

# **Sensitivity Analysis on Least-Squares American Options Pricing**

**Miguel Ângelo Maia Ribeiro**

Thesis to obtain the Master of Science Degree in

## **Engineering Physics**

Supervisors: Prof. Cláudia Rita Ribeiro Coelho Nunes Philippart  
Prof. Rui Manuel Agostinho Dilão

### **Examination Committee**

Chairperson: Prof. Full Name

Supervisor: Prof. Full Name 1 (or 2)

Member of the Committee: Prof. Full Name 3

**Month Year**



To my parents



## **Acknowledgments**

A few words about the university, financial support, research advisor, dissertation readers, faculty or other professors, lab mates, other friends and family...



## Resumo

Inserir o resumo em Português aqui com o máximo de 250 palavras e acompanhado de 4 a 6 palavras-chave...

**Palavras-chave:** palavra-chave1, palavra-chave2,...





## **Abstract**

Insert your abstract here with a maximum of 250 words, followed by 4 to 6 keywords...

**Keywords:** keyword1, keyword2,...



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# Nomenclature

## Greek symbols

$\alpha$	Angle of attack.
$\beta$	Angle of side-slip.
$\kappa$	Thermal conductivity coefficient.
$\mu$	Molecular viscosity coefficient.
$\rho$	Density.

## Roman symbols

$C_D$	Coefficient of drag.
$C_L$	Coefficient of lift.
$C_M$	Coefficient of moment.
$p$	Pressure.
$\mathbf{u}$	Velocity vector.
$u, v, w$	Velocity Cartesian components.

## Subscripts

$\infty$	Free-stream condition.
$i, j, k$	Computational indexes.
$n$	Normal component.
$x, y, z$	Cartesian components.
ref	Reference condition.

## Superscripts

*	Adjoint.
T	Transpose.



# Glossary

- CFD** Computational Fluid Dynamics is a branch of fluid mechanics that uses numerical methods and algorithms to solve problems that involve fluid flows.
- CSM** Computational Structural Mechanics is a branch of structure mechanics that uses numerical methods and algorithms to perform the analysis of structures and its components.
- MDO** Multi-Disciplinary Optimization is an engineering technique that uses optimization methods to solve design problems incorporating two or more disciplines.



# Chapter 1

## Introduction

### 1.1 Mathematical Finance

Mathematical finance, also known as quantitative finance, is a field of applied mathematics focused on the modeling of financial instruments. It is rather difficult to overestimate its importance since it is heavily used by investors and investment banks in everyday transactions. In recent decades, this field suffered a complete paradigm shift, following developments in computer science and new theoretical results that enabled investors to better price their assets. With the colossal sums traded daily in financial markets around the world, mathematical finance has become increasingly important and many resources are invested in the research and development of new and better theories and algorithms.

### 1.2 Derivatives

One of the subjects most studied by financial mathematicians is derivatives. In finance, a derivative is simply a contract whose value depends on other simpler financial instruments, known as *underlying assets*, such as stock prices or interest rates. They can virtually take any form desirable, so long as there are two parties interested in signing it and all government regulations are met.

The importance of derivatives has grown greatly in recent years. In fact, as of June 2017, derivatives were responsible for over \$542 trillion worth of trades, in the Over-the-Counter (OTC) market alone [1], as can be seen in Figure 1.1 (the OTC market refers to all deals signed outside of exchanges). This growth stalled after the 2008 global financial crisis due to new government regulations, implemented because of the role of derivatives in the market crashes [2].

### 1.3 Options

Of all kinds of derivatives, in this master thesis we will focus particularly on the most traded one: *options* [3].

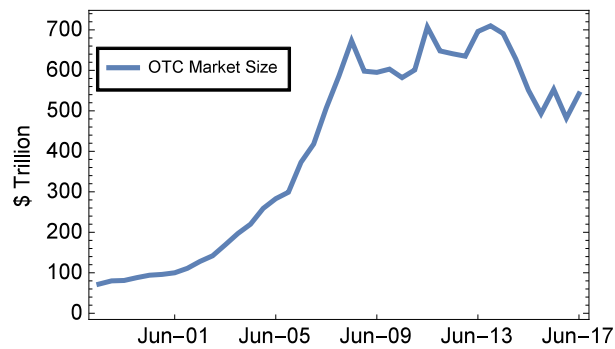


Figure 1.1: Size of OTC derivatives market since May 1996.

