Observational Astrophysics: Luminosity Period Measurement for α Ursae Minoris

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A light curve of for Polaris (α Ursae Minoris) was made from brightness measurements of photographs taken by a Planewave DCK 431mm telescope. The pulsation period of Polaris was measured to be $\mathrm{T_{exp}} = 3.882$ d, which comes within 2.19% of the accepted value of T = 3.969 d. From the pulsation period, the apparent magnitude of Polaris was found to be $\mathrm{M_{exp}} = -3.09$ which comes within 1.30% of the accepted value of $M_{\mathrm{polaris}} = -3.07 \pm 0.04$ [1]. The calculated distance to Polaris of $\mathrm{d_{exp}} = 103.1$ pc matches current estimates that range from 99 to 113 pc [2].

I. INTRODUCTION

On September 10, 1784, Edward Pigott noticed that the brightness of the star Eta Aquilae seemed to vary. He measured the brightness of this star over the next few days and found that its apparent magnitude was periodic [3], which made Pigott the first measure the pulsation period of a variable star.[1] A variable star (also called a pulsating star) is a star that dims and brightens as its surface expands or contracts, and can be found on the instability strip of the Hertzsprung-Russell diagram [4]. After discovering and analyzing data from over 2,000 pulsating stars, Henrietta Swan Leavitt found a relationship between the pulsation periods and the apparent magnitudes of classical Cepheid variables, a special subset of variable stars [5][6]. This meant that finding the period of brightness variations of a star could be used to estimate how far away from Earth the star is.

Polaris (α Ursae Minoris) is likely the closest classical Cepheid variable to Earth, which means that it is heavily studied. This experiment aims to find a light curve, find the period, and from the period calculate the apparent magnitude of Polaris. Using photographs taken by a telescope, the brightness of the star will then be analyzed using MaxIm DL. Measurements of average brightness will then be manipulated with Mathematica so as to find the pulsation period of Polaris. The period-luminosity equation will produce the absolute magnitude of Polaris. Finally, using an average value for apparent magnitude, the distance to Polaris will be measured.

II. THEORY AND EXPERIMENTAL DESIGN

A summary of relevant Astrophysics (A), classical Cepheid variables (B), and Polaris (C) will provide the background information necessary to explain the design of this experiment (D).

[1] A feat often mistakenly attributed to Pigott's collaborator, John Goodricke, who found the pulsation period of δ Cephei a few months later.

A. Magnitude

In Astrophysics, apparent magnitude and absolute magnitude have very precise definitions. Apparent magnitude, m, describes how bright each star appears in the sky (on a scale from 1 to 6; where 1 is the brightest star 6 is the dimmest star visible to the naked eye). Meanwhile absolute magnitude, M, is the apparent magnitude a star would have if it were located at a distance of 10 pc. The difference between the two is the distance modulus

$$m - M = 5\log_{10}(\frac{d}{10\ \mathrm{pc}}) \tag{1}$$

where d is the star's distance [7]. This equation leaves two unknown variables, M and d (presumably, m can be measured with a telescope). If it were somehow possible to find M, one would have a great method of estimating how far away various stars were, and how big the galaxy is.

B. Period-Luminosity Relation

A classical Cepheid variable is a specific kind of variable star. In particular, a classical Cepheid variable is a star that varies in brightness with a period of about 1 to 50 days, is a Population I star, and exhibits radial oscillations (often of up to millions of kilometers). These variations result in changing apparent magnitudes for a star. Using the period-luminosity relation credited to Leavitt, one could determine the luminosity or absolute magnitude of a Cepheid by knowing its period. At first, this equation was useful only for looking at Cepheid's in the Small Magnetic Cloud, but once it was "calibrated" by Ejnar Hertzsprung, it could be used to find the absolute magnitude of any Cepheid [8]. The period-luminosity equation is given by:

$$M_{\langle V \rangle} = -2.81\log_{10}P_d - 1.43 \tag{2}$$

where P_d is the pulsation period (in units of days) and $M_{\langle V \rangle}$ is the average absolute V magnitude [4].

