

# Cathode Ray Deflection

Miguel Conner and Neal Wool

*Department of Physics, Reed College, Portland, Oregon, 97202, USA*

(Dated: January 29, 2014)

A value for the charge to mass ratio of an electron was determined by measuring the vertical displacement of a cathode ray when subjected to a controlled magnetic field. The experimentally calculated result of  $(\frac{e}{m})_{exp} = (1.3 \pm .6) \times 10^{11} \frac{C}{kg}$  for the charge to mass ratio of the electron contains within its error bars the accepted value for this constant,  $\frac{e}{m} = 1.76 \times 10^{11} \frac{C}{kg}$ .

## I. INTRODUCTION

“What are these particles? are they atoms, or molecules, or matter in a still finer state of subdivision?” After analyzing the data from his cathode ray deflection experiment, Joseph John Thomson was left in a state of confusion. Thomson concluded that cathode rays were not “the ordinary chemical atoms, but ... primordial atoms, which we shall for brevity call corpuscles” [1]. Though the term corpuscles is no longer used, Thomson’s discovery of what is now called the electron revolutionized fundamental aspects of Physics and Chemistry. He showed that the atom was no longer the smallest indivisible particle it was believed to be, and his discovery won him the Nobel Prize in 1906 [2].

This experiment was based off of J. J. Thomson’s original cathode ray experiment, and uses the same basic setup albeit with improved technology and methods. Two crucial factors distinguished Thomsons experiment from similar attempts by other researchers of the time; the first was Thomson’s use of an evacuated tube that allowed electrons to be easily affected by the electric and magnetic fields, and the second was Thomson’s use of higher voltages. In Thomson’s original experiment, electrons are fired through an electron gun, two Helmholtz coils deflect the electrons, and this value is recorded [3]. By adjusting the current and the voltage, the electron path will change, and the value for  $\frac{e}{m}$  can be determined.

## II. EXPERIMENTAL DESIGN AND THEORY

A summary of the experimental setup will provide the backdrop necessary to concisely explain the theory and equataions behind this investigation. The experimental setup is outlined in Fig. (1).

Just as in J.J. Thomson’s experiment, measurement of the vertical displacement of a cathode ray (or stream of electrons) subject to a known magnetic and electric field can yield the charge to mass ratio of an electron. A 10 V DC power supply is routed through a current switch, and through both Helmholtz coils in series. Electrons are fired through an electron gun and then deflected by two Helmholtz coils. Increasing the value of the current affects the magnetic field inside the cathode ray tube, and increases the vertical displacement of the path of the cathode ray. Increasing the voltage affects the filament

(electron gun) and the electric field, and it therefore increases the intensity and the vertical displacement of the beam. The experiment was setup with a current switch, so that the direction of the magnetic field could be flipped in order to account for any deflection due to Earths magnetic field. An ammeter connected to this power supply in series measures the current flowing through the coils.

The high power voltage supply takes care of the supplying the adequate potential difference needed to accelerate the cathode rays and to produce an electric field. The electron gun fires a stream of electrons into a transparent, evacuated chamber known as the cathode ray tube. When current is flowing through the coils and there exist a voltage across the electron gun and the cathode ray tube, the cathode ray manifests itself as a visible light blue curve. The vertical displacement ( $d_-$  or  $d_+$ ) is measured at a distance of about 10 cm from the end of the electron gun ( $l = 10$  cm).

The end goal of the investigation is to produce a measurement for the charge to mass ratio of the electron, so it is fitting to begin with the equation that describes this parameter. This equation is given by

$$\frac{e}{m_e} = \frac{2V}{B^2 r^2}, \quad (1)$$

where  $e$  is the charge of the electron,  $m_e$  is it’s mass,  $V$  is the potential difference accross the electron gun,  $B$  is the magnetic field, and  $r$  is the radius of the curved path of the electron [4]. The value of  $V$  is the reading on the high voltage power supply. The other relevant component to Eq (1) is the strength of the magnetic field  $B$  due to two Helmholtz coils. The equation for this magnetic field is

$$B = \frac{8\mu_0 I N}{5\sqrt{5}R}, \quad (2)$$

where  $I$  is the constant current,  $N$  is the number of turns in each coil, and  $R$  is the radius of the Helmholtz coils [4]. The constant current  $I$  is measured with an ammeter, connected in series, between the switch and the 10 V DC power supply.

