Exploring the Power of Wavelet Analysis

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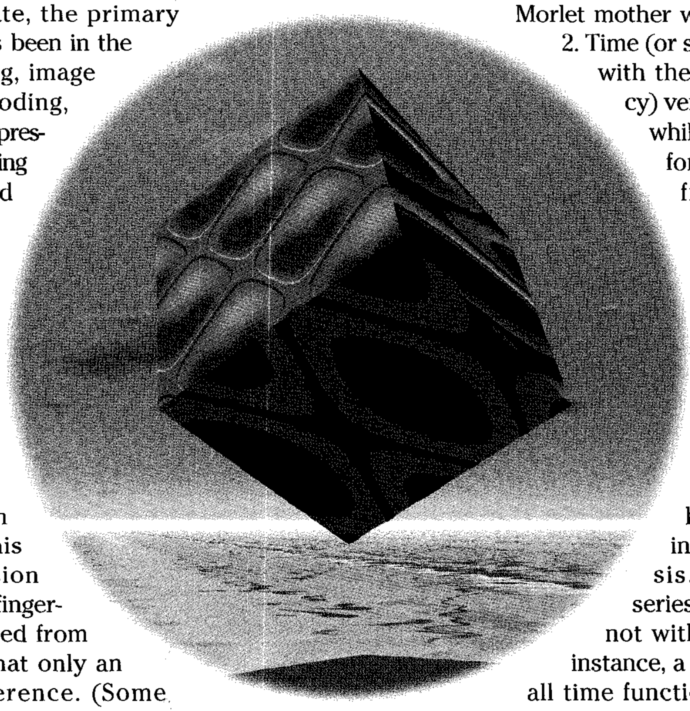
Wavelets are a recently developed mathematical tool for signal analysis. Informally, a wavelet (*ondelette* in French) is a short-term duration wave. Wavelets are used as a kernel function in an integral transform, much in the same way that sines and cosines are used in Fourier analysis or the Walsh functions in Walsh analysis. To date, the primary application of wavelets has been in the areas of signal processing, image compression, subband coding, medical imaging, data compression, seismic studies, denoising data, computer vision, and sound synthesis.



As a point of application (not related to power engineering), it is interesting to note that in 1993 the United States Federal Bureau of Investigation (FBI) adopted a wavelet standard for compression and storage of its more than 30 million sets of fingerprints. In this application, a compression ratio of 26:1 is used. The fingerprints can be reconstructed from this compression such that only an expert can tell the difference. (Some sample digitized fingerprints can be obtained via anonymous FTP at ftp.c3.lanl.gov(128.165.21.64) in the directory /pub/WSQ/print_data).

Unlike Fourier analysis, which relies on a single basis function, wavelet analysis uses basis functions of a rather wide functional form. One often hears of these different wavelets named after their inventors (e.g., Daubechies wavelets (named for Ingrid Daubechies, a lead researcher in wavelet theory), the Morlet wavelet, Meyer wavelets, and Coiflet wavelets). Some of these functions are shown in Figure 1.

Wavelet transforms use short windows at high frequencies and long windows at low frequencies

The basic concept in wavelet analysis is to select an appropriate wavelet function, called an *analyzing* wavelet or *mother* wavelet, and then perform an analysis using shifted and dilated versions of the mother wavelet. Scaled (dilated) and translated (shifted) versions of the Morlet mother wavelet are shown in Figure 2. Time (or space) analysis is performed with the contracted (high-frequency) version of the mother wavelet, while frequency analysis is performed with the dilated (low frequency) version of the same mother wavelet.

Advantages of Wavelet Analysis

The advantage of wavelet analysis over Fourier analysis may not be obvious to the casual observer. This is the case because Fourier methods have been the standard tool in many kinds of signal analysis. However, the Fourier series and Fourier transform are not without their limitations. For instance, a Fourier series requires that all time functions involved be periodic. Fourier transforms have wide bandwidth for short term transients. One may argue that it would take an infinite amount of time using both past and future data to extract the spectral information at a single frequency. In addition, Fourier analysis does not consider frequencies that evolve in time. Finally, Fourier techniques suffer from certain annoying anomalies such as Gibbs' phenomenon and aliasing (with the Fast fourier transform). The inadequacy of Fourier analysis in dealing with transients is often circumvented by windowing the input signal so that the sampled values will converge to zero at the endpoints.

