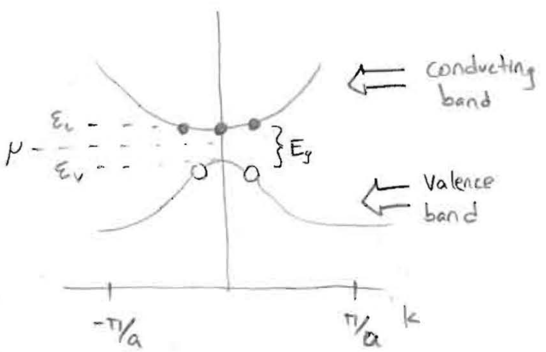
EXTRINSIC SEMICONDUCTORS

Last Time



Showed that for

$$\varepsilon_c - \mu \gg k_B T \text{ and}$$
$$\mu - \varepsilon_v \gg k_B T$$

density of e^- in cond band is:

$$n(T) = e^{-(\varepsilon_c-\mu)/k_BT} N(T)$$

density of holes in valence band:

$$p(T) = e^{-(\mu-\varepsilon_v)/k_BT} P(T)$$

and

$$n(T)\,p(T) = N(T)P(T)e^{-E_g/k_BT}$$

Intrinsic Semiconductor

means no doping and

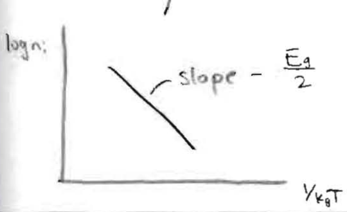
$$n = p \equiv n$$

and

$$n_i = \sqrt{N(T)P(T)}\, e^{-E_g/2k_BT}$$

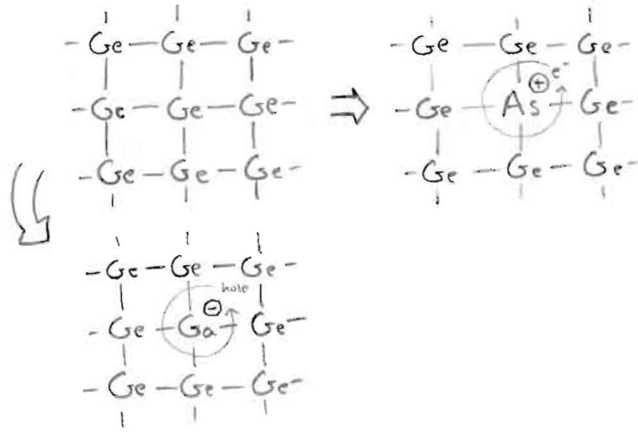
and

$$\mu_i = \frac{\varepsilon_c+\varepsilon_v}{2} + \frac{3}{4}k_BT\ln\left(\frac{m_v^*}{m_c^*}\right)$$

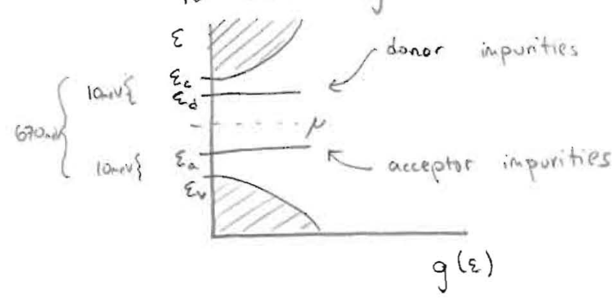


Doping

Substituting one atom in semiconductor with neighboring atom in periodic table



bound energy states are very close to band edges



$P_{docc} = \frac{1}{e^{(\varepsilon_d-\mu)/k_BT}+1}$

If $\varepsilon_d - \mu \gg k_BT$ and $\mu - \varepsilon_a \gg k_BT$, then prob of an impurity level being occupied by an electron / hole is negligibly small.