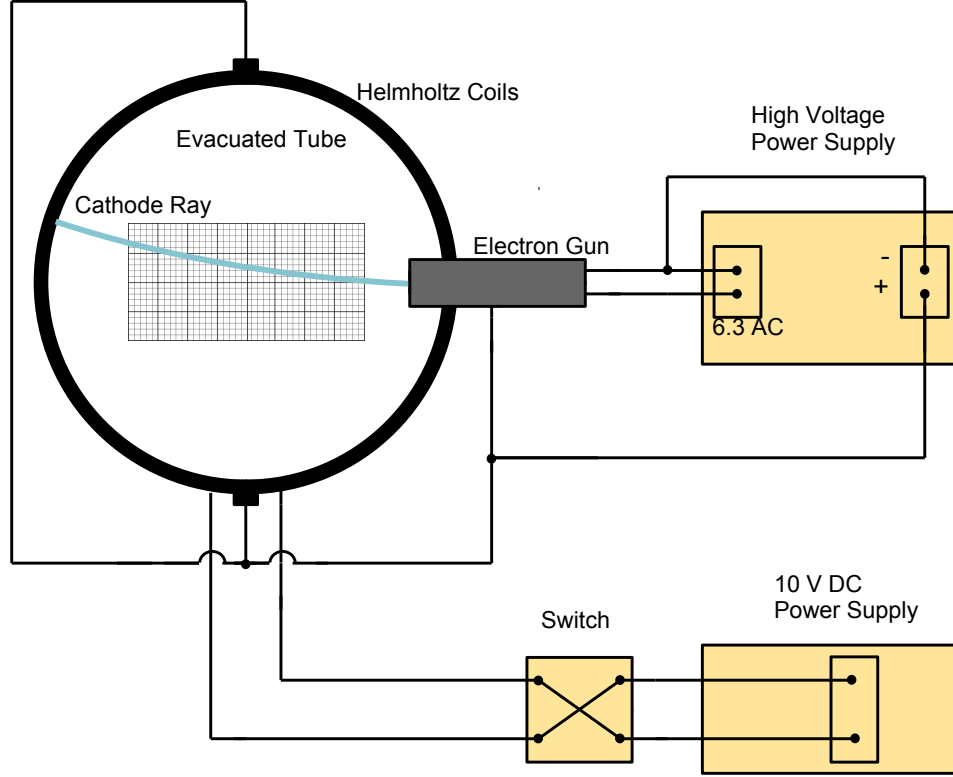


FIG. 1: A diagram of the experimental setup. The high-voltage power supply box controls the electron gun and the electric field, while the 10 V power supply controls the magnetic field.

Combining Eq. (1) and Eq. (2), the ratio between the charge of an electron  $e$  and its mass  $m_e$  is given by

$$\frac{e}{m_e} = \frac{250R^2V}{64\mu_0N^2\bar{r}^2I^2}, \quad (3)$$

where the term  $\bar{r}$  is a term that represents an average of measurements for the radius of the electron. This term is defined as

$$\bar{r} = \frac{2(r_+r_-)}{r_+ + r_-}, \quad (4)$$

where

$$r_+ = \frac{d_+^2 + l^2}{2d_+} \quad r_- = \frac{d_-^2 + l^2}{2d_-} \quad (5)$$

can be computed from measurements made in the experiment. The values  $d_-$  and  $d_+$  are the positive and negative displacements respectively of the cathode ray, and  $l$  is the distance traveled by the electron in the  $x$  direction at the point of height measurement. Now that the basics behind the experiment have been mentioned, it is key to point out a few minor details. When performing this experiment, the force due to the Earth's electric field was an uncontrolled factor that affected the electron's path.

