Example 7.4

Disease mapping: from foundations to multidimensional modeling

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This document reproduces the analysis made at Example 7.4 of the book: "Disease mapping: from foundations to multidimensional modeling" by Martinez-Beneito M.A. and Botella-Rocamora P., published by CRC press in 2019. You can watch the analysis made with full detail at this pdf document, or even execute it if you want with the material available at https://github.com/MigueBeneito/DMBook. Anyway, this pdf file should be enough for following most of the details of the analysis made for this example.

The statistical analysis below has been run in `R`, by additionally using the library `Rmarkdown`, so be sure that you have this software installed if you want to reproduce by yourself the content of this document. In that case we advise you to download first the annex material at https://github.com/MigueBeneito/DMBook, open with `Rstudio` the corresponding `.Rproj` file that you will find at the folder corresponding to this example and compile the corresponding `.Rmd` document. This will allow you to reproduce the whole statistical analysis below.

This document has been executed with real data that are not provided in order to preserve their confidentiality. Slightly modified data are provided instead, as described in Chapter 1 of the book. Thus, when reproducing this document you will not obtain exactly the same results, although they should be very close to those shown here.

Libraries and data loading

```
# Libraries loading
#-----------------
if (!require(RColorBrewer)) {
    install.packages("RColorBrewer")
    library(RColorBrewer)
}
if (!require(rgdal)) {
    install.packages("rgdal")
    library(rgdal)
}
if (!require(pbugs)) {
    if (!require(devtools)) {
        install.packages("devtools")
        devtools::install_github("fisabio/pbugs")
    } else {
        install_github("fisabio/pbugs")
    }
}

# Data loading
#------------
# For reproducing the document, the following line should be changed to
# load('../Data/ObsOral-ET-mod.Rdata') since that file contains the
# modified data making it possible to reproduce this document.
load("../Data/ObsOral-ET.Rdata")
# load('../Data/ObsOral-mod.Rdata')
```

```
load("../Data/ExpOral-ET.Rdata")
load("../Data/VR.Rdata")
```