As the name implies, an option contract grants its buyer the *option* to buy (in the case of a *call* option) or sell (for *put* options) its underlying asset at a future date, known as the *maturity*, for a fixed price, known as the *strike price*.

The description above pertains only to the most traded type of option -*European* options. In this thesis, unless stated otherwise, all options will be assumed European. There are, however, other types that enable exercising at dates other than maturity. The most well known example is American options, that enable their buyer to exercise at any point in time until the expiration. Other types, commonly known as *exotic*, will be studied in more detail in the following sections.

It's important to emphasize the fact that an option grants its buyer the right to do something. If *exercising* the option would lead to further losses, its buyer can simply decide to let the expiration date pass. This is indeed the most attractive characteristic of options.

### 1.3.1 Why Options are Important

Options are very useful tools to all investors. To hedgers (i.e. investors that want to limit their exposure to risk), options provide safety by fixing the future price of their underlying asset (e.g. if hedgers want to protect themselves against a potential price crash affecting one of their assets, they can buy put options on them. Now, even if the value their asset decreases significantly, their losses are contained because they can always exercise the options and sell the asset at the option's higher strike price).

To speculators (i.e. investors that want to take advantage of the uncertainty of future markets by speculating on their outcome), options grant access to much higher profits (e.g. if speculators strongly believe that the value of a given asset will greatly increase in the future, they can buy call options on that asset. If their prediction proves right, they can buy that asset for the option's lower strike price).

Due to all their advantages, and unlike some other types of derivatives, options have a cost. Finding the ideal price for an option is a fundamental concern to investors, since knowing its true value can give them a chance to take advantage of an under or overpriced option. Finding this price can be very difficult for some options, however. Though a lot of research has been done towards this goal, a great deal more is still required.

## **1.4 Topic Overview**

Provide an overview of the topic to be studied...

## **1.5 Objectives**

Explicitly state the objectives set to be achieved with this thesis...

## **1.6 Thesis Outline**

Briefly explain the contents of the different chapters...





## Chapter 2

# Background

### 2.1 Call and Put Options

As stated before, call and put options enable their buyer to buy and sell the underlying stock at the maturity for the fixed strike price. In the case of a call option, if at the maturity the price of my asset is greater than the strike, I can buy it for the latter and immediately sell it for its higher market price. Thus, the payoff of my option would be the difference of these two values. On the other hand, if the price of the asset decreases past the strike at the maturity, I should let my option expire, since I can buy the asset for the lower market price rather than the higher strike. In this case, the payoff of the option would be zero. The same reasoning can be made for put type options. The payoff function of these two types of options can then simply be deduced as

$$\begin{aligned}\text{Payoff}_{\text{call}} &= \max(S(T) - K, 0); \\ \text{Payoff}_{\text{put}} &= \max(K - S(T), 0),\end{aligned}\tag{2.1}$$

where  $K$  is the strike price and  $S(T)$  is the asset price,  $S(t)$ , at the maturity,  $T$ . These functions are represented in Figure 2.1.

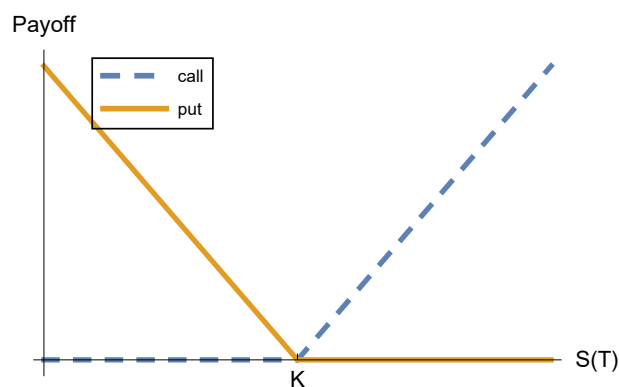


Figure 2.1: Payoff functions of *call* and *put* options.

## 2.2 Black-Scholes-Merton Formulae

Due to their high importance, options have been studied in great detail in the past. Probably the most important result in this field came from Fischer Black, Myron Scholes and Robert Merton, who developed a mathematical model to price options [4] - the famous Black-Scholes-Merton model - still in use in present days [5]. The last two actually earned the 1997 Nobel prize in Economics for this development.