This problem was dealt with through use of the current switch. The solution involves measuring the displacement with the Earth's magnetic field and the magnetic field produced by the coils in the *same* directions ( $d_+$ ), and the displacement with the direction of the field produced by the coils and the field produced by the Earth in *opposite* directions ( $d_-$ ). The result is an averaged value that can be found using Eq. (4). It should also be noted that the experiment was set up so that the cathode ray is traveling in a direction perpendicular to the direction of Earth's magnetic field. The cathode ray was traveling West or East, so that the effect on the electron due to the Earth's magnetic field was strictly vertical.

### III. DATA ANALYSIS

The data for this experiment consists of displacement measurements of the cathode ray at different voltages and currents. Data for four different voltage values, and six different current values, with the associated measurements for positive and negative deflection values, are shown in Tables I through IV.

The data points from Tables I-IV were used to calculate a value  $s$ , which is a part of the equation for the charge to mass ratio of the electron. All of the statistical errors

TABLE I: Current readings, positive deflection values ( $d_+$ ), and negative deflection values ( $d_-$ ) at 4000V. Error in Current: ( $\pm 0.02mA$ ); error in ( $d_+$ ) and ( $d_-$ ): ( $\pm 0.175cm$ )

Trial	Current (A)	$d_+$ (m)	$d_-$ (m)
1	0.2391	0.024	0.0225
2	0.2016	0.02	0.019
3	0.1628	0.016	0.015
4	0.1208	0.012	0.0105
5	0.0781	0.008	0.0065
6	0.0367	0.004	0.0025

TABLE II: Current readings, positive deflection values ( $d_+$ ), and negative deflection values ( $d_-$ ) at 4500V. Error in Current: ( $\pm 0.02mA$ ); error in ( $d_+$ ) and ( $d_-$ ): ( $\pm 0.175cm$ )

Trial	Current (A)	$d_+$ (m)	$d_-$ (m)
1	0.255	0.024	0.0225
2	0.2157	0.02	0.0185
3	0.172	0.016	0.0145
4	0.1275	0.012	0.01
5	0.0835	0.008	0.007
6	0.038	0.004	0.0025

are combined in  $s$ , so by taking the weighted average we eliminate as much error as possible.

The value of  $s$  is given by:

$$s = \frac{2V}{\bar{r}^2 I^2} \quad (6)$$

where  $\bar{r}$  is given by Eq. (4) and Eq. (5). After finding the  $s$  values for each voltage, it was also necessary to find the error in  $s$ .

The value of  $s$  is only a simple way to keep track of the changing statistical errors in the data, and makes the calculation of  $\frac{e}{m}$  easier. The computed value of  $s$  relates to the equation for the charge and mass ratio of the electron in the following way:

$$\frac{e}{m} = \frac{125R^2}{64\mu_0 N^2} s \quad (7)$$

where  $R$  is the radius of the coils (a value that has already been measured and stays constant) and  $N$  is the number of turns in the coil (which has also been measured and also stays constant). The weighted average  $\langle s \rangle_w$  shall be take the place of  $s$  in Eq. (7).

The value for  $\Delta s$  is calculated using the values of  $\Delta V$ ,  $\Delta I$ , and  $\Delta d$ . The error in the voltage in is based off of the spacing of the tick marks in the voltage generator, 50 V. The error the measurement of the current,  $\Delta I$ , is all based off of fluctuations from the ammeter, about 0.02 mA. Finally, the error in  $d$  was based of off the 0.1 cm tick mark spacing and the width of the beam divided by two. In total, this gives  $d$  an error of 0.175 cm. These