FIG. 1: The telescope used to photograph Polaris. Owned by iTelescope, “T21” is a Planewave DCK 431mm Imaging and Science Platform (Adapted from http://www.itelescope.net/telescopet21/).

To summarize, Eq. (2) gives the absolute magnitude as a function of the pulsation period. The absolute magnitude was one of the unknown variables in Eq. (1), so combining the two equations, one can calculate how far away a star is from Earth knowing only the pulsation period and the apparent magnitude of the star.

C. Polaris

Polaris, or α Ursae Minoris, is the brightest star in the Ursa Minor constellation. The most recent data gives an absolute magnitude of $M_{\text{polaris}} = -3.07 \pm 0.04$ and a period of $\mathrm{T}_{\text{polaris}} = 3.969$ d [1]. A value of $m_{\text{polaris}} = 1.98$ can can be used as a benchmark for the average apparent magnitude across it’s pulsation period [10].

D. Design

Photographs of Polaris were taken using iTelescope’s Planewave DCK 431mm Imaging and Science Platform (telescope 21), based in Mayhill, New Mexico [9].

Four photographs were taken across a span of two weeks, each photograph chosen at a specific time, so that the data points were evenly distributed across Polaris’ pulsation period. Polaris has a pulsation period of about four days, and by the definition of a periodic function:

$$\mathrm{M}(t) = \mathrm{M}(t + \mathrm{T}) \tag{3}$$

which says that the apparent magnitude at some time t should equal the apparent magnitude at time t + T,

TABLE I: MaxIm DL brightness measurements for Polaris. The time the photograph was take given in mm/dd format, and the time it was taken. The rest of the columns show the corresponding maximum, minimum, average, and standard deviation of the brightness. All dates in 2014.

Taken (date; time)	Max	Min	Avg	Std Dev
2/13 01:43:58	60381	6689	13616	11233
2/13 20:38:01	60842	6130	18541	14391
2/19 01:51:03	60322	3708	9508	10388
2/20 21:44:37	60425	1143	12258	14986

where T is the pulsation period (T $\approx$ 4 d, here). Thus, in order to get an even distribution of data points and therefore a good understanding of the pulsation period, one would need to make various measurements across one cycle.

With this in mind, the first two photographs were taken about 24 hours from each other: on 2/13 in the early morning and on 2/13 at night. Circumstances dictated that the next opportunity to take pictures would be 7 days later, so it was decided that the next two measurements would be made on 2/19 in the morning and on 2/20 at night, so as to avoid taking photos of Polaris at the same stage in it’s cycle (which would have occurred had the photos been taken on 2/17 or 2/21). The picture for on the 2/20 at night essentially repeated (or came close to) the picture taken on 2/13 in the morning (almost exactly two periods before), but this could not be helped since various other factors such as weather, brightness, humidity, etc. constrained the timing of the photographs.

MaxIm DL can extract brightness data from stellar photographs. Though the brightness units in this program are not specified, as long as a standardized method can be applied to the brightness of each cycle the data points can be plotted and an accurate period can be found. The units of brightness are unrelated to the period, only the general periodic trend matters.

III. DATA ANALYSIS

The images were opened in MaxIm DL as “.FITS” files (specific to scientific data). The “remove bloom” button was clicked so as to standardize the brightness and remove unnecessary glare from the star (settings were min: 12411 and max: 19556). After clicking the “Information” button and selecting the information method as “Area,” a 216 by 213 pixel box was made around the star. From here, the brightness measurements for Polaris were collected. It should be noted that these values were not given in the units of of apparent magnitude, but in an unspecified unit (since the program was not calibrated). The processed images are shown in Fig. (2).