Analysis changes from intrinsic case only in

$$n - p = N_d - N_a$$

density of donor impurities; density of acceptor impurities

from law of mass action

$$np = n_i^2$$
$$n(n-(N_d-N_a)) = n_i^2$$

$$\boxed{n = \frac{N_d-N_a}{2} + \sqrt{\left(\frac{N_d-N_a}{2}\right)^2 + n_i}}$$

$$p = n - (N_d - N_a)$$

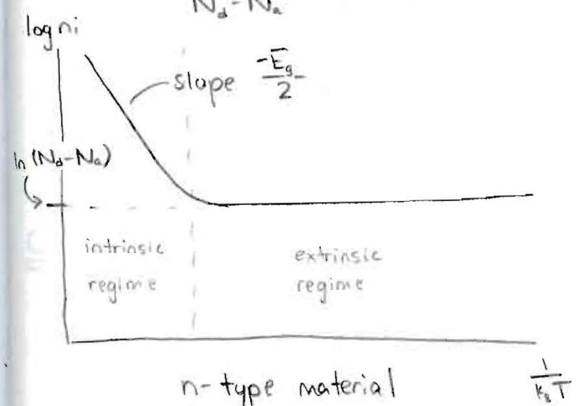
$$\boxed{p = -\frac{N_d + N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}}$$

if $N_d - N_a \ll n_i$, this is a small change to n, however if $N_d - N_a \gg n_i$...

Assume $N_d > N_a$ (n-type material) then

$$n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\sqrt{1 + \left(\frac{2n_i}{N_d - N_a}\right)^2} \approx N_d - N_a$$

$$p = -\frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\left(1 + \frac{1}{2}\left(\frac{2n_i}{N_d - N_a}\right)^2 + \dots\right) \approx \frac{n_i}{N_d - N_a} n_i \ll n_i$$



n-type material

Chemical Potential

note in general

$$n = e^{-(\varepsilon_c - \mu)/k_B T} N(T) = e^{+(\mu - \mu_i)/k_B T} \underbrace{e^{-(\varepsilon_c - \mu_i)/k_B T} N(T)}_{n_i} = n_i e^{(\mu - \mu_i)/k_B T}$$

$$p = n_i e^{-(\mu - \mu_i)/k_B T}$$

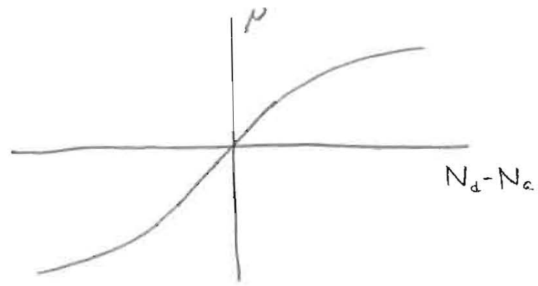
and

$$n - p = N_d - N_a$$

$$n_i \underbrace{\left[e^{(\mu - \mu_i)/k_B T} - e^{-(\mu - \mu_i)/k_B T}\right]}_{2\sinh\left[\frac{\mu - \mu_i}{k_B T}\right]} = N_d - N_a$$

and

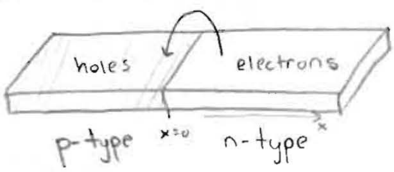
$$\boxed{\mu = \mu_i + k_B T \sinh^{-1}\left[\frac{N_d - N_a}{2n_i}\right]}$$



pn Junction

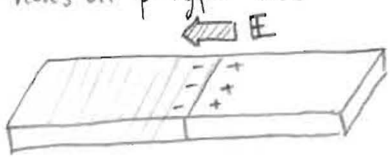
Inhomogenous Semiconductor

change doping as funct of position



	p-type	n-type
impurities:	N_a	N_d
	$N_d \approx 0$	$N_a \approx 0$
extrinsic region:	$p \approx N_a$	$p \approx 0$
	$n \approx 0$	$n \approx N_d$

electrons on the n-type side will annihilate holes on p-type side



sets up electric field $\mathbb{E}$ that stops flow of any more charge

Calculate Size of Depletion Region

on n-type side, normally

$$\underbrace{n}_{\text{density of cond. electrons}} = e^{-(\varepsilon_c - \mu)/k_BT} N(T)$$

but we need to include the energy from our electric field

$$n = e^{-(\varepsilon_c \underbrace{- e\phi(x)}_{\text{electrostatic energy}} - \mu)/k_BT} N(T)$$

