H) + q_L * (1 - \alpha_L)] - F$ s.t. $0 \le p \le 1, 0 \le r \le 1$

We get the results by solving above KKT conditions. **Proposition 2**. When the hotel is risk neutral, the optimal pricing strategies, the ratio of refund and the corresponding payoffs of the hotel are given as follows:

$$p = \frac{\beta + 1}{2}$$

$$r = 1$$

$$\pi = \frac{(1 - \beta)^2 [\alpha_L * k + \alpha_H * (1 - k)]}{4} - F$$

the corresponding quantity of demand for customers with low arriving probability, customers with high arriving probability and total quantity of demand are given as follows:

$$\begin{split} q_{\scriptscriptstyle H} &= \frac{(1-\beta)*(1-k)}{2} \\ q_{\scriptscriptstyle L} &= \frac{(1-\beta)*k}{2} \\ q &= \frac{1-\beta}{2} \end{split} \; .$$

Remark. When the hotel is risk neutral, to maximize the profit, the hotel should let the ratio of refund be 1, which means customers can get a full refund when they cancel the reservation.

From the above results solved by KKT conditions, the optimal ratio of refund is r=1 when the profit of the hotel is maximal. That is, the hotel will adopt the contract of full refund, which means the hotel allows customers to make reservation online and freely cancel their bookings.

Corollary 1. When the hotel is risk neutral, the total quantity of demand decreases with β ; the optimal prices increases with β ; the possible maximal profit increases with α_H and α_L , decreases with k and decreases with k.

TABLE I
The monotonicity of variables and parameters

	β	$\alpha_{\scriptscriptstyle H}$	$\alpha_{\scriptscriptstyle L}$	k
p	+	/	/	/
Q	-	/	/	/
π	-	+	+	-

Note: "+" means increasing; "-" means decreasing; "/" means not applicable

From the above results, we can find that when the hotel is risk neutral, the total quantity of demand and the optimal price only depends on the variable operational cost of each occupied room β but independent of α_L , α_H and k. The possible maximal profit increases with the arriving probabilities of customers. It is because the hotel's faced booking cancellation risk is lower when the customers' arriving probabilities increase, and hence it obtains more profit. When the proportion of customers with low arriving probability k increases, the hotel faces more risk and uncertainty and thus earns less profit. In addition, the possible maximal profit decreases with β .

B. Risk Non-Neutral Hotel

Proposition 1 suggests that for the hotel, booking contract with refund cancellation policy will finally bring the same *expected profit*, denoted as π^* , as the booking contract with deadline cancellation policy. However, the two types of booking contracts indeed have different profit variances. Therefore, for the risk non-neutral hotel, the two types of contracts of course will be different.

Proposition 3: in the case r<1, for a risk averse hotel, prepayment booking contract with refund cancellation policy is better than that with dead-line cancellation policy; whereas, for a risk seeking hotel, prepayment booking contract with deadline cancellation policy is better than booking contract with refund cancellation policy.

Note that when the decision makers are risk neutral, in optimum, both contracts will receive a same booking amount from the same customers. For the hotel, the expected revenue, denoted as π_i , generated from a particular customer, customer i, is the same with each other. As a result, to compare the variances between these two types of booking contracts is sufficient to compare the revenue variances brought by a customer who has booked a room from the hotel. When the customer has booked a room from the hotel, the hotel's revenue variance is determined by the customer's uncertain arriving results.

Under the booking contract with deadline cancellation policy, the hotel will get zero if the customer with noshow probability $(1-\alpha)$ cancels his booking before the booking cancellation deadline while gets p if the customer do not cancel his booking before the booking cancellation deadline. However, under the booking contract with refund cancellation policy, when r < I, the hotel will receive revenue (1-r)p > 0 if the customer is no-show and p otherwise. Therefore, we could easily find that the variance of hotel's revenue generated from customer i under the booking contract with deadline cancellation policy is larger than with refund policy. This is due to the fact that for any $x, y \in (0,1)$ and