This method was repeated exactly for all four pho-

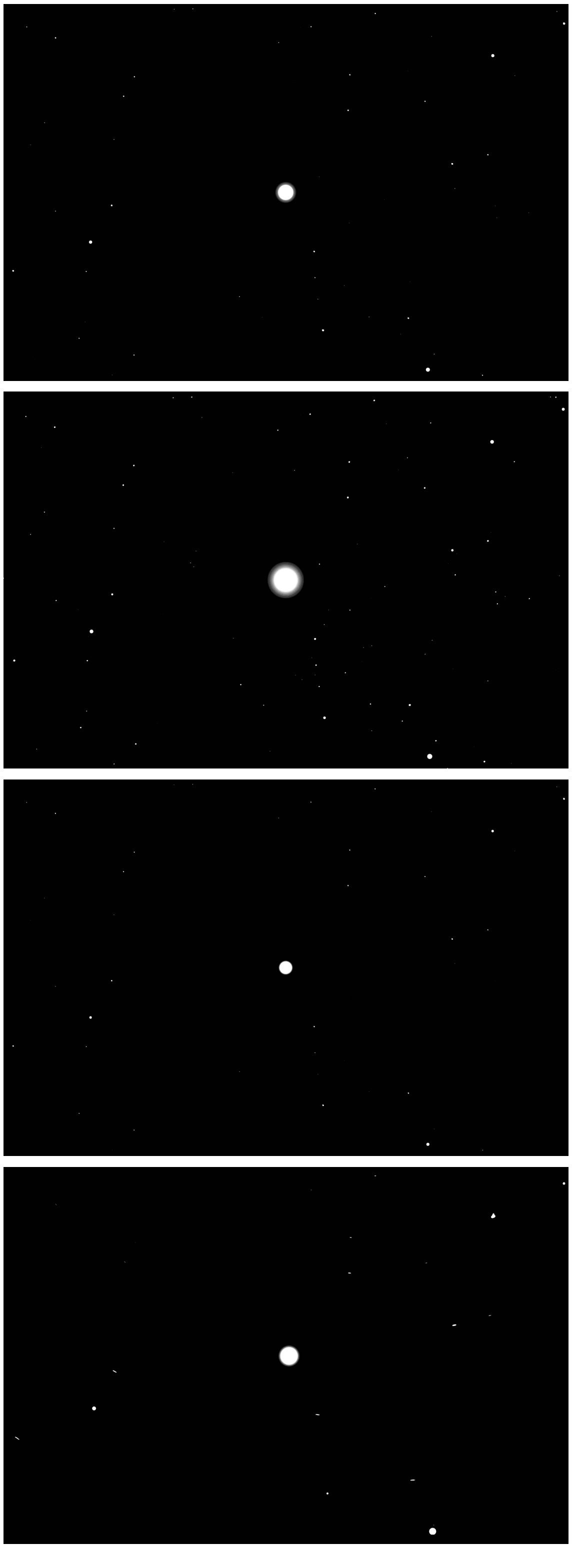


FIG. 2: The four edited pictures of Polaris from which data was extracted. The pictures are in chronological order from top to bottom; taken on: 2/13/2014 at 1:44, 2/13/2014 at 20:38, 2/19/2014 at 1:52, and 2/20/2014 at 21:45.

TABLE II: MaxIm DL time modifications for Polaris. Time and date conversion to time since reference point (2/13 at 0:00:00) and conversion to "modified" time which combines both pulsation cycles into one. All dates in 2014.

Taken (date; time)	Time Ref. (hrs)	"Modified" Time (hrs)
2/13 01:43:58	1.73	1.73
2/13 20:38:01	20.63	20.63
2/19 01:51:03	145.85	50.59
2/20 21:44:37	189.74	94.49

tographs. Data is shown in Table I.

The average brightness was used in the data analysis, though similar results can be found analyzing the period of the maximum and the minimum.

