Sensitivity Analysis on Least-Squares American Options Pricing

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I. INTRODUCTION

The stock market has suffered a complete paradigm shift in since past decades. Recent developments in computer science and mathematical finance have greatly enhanced our abilities to predict and take advantage of stock price changes. It shouldn't come as much of a surprise that there has been an ever growing desire from investors to take advantage of these new developments to increase potential profits.

With the colossal sums handled daily in the stock market, even a small improvement on the predictive abilities of a given algorithm can lead to great increases in profits for investors. Thus, it should be clear that a great amount of resources should be invested in the research and development of these algorithms. An investor that does not follow this strategy is bound to lose major profits when compared with his better prepared counterparts.

A. Derivatives and Options

Due to the developments in stock forecasting, our knowledge of derivatives has also greatly increased. A derivative is simply a contract whose value depends on other simpler financial instruments, like stocks or interest rates. Derivatives can virtually take any form desirable, so long as there are two parties interested in taking a part in it. In this work we will focus on the most commonly traded type - options - which shall be addressed later.

The derivatives market has become increasingly important in recent times. In fact, as of June 2017, derivatives are responsible for over 542 trillion US dollars worth of trades annually, in the Over-the-Counter (OTC) market alone, as can be seen on FIG. 1 (the OTC market refers to all deals signed outside of stock exchanges). Though the market size peaked in 2013 with over 710 trillion US dollars, it seems to have shrinked in the last decade, which might be attributed to the global financial crisis of 2007.
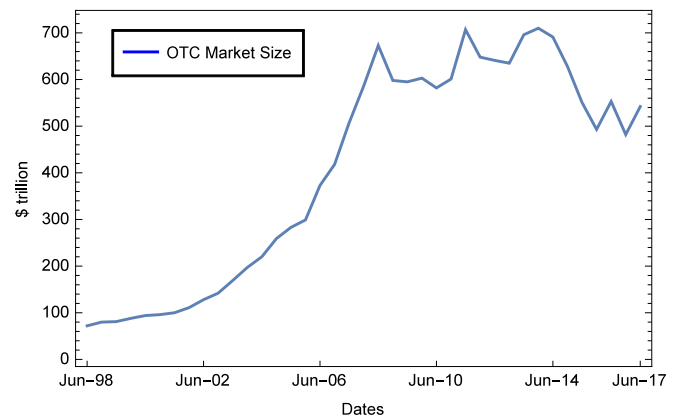

FIG. 1: Size of OTC derivatives market.
Source: stats.bis.org/statx/srs/table/d5.1 (on 16/11/2017)

Among the many types of derivatives, the most commonly traded are options.

In simple terms, a *call option* grants the investor the right to buy the underlying asset (e.g. stock) for a fixed price, known as the *strike price*, by a certain date, known as the *expiration date*.
On the other hand, a *put option* grants the investor the right to sell the underlying asset for the strike price, by the expiration date.

Options have several advantages that make them especially appealing to investors.
To hedgers (i.e. investors that want to limit their exposure to risk), options provide safety by fixing the future price of a stock. If this investor is afraid of a price crash, by buying put options he ensures that future losses are contained (though if the crash doesn't occur, the hedger loses his option investment).
To speculators (i.e. investors that take advantage of the uncertainty of future asset prices by betting on their outcome), options grant access to much larger profits when the market forecast proves true (though also greater losses when it doesn't). In short, options magnify consequences: a good result turns into a large profit whereas a bad prediction results in a loss of investment.

Because of these advantages, an investor has to pay a certain fee to acquire an option. This pricing problem is a fundamental concern to investors. If an option is under-priced by a seller, an informed investor can earn great profits by taking advantage of it. The price of an option and what influences it will be the main focus of this work.

B. European and American Options

Both call and put options can be further separated into several types. Among these European and American options are by far the most commonly traded. The holder of an *European option* can exert his right to buy/sell (call/put) the underlying assets, also known as *exercising*, only at the specified expiration date. An *American option* can be exercised at any point up to this very same date.

Due to their high importance, options have been studied in detail in the past. Possibly the most important result in this field came from Robert Merton and Myron Scholes who earned the 1997 Nobel prize in Economics for developing a mathematical model to price European options - the famous Black-Scholes formula - still used nowadays, though with some modifications.

Pricing American options, however, poses a greater challenge. Because they can be exercised at any time, the underlying assets must be closely monitored so as to attempt to exercise at an optimal point. Furthermore, unlike European options, no analytic pricing model currently exists for this type of derivatives. Several numerical models have been proposed in the past in an attempt to approximate the price of these options. These methods shall be discussed in detail in the following chapters.

American options are still nonetheless very much used by investors, so it is absolutely critical to understand which variables influence these types of derivatives and by what amount. With this knowledge, one can better prepare oneself to market changes and even mitigate potential risks.

II. OBJECTIVES

The main objective of this thesis is to study which factors affect the price of an option and in what extent.

We will begin by replicating the model developed by Longstaff and Schwartz to price American options. Their method is based upon a Monte Carlo generation of stock price paths with a set of market variables such as stock volatility and interest rate (using the Black-Scholes stochastic deferential equation). To these results, a least squares regression is then applied iteratively to generate an optimal stopping decision matrix with all the stock price paths generated.

With this method, we shall then apply a variation-based sensitivity analysis, as developed by Sobol and Saltelli. This analysis outputs a weight for each of the variables used that corresponds to their influence in the variance of the final option price. Thus, if a variable has a large weight, its variance has a large impact in the variance of the option price.

As a next step, we shall modify the Longstaff-Schwartz method to closely resemble real-world stocks. We will, for example, implement the GARCH(1,1) model for the stock price volatility to account for daily changes on its value. We could also try to implement interest rate models for the same reason. A further study of which models better suit our needs is still necessary.

Finally, we will apply our model to real-world stock prices, publicly available, and try to predict option prices.

III. STATE OF THE ART

Models to price American options, derived from the results of Black and Scholes, have also been proposed, among which the work of Longstaff and Schwartz are paramount.

Nowadays, computers have become fundamental tools in the study of stock

IV. OBJECTIVES

V. STATE OF THE ART

VI. COMMENTED BIBLIOGRAPHY

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VII. TIMETABLE