R function for calculating the DIC criterion of the models fitted

The function below computes the DIC criterion for disease mapping models fitted with `WinBUGS`. It returns DIC values comparable to those reported by `INLA`, in contrast to `WinBUGS`. See annex material for Example 4.3.

```
# Arguments: Simu.sSMRs: matrix of dimensions n.IterXn.Units where
# n.Iter are the number of MCMC iterations saved and n.Units the number
# of spatial units in the analysis. You will typically find this as a
# submatrix of the sims.matrix element of any bugs object. O: Vector of
# length n.Units with the observed deaths per spatial unit. E: Vector
# of length n.Units with the expected deaths per spatial unit.
DICPoisson = function(Simu.sSMRs, O, E) {
    mu = t(apply(Simu.sSMRs/100, 1, function(x) {
        x * E
    }))
    D = apply(mu, 1, function(x) {
        -2 * sum(O * log(x) - x - lfactorial(O))
    })
    Dmean = mean(D)
    mumean = apply(Simu.sSMRs/100, 2, mean) * E
    DinMean = -2 * sum(O * log(mumean) - mumean - lfactorial(O))
    # if(save==TRUE){return(c(Dmedia,Dmedia-DenMedia,2*Dmedia-DenMedia))}
    cat("D=", Dmean, "pD=", Dmean - DinMean, "DIC=", 2 * Dmean - DinMean,
        "\n")
}
```

```
# WinBUGS code for the autoregressive Spatio-temporal model
Autoregressive = function() {
    for (i in 1:nmuni) {
        for (j in 1:nperiods) {
            Obs[i, j] ~ dpois(lambda[i, j])
            # Modelling of the mean for every municipality and period
            log(lambda[i, j]) <- log(Exp[i, j]) + log.theta[i, j]
            # log-sSMR for every municipality and period
            log.theta[i, j] <- inter.mean + sd.inter * inter[j] + ST[i,
                j]
        }
    }

    # Spatio-temporal effect for the first period
    for (i in 1:nmuni) {
        ST[i, 1] <- pow(1 - ro * ro, -0.5) * BYM[i, 1]
        BYM[i, 1] <- sd.het * psi[i, 1] + sd.spat * phi[1, i]
        psi[i, 1] ~ dnorm(0, 1)
    }
    phi[1, 1:nmuni] ~ car.normal(map[], w[], nvec[], 1)
    # Spatio-temporal effect for the subsequent periods
    for (j in 2:nperiods) {
        for (i in 1:nmuni) {
```

```r
            ST[i, j] <- ro * ST[i, j - 1] + BYM[i, j]
            BYM[i, j] <- sd.het * psi[i, j] + sd.spat * phi[j, i]
            psi[i, j] ~ dnorm(0, 1)
        }
        phi[j, 1:nmuni] ~ car.normal(map[], w[], nvec[], 1)
    }

    # Prior distribution for the mean risk for every municipality and
    # period
    inter.mean ~ dflat()
    # Prior distribution for the global time trend
    inter[1:nperiods] ~ car.normal(mapT[], wT[], nvecT[], 1)
    # Prior distribution for the precision parameters in the model
    sd.inter ~ dunif(0, 5)
    sd.het ~ dunif(0, 5)
    sd.spat ~ dunif(0, 5)
    # Prior distribution for the temporal dependence parameter
    ro ~ dunif(-1, 1)
}

nperiods.12 = 12
adjT.12 = c(rbind(2:nperiods.12, 1:(nperiods.12 - 1)))
numT.12 = c(1, rep(2, nperiods.12 - 2), 1)
indexT.12 = c(1, cumsum(numT.12))

data = list(Obs = ObsOral, Exp = ExpOral, nmuni = length(VR.wb$num), nperiods = nperiods.12,
    w = rep(1, length(VR.wb$adj)), nvec = VR.wb$num, map = VR.wb$adj, wT = rep(1,
        length(adjT.12)), nvecT = numT.12, mapT = adjT.12)
inits = function() {
    list(ro = runif(1, -1, 1), inter.mean = rnorm(1, 0, 1), sd.inter = runif(1,
        0, 0.5), sd.het = runif(1, 0, 0.5), sd.spat = runif(1, 0, 0.5),
        psi = matrix(rnorm(nperiods.12 * data$nmuni, 0, 1), ncol = nperiods.12,
            nrow = data$nmuni), phi = matrix(rnorm(nperiods.12 * data$nmuni,
            0, 1), nrow = nperiods.12, ncol = data$nmuni))
}
param = c("log.theta", "inter.mean", "sd.inter", "sd.het", "sd.spat", "ro")
ResultOral = pbugs(data = data, inits = inits, parameters = param, model.file = Autoregressive,
    n.iter = 5000, n.burnin = 1000, DIC = F, n.chains = 3, bugs.seed = 1)
# Computing time
ResultOral$exec_time
```

```
## Time difference of 57.99719 mins
```

```r
# Result summaries
summary(ResultOral$summary[, "Rhat"])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.9995  1.0008  1.0024  1.0033  1.0048  1.0305
```

```r
summary(ResultOral$summary[, "n.eff"])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    70.0   440.0   860.0   723.7  1000.0  1000.0
```

```r
round(ResultOral$summary["ro", ], 3)
```

```
##    mean      sd    2.5%     25%     50%     75%   97.5%    Rhat   n.eff
##   0.961   0.025   0.905   0.950   0.967   0.978   0.993   1.019 180.000
```

```
# DIC
DICPoisson(100 * exp(ResultOral$sims.matrix[, grep("log.theta", dimnames(ResultOral$sims.matrix)[[2]])]
    t(ObsOral), t(ExpOral))
```

```
## D= 7222.411 pD= 110.3808 DIC= 7332.792
```

