Generalized Fuzzy Logic for Incomplete Information

P. Venkata Subba Reddy
Department of Computer Science and Engineering
Sri Venkateswara University
Tirupati, India
e-mail:pvsreddy@hotmail.co.in

***Abstract*— Zadeh defined fuzzy sets for incomplete information with single fuzzy membership . REN Ping has defined Generalized fuzzy set with two fold membership functions "True" and "False". In this paper Zadeh fuzzy logic is extended to REN Ping generalize fuzzy logic for incomplete information. Generalized fuzzy logic , fuzzy inference and fuzzy reasoning are discussed using Feneralized fuzzy sets. Generalized Fuzzy Certainty Factor(GFCF) is studied as the difference of "True " and "False" fuzzy membership functions to eliminate the conflict of evidence in Incomplete Information. The fuzzy truth variables are also discussed for Generalised fuzzy sets.**

Keywords*— *Generalized fuzzy logic, Generalized fuzzy inference and reasoning, Generalized fuzzy certainty factor

I. INTRODUCTION

Different methods are proposed to deal with incomplete information such as fuzzy logic[22], Bayesian Statistics[14] , Dempster-Shafer Theory [1] , Certainty Factor [1], Many-Valued Logic[12] , fuzzy Statistics [14] , Possibility theory[4]. Many theories based on probable of the likelihood , where as fuzzy logic deals with belief for incomplete information. The fuzzy sets, fuzzy inference and fuzzy reasoning are studied to deal with incomplete information with single membership function [22]. The fuzzy set with two fold membership function will give more evidence to deal with the incomplete information rather than single membership function. REN Ping [11] defined generalized fuzzy set with two fold membership function "True" and "False" and Generalized fuzzy logic is studied.

In the following, Zadeh fuzzy logic of single membership function is discussed. And also Zadeh fuzzy logic is extended to REN Ping Generalized fuzzy logic for two fold fuzzy sets "True " and "False". The Generalized fuzzy conditional inference and reasoning are studied based on Zadeh fuzzy logic. The Generalized Fuzzy Certainty Factor is defined by the difference between "True" and "False " membership function to made as single fuzzy membership function. The fuzzy truth variables are studied for Generalized fuzzy sets.

II. FUZZY LOGIC, FUZZY INFERENCE AND FUZZY REASONING

Zadeh[22] has introduced fuzzy set as a model to deal with imprecise, inconsistent and inexact information. fuzzy set is a class of objects with a continuum of grades of membership.

The fuzzy set A of X is characterizedas its membership function A $= \mu_A(x)$ and ranging values in the unit interval [0, 1]

$\mu_A(x)$: X →[0, 1], x Є X, where X is Universe of discourse.

A $= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$, "+" is union

For instance, the fuzzy proposition "x is Young"

Young = { 0.95/10 + 0.9/20 + 0.85/30+0.8/40 +0.55/50 + 0.4/60+0.3/70 +0.2/80+0.1/90 }

not Young ={ 0.05/10 + 0.1/20 + 0.15/30+0.2/40 +0.45/50 + 0.6/60+0.7/70 +0.8/80+0.9/90 }

For instance the fuzziness of "Rama who is 40 years old is YOUNG" is 0.8

The Graphical representation of young and not Young is shown in fig.1

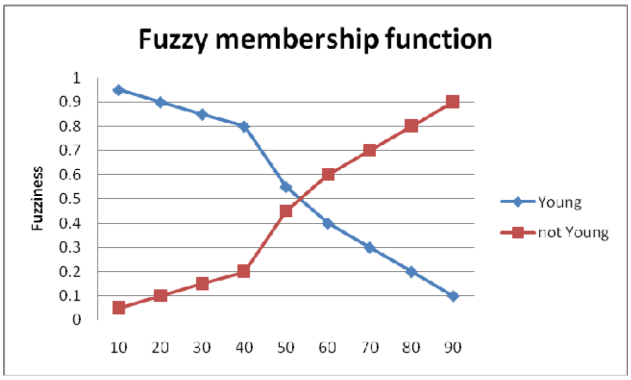

Fig.1. Fuzzy membership function

The fuzzy logic is defined as combination of fuzzy sets using logical operators. Some of the logical operations are given below

