Universidade do Minho
Escola de Ciências

Departamento de Matemática

Cálculo

Mestrado Integrado em Engenharia Informática

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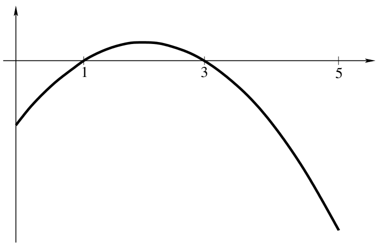
Exercício 8.1 Considere a função $f : [0,5] \to \mathbb{R}$ representada graficamente na figura ao lado e seja $F : [0,5] \to \mathbb{R}$ uma função primitiva de f.



a) Encontre os pontos críticos de F.

b) Classifique os pontos críticos de F.

Exercício 8.2 Calcule os seguintes integrais indefinidos:

a) $\displaystyle\int (3x^2 - 2x^5)\, dx$

b) $\displaystyle\int (\sqrt{x}+2)^2\, dx$

c) $\displaystyle\int (2\theta + 10)^{20}\, d\theta$

d) $\displaystyle\int x^4(x^5+10)^9\, dx$

e) $\displaystyle\int y^2 e^{y^3}\, dy$

f) $\displaystyle\int \sqrt{2x+1}\, dx$

g) $\displaystyle\int \frac{2x+1}{x^2+x+3}\, dx$

h) $\displaystyle\int \frac{t}{3-t^2}\, dt$

i) $\displaystyle\int \frac{1}{4-3\,x}\, dx$

j) $\displaystyle\int \operatorname{th} x\, dx$

k) $\displaystyle\int \frac{1}{e^{3x}}\, dx$

l) $\displaystyle\int \frac{-7}{\sqrt{1-5x}}\, dx$

m) $\displaystyle\int \frac{\sqrt{1+3\,\ln a}}{a}\, da$

n) $\displaystyle\int z\ \operatorname{sen} z^2\, dz$

o) $\displaystyle\int \frac{1}{x(\ln^2 x+1)}\, dx$

p) $\displaystyle\int \left(\frac{2}{x}-3\right)^2 \frac{1}{x^2}\, dx$

q) $\displaystyle\int \operatorname{sen}(\pi - 2\,x)\, dx.$

Exercício 8.3 Usando primitivação por partes calcule:

a) $\displaystyle\int \ln x\, dx$

b) $\displaystyle\int x\ \operatorname{sen}(2x)\, dx$

c) $\displaystyle\int \operatorname{arctg} x\, dx$

d) $\displaystyle\int x \cos x\, dx$

e) $\displaystyle\int \ln(1-x)\, dx$

f) $\displaystyle\int x\ \ln x\, dx$

g) $\displaystyle\int x^2 \operatorname{sen} x\, dx$

h) $\displaystyle\int x\ \operatorname{sen} x\ \cos x\, dx$

i) $\displaystyle\int \ln^2 x\, dx$

j) $\displaystyle\int e^x \cos x\, dx$

k) $\displaystyle\int \operatorname{arcsen} x\, dx$

l) $\displaystyle\int e^{\operatorname{sen} x}\ \operatorname{sen} x\ \cos x\, dx$

m) $\displaystyle\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}\, dx$

n) $\displaystyle\int x\ \operatorname{arctg} x\, dx$

o) $\displaystyle\int x^2\ \ln x\, dx$

p) $\displaystyle\int \operatorname{sen}(\ln x)\, dx$

q) $\displaystyle\int \operatorname{ch} x\ \operatorname{sen}(3x)\, dx$

r) $\displaystyle\int x^3\, e^{x^2}\, dx$

Exercício 8.4 Determine F, uma primitiva da função f, sabendo que $F(1) = 0$. A solução encontrada é única?

a) $f(x) = \operatorname{sen} x\ \cos x$

b) $f(x) = \operatorname{sen}(2x)\ \cos x$

c) $f(x) = \operatorname{sen}^2 x$

d) $f(x) = \dfrac{1}{x}, x > 0$

Exercício 8.5 Sendo $f : \mathbb{R} \to \mathbb{R}$ definida por $f(x) = x^2\ \operatorname{sen} x$, encontre a primitiva de f cujo gráfico passa pelo ponto $(\frac{\pi}{2}, \pi)$.