This model states that the price of an European call or put option, whose underlying asset is a stock, follows the partial differential equation (PDE)

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0, \quad (2.2)$$

where  $V$  is the price of the option,  $S$  is the price of the underlying stock,  $r$  is the risk-free interest rate and  $\sigma$  is the stock price volatility.

The risk-free interest rate,  $r$ , is the interest an investor would receive from a risk-free investment. An investor should never invest in risky products whose expected return is lower than this interest, since there's the alternative of investing without risk. In general, this rate changes slightly with time, but Black *et al.* assumed, in their original model, that it remains constant throughout the option duration. Some later developments dealt with this shortcoming, but because option prices do not significantly depend on this value [5], in the remainder of this thesis we shall assume it is constant and known.

Finally, the volatility,  $\sigma$ , is a measure of uncertainty and will be studied in more detail in section 2.3.

One important assumption of this model is that stock prices follow a Geometric Brownian Motion (GBM), defined as

$$dS(t) = rS(t)dt + \sigma S(t)dW(t), \quad (2.3)$$

with  $\{W(t), t > 0\}$  defining a one-dimensional Brownian motion.

Pricing options is fairly straightforward - we simply need to solve the PDE in eq. (2.2) in a similar fashion to the initial value problem for the diffusion equation [6]. The results published in the original article by Black *et al.* state that, at time  $t$ , call and put options can be valued as

$$\begin{aligned} C(S(t), t) &= N(d_1)S(t) - N(d_2)Ke^{-r(T-t)}; \\ P(S(t), t) &= -N(-d_1)S(t) + N(-d_2)Ke^{-r(T-t)}, \end{aligned} \quad (2.4)$$

where  $N(\cdot)$  is the cumulative distribution function of the standard normal distribution,  $T$  is the maturity time, and  $d_1, d_2$  are given by

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T-t}} \left[ \ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right]; \\ d_2 &= d_1 - \sigma\sqrt{T-t}. \end{aligned} \quad (2.5)$$

From eq. (2.4) we can derive a relationship between  $C(S, t)$  and  $P(S, t)$ , known as the *put-call parity*

$$C(S(t), t) = S(t) - Ke^{-r(T-t)} + P(S(t), t). \quad (2.6)$$

Because of this duality, we can always obtain the prices of put options from call options with the same underlying asset, maturity and strike. For this reason, unless otherwise stated, all options will be assumed calls in the following sections.

## 2.3 Volatility

Volatility is a measure of the uncertainty of future stock prices changes. In other words, a high volatility will lead to great future fluctuations in the stock price, whereas a stock with low volatility is more stable.

Of all the parameters in the Black-Scholes formula, (2.2), volatility is the only one we can't easily observe from market data. Furthermore, unlike the interest rate, volatility has a great impact on the behavior of stock prices and consequently on the price of options. These two factors make volatility one of the most studied subjects in option pricing.

It should be noted that there are several types of volatility, depending on what is being measured. Some of these types will be approached in the next subsections.

### 2.3.1 Implied Volatility

The *implied volatility* can be described as the value of stock price volatility that, when input into the Black-Scholes pricer in eq. (2.4), outputs a value equal to the market price of a given option. In other words, it would be the stock volatility that the seller/buyer of the option assumed when pricing it (given that the Black-Scholes model was used).

Because eq. (2.4) is not invertible, we need to use some numerical method (e.g Newton's method) to find the solution to the equation

$$C(\sigma_{imp}, S(t), t) - \bar{C} = 0, \quad (2.7)$$

where  $C(\sigma_{imp}, \cdot)$  corresponds to the solution of eq. (2.4) using the implied volatility  $\sigma_{imp}$  and  $\bar{C}$  is the price of the option observed at the market.

We can obtain the implied volatility of an option from its price or the price from its implied volatility, because eq. (2.4) is monotonic in the volatility. This duality is so fundamental that exchanges sometimes sell their options providing only the implied volatility instead of the price [5].