Let A, B and C are fuzzy sets. The operations on fuzzy sets are given as

Negation

If x is not A

$A' = 1 - \mu_A(x)/x$

Conjunction

x is A and y is B→ (x, y) is A x B

A x B=min($\mu_A(x)$, $\mu_B(y)$}(x,y)

If x=y

x is A and y is B→ (x, y) is A ΛB

AΛB=min($\mu_A(x)$, $\mu_B(y)$}/x

Disjunction

x is A or y is B→ (x, y) is A' x B'

$A' \times B' = \max(\mu_A(x), \mu_B(y)\}(x,y)$
If x=y
x is A and y is B→ (x, y) is A V B
$AVB = \max(\mu_A(x), \mu_B(y)\}/x$

Implication

if x is A then y is B
$A \rightarrow B = \min\{1, 1-\mu_A(x) + \mu_B(y)\}/(x,y)$
If x is A then y is B else y is C= A x B + A' x C
$= \{\min(\mu_A(x), \mu_B(y), \min(1-\mu_A(x), \mu_C(y)\}(x,y)$

if x is A then x is B else y is C may be separated in to two clauses "if x is A then y is B" and "if x is not A then y is C"[16]

$A \rightarrow B = \min\{1, 1-\mu_A(x) + \mu_B(y)\}/(x,y)$ if A
$A' \rightarrow C = \min\{1, \mu_A(x) + \mu_C(y)\}/(x,y)$ if not A

Composition

$A \text{ o } R = \min_x \{\mu_A(x), \mu_R(y)\}/(x,y)$, where R=A→B
If x = y
$A \text{ o } B == \min\{\mu_A(x), \mu_B(y)\}/(x,y)$

The fuzzy propositions may contain quantifiers like “very”, “more or less” . These fuzzy quantifiers may be eliminated as

Concentration

$\mu_{very}(x) = \mu_A(x)^2$

Diffusion

$\mu_{\text{more or less}}(x) = \mu_A(x)^{0.5}$

Fuzzy Truth Variables

x is A is T
x is $\mu_A(x)^{-1}$oT, where $\mu_A(x)^{-1}$ inverse of compatibility function A and T is truth variable.

The fuzzy inference is a drawing conclusion from fuzzy propositions using fuzzy inference rules[21]

Some of the fuzzy inference rules are given bellow

R1: x is A
x is A and y are B
____
Y is A o B

R2: x is A
x is A or y is B
____
Y is A + B
Where “ +” union.

R3: x and y are A
y and z are B
____
y and z are A o B

R4: x is A
if x is A then y is B
____
y is A o (A→B)

R5: x is A is T1
if x is A then y is B is T2
____
y is A o (A→B) is T1 o T2
where T1 and T2 are truth variables.

R6: x is A if A
if x is A then y is B then y is C
____
y is A o (A→B)

R7: x is A' if not A
if x is A then y is B then y is C
____
z is A' o (A'→C)

R6: x is A is T1 if A
if x is A then y is B then y is C is T2
____
y is A o (A→B) is T1 o T2

R7: x is A' is T1 if not A
if x is A then y is B then y is C is T2
____
z is A' o (A'→C) is T1 o T2
where T1 and T2 are truth variables.

III. GENERALIZED FUZZY LOGIC FOR INCOMPLETE INGORMATION

REN Ping[11] studied Generalized fuzzy logic with two fold fuzzy set with logical operations. Zadeh fuzzy logic is exted to Generalized fuzzy logic in the following.

A. Generalized fuzzy set

The fuzzy set for proposition “ x is A” is defined as
$A = \mu_A(x)$, where A is fuzzy set and x Є X, $\mu_A(x)$ is fuzzy membership function.

REN Ping [11] define fuzzy set with two fold membership funxtion using True and False.

