Names: ______________________________

On my honor, I/we have not given, nor received,
nor witnessed any unauthorized assistance on this work.

Signature: ______________________________

Signature: ______________________________

Signature: ______________________________

Signature: ______________________________

Signature: ______________________________

Collaboration Stmt - Reference any resources (beyond those listed on the Sprint Resources page) which you used in completing this assignment:

Propositional Logic:

Question:	1	2	3	4	5	Total
Points:	10	16	10	7	10	53
Score:						

Set Theory:

Question:	6	7	Total
Points:	16	6	22
Score:			

Applications:

Question:	11	12	Total
Points:	10	10	20
Score:			

Data Representation:

Question:	8	9	10	Total
Points:	10	6	7	23
Score:				

TOTAL GRADE: __________/ 100%

Propositional Logic

1. Let h = "Maria is healthy", w = "Maria is wealthy", and z = "Maria is wise."

   Write each of the following compound statements:

   (a) (2 points) Maria is healthy and wealthy but not wise.

   (b) (2 points) Maria is not healthy but she is wealthy and wise.

   (c) (2 points) Maria is neither healthy, wealthy, nor wise.

   (d) (2 points) Maria is neither healthy nor wise, but she is wealthy.

   (e) (2 points) Maria is wealthy, but she is not both healthy and wise.

2. Draw a truth table for each of the following:

   (a) (2 points) $p \wedge \neg q$

(b) (4 points) $p \vee (q \wedge \neg r)$

(c) (2 points) $\neg(p \wedge q)$

(d) (4 points) $p \wedge \neg(q \rightarrow r)$

(e) (4 points) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

3. Prove that
$(\neg p \vee q) \vee (p \wedge \neg q)$
is a tautology in both of the following ways:
(a) (4 points) Using a truth table

(b) (6 points) Provide a more formal proof using the laws of Boolean Algebra

4. (7 points) The equivalences $p \vee (p \wedge q) \equiv p$ and $p \wedge (p \vee q) \equiv p$ are often known as the **Absorption Laws** of Boolean Algebra. Using other Boolean Algebra laws, prove that these equivalences are true.

5. Use a truth table to establish the truth of each statement:

   (a) (5 points) A conditional statement is not logically equivalent to its inverse.

(b) (5 points) A conditional statement and its contrapositive are logically equivalent.

Set Theory

6. If the set $A = \{a, b, c\}$ and the set $B = \{c, d, e\}$ find:

(a) (1 point) $|A|$ ______________

(b) (2 points) $\mathcal{P}(A)$ ____

(c) (1 point) $|\mathcal{P}(A)|$ ______________

(d) (2 points) $A \times B$ ____

(e) (2 points) $A \cup B$ _________________________________

(f) (2 points) $A \cap B$ ______________

(g) (2 points) $A \setminus B$ (often seen as $A - B$) ______________

(h) (2 points) $B \setminus A$ (often seen as $B - A$) ______________

(i) (2 points) Draw a Venn diagram of A and B.

7. Let:
$A = \{2, 4, 6\}$
$B = \{6, 2, 4\}$
$C = \{1, 2, 3, 4\}$

Mark each of the following statements as either True or False. If a statement is false, give a brief reason.

(a) (1 point) A $\subseteq$ B ______

(b) (1 point) B $\subseteq$ A ______

(c) (1 point) C $\not\subseteq$ A ______

(d) (1 point) A $\subseteq$ C ______

(e) (1 point) A $\not\subseteq$ B ______

(f) (1 point) A $\subset$ C ______

Data Representation

8. Convert the following numbers from decimal (base 10) to binary (base 2).

(a) (1 point) 10

(b) (2 points) 31

(c) (2 points) 66

(d) (2 points) 201

(e) (3 points) 2679

9. Convert the following binary numbers (base 2) to hexadecimal (base 16). Show your work for full credit.

   (a) (1 point) `1101`

   (b) (1 point) `10011011`

   (c) (2 points) `111100111011`

(d) (2 points) `1010011111000001`

10. Compute the following results. Give you answers in hexadecimal format. Show your work for full credit.

(a) (3 points) `0xC7 | 0xA8`

(b) (3 points) `0xC7 & 0xA8`

(c) (1 point) `~0xC7`

Applications

11. You are the manager of a programming team. One of your programmers has changed several if statements. Prove (without using truth tables) whether or not the new code is logically equivalent to the old code. (For those who have not programmed before, the code-to-logic mapping is given.)

   (a) (5 points) Old code:
   ```
   if (x < 2 || !(1 < x && x < 3))
   ```

   New code:
   ```
   if (x <= 1 || x < 2 || x >= 3)
   ```

   Let:
   ```
   a = x < 2
   b = x > 1
   c = x < 3
   ```

   Then:
   Old code is $a \vee \neg(b \wedge c)$
   New code is $\neg b \vee a \vee \neg c$

(b) (5 points) Old code:

```
if (x < y) {
  w = 4;
}
if (x < y) {
        z = 8;
 }
```

New code:

```
if (x < y) {
  w = 4;
  z = 8;
}
```

Let:

```
a = x < y
b = w = 4
c = z = 8
```

Then:

Old code is $(a \to b) \wedge (a \to c)$

New code is $a \to (b \wedge c)$

12. (10 points) Consider the two pieces of code:

    Snippet 1: `if(1 < x && x < 5) { ... }`

    and

    Snippet 2: `if(!(x >= 1 || x >= 5)) { ... }`

    As in the previous problems, use formal logic to show that these two snippets of code are logically equivalent. Be sure to clearly define your propositions as there are multiple correct ways to complete this problem correctly.