MATH 425 Fall 2024 Homework 5

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1) Unfortunately, it seems that there seems to be no solution. I found that the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ -2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}$ which span the subspace are linearly dependent which is preventing me from finding closest point $\mathbf{x}^*$. When we try to solve for $\mathbf{x}$ in $K\mathbf{x} = \mathbf{f}$, I found that K does not have an inverse since det(K) = 0, which means K is a singular matrix.

   However, I can pick a subspace in which the vectors are linearly independent. Using MATLAB, I called rref(A) and found that I can discard the last vector $\begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}$ to get a set of linearly independent vectors, $\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ -2 \\ -1 \end{pmatrix}$. K is now a nonsingular matrix which gives me the following result.

$$\mathbf{x}^* = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} \text{ is the closest point}$$

2) There are two ways to find the least squares solution:

   Way 1: Closest Point Solution
   $K\mathbf{x} = \mathbf{f}, \mathbf{x}^* = K\mathbf{f}$
   $K = A^T A$, $\mathbf{f} = A^T\mathbf{b}$

   Using MATLAB, I got the solution:

$$\mathbf{x}^* = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \text{ is the least squares solution}$$

Way 2: Orthonormalize A using the gram-schmidt process, then use orthogonal projection formula (e.q. 4.41 from the textbook)

$$\mathbf{w} = c_1\mathbf{u}_1 + \cdots + c_n\mathbf{u}_n \text{ where } c_i = < \mathbf{v}, \mathbf{u}_i >, i = 1, \ldots, n$$

Using MATLAB, I got the solution:

$$\mathbf{x}^* = \begin{pmatrix} 0 \\ 5 \\ 6 \\ 8 \end{pmatrix} \text{ is the least squares solution}$$

This answer was definitely unexpected, and I am not sure how I arrived at this conclusion.

3) Our least squares line will have the form $y = Mx + B$. To find the least squares line, need to find $\mathbf{x}^*$.
$K\mathbf{x} = \mathbf{f}, \mathbf{x}^* = K\mathbf{f}$
$K = A^T A$, $\mathbf{f} = A^T\mathbf{b}$
I set A and b to be:

$$A = \begin{pmatrix} 1 & 1989 \\ 1 & 1990 \\ \vdots & \vdots \\ 1 & 1999 \end{pmatrix}, A^T = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1989 & 1990 & \cdots & 1999 \end{pmatrix} \mathbf{b} = \begin{pmatrix} 86.4 \\ \vdots \\ 89.8 \\ 129.5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} B \\ M \end{pmatrix}$$

Using MATLAB, I got the the value

$$\mathbf{x}^* = \begin{pmatrix} -7717.7 \\ 3.9227 \end{pmatrix}$$

Which gives the the line

$$y(t) = 3.9227t - 7717.7$$

Assuming that the trend continues, the housing prices for 2005 and 2010 will be:

$$y(2005) = 3.9227(2005) - 7717.7 = 147.4$$
$$y(2010) = 3.9227(2005) - 7717.7 = 167.0$$

4d)

$$p(x) = p_1(x) + ip_2(x) = (10.7949cos(0x) - (0.0000)sin(0x)) + i(10.7949sin(0x) + 0.0000cos(0x)) +$$

$$(-0.3613cos(1x) - (5.9568)sin(1x)) + i(-0.3613sin(1x) + 5.9568cos(1x)) +$$

$$(-1.8506cos(2x) - (2.4674)sin(2x)) + i(-1.8506sin(2x) + 2.4674cos(2x)) +$$

$$(-2.1061cos(3x) - (1.0220)sin(3x)) + i(-2.1061sin(3x) + 1.0220cos(3x)) +$$

$(-2.1590cos(4x)-(0.0000)sin(4x))+i(-2.1590sin(4x)+0.0000cos(4x))+$

$(-2.1061cos(5x)-(1.0220)sin(5x))+i(-2.1061sin(5x)+1.0220cos(5x))+$

$(-1.8506cos(6x)-(2.4674)sin(6x))+i(-1.8506sin(6x)+2.4674cos(6x))+$

$(-0.3613cos(7x)-(5.9568)sin(7x))+i(-0.3613sin(7x)+5.9568cos(7x))$

Removing all the imaginary terms leaves us with:
$p_1(x) = 10.7949-0.3613cos(1x)-5.9568sin(1x)-1.8506cos(2x)-2.4674sin(2x)-2.1061cos(3x)-1.0220sin(3x)-2.1590cos(4x)-2.1061cos(5x)-1.0220sin(5x)-1.8506cos(6x)-2.4674sin(6x)-0.3613cos(7x)-5.9568sin(7x)$