write this as

$$n = N(T) \frac{e^{(e\phi(x) - e\phi(\infty))/k_BT}}{e^{(\varepsilon_c - e\phi(\infty) - \mu)/k_BT}}$$

$$= e^{(e\phi(x) - e\phi(\infty))/k_BT} N_d$$

similarly

$$p = e^{(e\phi(-\infty) - e\phi(x))/k_BT} N_a$$

recall the law of mass action

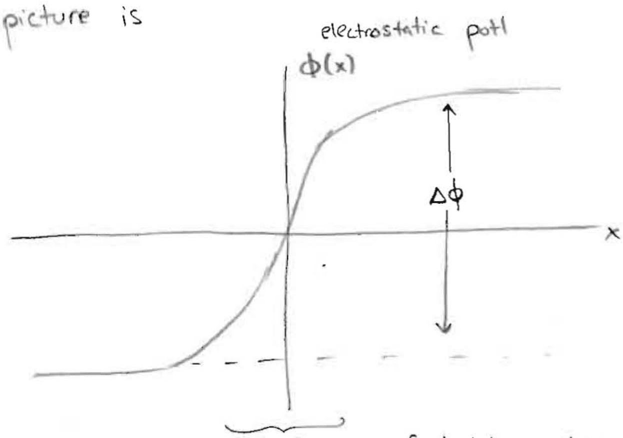
$$np = e^{-E_g/k_BT} N(T) P(T)$$

$$e^{-(e\phi(\infty) - e\phi(-\infty))/k_BT} N_d N_a = e^{-E_g/k_BT} N(T) P(T)$$

Let $\Delta\phi = \phi(\infty) - \phi(-\infty)$

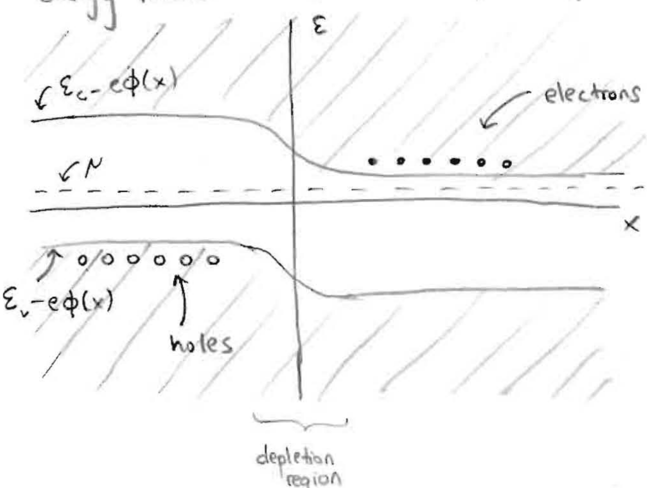
$$\boxed{e\Delta\phi = E_g + k_BT \ln\left(\frac{N_d N_a}{N(T)P(T)}\right)}$$

picture is



d? ~ size of depletion region

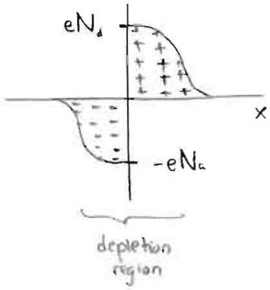
energy levels



to find d, we need to solve Laplace's equation

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{1}{\varepsilon}\left[\underbrace{eN_d - eN_a}_{\text{concent. of impurities (ionized)}} - en + ep\right]$$

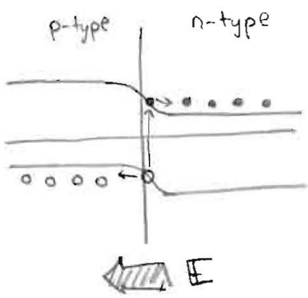
$$= -\frac{e}{\varepsilon}\left[N_d(x) - N_a(x) - e^{(\phi(x) - \phi(\infty))/k_BT} N_d(\infty) - e^{(\phi(-\infty) - \phi(x))/k_BT} N_a(-\infty)\right]$$



essentially, all e^- and holes gone from this region leaving ionized impurities

$$d = \sqrt{\frac{2\varepsilon\Delta\phi}{e}\left(\frac{1}{N_d}+\frac{1}{N_a}\right)}$$