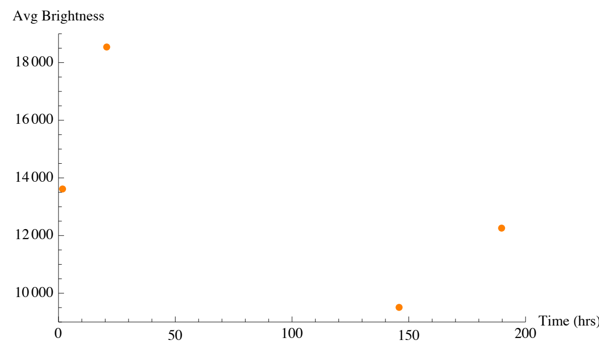


FIG. 3: The average brightness of Polaris at four different times. Measured from "reference time." The average brightness is in unspecified units.

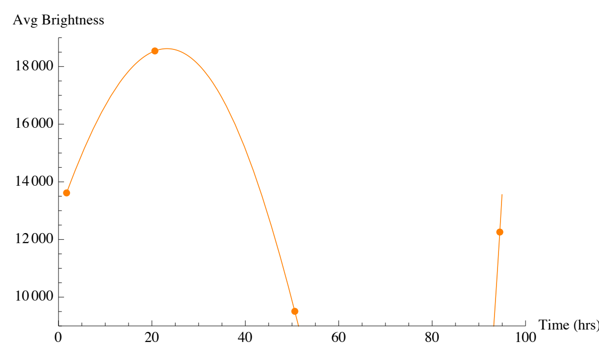


FIG. 4: The average brightness of Polaris at four different times. The last two data points have been "shifted" back by one period. The best fit line (as computed by Mathematica) is shown with the data points. The average brightness is in unspecified units.

The time and date the photograph was taken were converted into hours since 2/13 at 0:00:00, to create a "zero point" for the time, and plotted in Fig. (3). Since the data was collected over almost nine days and the period of the star was only four days, some of the measurements were made across two different cycles. In order to model one period, the two periods were combined into one. This was accomplished by subtracting the accepted pulsation period of Polaris from the second two data points (the two data points that were in the second cycle), and the result is shown in Fig. (4). (The adjusted times are shown in Table II.) To find the period, a best fit line was found for the "modified" data of one cycle, and the point was found when the line repeated itself. The pulsation period was found to be about 3.88 days.

From the pulsation period, Eq. 2 was used to find an absolute magnitude of −3.09. Finally, with the help of Eq. 1 and using an apparent magnitude of $m = 1.98$ found in the literature,[2] the distance between Earth and Polaris was found to be 103.1 pc (or 336.1 ly).

IV. CONCLUSION

The calculation for the pulsation period of Polaris using the average brightness (as found by MaxIm DL) from the four pictures. The period was found to be:

$$T_{\mathrm{exp}} = 3.882\ \mathrm{d}$$

which is incredibly close to the accepted value of $T_{\mathrm{polaris}} = 3.969$ d. A period of almost four days confirms that Polaris is a classical Cepheid variable. From the determined pulsation period, the apparent magnitude was calculated to be:

$$M_{\mathrm{exp}} = -3.09$$

and also lies near the accepted absolute magnitude of $M_{\mathrm{polaris}} = -3.07 \pm 0.04$.

The distance to Polaris was calculated using the absolute magnitude measured from this data, and an apparent magnitude borrowed from the literature, and it was found that

$$d_{\mathrm{exp}} = 103.1\ \mathrm{pc}$$

falls between two recent estimates of 99 and 113 pc.

The most significant source of error in this experiment comes from the "quick fix" that was made in "pasting together" brightness values from two different periods into one period. This approximation could easily be eliminated if the photographs were to have been taken regularly across a single period. Additionally, more pictures would provide more data points which would theoretically produce a better result. Although difficult to control, performing this experiment with clear skies and a new moon might also yield a slightly better pictures, and therefore a better result.

