

Instructions - Complete each problem by hand. You can use Python to validate your results or as a calculator, but you can't just simulate your way to an answer for full points. Submit your work through Canvas in one or more files in any format that works for you. Scans/photos of hand written documents are fine if that's what you prefer, as long as I can read it.

Problem 1 (20 points)

A standard 6-sided die is rolled 1,000 times. Let random variables S be defined as the sum of the rolls and A be the average of the rolls.

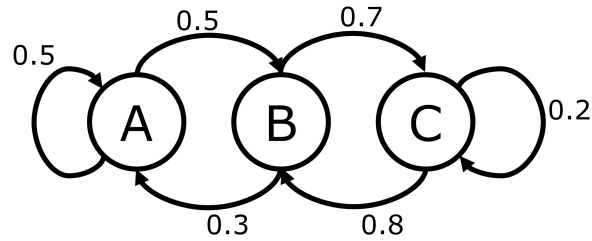
[a] What is the value of $E[A]$?

[b] What is the value of $Var[A]$?

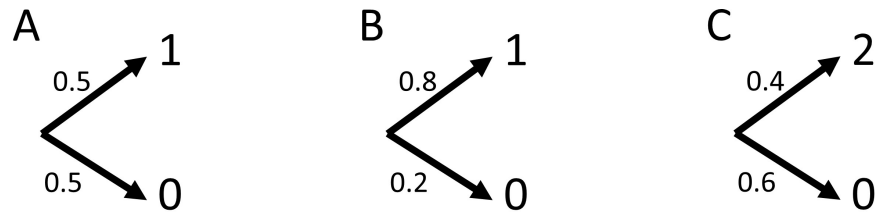
[c] What is a reasonable value for $P(3,500 \leq S \leq 3,550)$?

Problem 2 (25 points)

Consider a Hidden Markov Model with three known states:



Each state can produce the numbered observable outputs as shown here with their probabilities:



Finally, assume the system is guaranteed to start in State A.

- [a] Write out the state transition matrix.
- [b] What is the probability of observing outputs $[1, 0, 2]$?
- [c] What is the probability of observing outputs $[0, 0, 0]$?
- [d] If outputs $[1,1]$ are observed, what is the most likely sequence of hidden states?

Problem 3 (15 points)

Let $f(x) = 4/x^5$ for $x \geq 1$ and 0 everywhere else. Use the Chebyshev Inequality to give a bound for $P(X \leq 3)$. Be sure to indicate if it is a lower or upper bound.