Solar Cell

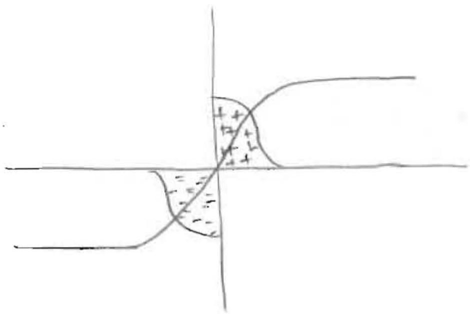


generates a current

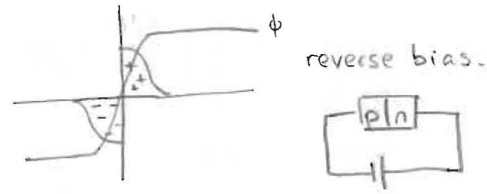
$\xleftarrow{\quad j \quad}$

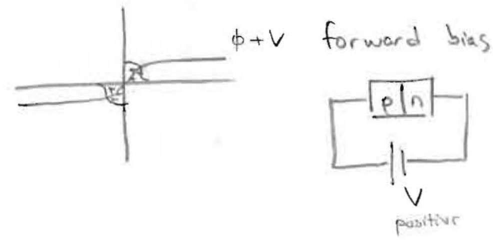
CURRENT

pn Junction

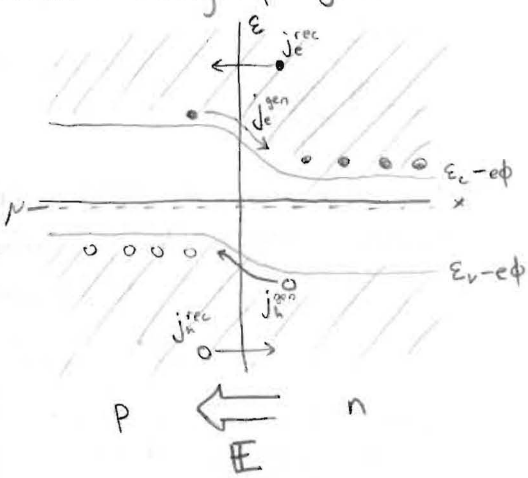


p | n
at equilibrium





Current through pn junction



there are four currents (two types)

- <u>generation currents</u> the density of minority carriers is, on p side,

$$n = \frac{n_i^2}{N_a} = e^{-E_g/k_BT} N(T) P(T) \frac{1}{N_a} \propto e^{-E_g/k_BT}$$

no dep on applied voltage

- <u>recombination currents</u> the density of thermally-excited majority carriers is, on n side

$$n_{therm} \cong e^{-(\varepsilon_c - e\phi(\infty) - eV)/k_BT} N$$

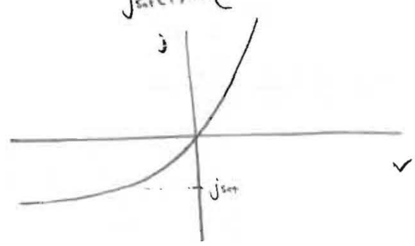
$\approx E_g$

$$\propto e^{-e\Delta\phi/k_BT} e^{+eV/k_BT} = e^{-E_g/k_BT} e^{eV/k_BT}$$

the total current is

$$j = j_e^{gen} + j_h^{gen} + j_e^{rec} + j_h^{rec} = j_{sat}(T)\left(e^{eV/k_BT} - 1\right)$$

$$j_{sat}(T) \propto e^{-E_g/k_BT}$$



MAGNETISM

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electrons have two types of magnetic moments

Spin

$$\mathbb{S} = \hbar \sigma$$

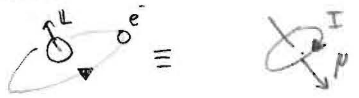
$$-e \equiv \mu = \overset{\approx 2}{g} \frac{e}{2m} \mathbb{S} = g \underbrace{\frac{e\hbar}{2m}}_{\text{Bohr magneton } \mu_B} \sigma$$

so from electron spin with

$$\mathbb{S} = \pm \frac{\hbar}{2}$$

$$\mu = \pm \mu_B$$

orbital



where

$$\ell = \frac{1}{\hbar} \mathbb{L}$$

$$\mu_o = \frac{e}{2m} \mathbb{L} \equiv \frac{e\hbar}{2m} \ell = \mu_B \ell$$

by magnetism, we mean the ordering of these magnetic moments

Two Classes

- para- and diamagnetism, where moments order due to ext. field
- ferro- and antiferromagnetism where moments spontaneously order

