CSC MATH 425 Homework 2

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1.c Yes, my function *myRank()* computes the rank of A to be 3.

2.a An example of a 4×4 strictly column diagonally dominant matrix is:

$$\begin{bmatrix} 3 & 1 & 1 & 0 \\ 2 & 4 & 1 & 0 \\ 0 & 1 & 5 & 2 \\ 1 & 1 & 3 & 6 \end{bmatrix}$$

In this matrix, the magnitude of every diagonal entry is greater than the sum of every non-diagonal entry in their column.

2.b Using this inequality,

$$|a_{jj}| > \sum_{i \neq j}^{n} |a_{ij}|$$

We can show that Gaussian elimination with partial pivoting on the array from 2.a will have zero row interchanges. When we use partial pivoting, the algorithm looks for values below a_{jj} and finds that there are no rows whose value is greater than the current pivot. No row interchanges occur, and algorithm moves on to the next diagonal.

1. In the first column we see that:
   $a_{11} > a_{12}$, $a_{11} > a_{13}$, $a_{11} > a_{14}$.
   No row interchanges occur.
2. In the second column we see that:
   $a_{21} > a_{22}$, $a_{21} > a_{23}$, $a_{21} > a_{24}$.
   No row interchanges occur.
3. Third column:
   $a_{31} > a_{32}$, $a_{31} > a_{33}$, $a_{31} > a_{34}$.
   No row interchanges occur.
4. Fourth column:
   $a_{41} > a_{42}$, $a_{41} > a_{43}$, $a_{41} > a_{44}$.
   No row interchanges occur.

2.c This example is used in Matlab and the results indeed show that zero row interchanges have occured.