$$\pi_i = x * 0 + (1-x) * p = y * (1-r)p + (1-y) * p ,$$
 we have

$$x*(0-\pi_i)^2 + (1-x)*(p-\pi_i)^2 > y*[(1-r)p-\pi_i] + (1-y)*(p-\pi_i)^2.$$

IV. CONCLUSION

The online reservation system has been made full use of in hospitality industry. Pricing strategies for different types of reservation become one of the essential part of hotel revenue management. This paper studies the pricing strategies of hotel's online reservation system and hotel revenue management based on the Stackelberg game analysis. We segment the customers market into two parts according to the difference in arriving probabilities, the customers with high arriving probabilities and customers with low arriving probability. Then learned two different types of hotel booking contracts that differ in cancellation policies. In addition, we take the condition that hotel may has different attitudes to risk into account and obtain some

conclusions about pricing strategies and revenue management.

When the hotel is risk neutral, the two types of cancellation contract are indeed equivalent if the customer's booking cancellation rate is uniformly distributed in the time interval between its booking and expected check-in time. Besides, the total demand and the optimal price both are only depending on the variable cost. Therefore, to sell the rooms as many as possible and obtain more profit, hotels should reduce the variable cost of per room and guarantee that customers will get a full refund when they cancel the reservation. However, for a risk averse hotel, booking contract with refund cancellation policy is better than that with deadline cancellation policy whereas for a risk seeking hotel, booking contract with deadline cancellation policy is better.

On the other hand, there are also some questions remain incomplete and imperfect in this paper that worth further study. We only considered the situation that one hotel kept a monopoly in the market. However, there is fierce competition in hospitality industry in practice. It is almost impossible that one hotel would keep a monopoly. Since considering the competition will make the model too complicated to solve, this paper has not taken competition into consideration, which worth further analysis in the future study. Second, this paper assume that all customers book one type of room through the ORS. Though there are some hotel does only have one type of room, but in practice, many hotels may offer two or even more types of room. Third, this paper consider two most common contracts on ORS, which are booking with payment and booking without payment, and combine them. There might also exists other promotion such as discount coupon that may change the demand of consumers. It is worth further study.

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APPENDIX

Proof of Proposition 2

The hotel's profit $\pi = q^*\alpha^*(P-\beta) + p^*(1-r)(1-\alpha)^*q - F$ is jointly concave in (p,r). Applying the demand of consumers with high arriving probabilities and consumers with low arriving probabilities $q_{H} = (1-p^*((1-\alpha_{H})^*(1-r) + \alpha_{H})^{\prime}\alpha_{H})^*(1-k)$ and $q_{L} = (1-p^*((1-\alpha_{L})^*(1-r) + \alpha_{L})^{\prime}\alpha_{L})^*k$ in the expression of profit

 $\pi = (q_{_{\! H}} * \alpha_{_{\! H}} + q_{_{\! L}} * \alpha_{_{\! L}}) * (P - \beta) + p * (1 - r) * [q_{_{\! H}} * (1 - \alpha_{_{\! H}}) + q_{_{\! L}} * (1 - \alpha_{_{\! L}})] - F$ $st.0 \le p \le 1, 0 \le r \le 1$

$$\begin{aligned} & \text{Solving} \begin{cases} \frac{\partial \pi}{\partial r} = 0 \\ \frac{\partial \pi}{\partial p} = 0 \end{cases}, & \text{we can get:} & \begin{cases} p = 0 \\ r = \frac{1}{\alpha_{l} * k - \alpha_{l} - \alpha_{l} * k + 1} \end{cases} \text{ and } \begin{cases} p = \frac{1 + \beta}{2} \\ r = 1 \end{cases} \\ & \text{Since } p > 0 \text{, the final results are } \begin{cases} p^* = \frac{1 + \beta}{2} \\ r^* = 1 \end{cases}. & \text{By apply-} \end{cases}$$

Since p > 0, the final results are r = 1. By applying them in the expressions, we can get the possible optimal profit, the demand of two types of consumers and the total demand

$$\pi^* = \frac{(1-\beta)^2 [\alpha_L^* k + \alpha_H^* * (1-k)]}{4} - F$$

$$q^*_H = \frac{(1-\beta)^* (1-k)}{2}$$

$$q^*_L = \frac{(1-\beta)^* k}{2}$$

$$q^* = \frac{1-\beta}{2}$$