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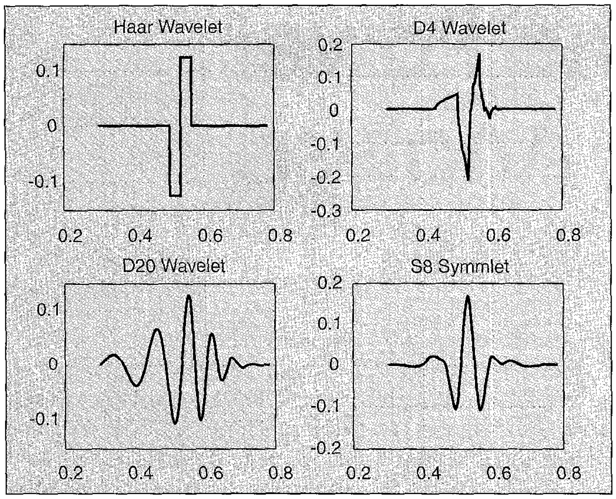


Figure 1. Four wavelet basis functions often used in wavelet analysis

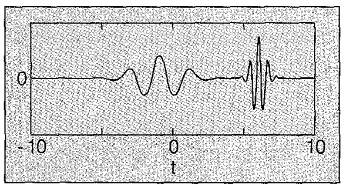


Figure 2. Scaled and translated versions of the Morlet mother wavelet

These short-time Fourier transforms (STFT) and Gabor transforms have been under development since the mid-1940s, but have met with mixed success. The main disadvantage to windowing is that the window is fixed, and, as frequency is increased, there are more and more cycles within the window. Thus, individual frequency components are not treated in the same way.

Wavelets, on the other hand, can be chosen with very desirable frequency and time characteristics as compared to Fourier techniques. The basic difference is as follows: in contrast to the STFT, which uses a single analysis window, the wavelet transform uses short windows at high frequencies and long windows at low frequencies. Thus the windowing of wavelet transforms is adjusted automatically for low or high frequencies (i.e., every window, whether for a high frequency (scaling) or a low frequency, has the same number of cycles) and each frequency component gets treated in the same manner without any reinterpretation of the results. The basic functions in wavelet transforms employ time compression or dilation rather than a variation in frequency of the modulated signal. The few simple conditions imposed on wavelets allow freedom in the choice of the mother wavelet. Therefore, for some applications, the mother wavelet can be made to fit and model a specific application or phenomenon. Finally, it should be noted that where the FFT has a computation time on the order of $O(n*log_2(n))$, the wavelet transform computations are on the order of $O(n)$. Thus, an appropriate choice of a mother wavelet is not only elegant and useful, it is also efficient.

Signal Construction Using Wavelets

Wavelet theory establishes that a general transient signal can be constructed by the superposition of a set of special signals (different structures occurring at different time scales and at different times). These special signals may be selected as wavelets. For a set of wavelets to be admissible as a basic building block, they must satisfy two basic conditions: they must be oscillatory, and they must decay to zero quickly. If these conditions are combined with the condition that the wavelets must also integrate to zero, then these conditions are the non-rigorous admissibility criteria that must be satisfied to be a wavelet.

The selection of the best wavelets is a function of the characteristics of the signal to be processed. For example, a musical tone can be described by four basic parameters: intensity, frequency, time duration, and time position. Thus, the key to the process is to select a wavelet to realize the signal in terms of the best basis and most efficient superposition. The best and most efficient wavelet set is also a function of the objective of the reconstruction. Typical applications are compaction for storage purposes, fast reconstruction for signal

