Cálculo:

pendiente

Ecuación de Onda:

1. Fórmula de D'Alambert ($U = \mathbb{R}^n$): $u(x;r,t) = \frac{1}{2}[g(x+t) - g(x-t)] + \frac{1}{2}\int_{x-t}^{x+t} h(y)\,\mathrm{d}y$
2. Medias Esféricas: $U(r,t;x) = ⨍_{\partial B(x,r)} u(y,t)\,\mathrm{d}y$, $U_{tt} - U_{rr} - \frac{n-1}{r}U_r = 0$ s.a. $U = G, U = H$ en $\mathbb{R}_+ \times \{t = 0\}$
3. Fórmula de Kirchhoff: $u(x,t) = ⨍_{\partial B(x,t)} th(y) + g(y) + Dg(y)\cdot(y-x)\,\mathrm{d}S(y)$, válida sólo para $n = 3$.
4. Fórmula de Poisson 2D: $u(x,t) = \frac{1}{2}⨍_{B(x,t)} \frac{tg(y)+t^2h(y)+tDg(y)\cdot(y-x)}{(t^2-|y-x|^2)^{\frac{1}{2}}}\,\mathrm{d}S(y)$
5. Fórmulas de Poisson 3D: $u(x,t) = \partial_t\Big(t⨍_{\partial B(x,t)} g\,\mathrm{d}S\Big) + \frac{1}{2r}⨍_{\partial B(x,t)} h(y)\,\mathrm{d}S(y)$ $= ⨍_{\partial B(x,t)} th(y) + g(y) + Dg(y)\cdot(y-x)\,\mathrm{d}S(y)$
6. Solución homogénea general n impar:
$$u(x,t) = \frac{1}{\gamma_n}\left[\partial_t\left(\frac{1}{t}\partial_t\right)^{\frac{n-3}{2}}\left(t^{n-2}⨍_{\partial B(x,t)} g\,\mathrm{d}S\right) + \left(\frac{1}{t}\partial_t\right)^{\frac{n-3}{2}}\left(t^{n-2}⨍_{\partial B(x,t)} h\,\mathrm{d}S\right)\right],$$
$$\gamma_n = 1\cdot 3\cdot 5\cdots(n-2)$$
7. Solución homogénea general n par:
$$u(x,t) = \frac{1}{\gamma_n}\left[\partial_t\left(\frac{1}{t}\partial_t\right)^{\frac{n-2}{2}}\left(t^n⨍_{B(x,t)} \frac{g(y)}{(t^2-|y-x|^2)^{\frac{1}{2}}}\,\mathrm{d}y\right) + \left(\frac{1}{t}\partial_t\right)^{\frac{n-2}{2}}\left(t^n⨍_{B(x,t)} \frac{h(y)}{(t^2-|y-x|^2)^{\frac{1}{2}}}\,\mathrm{d}y\right)\right],$$
$$\gamma_n = 2\cdot 4\cdot 6\cdots n$$
8. Solución no-homogénea: $u(x,t) = \int_0^t u(x,t;s)\,\mathrm{d}s$, donde para $t \geq s$, $u_{tt}(\cdot;s) - \Delta u(\cdot;s) = 0,$ s.a. $u(\cdot;s) = 0, u_t(\cdot;s) = f(\cdot,s)$ en $t = s$ La solución general se obtiene sumando homogénea + inhomogenea
9. Energía: $E(t) = \frac{1}{2}\int_U (u_t^2 + |Du|^2)\,\mathrm{d}x$. $\dot{E} = \int_U u_t(u_{tt} - \Delta u)\,\mathrm{d}x + \int_{\partial U} u_t Du\cdot\nu\,\mathrm{d}S$

Espacios de Sobolev