Question 3: Assume you have a prior believe that the value of lamba in a Poisson process is uniformly distributed between 5 and 10 arrivals per minute. You then observe the process and count 7 arrivals in the first minute and 9 arrivals in the second minute. Use Bayes theorem to determine the posterior distribution of values of lambda.

Prior believe → λ = uniform (5, 10) arrivals / minute

~~t = minute~~

~~$\mu = \lambda / t$ = arrivals~~

Observation → 7 arrivals in the first minute

9 arrivals in the second minute

Bayes Theorem → $\underbrace{pr(B|A)}_{\text{posterior}} = \dfrac{\overbrace{pr(A|B)}^{\text{Likelyhood}}\ \overbrace{pr(B)}^{\text{Prior}}}{p(A)}$

$$pr(\lambda | \text{Observation}) = \frac{pr(\text{Observation} | \lambda)\ pr(\lambda)}{pr(\text{Observation})}$$

$$pr(\text{Observation}) = \underbrace{\frac{1}{\lambda}}_{\text{probability of getting 7 in the first min}} \cdot \overbrace{\frac{1}{\lambda}}^{\text{Probability of getting a 9 in the second min}} = \frac{1}{\lambda^2}$$

pr(λ) = Prior belief = uniform(5, 10)

pr(Observation | λ) = pr(7 | uniform(5,10)) · pr(9 | uniform(5,1

$= 1/5 \cdot 1/5 = 1/25$

$$pr(\lambda \mid \text{observation}) = \frac{\frac{1}{25} \cdot \text{uniform}(5,10)}{\frac{1}{\lambda^2}}$$

$$\boxed{pr(\lambda \mid \text{observation}) = \frac{\lambda^2 \, \text{uniform}(5,10)}{25}}$$