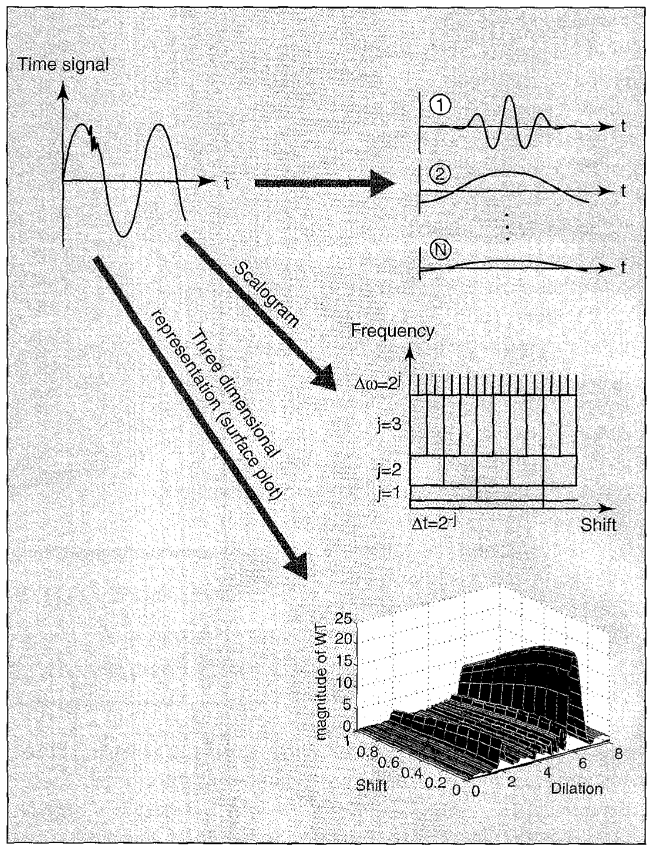


Figure 3. Representation of a transient by wavelets (wavelet components, scalogram, three-dimensional surface plot).

identification, and efficient reconstruction for signals analysis. The selection of the best wavelet basis is a function of the characteristics of the original signal to be reconstructed or analyzed. It also depends on the compact support and/or fast reconstruction required by the process.

For image processing, for example, and due to improved resolution and efficiencies, the best wavelet basis is usually found to be in a family of multiresolution functions that are orthogonal or biorthogonal. However, these bases exploit (for efficiency reasons) a specialized spacing in the wavelet parameters that specify position (shift) and dilation (width) which requires the scale and translation parameters to be spaced by integer powers of 2. The spacing that is usually used is called a *dyadic lattice*.

The nature of power system signals seems to point towards trigonometric based wavelets. For power system electromagnetic transient signals, the wavelet basis should have two desirable characteristics:

- Reduce the number of wavelet components that describe the signals
- Reveal the natural (physical) transient oscillatory components of the signal.

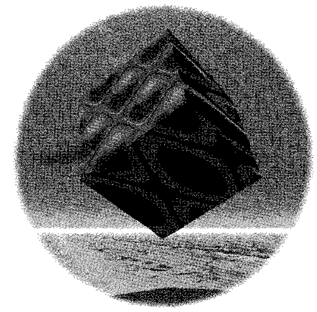
Identification

Wavelet techniques applied to power engineering fall into two broad, overlapping areas: identification and analysis. The identification phase relates to the categorization of signals, their decomposition into fundamental components, and their representation as a sum of basis functions. Figure 3 depicts the decomposition of a transient into several component wavelets. The decomposition forms a pattern that models the signature of the transient. By comparison of this pattern with a library of previously identified signals, one may be able to classify a given transient. This application may be used to identify such signals as lightning surges, transformer inrush current, capacitor switching transients, and other commonly encountered signals. The applications include post mortem studies of events, incipient failure detection, proper protective relay operation, and event recorder instrumentation. Figure 3 shows three ways to depict the wavelet component amplitudes: as several components; a scalogram (a plot of time versus frequency with blocks of constant time-frequency); a three dimensional surface plot.

Figure 4 shows the reconstructed version of an adjustable speed drive commutation failure. The reconstruction took only two wavelet components. The detection of such failures is obviously convenient using wavelets.