The effectiveness of this experiment could be improved if given apparent magnitudes were used to calibrate the photographs in MaxIm DL. This would allow one to extract the apparent magnitude of the star and calculate the distance to the star without relying on separate sources, as was done here. Finally, a better understanding of error in the realm of Astrophysics might provide insight into the accuracy of the results, whether this accuracy was due to blind luck or to the combination of careful analysis and a well-designed experiment.

[2] Extracting an apparent magnitude from the image can be done with MaxIm DL, but one must first calibrate the program using the apparent magnitude of another star in the image.

Appendix

Part of this experiment also involved setting up a the Orion AstroView 6 EQ telescope. This experience was beneficial because it provided the experimenter with working, intuitive knowledge of a telescope. Concepts such as the counterweight, the layout of the lenses and mirrors in the tube, the polar axis finder, and the fine adjustment knobs which were initially unfamiliar helped the experimenter appreciate the power of the remote telescope used in this study.

[1] D. G. Turner, V. V. Kovtyukh, I. Usenko, N. Gorlova, Astrophys. Lett. (2012), arXiv:1211.6103v1 [astro-ph.SR].

[2] N.R. Evans, D.D. Sasselov, C.I. Short, Astron. J. **567** (2002), 1121.

[3] K. Crosswell, Sky & Telescope (1997), p 90.

[4] B. W. Carroll and D. A. Ostlie, *An Introduction to Modern Astrophysics*, 2nd ed. (Addison-Wesley, San Francisco, CA, 2007), pp 483-490.

[5] H.S. Leavitt, Annals of Harvard College Observatory **60** (1908).

[6] H. S. Leavitt, E. C. Pickering, Harvard College Observatory Circular **173** (1912).

[7] B. W. Carroll, D. A. Ostlie, *An Introduction to Modern Astrophysics*, 2nd ed. (Addison-Wesley, San Francisco, CA, 2007), pp 60-62.

[8] J.D. Fernie, Pub. Astron. Soc. Pac **81**, 707 (1969).

[9] iTelescope.Net Pty Ltd, "Telescope 21" 24 Feb 2014 http://www.itelescope.net/telescopet21/

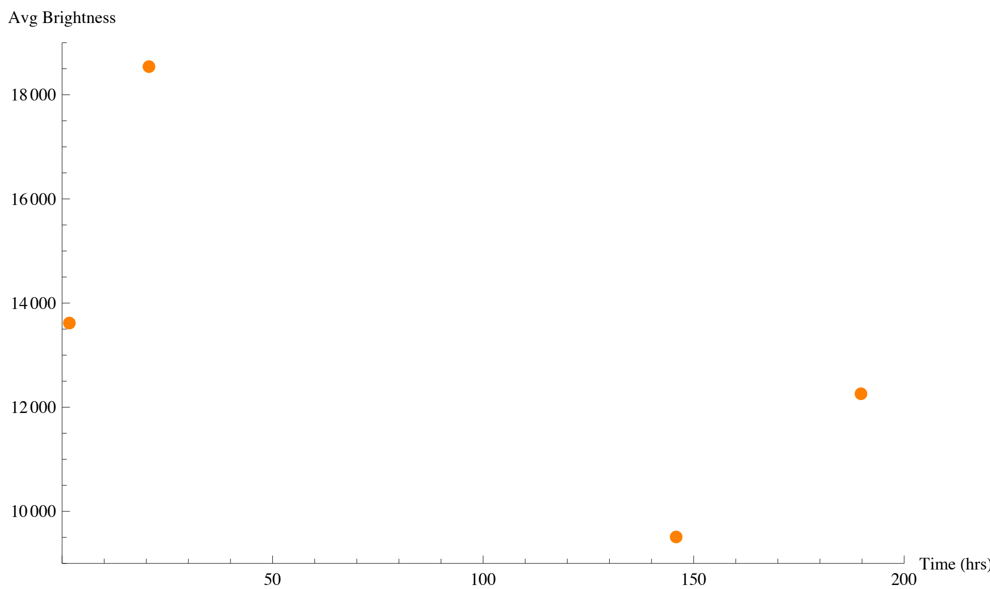
[10] N. R. Evans *et al.*, Astron. J. **136**, 1137 (2008).