One interesting property of implied volatility is that, in the real-world, it varies with the strike price and maturity. In principle, if investors really used the Black-Scholes model to price their options, two options with the same underlying asset should have the same implied volatility, despite their strike prices or maturities. However, when observing market data, this is not the case. The observed implied volatilities form two possible shapes in a scatter plot, known as *smile* and *skew*. An implied volatility smile shows greater  $\sigma_{imp}$  for options with strikes different from the current stock price and the minimum where the strike equals the stock price. A skew, on the other hand, only shows greater  $\sigma_{imp}$  in one of the directions (i.e. for strikes either higher or lower than the current stock price). You can observe both these phenomena in Figure 2.2. We can therefore conclude that options with strikes different from the current stock price are overpriced. The reason behind this odd market behavior is the simple demand-supply

rule [5]. On the one hand, some investors are risk-averse and want to hedge their losses in case of a market crash (as explained in subsection 1.3.1). They don't mind paying a higher price for an option if this means they would be safe from crashes. For this reason, the prices of low strike (call) options increase which drives their implied volatility up. On the other hand, some other investors are risk-seekers and want to take advantage of possible sudden price increases, buying the stocks for lower prices. They don't mind paying higher prices for the options and this drives the prices of high strike options (and, consequently, their implied volatility) up.

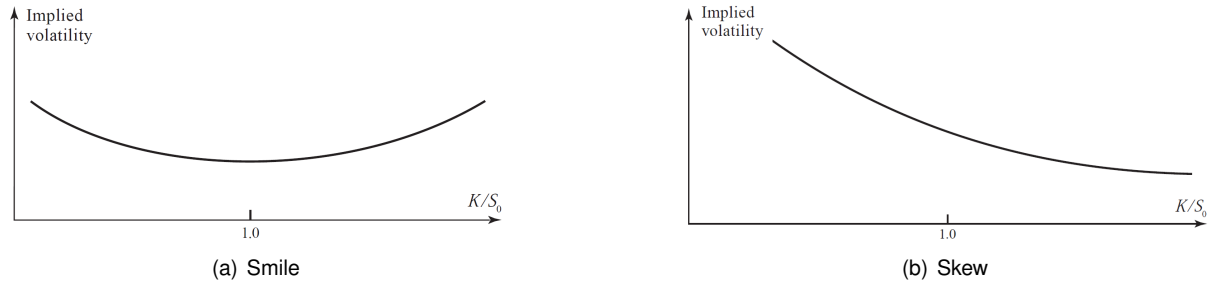


Figure 2.2: Example of an implied volatility smile and skew. [source=Hull](#)

## 2.3.2 Local Volatility

In their original work, Black *et al.* assumed that volatility is constant throughout the whole contract. From market data, it can be clearly seen that this is not the case. There may be times where new information reaches the market and trading increases, driving volatility up. The opposite is also true.

The model in eq. (2.4) is clearly not enough to truly grasp real-world trading. We should have a model where volatility is dynamic, measuring the amount of randomness in the stock price at any given time. The geometric Brownian motion from eq. 2.3 would thus become

$$dS(t) = rS(t)dt + \sigma(t)S(t)dW(t),$$

where  $\sigma(t)$  is now some function of time.

However, as we saw in subsection 2.3.1 for implied volatility, the market's view of volatility also varies with the strike price. A simple dynamic volatility model is thus insufficient. The local volatility should then be a function of both time and strike price. We are left with a new GBM, given by

$$dS(t) = rS(t)dt + \sigma(K, t)S(t)dW(t), \quad (2.8)$$

where  $\sigma(K, t)$  depends now on  $K$  and  $t$ .

The local volatility function is very difficult to measure. Some models have been proposed to model it. We shall approach them in section 2.4.

## 2.4 Dupire's Formula

## 2.5 Theoretical Overview

Some overview of the underlying theory about the topic...

## 2.6 Theoretical Model 1

The research should be supported with a comprehensive list of references. These should appear whenever necessary, in the limit, from the first to the last chapter.

A reference can be cited in any of the following ways:

- Citation mode #1 - [7]
- Citation mode #2 - Jameson et al. [7]
- Citation mode #3 - [7]
- Citation mode #4 - Jameson, Pierce, and Martinelli [7]
- Citation mode #5 - [7]
- Citation mode #6 - Jameson et al. 7
- Citation mode #7 - 7
- Citation mode #8 - Jameson et al.
- Citation mode #9 - 1998
- Citation mode #10 - [1998]

Several citations can be made simultaneously as [8, 9].