Given some Universe of discourse X, the proposition “ x is A” is definedas its two fold fuzzy membership function as
$\mu_A(x) = \{\mu_A^{True}(x), \mu_A^{False}(x)\}$
or
$A = \{\mu_A^{True}(x), \mu_A^{False}(x)\}$
Where A is Generalized fuzzy set and x Є X ,
$0 <= \mu_A^{True}(x) <= 1$ and, $0 <= \mu_A^{False}(x) <= 1$
$A = \{ \mu_A^{True}(x_1)/x_1 + \ldots + \mu_A^{True}(x_n)/x_n$,
$\mu_A^{False}(x_1)/x_1 + \ldots + \mu_A^{True}(x_n)/x_n$, x_i Є X, “+” is union
$\mu_A^{True}(x) + \mu_A^{False}(x) < 1$,
$\mu_A^{True}(x) + \mu_A^{False}(x) > 1$ and
$\mu_A^{True}(x) + \mu_A^{False}(x) = 1$
are interpreted as redundant, insufficient and sufficient Knowledge respectively.

For example ‘ x is Young” , Young may be given as
$Young = \{\mu_{Young}^{True}(x), \mu_{Young}^{False}(x)\}$
={.95/10+0.9/20+0.8/30+0.6/40+0.4/50+0.3/60+0.2/70+0.15/80+0.1/90,0.3/10+0.25/20+02/30+0.15/40+0.1/50+0.06/60+0.0470+0.02/80+0.01/90]

For instance the fuzziness of “Rama who is 40 years old is YOUNG” is {0.8,02},

The Graphical representation of fuzzy membership functions “True” and “False” of “ Young” are shown in fig.2

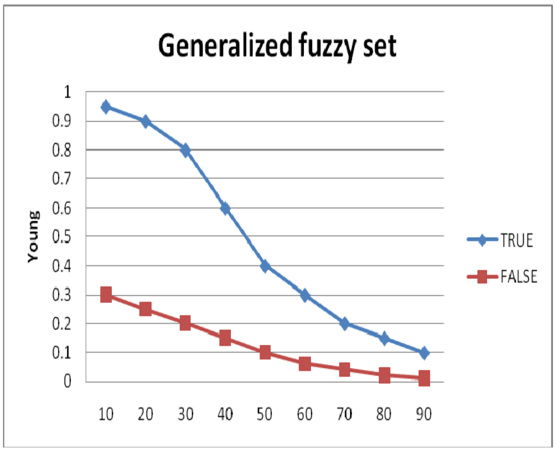

Fig.2.Generalized fuzzy set

The logical operations on Generalized fuzzy set defined as [11]

$A' = \{\mu_A^{False}(x), \mu_A^{True}(x)\}$

$AVB=\{\max(\mu_A^{True}(x), \mu_B^{True}(x)), \min(\mu_A^{False}(x), \mu_B^{False}(x))\}$

$A\Lambda B=\{\min(\mu_A^{True}(x), \mu_B^{True}(x)), \max(\mu_A^{False}(x), \mu_B^{False}(x))\}$

The situation like "very Young" and " more or less" may be defined as

$\mu_{very\ young}(x)=\{\mu_{very\ young}^{True}(x), \mu_{very\ yong}^{False}(x)\}$

$\mu_{more\ or\ less\ young}(x)=\{\mu_{more\ or\ less\ young}^{True}(x), \mu_{more\ or\ less\ young}^{False}(x)\}$

where very and more or less are fuzzy quantifiers.

B. Extention of Zadeh fuzzy logic to Generalized fuzzy logic

Since formation of the generalized fuzzy set simply as two fold fuzzy set, Zadeh fuzzy logic is extended to these generalized fuzzy sets.

The fuzzy logic is defined as combination of fuzzy sets using logical operators. Some of the logical operations are given below

Suppose A, B and C are fuzzy sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Negation

$A'= \{1- \mu_A^{True}(x), 1- \mu_A^{False}(x) \}/x$

Disjunction

$AVB=\{ \max(\mu_A^{True}(x), \mu_A^{True}(y)), \max(\mu_B^{False}(x), \mu_B^{False}(y))\}(x,y)$

Conjunction

$A\Lambda B=\{ \min(\mu_A^{True}(x), \mu_A^{True}(y)), \min(\mu_B^{False}(x), \mu_B^{False}(y))\}/(x,y)$

Implication

$A\rightarrow B= \{\min (1, 1- \mu_A^{True}(x) + \mu_B^{True}(y), \min(1, 1- \mu_A^{False}(x) + \mu_B^{False}(y)\}(x,y)$