Classical Electrodynamics

Define

$\mathbb{M} \equiv$ magnetization = dipole moment per unit volume

$\mathbb{H} \equiv$ auxiliary field $= \frac{1}{\mu_0} \mathbb{B} - \mathbb{M}$

where $\mu_0 \mathbb{H}$ is "external" field

Define magnetic susceptibility

$$\boxed{\chi = \frac{\partial M}{\partial H} \approx \mu_0 \frac{\partial M}{\partial B}}$$

for dia, paramagnetic

Linear Materials

$$\mathbb{M} = \chi \mathbb{H}$$

independent of $\mathbb{H}$ in linear material

define

$\chi > 0$ paramagnet

$\chi < 0$ diamagnet

Quantum Mechanics

Hamiltonian of an e^- in a magnetic field is

vec pot'l, scalar pot

$$\mathcal{H} = \frac{1}{2m} (\mathbb{p} + e\mathbb{A})^2 - e\phi + \frac{ge}{2m} \mathbb{B} \cdot \mathbb{S}$$

If $\mathbb{B}$ is uniform, $\mathbb{B} = B_0 \hat{z}$, then

$$\mathcal{H} = \mathcal{H}_0 + \mu_B \mathbb{B} \cdot (\ell + g\sigma)$$

terms indep of $\mathbb{B}$

$$+ \frac{e^2}{2m} \frac{1}{4} |\mathbb{B} \times \mathbb{r}|^2$$

using gauge where

$$\mathbb{A} = \frac{1}{2} \mathbb{B} \times \mathbb{r}$$

$$\Downarrow$$

$$\nabla \times \mathbb{A} = \mathbb{B}$$

Paramagnetism

Last Time

e^- have two sources of magnetic moments

$$\mu_s = -g\frac{e}{2m}\mathbb{S} = -g\mu_B\sigma$$

$$\mu_o \cong -\frac{e}{2m}\mathbb{L} = -\mu_B \ell$$

Magnetization

$$\mathbb{M} = \frac{\text{dip moment}}{\text{volume}}$$

and

$$\chi = \frac{\partial M}{\partial H} \approx \mu_o \frac{\partial M}{\partial B}$$

Quantum Mechanics

$$\mathcal{H} = \mathcal{H}_o + \underbrace{\mu_B \mathbb{B}\cdot(g\sigma + \ell)} + \frac{e^2}{8m}|\mathbb{r}\times\mathbb{B}|^2$$

energy is lower if σ and ℓ are antialigned with $\mathbb{B}$ meaning μ_s and μ_o tend to align with $\mathbb{B}$

Statistical Mechanics

the magnetization for system in n^{th} eigenstate is

$$\boxed{M_n = \frac{-1}{V}\frac{\partial E_n(B)}{\partial B}}$$

and for system at temp T

$$M = \frac{1}{Z}\sum_n{}' M_n e^{-E_n/k_BT}$$

$$= \frac{1}{Z}\left(+\frac{1}{V}\right)k_BT\frac{\partial}{\partial B}\sum_n^{\infty}{}' e^{-E_n/k_BT}$$

$$= \frac{1}{V}\frac{\partial}{\partial B}\underbrace{k_BT\ln\left(\sum_n{}' e^{-E_n/k_BT}\right)}_{\equiv -F}$$

so that mag at temp T is

$$M = -\frac{1}{V}\frac{\partial F}{\partial B}$$

where

$$e^{-F/k_BT} = \sum_n{}' e^{-E_n/k_BT}$$

Curie's Law

consider single spin ½ particle and ignore orbital DOF

$$\mathcal{H} = g\mu_B\mathbb{B}\cdot\sigma$$

$g\approx 2$, $\mathbb{B} = B_o\hat{z}$

$$= g\mu_B B_o \sigma_z \quad (\sigma_z = \pm ½)$$

$$= \begin{cases} \mu_B B_o & \text{up spin} \\ -\mu_B B_o & \text{down spin} \end{cases}$$