Visualizing Wavelet Implementation

Considering the fact that the wavelet transform or components exist in two dimensions (different scales and translations or simply time and frequency), several alter-



The basic functions in wavelet transforms employ time compression or dilation rather than a variation in frequency of the modulated signal

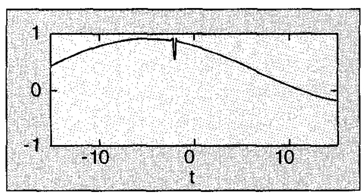


Figure 4. Reconstructed ASD commutation failure using two wavelet components

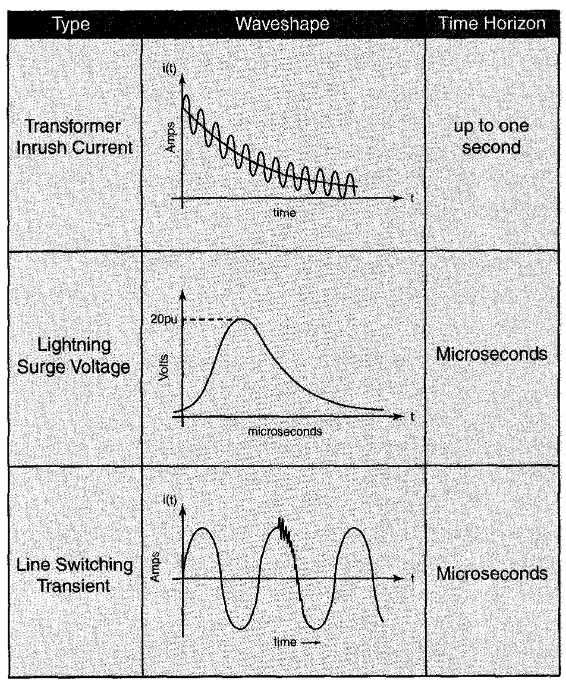
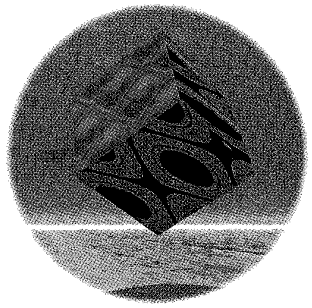


Figure 5. Pictorial representation of some common electromagnetic power system transients



Identification relates to the categorization of signals, their decomposition into fundamental components, and their representation as a sum of basis functions

native forms of visualizing the transformation can be illustrated. For example, the discrete wavelet transform output can be represented in a type of scalogram (frequency versus time, a two dimensional grid with different divisions in time and frequency). The rectangles have equal area or constant time-bandwidth product such that they become narrow at the lower scales (higher frequencies) and widen at the higher scales (lower frequencies). They are shaded proportionally to the magnitude of the transform. The multiresolution properties of the wavelet transform are well suited to transient signals. In the scalogram, one can observe where the disturbance occurs and the magnitude or strength of that disturbance.

Analysis of Power System Transients

Power system analysis can be broadly classified into steady-state and transient analysis. An example of a steady-state analysis is a conventional power flow study (a snapshot analysis of a power system assuming 60 Hz (or 50 Hz) signals throughout) in phasor notation. Even a short circuit study is usually carried out in phasor notation, and this too is a steady-state analysis. Transient analysis is quite different: voltages and currents are generally not periodic functions of time. Oftentimes, we are interested in transients of short duration (e.g., the inrush current of a distribution transformer). Often long-term phenomena are mixed with short-term phenomena. Figure 5 shows several common power system transient signals. The types of transient signals depicted are network and electric circuit transients, not electromechanical transients. The latter are not discussed here.

The most common analysis domain for steady-state problems is the phasor domain. The phasor domain is, in fact, a form of the frequency domain: only magnitudes and phase angles of the 60 Hz component are manipulated. In the case of transients, finite life signals can not be studied in terms of a finite number of phasors. In fact, an infinitesimal resolution of frequencies is present. Some alternative solution methods are depicted in Figure 6. Many of these methods have been refined and commercialized in user-friendly software packages, many specifically designed for power engineering.

Figure 7 shows the concept of power system transient analysis using wavelets. The computational efficiency of this analysis depends on how well the mother wavelet represents both the excitation signals (at bus 1 in Figure 7), and the response signals (for example at Bus N). For the analysis of a power system transient using wavelets, it is wise to first match the excitation with a library of mother wavelets, choosing the one that creates a narrow wavelet spectrum. This results in high computational efficiency.