If A then B else C =

$A\rightarrow B = \{\min (1, 1- \mu_A^{True}(x) + \mu_B^{True}(y), \min(1, 1- \mu_A^{False}(x) + \mu_B^{False}(y)\}/(x,y)$ if A

$A'\rightarrow C = \min (1, \mu_A^{True}(x) + \mu_C^{True}(y), \min(1,\mu_A^{False}(x) + \mu_C^{False}(y)\}(x,y)$ if not A

Composition

$A \circ R= \{\min_x (\mu_A^{True}(x), \mu_A^{True}(x)), \min_x(\mu_R^{False}(x), \mu_R^{False}(x))\}/y$

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

Concentration

"x is very A

$\mu_{very\ A}(x) = \{ \mu_A^{True}(x)^2, \mu_A^{False}(x)\mu_A(x)^2 \}$

Diffusion

"x is more or less A"

$\mu_{more\ or\ less\ A}(x) = (\mu_A^{True}(x)^{1/2}, \mu_A^{False}(x)\mu_A(x)^{0.5}$

$A = \{ 0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 +0.5/x_5 , 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 +0.6/x_5\}$

$B = \{ 0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0..5/x_4 +0.6/x_5 , 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 +0.7/x_5\}$

$A V B = \{ 0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 +0.6/x_5 , 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 +0.7/x_5\}$

$A \Lambda B = \{ 0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 +0.5/x_5 , 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 +0.6/x_5\}$

$A' = \text{not } A= \{ 0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 +0.5/x_5 , 0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 +0.4/x_5\}$

$A\rightarrow B = \{ 1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 +1/x_5 , 1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 +1/x_5\}$

$A \circ B = \{ 0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 +0.5/x_5 , 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 +0.6/x_5\}$

$\mu_{very\ A}(x) = \{ \mu_A^{True}(x)^2, \mu_A^{False}(x)\mu_A(x)^2 \}$

$= \{ 0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 +0.25/x_5 , 0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 +0.36/x_5$

$\mu_{more\ or\ less\ A}(x) = (\mu_A^{True}(x)^{1/2}, \mu_A^{False}(x)\mu_A(x)^{1/2} \}$ Diffusion

$= \{ 0.89/x_1 + 0.94/x_2 + 0.83/x_3 + 0.77/x_4 +0.70/x_5 , 0.63/x_1 + 0.54/x_2 + 0.63/x_3 + 0.83/x_4 +0.77/x_5\}$

C. Generalized fuzzy inference and Reasoning

The fuzzy reasoning is drawing conclusions from fuzzy propositions using fuzzy inference rules[21]. Some of the fuzzy inference rules are given below for the propositions with two fold fold membership function

I1: x is A
x and y are B

---

y is AoB

$A \circ B= \{\min_x (\mu_A^{True}(x), \mu_A^{True}(x,y)), \min_x(\mu_B^{False}(x), \mu_B^{False}(x,y))\}$

x is very small
x and y are Approximately equal

---

y is very small o Approximately equal

I2: x and y are A
x and z are B

---

y and z are A o B

$A\circ B = \{\min (\mu_A^{True}(x,y), \mu_A^{True}(y,z)), \min (\mu_B^{False}(x,y), \mu_B^{False}(y,z))\}$

x and y are more or less equal
y and z are Approximately equal if ` A

---

y and z are more or less equal o Approximately equal Implication

I3: x is A
if x is A then y is B Composition

$y \text{ is } A \circ (A\rightarrow B)$

A o (A→B) =Ao{min (1, 1- $\mu_A^{True}(x)$ + $\mu_B^{True}(y)$, min (1, 1- $\mu_A^{False}(x)$ + $\mu_B^{False}(y)$}
x is very small
If x small then y is medium

---

y is very small o (small → medium)
I4: x is A if A
if x is A then y is B else y is C

---

y is A o (A→B)
A o (A→B) =Ao {min (1, 1- $\mu_A^{True}(x)$ + $\mu_B^{True}(y)$, min (1, 1- $\mu_A^{False}(x)$ + $\mu_B^{False}(y)$}
x is very small
If x small then y is medium else z is large

---

y is very small o (small → medium)
I5: x is A' if not A
if x is A then y is B else y is C