then

$$F = -k_BT\ln\left(e^{\mu_B B_o/k_BT} + e^{-\mu_B B_o/k_BT}\right)$$

$$= -k_BT\ln\left(2\cosh\left(\frac{\mu_B B_o}{k_BT}\right)\right)$$

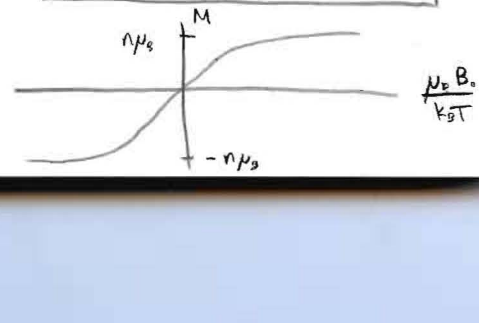
for N independent spin ½ particles

$$F = -Nk_BT\ln\left(2\cosh\left(\frac{\mu_B B_o}{k_BT}\right)\right)$$

the magnetization is

$$M = -\frac{1}{V}\frac{\partial F}{\partial B}$$

$$\boxed{M = n\frac{1}{\mu_B}\tanh\left(\frac{\mu_B B_o}{k_BT}\right)}$$



$$\chi \simeq \mu_0 \frac{\partial M}{\partial B_0} = n \frac{\mu_0 \mu_B^2}{k_B T} \operatorname{sech}^2\left(\frac{\mu_B B_0}{k_B T}\right)$$

at fields such that $\mu_B B_0 \ll k_B T$

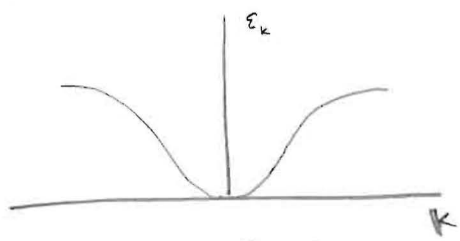
$$\boxed{\chi \simeq n \frac{\mu_0 \mu_B^2}{k_B T}}$$

Curie's law

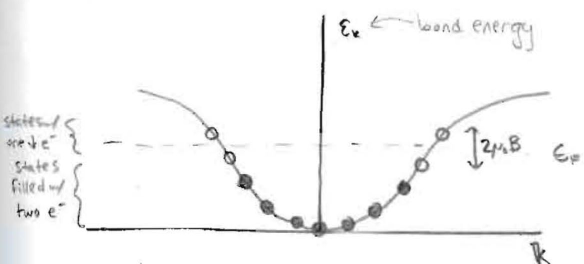
note

$$\frac{\mu_B}{k_B} = 0.67 \ \frac{\text{Kelvin}}{\text{Tesla}}$$

Pauli Paramagnetism



in absence of B, ground state has all orbitals filled up to ε_F with two e^-'s.



can then write

$$n_\downarrow = \frac{1}{2}\int_0^{\varepsilon_F + \mu_B B_0} g(\varepsilon_k)\, d\varepsilon_k \simeq \frac{1}{2}\int_0^{\varepsilon_F} g(\varepsilon_k)\, d\varepsilon_k - \frac{g(\varepsilon_F)}{2}\mu_B B_0$$

$$n_\uparrow = \frac{1}{2}\int_0^{\varepsilon_F - \mu_B B_0} g(\varepsilon_k)\, d\varepsilon_k \simeq \frac{1}{2}\int_0^{\varepsilon_F} g(\varepsilon_k)\, d\varepsilon_k - \frac{g(\varepsilon_F)}{2}\mu_B B_0$$

$$M = \underbrace{n_\downarrow}_{\text{density of down spins}} \underbrace{\mu_B}_{\text{moment of down spins}} + n_\uparrow(-\mu_B) = g(\varepsilon_F)\mu_B^2 B_0$$

so that

$$\chi_{pauli} = \mu_0 \frac{\partial M}{\partial B_0} = \mu_0 \mu_B^2 g(\varepsilon_F)$$

for free-e^- $g(\varepsilon_F)$

$$\boxed{\chi_{pauli} = \frac{3n}{2}\frac{\mu_0 \mu_B^2}{k_B T_F}} \simeq 10^{-6}$$