Variance decompositions

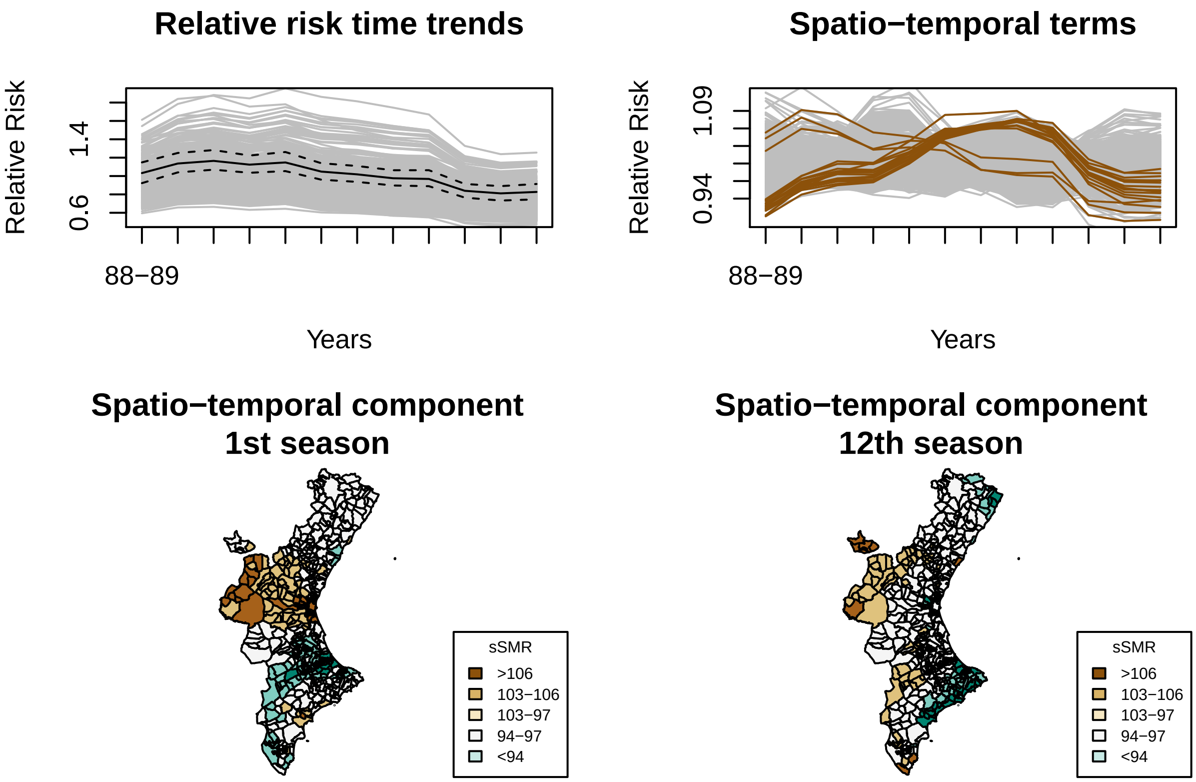
```
nIter = dim(ResultOral$sims.list$log.theta)[1]
components = matrix(nrow = nIter, ncol = 4)
for (i in 1:nIter) {
    m = mean(ResultOral$sims.list$log.theta[i, , ])
    S = apply(ResultOral$sims.list$log.theta[i, , ] - m, 1, mean)
    T = apply(ResultOral$sims.list$log.theta[i, , ] - m, 2, mean)
    ST = ResultOral$sims.list$log.theta[i, , ] - (m + matrix(rep(S, length(T)),
        ncol = length(T)) + matrix(rep(T, length(S)), ncol = length(T),
        byrow = T))
    components[i, ] = c(m, var(S), var(T), var(as.vector(ST)))
}
aux = apply(components, 2, mean)
aux[2:4]/sum(aux[2:4])
```

```
## [1] 0.7309678 0.1607251 0.1083071
```