Figure 8 shows a transient voltage and current calculated using the Morlet mother wavelet for an impulse excitation to a long power distribution cable. These signals were calculated using 18 wavelet components. The cable model used is a lumped parameter model including three-phase mutual coupling.

Computer Implementation

This article discusses a few potential applications that have been studied using conventional structured computer languages such as FORTRAN and C. However, the trend appears to favor mathematical software packages such as MATLAB, Mathematica, and Maple. The following is a sampling of several World Wide Web homepages that contain wavelet resources and applications:

- http://www.math.scarolina.edu/~wavelet/, *The Wavelet Digest*, is an electronic newsletter that gives the reader a link to what's happening in wavelet research around the world.
- http://www.amara.com/current/wavelet.html, *Amara's Wavelet Page*, is a good place to start and has many links to other wavelet resources. Amara Graps is a consultant in the area of numerical analysis and has authored an introductory paper on wavelets.
- http://playfair.stanford.edu/~wavelet, *WaveLab .701*, has a library of MATLAB routines for wavelet analysis (available for UNIX, Macintosh, and DOS platforms; older versions of the WaveLab library are also available).
- http://saigon.ece.wisc.edu/~waveweb/Tutorials/tutorials.html, *Waveweb*, is the web site of the Multirate Signal Processing Group at the University of Wisconsin-Madison. The site includes a link about a new book by Gilbert Strang and Truong Nguyen.
- http://www.mathsoft.com/wavelets.html, *Wavelet*