---

z is A' o (A'→C)
A' o (A'→C) =A'o {min (1, $\mu_A^{True}(x)$ + $\mu_C^{True}(y)$, min (1, $\mu_A^{False}(x)$ + $\mu_C^{False}(y)$}
is not very small
If x small then y is medium else z is large

---

y is not very small o (not small → large)

IV. GRNERALIZED FUZZY CERTAINTY FACTOR

The fuzzy set A of X is characterizedas its membership function A = $\mu_A(x)$ and ranging values in the unit interval [0, 1]
$\mu_A(x)$: X →[0, 1], x ∈ X, where X is Universe of discourse.
$A=\mu_A(x) = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$,
"+" is union

The Certainty factor in Mycin defined as
CF[h,e]=MB[h,e]-MD[h,e], where MB[h,e] and MD[h,e] are Probabilities.

The Generalized Fuzzy Cirtanity Factor (GFCF) is defined as
CF[h,e]=MB[h,e]-MD[h,e],
where MB[h,e] and MD[h,e] are Probabilities
$\mu_A^{GFCF}(x) = \mu_A^{True}(x) - \mu_A^{False}(x)$
MB[x, A]= $\mu_A^{True}(x)$ and MD[x,A}= $\mu_A^{False}(x)$
MB[x,A] and MD[x,B] are fuzzy functions
MB[x,A]= ∫MB[h,e] $\mu_A^{True}(x)$ /x
MD[x,A]= ∫MB[h,e] $\mu_A^{False}(x)$ /xMB[x,A] and MD[x,B] are fuzzy probabilities

$$\mu_A^{GFCF}(x) = \mu_A^{True}(x) - \mu_A^{False}(x) \quad \mu_A^{True}(x) \geq \mu_A^{False}(x)$$
$$= 0 \quad \mu_A^{True}(x) < \mu_A^{False}(x)$$

The fuzzy certainty factor becomes single fuzzy membership function.
$\mu_A^{GFCF(x)}$:X→[0, 1], x ∈ X, where X is Universe of discourse.

The Generalized Fuzzy Certainty Factor(GFCF) will compute the conflict of evidence in the Uncertain Information.

For Example
$\mu_{Aoung}^{GFCF}(x)$ =
{.95/10+0.9/20+0.8/30+0.6/40+0.4/50+0.3/60+0.2/70+0.1/80 + 0.06/90}-
{0.2/10+0.2/20+0.15/30+0.08/40+0.09/50+0.04/60+0.02/70+0.01/80 +0.02/90}
=0.75/10+0.7/20+0.65/30+0.52/40+0.31/50+0.26/60+0.18/70 + 0.09/80+0.04/90}

For instance "Rama is Young" with fuzziness {0.9,0.2}

The GFCF of of "Rama who is 40 years old is YOUNG" is 0.7

The Graphical representation of GFCF is shown in fig.3

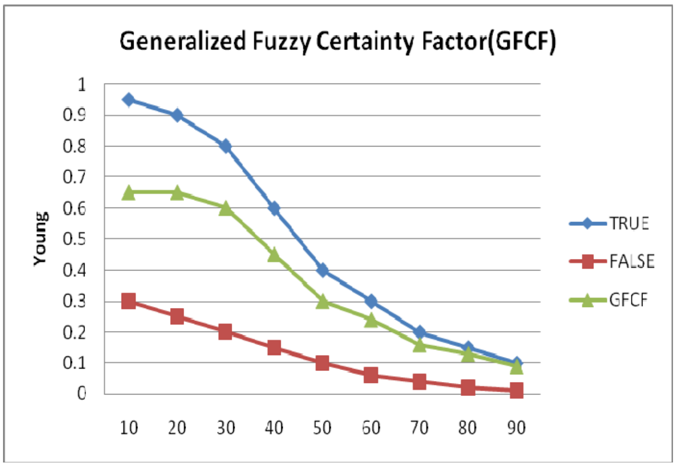

Fig.3. Generalized Fuzzy Certainty Factor

V. FUZZY TRUTH VARIABLES

Zadeh[21] defined fuzzy truth values for the proposition of type " x is A is T" where T is truth variable like true, false, very true etc.

The fuzzy inference is given as x is $\mu_A(x)^{-1}$oT where $\mu_A(x)^{-1}$ is inverse of compatibility function A and T is truth variable .