This is often the default bibliography style adopted (numbers following the citation order), according to the options:

```
\usepackage{natbib} in file Thesis_Preamble.tex,  
\bibliographystyle{abbrvnat} in file Thesis.tex.
```

Notice however that this style can be changed from numerical citation order to authors' last name with the options:

```
\usepackage[numbers]{natbib} in file Thesis_Preamble.tex,  
\bibliographystyle{abbrvnatsrtnat} in file Thesis.tex.
```

Multiple citations are compressed when using the `sort&compress` option when loading the `natbib` package as `\usepackage[numbers,sort&compress]{natbib}` in file `Thesis_Preamble.tex`, resulting in citations like [10–13].

## 2.7 Theoretical Model 2

Other models...

## Chapter 3

# Implementation

Insert your chapter material here...

### 3.1 Numerical Model

Description of the numerical implementation of the models explained in Chapter 2...

### 3.2 Verification and Validation

Basic test cases to compare the implemented model against other numerical tools (verification) and experimental data (validation)...





# Chapter 4

## Results

Insert your chapter material here...

### 4.1 Problem Description

Description of the baseline problem...

### 4.2 Baseline Solution

Analysis of the baseline solution...

### 4.3 Enhanced Solution

Quest for the optimal solution...

#### 4.3.1 Figures

Insert your section material and possibly a few figures...

Make sure all figures presented are referenced in the text!

#### Images



Figure 4.1: Caption for figure.



(a) Airbus A320



(b) Bombardier CRJ200

Figure 4.2: Some aircrafts.

Make reference to Figures 4.1 and 4.2.

By default, the supported file types are *.png, .pdf, .jpg, .mps, .jpeg, .PNG, .PDF, .JPG, .JPEG*.

See [http://mactex-wiki.tug.org/wiki/index.php/Graphics\\_inclusion](http://mactex-wiki.tug.org/wiki/index.php/Graphics_inclusion) for adding support to other extensions.

## Drawings

Insert your subsection material and for instance a few drawings...

The schematic illustrated in Fig. 4.3 can represent some sort of algorithm.

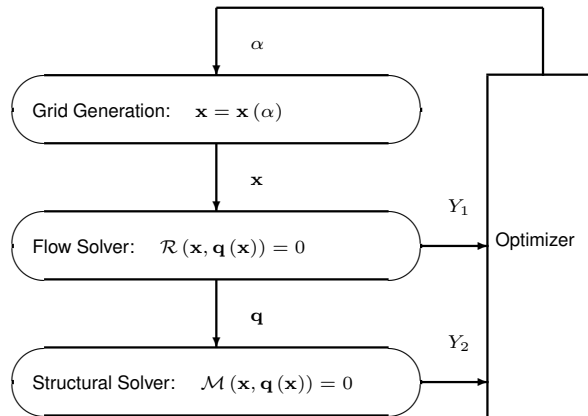


Figure 4.3: Schematic of some algorithm.

## 4.3.2 Equations

Equations can be inserted in different ways.

The simplest way is in a separate line like this

$$\frac{dq_{ijk}}{dt} + \mathcal{R}_{ijk}(\mathbf{q}) = 0. \quad (4.1)$$

If the equation is to be embedded in the text. One can do it like this  $\partial\mathcal{R}/\partial\mathbf{q} = 0$ .

It may also be split in different lines like this

$$\begin{aligned} &\text{Minimize} && Y(\alpha, \mathbf{q}(\alpha)) \\ &\text{w.r.t.} && \alpha, \\ &\text{subject to} && \mathcal{R}(\alpha, \mathbf{q}(\alpha)) = 0 \\ &&& C(\alpha, \mathbf{q}(\alpha)) = 0. \end{aligned} \tag{4.2}$$

It is also possible to use subequations. Equations 4.3a, 4.3b and 4.3c form the Navier–Stokes equations 4.3.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0, \tag{4.3a}$$

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j + p \delta_{ij} - \tau_{ji}) = 0, \quad i = 1, 2, 3, \tag{4.3b}$$

$$\frac{\partial}{\partial t} (\rho E) + \frac{\partial}{\partial x_j} (\rho E u_j + p u_j - u_i \tau_{ij} + q_j) = 0. \tag{4.3c}$$

### 4.3.3 Tables

Insert your subsection material and for instance a few tables...