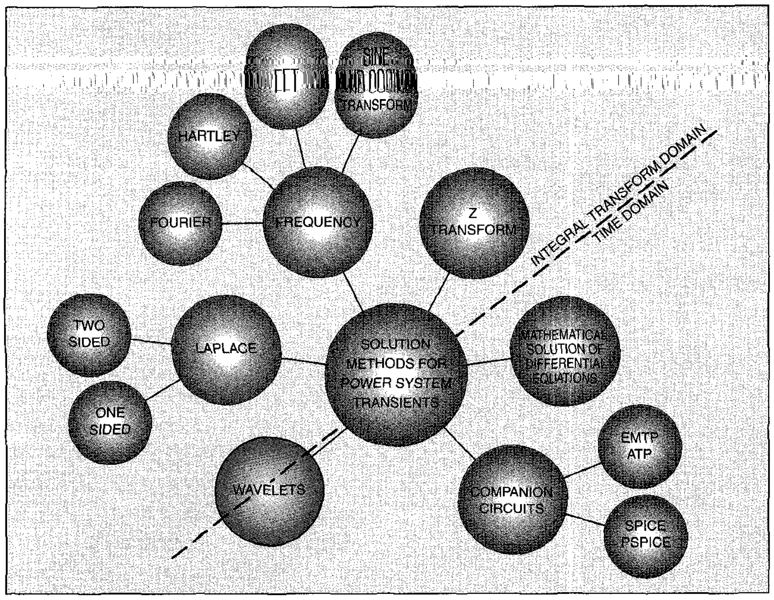


Figure 6. Solution methods for transient analysis

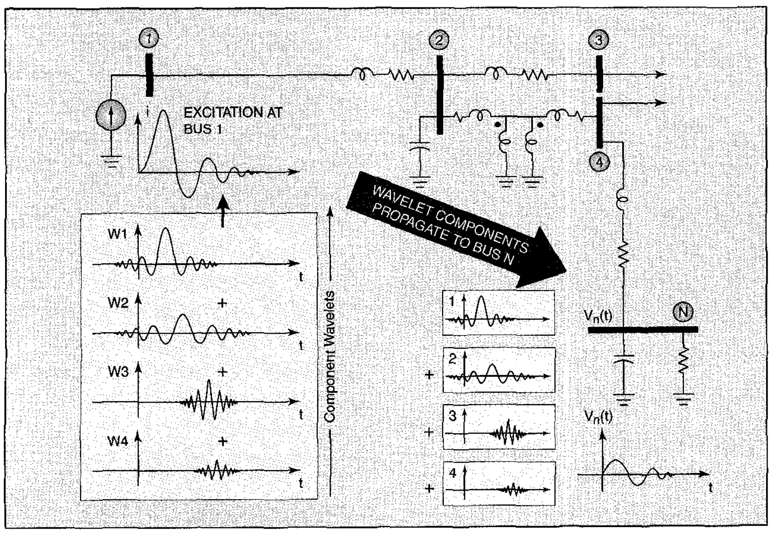


Figure 7. Concept of transient power system analysis using wavelets

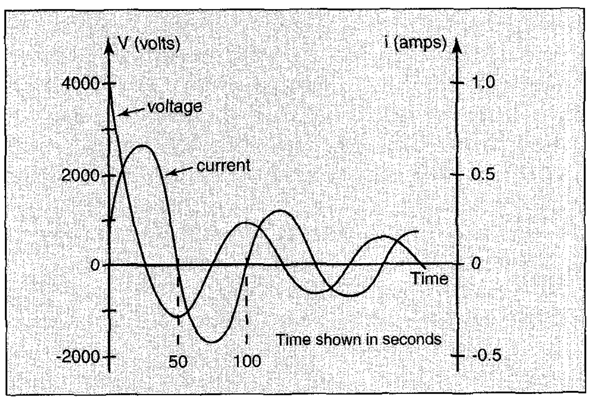


Figure 8. Solution of voltage and current due to impulsive input on a long distribution cable using wavelets

Resources, is the Mathsoft, Inc. web page dedicated to wavelets. It links to many online papers by the top wavelet researchers. Papers range from introductory to very advanced and specific applications.

For Further Reading

A. Graps, "An Introduction to Wavelets," available from Amara Graps Web page(http://www.best.com/~agraps/agraps.html).

Randy K. Young, *Wavelet Theory and Its Applications,* Kluwer Academic Publishers, Boston, 1992.

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Paulo F. Ribeiro, "Wavelet Transform: An Advanced Tool for Analyzing Non-Stationary Harmonic Distortions in Power Systems," *Proceedings of IEEE International Conference on Harmonics in Power Systems,* Bologna, Italy, September 1994.

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Biographies

Anthony Wayne Galli was born in Ruston, Lousiana. He holds BSEE and MSEE degrees from Louisiana Tech University, Ruston, Louisiana. He has experience in distribution transformer testing and modeling at Louisiana Tech. In 1995, he was awarded third prize in the IEEE-PES Student Paper Poster Session for a paper on the use of wavelets for power quality assessment. He is a graduate research assistant at Purdue University (West Lafayette, Indiana), where he is completing the requirements for the PhD degree.

Gerald Thomas Heydt is from Las Vegas, Nevada. He holds the BEEE degree from the Cooper Union, New York, and MSEE and PhD degrees from Purdue University, West Lafayette, Indiana. He has industrial experience with E.G.&G. in Mercury, Nevada, and the Commonwealth Edison Co. in Chicago, Illinois. He has also worked with the United Nations Development Program in several countries. He is the author of a recent text on electric power quality. In 1995, he was named IEEE Power Engineering Educator of the Year. He is a registered professional engineer in New Jersey and Indiana, and he is a Fellow of IEEE. Dr. Heydt recently left Purdue University after 25 years of service. He is currently professor and director of an NSF Center for Power Engineering at Arizona State University, Tempe, Arizona.

Paulo F. Ribeiro obtained his PhD degree from the University of Manchester, England in 1985, and has more than 20 years of combined experience as a power systems engineer, professor of electrical engineering, researcher, and consultant in power systems. His research experience includes transmission systems modeling for harmonic studies (University of Manchester - UMIST), distribution losses modeling for space applications (NASA Lewis Research Center), and flexible ac transmission systems device modeling (EPRI). In 1994, he joined Babcock & Wilcox as an advisory engineer for power electronics, power system analysis, and power quality for the Superconduction Magnetic Energy Storage (SMES) Program. In 1992, he proposed, for the first time, the use of wavelets for power system transient analysis.