TABLE III: Current readings, positive deflection values ( $d_+$ ), and negative deflection values ( $d_-$ ) at 5000V. Error in Current: ( $\pm 0.02mA$ ); error in ( $d_+$ ) and ( $d_-$ ): ( $\pm 0.175cm$ )

Trial	Current (A)	$d_+$ (m)	$d_-$ (m)
1	0.2694	0.024	0.0225
2	0.2261	0.02	0.018
3	0.18	0.016	0.0145
4	0.1335	0.012	0.01
5	0.0881	0.008	0.0065
6	0.0381	0.004	0.002

TABLE IV: Current readings, positive deflection values ( $d_+$ ), and negative deflection values ( $d_-$ ) at 5500V. Error in Current: ( $\pm 0.02mA$ ); error in ( $d_+$ ) and ( $d_-$ ): ( $\pm 0.175cm$ )

Trial	Current (A)	$d_+$ (m)	$d_-$ (m)
1	0.2382	0.024	0.022
2	0.2386	0.02	0.018
3	0.1898	0.016	0.0145
4	0.1391	0.012	0.01
5	0.0925	0.008	0.0065
6	0.0407	0.004	0.002

error were combined by means of error propagation to give  $\Delta s$ .

The data yields a weighted value for  $s$  of:

$$\langle s \rangle_w = (2.3 \pm 0.1) \times 10^6 \frac{V^2}{m^2 A^2}$$

With  $s$  computed, it is now appropriate to find  $\frac{e}{m}$ . In Eq. (7) there are three variables,  $R$ ,  $N$  and  $s$ , so  $\Delta \frac{e}{m}$  will be based of  $\Delta R$ ,  $\Delta l$ , and  $\Delta s$  (see Appendix).  $R$  is the radius of the coils, as calculated by averaging the inner radius and outer radius of the coils.  $\Delta R$  was given a value of 2 cm because of the error involved in assuming the averaged value of the coils is the true radius. The error in  $l$  should also be considered, though it plays a less significant role in the course of the error calculation.  $l$  is the 10 cm horizontal distance the cathode ray travels from the point of exit form the electron gun to the point of vertical measurement. The error in  $l$  was found to be 0.1 cm. The weighted value  $\langle s \rangle_w$  shall be used for  $s$  and  $\Delta s$ .

Using Eq. (7), the experimentally determined value for the charge to mass ratio of the electron was found to be:

$$\left( \frac{e}{m} \right)_{exp} = (1.3 \pm .6) \times 10^{11} \frac{C}{kg}$$

and is represented by the solid brown line, with an error margin represented by the beige shaded region in Fig. (2). The accepted value of  $\frac{e}{m} = 1.76 \times 10^{11} \frac{C}{kg}$  lies inside the boundaries of the error rectangle, and is represented by the dashed black line.

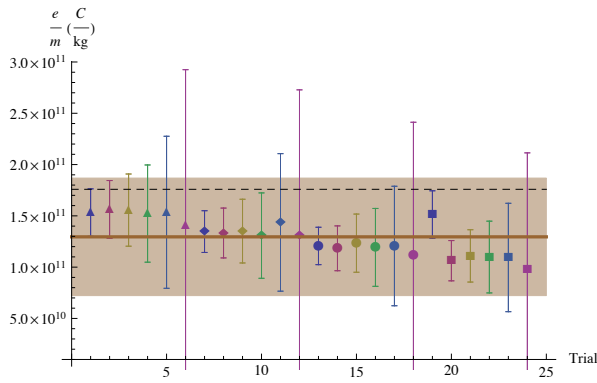


FIG. 2: A figure showing the experimental value (brown line), its associated errors (shaded region), and the accepted value (black dashed line) for the ratio between the charge of an electron and its mass. The data points represent the original data after calculations (calculated using Eq. (3)). Each data point shape corresponds to a specific voltage; triangle is 4000 V, diamond is 4500 V, circle is 5000 V, and square is 5500 V.

