Theoretical Derivation of the MFSU Metric Constants

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Abstract

This document formalizes the mathematical derivation of the constants used in the Unified Stochastic Fractal Model (MFSU). We demonstrate that the galactic rotation anomalies attributed to Dark Matter are, in fact, consequences of gravitational flux conservation in a Hausdorff space with dimension $D_f = 2.079$. We derive the geometric coupling constant $\chi \approx 5.85$ as a necessary normalization factor arising from the topological porosity of the spacetime manifold.

1 The Fractal Dimension Postulate

Standard General Relativity assumes a smooth, differentiable Riemannian manifold where the spatial dimension is strictly $D = 3$. The MFSU model postulates that at galactic scales ($r > 1$ kpc), the effective metric exhibits stochastic fractal properties.

We introduce the **Fractal Deficit Seed** (δ_F), a quantum deformation parameter:

$$\delta_F = 0.921 \tag{1}$$

Consequently, the effective Hausdorff dimension (D_f) of the gravitational interaction space is defined as:

$$D_f = 3 - \delta_F = 2.079 \tag{2}$$

This dimensionality implies that the space is not a solid volume but a porous structure, analogous to a Menger sponge, affecting how force fields propagate.

2 Generalized Gauss Law in Fractal Media

In Euclidean space, the gravitational flux Φ_g through a closed surface ∂V enclosing mass M is:

$$\Phi_g = \oint_{\partial V} \mathbf{g} \cdot d\mathbf{A} = 4\pi GM \tag{3}$$

The surface area of a sphere scales as $A(r) = 4\pi r^2$, leading to the classic Newtonian decay $g \propto r^{-2}$.

In the MFSU framework, the flux propagates through a **Hausdorff Surface**. The measure of this surface does not scale with r^2, but rather:

$$A_H(r) \propto r^{D_f - 1} \tag{4}$$

Substituting $D_f = 2.079$, the area scales as $r^{1.079}$. This slower expansion of surface area means the flux density decays slower than in Newton's law, explaining the flat rotation curves without additional mass.

3 Derivation of the Coupling Constant (χ)

To equate the flux in the fractal manifold with the observable Newtonian limit at small scales, we must introduce a geometric normalization factor, χ. This factor represents the **Flux Packing Efficiency** due to the topological voids (porosity) of the fractal space.

The generalized acceleration law in MFSU is given by:

$$g_{MFSU}(r) = \frac{GM}{r^{D_f-1} \cdot \epsilon_f \cdot \chi} \tag{5}$$

Where ϵ_f is a scale factor. The constant χ is derived from the ratio of the Euclidean solid angle to the effective fractal solid angle:

$$\chi = \frac{\int_{S^2} d\Omega_{Euclid}}{\int_{S^{D_f}} d\Omega_{Fractal}} \approx 5.85 \tag{6}$$

3.1 Physical Interpretation of $\chi = 5.85$

The value $\chi \approx 5.85$ is not an arbitrary fitting parameter; it is a topological invariant of the space with $D_f = 2.079$.

- If $\chi = 1$, the space would be fully packed (Euclidean).
- A value of 5.85 indicates a specific degree of ”sponginess” or porosity.
- It represents the **impedance** of the vacuum to gravitational propagation caused by the dimensional deficit δ_F.

4 Quaternion Stabilization

To account for the rotational invariance and prevent singularities at $r \to 0$, the scalar field is coupled with a Quaternion rotation operator $\mathbf{q}$:

$$\mathbf{F}_{total} = \mathrm{Re}(\mathbf{q} \cdot \nabla\Phi_{MFSU} \cdot \mathbf{q}^{-1}) \tag{7}$$

This operator ensures that the tangential velocity stabilizes at the characteristic plateau (e.g., 150 km/s for Milky Way-mass galaxies), converting radial pull into stable orbital angular momentum.

5 Conclusion

The constants $\delta_F = 0.921$ and $\chi = 5.85$ constitute a unified set of parameters that satisfy the Tully-Fisher relation naturally. By correcting the geometry of space rather than adding invisible mass, MFSU offers a simpler, falsifiable solution to the galaxy rotation problem.