For instance, the inference for "x is young is very true" is given as

x is $\mu_{young}(x)^{-1}$o very true

Generalizef fuzzy set is straight forward to fuzzy truth variables.

The proposition "x is A is T " may be represented as $\mu_A(x)^{-1}$oT , where A ={ $\mu_A^{True}(x)$, $\mu_A^{False}(x)$}

The truth variables apply on respective true or false functions

For instance "x is young is true" is given as

A ={ $\mu_{young}^{True}(x)$, $\mu_{young}^{False}(x)$}

The fuzzy truth vales may contain quantifiers

"x is A is very true" is given as
A ={ $\mu_{very\ A}^{True}(x)$, $\mu_A^{False}(x)$}

"x is A is very false" is given as
A ={ $\mu_A^{True}(x)$, $\mu_{very\ A}^{False}(x)$}

For instance, "Rama is young is very true" is given as
A ={ $\mu_{very\ young}^{True}$(Rama) , μ_{young}^{False}(Rama)}

"Rama is young is very false" is given as
A ={ μ_{young}^{True}(Rama) , $\mu_{very\ young}^{False}$(Rama)}

VI. GENERALIZED FUZZY LOGIC APPLICATION TO MEDICAL DIAGNOSIS

The fuzzy set with two fold membership function is applicable to where the commonsense knowledge is applied. Medical diagnosis is considered as an example to deal with commonsense knowledge.

EMYCIN[1] is Medical expert system shell in which medical diagnosis is directly defined with two fold fuzzy membership function.

The Certainty Factor defined in MYCIN [1]

CF[h,e]= MB[h,e]-MD[h,e]

The Generalized Fuzzy Certainty Factor(GFCF) is defined based on MB[h,e] and MD[h,e],

MB[x,A]= $\mu_A^{True}(x)$ and

MD[x,A]=$\mu_A^{False}(x)$ are fuzzy functions.

$\mu_A^{GFCF}(x) = \{ \mu_A^{True}(x) - \mu_A^{False}(x) \}$

for instance " x has fever"

The GFCF for fever given as

$\mu_{fever}^{GFCF}(x) = \{ \mu_{fever}^{True}(x) - \mu_{fever}^{False}(x) \}$

Consider the rule in medical diagnosis

If the patient hasfever
and rash
and body ache
and chills
Then the patient has chickenpox

For instance, Fuzziness may be given for symptoms and diagnosis as

IF the patient fever (0.9, 0.2)
AND rash(0.8, 0.2)
AND body ache(0.7, 0.1)
AND chills(0.9, 0.1)
THEN the patient has chickenpox(0.7, 0.2)
Cf1.7,cf2.6,cf3=.6,cf4=.8 and cf5=.5

Using LISP Programming the GFCF may be computed as

```
(defun CF ( cf1 cf2 cf3 cf4 cf5)
( min 1 ( + (- 1 (min cf1 cf2 cf3 cf4)) cf5))) ( CF .7 .6 .6 .8
.5 ) is given as 0.9
```

The above fuzzy rule may be interpreted in EMYCIN as

```
IF (AND
(SAME CNTEXT FEVER)
(SAME CNTEXT RASH )
(SAME CNTEXT BODY-ACHE) THEN
(CONCLUDE CNTEXT CHICKEN-POX TALLY 0.9)
```

The EMYCIN will diagnose chicken-pox with fuzzy certainty factor 0.9.

VII. CONCLUSION

Various theories are proposed to deal with uncertainty. Many theories based on Probability of the likelihood, where as fuzzy logic based on belief. Zadeh fuzzy logic is extended to REN Ping generalized fuzzy set. The Generalized fuzzy logic, fuzzy Inference and fuzzy reasoning are studied based on Zadeh fuzzy logic . The Generalized fuzzy Certainty Factor is studied to define as single fuzzy membership function to compute the conflict of evidence in the incomplete information. The fuzzy truth variables are also studied for Generalized fuzzy set. The generalized fuzzy set theory is interpreted in EMYCIN for fuzzy medical diagnosis as an application.

ACKNOWLEDGMENT

The Author expressing thanks to referees and Program Chair Bernadette Bouchon accepted for presentation at the 2013 IEEE International Conference and Reviewers for their valuable suggestions.