The experimental error is significant in this investigation, and can be attributed a few factors. The largest source of error in this investigation was  $\Delta R$ , the error associated with measuring the radius of the coil. This error was large because assigning the radius a single value implies that the effects of the width of the coil are being ignored. The values for the outer radius of  $R$  ( $R_O = 7.5$  cm) and the inner radius ( $R_I = 6.15$  cm) were averaged to give a value of  $R = 6.8$  cm. The other major portion of the error associated in this experiment comes from  $\Delta s$ , which is based on errors in voltage, current, and measurement of  $d$ . (A side by side comparison between the magnitude of error due to  $R$  and the magnitude of the error due to  $s$  is shown in the Appendix). Finally, it is also important to note that measurements for  $d$  are on the order of tenths of a centimeter, and can often be hard to determine with accuracy, especially when the measurements themselves are of just a few centimeters.

## IV. CONCLUSION

The charge to mass ratio of the electron was found to be:

$$\left(\frac{e}{m}\right)_{exp} = (1.3 \pm .6) \times 10^{11} \frac{\text{C}}{\text{kg}}$$

so that the accepted value of  $\frac{e}{m} = 1.76 \times 10^{11} \frac{\text{C}}{\text{kg}}$  lies within the error bars of the experimentally determined result. The most significant source of error in this experiment comes from the approximation of the radii of the coils,  $\Delta R$ . This error could not be decreased without significantly complicating the data analysis; the single measurement of the radius meant that calculating the strength of the magnetic field was a matter of inputting a simple value into the equation rather than integrating across the length of the coil. Even the data points with a large error were used when computing the weighted average, because the weighted average already takes care of assigning less importance to results with larger error.

## Appendix

The statistical error was calculated by means of error propagation, which combined  $\Delta R$ ,  $\Delta s$ , and  $\Delta l$ . This looks like

$$\Delta \frac{e}{m} = \sqrt{\left(\frac{\partial \frac{e}{m}}{\partial R} \Delta R\right)^2 + \left(\frac{\partial \frac{e}{m}}{\partial s} \Delta s\right)^2 + \left(\frac{\partial \frac{e}{m}}{\partial l} \Delta l\right)^2}. \quad (1)$$

However, the term  $\frac{\partial \frac{e}{m}}{\partial l}$  is quite difficult to compute. Instead, the  $\Delta l$  term was computed by plugging in the values of  $l + \Delta l$  and  $l - \Delta l$  to see how they affected to overall value of  $\frac{e}{m}$ . This error was then added in quadrature along with the other two errors, and was found to be much less significant than the other two errors. With the error and data taken into account, the errors due to each term are:

$$\begin{aligned} \left(\frac{\partial \frac{e}{m}}{\partial R} \Delta R\right)^2 &= 3.2 \times 10^{21} \\ \left(\frac{\partial \frac{e}{m}}{\partial s} \Delta s\right)^2 &= 4.0 \times 10^{19} \\ \left(\frac{\partial \frac{e}{m}}{\partial l} \Delta l\right)^2 &= 10000, \end{aligned}$$

so the  $\Delta R$  term clearly affects the error the most, while the  $\Delta l$  term is negligible.

- 
- [1] J. J. Thomson, "On the Cathode Rays," Proceedings of the Cambridge Philosophical Society **9**, 243 (1897).  
[2] J. J. Thomson, "Nobel Lecture: Carriers of Negative Electricity". Nobelprize.org. 21 Apr 2013 [http://www.nobelprize.org/nobel\\_prizes/physics/laureates/1906/thomson-lecture.html](http://www.nobelprize.org/nobel_prizes/physics/laureates/1906/thomson-lecture.html)

- [3] J. J. Thomson, "Cathode Rays," The Electrician **39**, 104 (1897).  
[4] D. C. Giancoli, *Physics for Scientists & Engineers*, 4th ed. (Prentice Hall, Upper Saddle River, NJ, 2008), pp. 721–723, 744–745.