Start by inputting the data collected from MaxIm DL, and a make plot of average brightness vs. time.

```
(*data is a 5-tuple with {Time (sec)/(sec/hr), Max, Min, Avg, St Dev}
   set 2/13 0:00:00 as time zero (hrs)*)
```

$$\text{data} = \left\{\left\{\frac{6238.}{3600}, 60381, 6689, 13616, 11233\right\}, \left\{\frac{74281.}{3600}, 60842, 6130, 18541, 14391\right\}, \left\{\frac{525063.}{3600}, 60322, 3708, 9508, 10388\right\}, \left\{\frac{683077.}{3600}, 60425, 1143, 12258, 14986\right\}\right\};$$

```
AvgPlot = ListPlot[Table[{data[[n, 1]], data[[n, 4]]}, {n, 1, 4}],
  PlotRange → {{0, 200}, {9000, 19000}}, AxesLabel →
   {"Time (hrs)", "Avg Brightness"}, PlotStyle → {Orange, PointSize[.015]}]
```



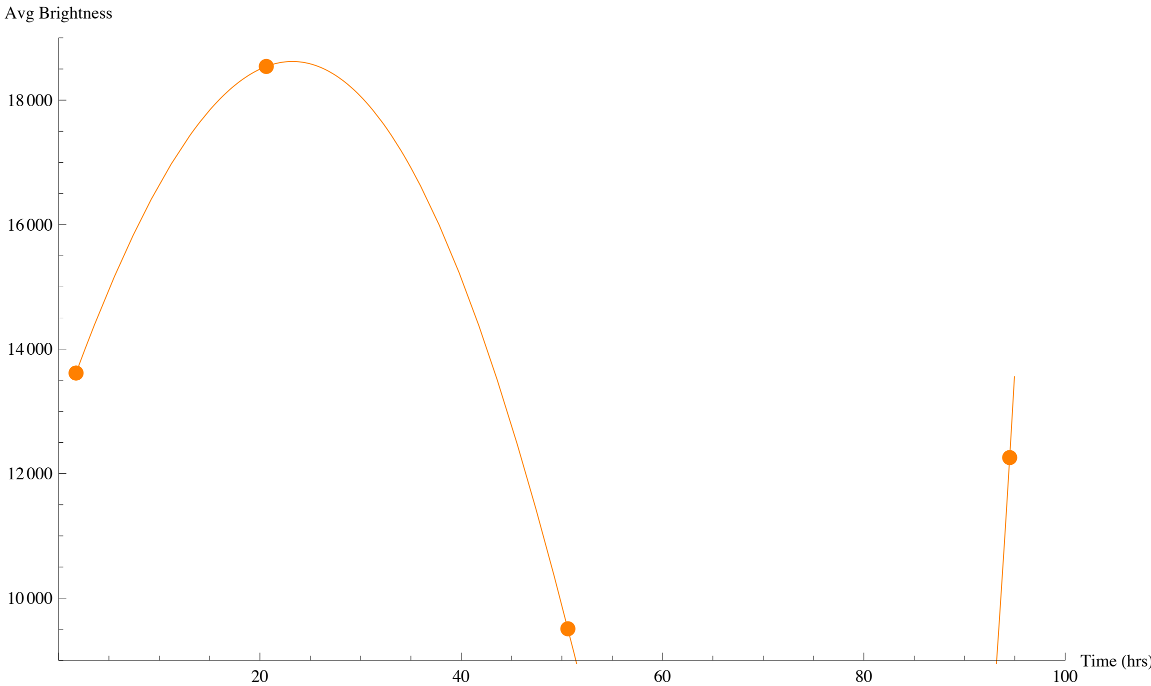
Unfortunately our data is from two separate periods. The first two data points from one cycle, and the last two from the second cycle. We can cheat a little and try to combine the two cycles into one cycle by subtracting the total period (the accepted value) of one cycle from the time of the last two data points. Since we are now looking at data points from one period, we can also include a best fit line. (We couldn't do a fit before because *Mathematica* assumed the data points all made up one period, instead of two. By crunching the data into one period, we circumvent this issue.)

```
(*dataMinusPeriod is the same 5-tuple, but this time,
a the period of one pulsation has been subtracted ONLY from the
 last two data points, so as to mimic the effect of one cycle. T =
 (3.969 days)(24 hrs/day)= 95.3299 hrs.*)
```