REFERENCES

[1] B.G. Buchanan and E.H. Shortliffe, Rule-Based Expert System: The MYCIN Experiments of the Stanford Heuristic Programming Project, Readings, Addition-Wesley, M.A, 1984.

[2] B. Bouchon-Meunier, R.R. Yager, L.A. Zadeh (eds), Uncertainty in Intelligent and Information Systems, Advances in Fuzzy Systems—Applications and Theory, World Scientific, Singapore, 2000.

[3] Dubois and H. Prade, "On several representations of an uncertain body of evidence", In: Fuzzy information and decision processes, M.M.Gupta and E. Sanchez (Eds.), North-Holland, 1982,pp.167-181.

[4] Dubois and H. Prade, Representation and combination of uncertainty with belief functions and possibility measures, Computational Intelligence vol.4, pp.244-264, 1999.

[5] Dubois and H. Prade, , Non-Standard Theories of Uncertainty in Knowledge Representation and Reasoning, KR, 1994,pp. 634-645.

[6] Dubois, H. Fargier and H. Prade, Comparative uncertainty, belief functions and accepted beliefs, UAI, pp.113-120.

[7] D. Dubois and H. Prade, On the use of aggregation operations in information fusion processes, Fuzzy Sets and Systems, vol.142(1) , pp.143-161,2004.

[8] A Kaufmann and M.M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Applications, New York: Von Nostrand , 1985.

[9] G.J. Klir, Generalized information theory: aims, results, and open problems, Reliability Engineering and System Safety, vol.85(1-3), pp.21-38, 2004.

[10] E.H. Mamdani and S. Assilian. An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller, Int. J. Man-Machine Studies, vol.7,pp.1-13, 1975.

[11] REN Ping, "Generalized Fuzzy Sets and Representation of Incomplete Knowledge", Fuzzy Sets and Systems, vol.36,pp. 91-96, 1990.

[12] N. Rescher, Many-Valued Logic, McGrow-Hill, New York, 1969.

[13] H. Prade, " A Computational Approach to Approximate and Plausible Reasoning with Applications to Expert Systems", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol.7,pp. 3-29, 1985.

[14] Shafer, G, A Mathematical Theory of Evidence , Priceton, NJ, University Press, 1976.

[15] Shafer, G, and J. Pearl,Readings in Uncertainy Reasoning, Los Altos, CA, Morgan-Kaufmann, 1990.

[16] P. Venkata Subba reddy "Fuzzy Conditional Inference for Mediiagnosis", Second International Conference on Fuzzy Theory and Technology, Proceedings, Abstract and Summaries of FT&T1993, University of North-Carolina, Duke University, October 13-16, USA,,1993, pp.193-195.

[17] P. Venkata Subba reddy and M. Syam Babu, 'Some Methods of Reasoning for Conditional Propositions", Fuzzy Sets and Systems, vo.52,pp.229-250,1992.

[18] P. Venkata Subba Reddy, "Generalized Fuzzy Sets in Various Situations for Incomplete Knowledge", First International Conference on Fuzzy Theory and Technology, Proceedings, Abstract and Summaries of FT&T1992, University of North-Carolina, Duke University, October 14-16, 1992 ,USA,pp.181-183.

[19] R.R. Yager, Uncertainty representation using fuzzy measures, IEEE Trans. on Systems, Man and Cybernetics, Part B. vol.32,pp.213-20,2002.

[20] J. Yen and R. Langari, Fuzzy Logic: Intelligence, Control and Information, Prentice Hall, 1980.

[21] L. A Zadeh, "Calculus of Fuzzy Restrictions", In Fuzzy Sets and their Applications to Cognitive and Decision Processes, L. A. Zadeh, King-Sun FU, Kokichi Tanaka and Masamich Shimura (Eds.), Academic Press, New York, 1975,pp.1-40.

[22] L.A. Zadeh, Fuzzy sets, *In Control* , vol.1, pp.338-353, 1965.

[23] L.A. Zadeh, Generalized theory of uncertainty (GTU, Principal concepts and ideas Computational Statistics & Data Analysis,vol.51,pp.15-46.,2006.