

CSC 520 Fall 2023
Midterm #2 – Complexity
125 points/10 extra credit

Submit a .zip archive containing 4 files.

- A .txt file containing only your honor code affirmation, replacing <name> with your actual name: **On my honor as an SFSU student, I, <name>, have neither given or received inappropriate help with this midterm exam.**
- A .py Python source code files containing your solutions to problems 1 & 2.
- A plain text .txt file containing your solution to problem 3.

1. 50 points/10 extra credit points for complete correct solution.

ConvertCliqueToHalfIndependentSet-to_complete.py, included with the test materials, is a template for the conversion procedure used in the polyreduction $\text{HasClique} \leq_p \text{HasIndependentSet}$ (problems defined below). Follow the instructions in the source code to complete this procedure, and submit the result. As always, matching expected and actual results in the test cases is a good sign. There's not a lot of coding, but deriving a formula for count_new_nodes requires some thought. (This is the number of new nodes, if any, to be added to a HasClique instance to create a HalfIndependentSet instance. How those new nodes are configured depends on the HasClique instance.)

Bear in mind that the **overall goal is to map an instance I of HasClique onto an instance I' of HalfIndependentSet, such that I' is a positive instance of HalfIndependentSet iff I is a positive instance of HasClique.** You may need to add nodes to generate I' — how would you add a node so that it would never be part of an independent set? So that it would always be added to any independent set in I' ?

A **clique** is a subset of the nodes in an unweighted, undirected graph (specified as a white space delimited sequence of edges), such that there is **an edge connecting every two nodes** in the subset.

An instance of HasClique is an unweighted, undirected graph specified as a series of edges, separated by a semicolon (;) from an integer > 1 . For example, 'a,b a,c b,c d,e;3' is an instance of HasClique. We call the graph portion of the instance g , and the integer clique_size.

An **instance of HasClique is a positive instance iff g contains a clique at least as large as clique_size.** For example, 'a,b a,c b,c d,e;3' is a positive instance because $\{a, b, c\}$ is a clique of size 3. 'a,b a,c b,c d,e;4' is a negative instance because the graph contains no clique of size 4. 'a,b b,c d,e;3' is a negative instance because with the deletion of edge a,c , $\{a, b, c\}$ is no longer a clique.

An **independent set** is a subset of the nodes in an unweighted, undirected graph, specified as a white space delimited sequence of edges, such that there are **no edges between any two nodes in the subset**. For example, the graph "a,a b,c b,d b,e c,d c,e d,e" has an independent set of size 2, $\{a,b\}$ since there is no link between nodes a and b . Note that loop back nodes are disregarded when looking for independent sets.

An instance of HalfIndependentSet is also an unweighted, undirected graph specified as a series of edges. For example, "a,d a,e a,f b,d b,e b,f c,c c,f e,f".

A **HalfIndependentSet instance is a positive instance iff at least half the nodes graph form an independent set.** "a,d a,e a,f b,d b,e b,f c,c c,f e,f" is a positive instance because $\{a, b, c\}$ forms an independent set of size 3, out of a total of 6 graph nodes. "a,a a,c a,d a,e b,b b,c b,d b,e c,d c,e d,e" is a negative instance, because it has no independent sets larger than 2, which is one less than twice the number of graph nodes.

2. 50 points. VfyPairNodeCover-to_complete.py, included with the test materials, is a template for a verifier for the PairNodeCover problem (defined below). Follow the instructions in the source code to complete this verifier, and submit the result.

A node cover is a subset of the nodes in a graph such that every edge in the graph has an endpoint in one of the nodes of the subset. For example, $\{a\}$ covers 'a,b a,c a,d a,e'.

An instance of PairNodeCover is an unweighted, undirected graph specified as a series of edges. For example, 'a,b a,c a,d a,e' is an instance of PairNodeCover.

A PairNodeCover instance is a positive instance iff it has a node cover set whose size is ≤ 2 .

'a,b a,c b,c' is a positive instance because it is covered by $\{a,b\}$. 'a,b c,d e,f' is a negative instance because the minimum node cover set size is 3.

3. 25 points. Answer the questions below, and submit the results a plain text .txt file.

a. (5 points) Question one asked you to implement $\text{HasClique} \leq_P \text{HalfIndependentSet}$. Does this polyreduction show that $\text{HalfIndependentSet} \in \text{NP-hard}$? Explain.

The next questions refer to the decision problem NoWayD, whose instances are weighted graphs paired with thresholds. An instance of NoWayD is a positive instance iff every Hamilton circuit through the graph has a cost greater than the threshold.

b. (5 points) Could a working verifier for NoWayD take as a hint a list of all possible Hamilton circuits through the graph, and return 'correct' iff the cost of every circuit on the list exceeded the threshold? Explain.

c. (5 points) Is NoWayD $\in \text{NP-complete}$? Explain.

d. (5 points) Does there exist a polyreduction $\text{TspD} \leq_P \text{NoWayD}$, where TspD is the decision variant of the traveling salesman problem? Explain.

e. (5 points) Is NoWayD decidable? Explain.