Make sure all tables presented are referenced in the text!

Follow some guidelines when making tables:

- Avoid vertical lines
- Avoid “boxing up” cells, usually 3 horizontal lines are enough: above, below, and after heading
- Avoid double horizontal lines
- Add enough space between rows

Model	$C_L$	$C_D$	$C_{M_y}$
Euler	0.083	0.021	-0.110
Navier–Stokes	0.078	0.023	-0.101

Table 4.1: Table caption.

Make reference to Table 4.1.

Tables 4.2 and 4.3 are examples of tables with merging columns:

An example with merging rows can be seen in Tab.4.4.

If the table has too many columns, it can be scaled to fit the text width, as in Tab.4.5.

	Virtual memory [MB]	
	Euler	Navier–Stokes
Wing only	1,000	2,000
Aircraft	5,000	10,000
(ratio)	5.0×	5.0×

Table 4.2: Memory usage comparison (in MB).

	$w = 2$			$w = 4$		
	$t = 0$	$t = 1$	$t = 2$	$t = 0$	$t = 1$	$t = 2$
$dir = 1$						
$c$	0.07	0.16	0.29	0.36	0.71	3.18
$c$	-0.86	50.04	5.93	-9.07	29.09	46.21
$c$	14.27	-50.96	-14.27	12.22	-63.54	-381.09
$dir = 0$						
$c$	0.03	1.24	0.21	0.35	-0.27	2.14
$c$	-17.90	-37.11	8.85	-30.73	-9.59	-3.00
$c$	105.55	23.11	-94.73	100.24	41.27	-25.73

Table 4.3: Another table caption.

ABC	header			
	1.1	2.2	3.3	4.4
IJK	group	0.5		0.6
		0.7		1.2

Table 4.4: Yet another table caption.

Variable	a	b	c	d	e	f	g	h	i	j
Test 1	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
Test 2	20,000	40,000	60,000	80,000	100,000	120,000	140,000	160,000	180,000	200,000

Table 4.5: Very wide table.

#### 4.3.4 Mixing

If necessary, a figure and a table can be put side-by-side as in Fig.4.4



Legend		
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Figure 4.4: Figure and table side-by-side.



## **Chapter 5**

# **Conclusions**

Insert your chapter material here...

### **5.1 Achievements**

The major achievements of the present work...

### **5.2 Future Work**

A few ideas for future work...





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# Appendix A

## Vector calculus

In case an appendix is deemed necessary, the document cannot exceed a total of 100 pages...

Some definitions and vector identities are listed in the section below.

### A.1 Vector identities

$$\nabla \times (\nabla \phi) = 0 \tag{A.1}$$

$$\nabla \cdot (\nabla \times \mathbf{u}) = 0 \tag{A.2}$$



## **Appendix B**

# **Technical Datasheets**

It is possible to add PDF files to the document, such as technical sheets of some equipment used in the work.

### **B.1 Some Datasheet**

### BENEFITS

#### Maximum Light Capture

SunPower's all-back contact cell design moves gridlines to the back of the cell, leaving the entire front surface exposed to sunlight, enabling up to 10% more sunlight capture than conventional cells.

#### Superior Temperature Performance

Due to lower temperature coefficients and lower normal cell operating temperatures, our cells generate more energy at higher temperatures compared to standard c-Si solar cells.

#### No Light-Induced Degradation

SunPower n-type solar cells don't lose 3% of their initial power once exposed to sunlight as they are not subject to light-induced degradation like conventional p-type c-Si cells.

#### Broad Spectral Response

SunPower cells capture more light from the blue and infrared parts of the spectrum, enabling higher performance in overcast and low-light conditions.

#### Broad Range Of Application

SunPower cells provide reliable performance in a broad range of applications for years to come.

The SunPower™ C60 solar cell with proprietary Maxeon™ cell technology delivers today's highest efficiency and performance.

The anti-reflective coating and the reduced voltage-temperature coefficients provide outstanding energy delivery per peak power watt. Our innovative all-back contact design moves gridlines to the back of the cell, which not only generates more power, but also presents a more attractive cell design compared to conventional cells.



