Monday, August 22, 2022 9:41 AM

Random variable : variable whose value is random

unknown
unpredictable
no pattern
initial conditions effect

trials
distribution
independent / dependent
probability mass function
process

flip a coin twice count # heads

rv is # heads h

h	$p(h)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

h	pr
≤ 0	$\frac{1}{4}$
≤ 1	$3/4$
≤ 2	1

Expected value $\neq$ average
$\neq$ mean

$$E(x) = \sum_i x_i \cdot p(x_i) = 0 \cdot p(0) + 1 \cdot p(1) + 2 \cdot p(2)$$

$0 \cdot \frac{1}{4}$
$1 \cdot \frac{1}{2}$
$2 \cdot \frac{1}{4}$

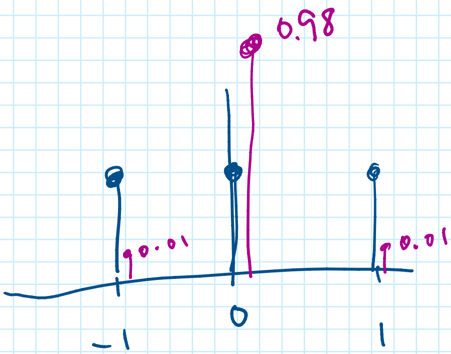
$= \boxed{1}$

Experiment 1 HH 2
2 TH 1

$$avg = \frac{2+1+1}{3} = \underline{\underline{1.33}}$$

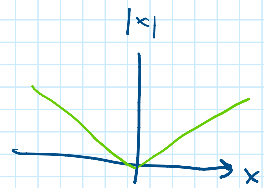
3 TH 1

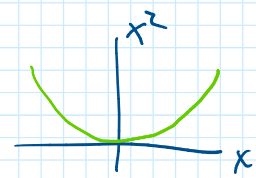
experimental average
mean



Variance tells us on average how much the rv's <u>vary</u> from the expected value

rv	pr	
-1	$\frac{1}{3}$	$(-1)^2$
0	$\frac{1}{3}$	0
1	$\frac{1}{3}$	$(1)^2$





$E(rv) = 0$

$E(rv) = 0$

$E(\cdot) = 0$

$$Var(x) = E\left((x - E(x))^2\right)$$

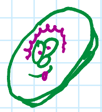
$$= E\left(x^2 - 2x E(x) + E^2(x)\right)$$

$$= E(x^2) - E(2x E(x)) + E(E^2(x))$$

$$-2\underbrace{E(x) \cdot E(x)}_{-2\ E^2(x)} + E^2(x)$$

$$\boxed{Var(x) = E(x^2) - E^2(x)}$$

h	p(h)	h-E(h)^2	h^2



$p(H) = 0.6$

nFlips $= 3$

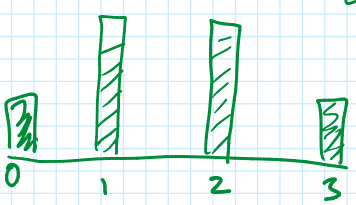
h : # heads

h	$p(h)$	$h-E(h)^2$	h^2
0	0.4^3	$(0-1.8)^2$	0
1	$3 \times 0.6 \times 0.4^2$	$(1-1.8)^2$	1
2	$3 \times 0.6^2 \times 0.4$	$(2-1.8)^2$	4
3	0.6^3	$(3-1.8)^2$	9

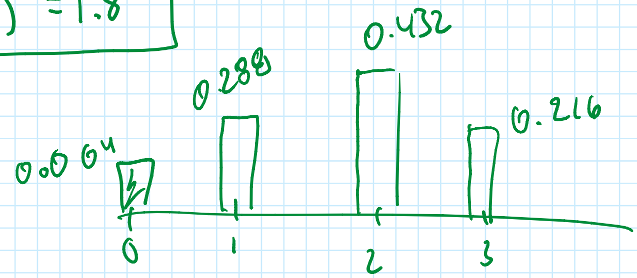
$3 \times 0.6 \times 0.4 \times 0.4$

$3 \times 0.6 \times 0.6 \times 0.4$

$$\boxed{E(h) = 1.8}$$



$p = \frac{1}{2}$



$E(h^2) = 3.96$

$$Var(h) = E(h^2) - E^2(h)$$

$$= 3.96 - (1.8)^2 = \boxed{0.72}$$

			h	p(h)	h^2	(h-E)^2
p			0	0.064	0	3.24
0.6			1	0.288	1	0.64
			2	0.432	4	0.04
			3	0.216	9	1.44
		E(h)		1.8		
		E(h^2)		3.96		
		E((h-E)^2)		0.72		
		E(h^2)-E(h)^2		0.72		

h	$Pr(h)$
0	$(1-p)^3$
1	$3p(1-p)^2$
2	$3p^2(1-p)$
3	p^3

$$E(h) = 0 \cdot (1-p)^3 + 1 \cdot (3p(1-p)^2) + 2 \cdot 3p^2(1-p) + 3 \cdot p^3$$

1 coin flip

h	Pr
0	$1-p$
1	p

$$E(h) = p$$

$$E(h^2) = p$$

$$Var(h) = p - p^2 = p(1-p)$$

3 flips

$$E(3 \text{ flips}) = E(h) + E(h) + E(h) = 3p$$

$$Var(3 \text{ flips}) = V(h) + V(h) + V(h) = 3p(1-p)$$

$$\frac{d}{dp} Var = 3(1-p) - 3p = 0$$

$$3 - 3p - 3p = 0$$

$$6p = 3$$

$$p = \frac{1}{2}$$