$$T = 24 * (3.969);\ \text{dataMinusPeriod} = \text{data} = \left\{\left\{\frac{6238.}{3600}, 60381, 6689, 13616, 11233\right\}, \left\{\frac{74281.}{3600}, 60842, 6130, 18541, 14391\right\}, \left\{\frac{525063.}{3600} - T, 60322, 3708, 9508, 10388\right\}, \left\{\frac{683077.}{3600} - T, 60425, 1143, 12258, 14986\right\}\right\};$$

```
(*fit finds a line of best fit for the the four plotted data points. *)
```

```
fit = Fit[Table[{dataMinusPeriod[[n, 1]], dataMinusPeriod[[n, 4]]}, {n, 1, 4}],
   {1, x, x^2, x^3, x^4, x^5, x^6, x^7}, x];
MaxPlotMinusPeriod = Show[ListPlot[
   Table[{dataMinusPeriod[[n, 1]], dataMinusPeriod[[n, 4]]}, {n, 1, 4}],
   PlotRange → {{0, 100}, {9000, 19 000}},
   AxesLabel → {"Time (hrs)", "Avg Brightness"},
   PlotStyle → {Orange, PointSize[.015]}],
  Plot[fit, {x, 1.73, 94.96}, PlotStyle → {Orange}]]
```




```
(*Now we want to look at the period of our periodic
 function. The first measurement for the maximum brightness was
 13616. When does our fit repeat these values? Lets find out:*)

Solve[fit == 13 616]
```

```
{{x → -102.69 - 55.8451 i}, {x → -102.69 + 55.8451 i}, {x → -4.46031 - 117.016 i},
 {x → -4.46031 + 117.016 i}, {x → 1.73278}, {x → 43.3536}, {x → 94.9816}}
```

Paying attention only to the real solutions for x, we find the first real solution to be our first data point. The second solution is about half the period. We care about the third solution, which says that T_{exp} = 94.91-1.73 hrs = 93.17 hrs, which is very close to the accepted value. To convert these values to days, and find the percent error:

$$T_{exp} = \frac{94.90646582863866\grave{} - 1.7327777777777698\grave{}}{24}$$

3.88224

$$T_{error} = \frac{\frac{T}{24} - T_{exp}}{\frac{T}{24}}$$

0.0218602

The period-luminosity equation gives us the apparent magnitude from the pulsation period.

```
M_V[P_] := -2.81 Log[10, P] - 1.43;
M_exp = M_V[T_exp]
```

```
-3.08532
```

We are left with an apparent magnitude of M_{exp}= -3.09.

To estimate the distance from Polaris, we will use our value M_{exp} and a value of apparent magnitude m taken from a reliable source.

```
d_exp[m_, M_] := (10) * 10^(.2 (m-M));
d_exp[1.98, M_exp]
```

```
103.054
```

```
3.26163344 * d_exp[1.98, M_exp]
```

```
336.124
```

This estimate says that Polaris is about $d_{\text{exp}} = 103.1$ pc away from Earth (in light years, d_{exp} = 336.1 ly). The measurements vary from 113 pc to 99 pc, but it is safe to say that our value came pretty close.