SunPower's High Efficiency Advantage

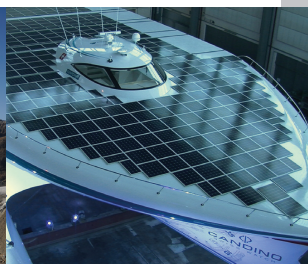
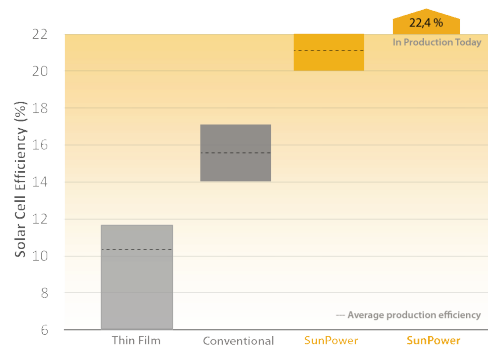


Photo courtesy of 3S Photovoltaics

### Electrical Characteristics of Typical Cell at Standard Test Conditions (STC)

STC: 1000W/m<sup>2</sup>, AM 1.5g and cell temp 25°C

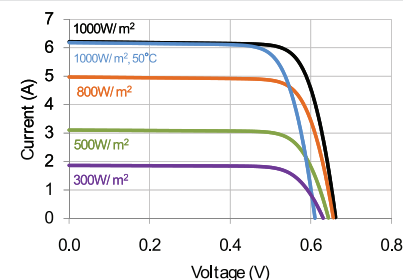
Bin	P <sub>mp</sub> (Wp)	Eff. (%)	V <sub>mp</sub> (V)	I <sub>mp</sub> (A)	V <sub>oc</sub> (V)	I <sub>sc</sub> (A)
G	3.34	21.8	0.574	5.83	0.682	6.24
H	3.38	22.1	0.577	5.87	0.684	6.26
I	3.40	22.3	0.581	5.90	0.686	6.27
J	3.42	22.5	0.582	5.93	0.687	6.28

All Electrical Characteristics parameters are nominal  
Unlaminated Cell Temperature Coefficients  
Voltage: -1.8 mV / °C Power: -0.32% / °C

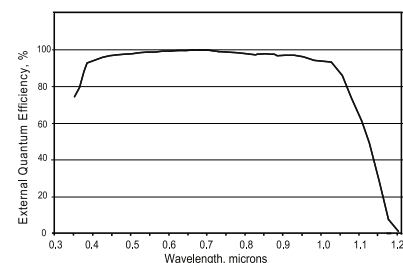
### Positive Electrical Ground

Modules and systems produced using these cells must be configured as "positive ground systems".

### TYPICAL I-V CURVE



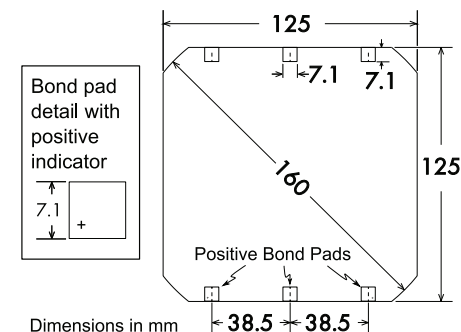
### SPECTRAL RESPONSE



### Physical Characteristics

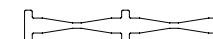
Construction:	All back contact
Dimensions:	125mm x 125mm (nominal)
Thickness:	165µm ± 40µm
Diameter:	160mm (nominal)

### Cell and Bond Pad Dimensions



Bond pad area dimensions are 7.1mm x 7.1mm  
Positive pole bond pad side has "+" indicator on leftmost and rightmost bond pads.

### Interconnect Tab and Process Recommendations



Tin plated copper interconnect. Compatible with lead free process.

### Packaging

Cells are packed in boxes of 1,200 each; grouped in shrink-wrapped stacks of 150 with interleaving. Twelve boxes are packed in a water-resistant "Master Carton" containing 14,400 cells suitable for air transport.

Interconnect tabs are packaged in boxes of 1,200 each.

### About SunPower

SunPower designs, manufactures, and delivers high-performance solar electric technology worldwide. Our high-efficiency solar cells generate up to 50 percent more power than conventional solar cells. Our high-performance solar panels, roof tiles, and trackers deliver significantly more energy than competing systems.