Plot

```
par(mfrow = c(2, 2))
par(mar = c(5, 4, 4, 2) + 0.1)
Ts = matrix(nrow = nIter, ncol = 12)
Ss = matrix(nrow = nIter, ncol = 540)
for (i in 1:nIter) {
    m = mean(ResultOral$sims.list$log.theta[i, , ])
    Ts[i, ] = apply(ResultOral$sims.list$log.theta[i, , ] - m, 2, mean)
    Ss[i, ] = apply(ResultOral$sims.list$log.theta[i, , ] - m, 1, mean)
}
aux = apply(ResultOral$sims.list$log.theta, c(2, 3), mean)

plot(exp(apply(Ts, 2, mean)), type = "l", axes = F, xlab = "Years", ylab = "Relative Risk",
    ylim = c(0.5, 1.9), main = "Relative risk time trends")
for (i in 1:540) {
    lines(exp(aux[i, ]), col = "gray")
}
lines(exp(apply(Ts, 2, mean)), type = "l")
lines(exp(apply(Ts, 2, quantile, 0.025)), type = "l", lty = 2)
lines(exp(apply(Ts, 2, quantile, 0.975)), type = "l", lty = 2)
axis(1, 1:12, labels = c("88-89", "", "92-93", "", "96-97", "", "00-01",
    "", "04-05", "", "08-09", ""))
axis(2, at = c(0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8))
box()
```

```
STs = array(dim = c(1002, 540, 12))
for (i in 1:nIter) {
    m = mean(ResultOral$sims.list$log.theta[i, , ])
    S = apply(ResultOral$sims.list$log.theta[i, , ] - m, 1, mean)
    T = apply(ResultOral$sims.list$log.theta[i, , ] - m, 2, mean)
    STs[i, , ] = ResultOral$sims.list$log.theta[i, , ] - (m + matrix(rep(S,
        length(T)), ncol = length(T)) + matrix(rep(T, length(S)), ncol = length(T),
        byrow = T))
}
aux.ST = apply(STs, c(2, 3), mean)

plot(exp(aux.ST[1, ]), type = "l", axes = F, xlab = "Years", ylab = "Relative Risk",
    ylim = c(0.9, 1.12), main = "Spatio-temporal terms")
for (i in 1:540) {
    lines(exp(aux.ST[i, ]), col = "gray")
}
for (i in 1:540) {
    if (mean(exp(Ss[, i])) > 1.5) {
        lines(exp(aux.ST[i, ]), col = brewer.pal(7, "BrBG")[1])
    }
}
axis(1, 1:12, labels = c("88-89", "", "92-93", "", "96-97", "", "00-01",
    "", "04-05", "", "08-09", ""))
axis(2, at = c(0.94, 0.97, 1, 1.03, 1.06, 1.09))
box()

par(mar = c(1, 1, 2, 1) + 0.1)
cut1 = cut(exp(aux.ST[, 1]), c(0, 0.94, 0.97, 1.03, 1.06, 10))
plot(VR.cart, col = brewer.pal(5, "BrBG")[5:1][cut1])
title("Spatio-temporal component\n1st season", cex = 0.75)
legend(x = "bottomright", fill = brewer.pal(7, "BrBG"), legend = c(">106",
    "103-106", "103-97", "94-97", "<94"), cex = 0.65, inset = 0.03, title = "sSMR")

cut2 = cut(exp(aux.ST[, 12]), c(0, 0.94, 0.97, 1.03, 1.06, 10))
plot(VR.cart, col = brewer.pal(5, "BrBG")[5:1][cut2])
title("Spatio-temporal component\n12th season", cex = 0.75)
legend(x = "bottomright", fill = brewer.pal(7, "BrBG"), legend = c(">106",
    "103-106", "103-97", "94-97", "<94"), cex = 0.65, inset = 0.03, title = "sSMR")
```


Relative risk time trends
Relative Risk
0.6
1.4
88−89
Years
Spatio−temporal terms
Relative Risk
0.94
1.09
88−89
Years
Spatio−temporal component
1st season
sSMR
>106
103−106
103−97
94−97
<94
Spatio−temporal component
12th season
sSMR
>106
103−106
103−97
94−97
<94