For this homework, include all code and computations in a MATLAB file named `math425hw5.m`. You will need to submit this file along with a document containing your answers which do not involve MATLAB. **Do not submit a zipped (compressed) folder**.

1. Find the closest point from $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ -2 \end{pmatrix}$ to the subspace spanned by

$$\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}.$$

Use any MATLAB command that you think is useful to do this computation.

2. Find the least squares solution to the system $A\mathbf{x} = \mathbf{b}$ in *two* ways using MATLAB where

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -2 & 3 \\ 1 & 5 & -1 \\ -3 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 0 \\ 5 \\ 6 \\ 8 \end{pmatrix}.$$

However, you are not allowed to use `b \ A`.

3. The median price (in thousands of dollars) of existing homes in the Minneapolis metropolitan area from 1989 to 1999 was:

year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
price	86.4	89.8	92.8	96.0	99.6	103.1	106.3	109.5	113.3	120.0	129.5

First find an equation of the least squares line for these data using MATLAB. Then use the result to estimate the median price of a house in the year 2005, and the year 2010, assuming the trend continues.

4. Let $f(x) = x^2$ on the interval $[0, 2\pi]$. In this exercise you will compute the discrete Fourier coefficients $c_0, c_1, \ldots, c_7$ of f from the sample vector

$$\mathbf{f} = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_7 \end{pmatrix}$$

where $f_j = f(j2\pi/8)$ for $j = 0, \ldots, 7$.

a) Compute the sample vector $\mathbf{f}$ in MATLAB.

b) Next compute the vectors $\omega_k = (\zeta_8^{0k}, \zeta_8^{1k}, \zeta_8^{2k}, \ldots, \zeta_8^{7k})^T$ for $k = 0, 1, \ldots, 7$ where $\zeta_8 = e^{i(2\pi/8)}$. In MATLAB the complex exponential can be computed by `exp(i*2*pi/8)`.

c) Now compute $c_k = \langle \mathbf{f}, \omega_k \rangle$ for $k = 0, \ldots, 7$. The complex inner product $\langle v, w \rangle$ is computed by `dot(v,w)` in MATLAB. **Correction: it appears that the complex inner product $\langle v, w \rangle = \sum_{i=1}^n v_i \overline{w_i}$ is not implemented as `dot(v,w)`. Strangely, you need to use `dot(w,v)`. Note the switch in the arguments. This is the only way to force complex conjugation on the coordinates of w.** However, do not forget to scale the result by 1/8 since we are using the scaled inner product for the discrete Fourier transform.

d) With the discrete Fourier coefficients you have computed, set $p(x) = c_0 \cdot 1 + c_1 e^{ix} + c_2 e^{i2x} + \cdots + c_7 e^{i7x}$ where $e^{ijx} = \cos(jx) + i\sin(jx)$. Write $p(x) = p_1(x) + ip_2(x)$ where $p_1(x)$ and $p_2(x)$ are real-valued functions. Compute $p_1(x)$. This is the function that reconstructs $f(x)$.
e) Find out how you can plot the graph of a function in `MATLAB`. Then plot the graphs of $f(x)$ and $p_1(x)$ on the interval $[0, 2\pi]$. What do you see?
f) Now we will compute a modified version of a reconstruction of the signal $f(x) = x^2$ on $[0, 2\pi]$. The discrete Fourier coefficients we will compute are $c_{-4}, c_{-3}, \ldots, c_3$. For this you will need ω_k for $k = -4, -3, \ldots, 3$. And $c_k = \langle \mathbf{f}, \omega_k \rangle$ for $k = -4, -3, \ldots, 3$. Again do not forget to scale the inner product by $1/8$.
g) Let $q(x) = c_{-4}e^{i(-4)x} + c_{-3}e^{i(-3)x} + \cdots + c_3 e^{i3x}$. Write $q(x) = q_1(x) + iq_2(x)$, and compute $q_1(x)$.
h) Plot the graph of $f(x)$ and $q_1(x)$ on the interval $[0, 2\pi]$. Now what do you observe?