Exercício 8.6 Calcule os seguintes integrais indefinidos usando a substituição indicada:

a) $\displaystyle\int x\sqrt{x-1}\,dx, \quad x = t^2+1$

b) $\displaystyle\int \sqrt{1-x^2}\,dx, \quad x = \operatorname{sen} t$

c) $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx, \quad x = \ln t$

d) $\displaystyle\int \sqrt{1+x^2}\,dx, \quad x = \operatorname{sh} t$

Exercício 8.7 Calcule os seguintes integrais indefinidos:

a) $\displaystyle\int \frac{3x^2-4x-1}{(x^2-1)(x-2)}\,dx$

b) $\displaystyle\int \frac{2x^2+x+1}{(x-1)(x+1)^2}\,dx$

c) $\displaystyle\int \frac{4x^2+x+1}{x^3-x}\,dx$

d) $\displaystyle\int \frac{27}{x^4-3\,x^3}\,dx$

e) $\displaystyle\int \frac{x^4-8}{x^3-2\,x^2}\,dx$

f) $\displaystyle\int \frac{x+3}{(x-2)\,(x^2-2\,x+5)}\,dx$

Exercício 8.8 Calcule os seguintes integrais indefinidos.

a) $\displaystyle\int \frac{x}{x^2-1}\,dx$

b) $\displaystyle\int \frac{x}{\sqrt{x^2-1}}\,dx$

c) $\displaystyle\int \frac{1}{x}\ \operatorname{sen}(\ln x)\,dx$

d) $\displaystyle\int \frac{-3}{x\,(\ln x)^3}\,dx$

e) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

f) $\displaystyle\int \frac{e^x}{1-2e^x}\,dx$

g) $\displaystyle\int \frac{1}{\cos^2(7x)}\,dx$

h) $\displaystyle\int \left(\sqrt{2x-1}-\sqrt{1+3x}\right)\,dx$

i) $\displaystyle\int \frac{1}{x}\,\left(1+\ln^2 x\right)\,dx$

j) $\displaystyle\int \frac{2+\sqrt{\operatorname{arctg}(2x)}}{1+4x^2}\,dx$

k) $\displaystyle\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx$

l) $\displaystyle\int \frac{\operatorname{sen} x}{\sqrt{1+\cos x}}\,dx$

m) $\displaystyle\int \frac{1}{(2+\sqrt{x})^7\,\sqrt{x}}\,dx$

n) $\displaystyle\int \operatorname{tg}^2 x\,dx$

o) $\displaystyle\int \frac{x+\operatorname{arcsen}^4(3x)}{\sqrt{1-9\,x^2}}\,dx$

p) $\displaystyle\int \frac{x\,e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}\,dx$

q) $\displaystyle\int \frac{1}{\cos^2 x\ \operatorname{sen}^2 x}\,dx$

r) $\displaystyle\int \cos^2 x\ \operatorname{sen}^2 x\,dx$

s) $\displaystyle\int \frac{1}{1+e^x}\,dx$

t) $\displaystyle\int \frac{1}{x\sqrt{x^2-1}}\,dx$

u) $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$

Exercício 8.9 Em cada alínea, determine a única função $f:\mathbb{R}\to\mathbb{R}$, duas vezes derivável, tal que:

a) $f''(x) = 4x-1, \quad x\in\mathbb{R}, \quad f(1)=3$ e $f'(2)=-2$;

b) $f''(x) = \operatorname{sen} x\cos x, \quad x\in\mathbb{R}, \quad f(0)=0$ e $f'(0)=1$.