Name: _____

On my honor, I/we have not given, nor received,
nor witnessed any unauthorized assistance on this work.

Signature: _____

Signature: _____

Signature: _____

Signature: _____

Signature: _____

Propositional Logic:

Question:	1	2	3	4	5	6	Total
Points:	3	10	25	15	5	10	68
Score:							

Set Theory:

Question:	7	8	Total
Points:	14	8	22
Score:			

Data Representation:

Question:	9	10	11	12	Total
Points:	10	6	8	8	32
Score:					

Applications:

Question:	13	14	Total
Points:	16	10	26
Score:			

TOTAL GRADE: _____/ 148

Propositional Logic

1. (3 points) What is the negation of the statement `1 < x < 5` ?

2. Let h = "Maria is healthy", w = "Maria is wealthy", and z = "Maria is wise."

   Write each of the following compound statements:

   (a) (2 points) Maria is healthy and wealthy but not wise.

   (b) (2 points) Maria is not healthy but she is wealthy and wise.

   (c) (2 points) Maria is neither healthy, wealthy, nor wise.

   (d) (2 points) Maria is neither healthy nor wise, but she is wealthy.

   (e) (2 points) Maria is wealthy, but she is not both healthy and wise.

3. Draw a truth table for each of the following:

   (a) (5 points) $p \wedge \neg q$

(b) (5 points) $p \vee (q \wedge \neg r)$

(c) (5 points) $\neg(p \wedge q)$

(d) (5 points) $p \wedge \neg(q \rightarrow r)$

(e) (5 points) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

4. Determine which of these are tautologies and which are contradictions by writing truth tables:

   (a) (5 points) $((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$

   (b) (5 points) $(\neg p \vee q) \vee (p \wedge \neg q)$

   (c) (5 points) $(p \to (q \vee r)) \leftrightarrow ((p \wedge \neg q) \to r)$

5. (5 points) Suppose that p and q are statements such that $p \to q$ is false. What is the truth value of $q \to p$? Explain how you know.

6. Use a truth table to establish the truth of each statement:
   (a) (5 points) A conditional statement is not logically equivalent to its inverse.
   (b) (5 points) A conditional statement and its contrapositive are logically equivalent.

Set Theory

7. If the set $A = \{a, b, c\}$ and the set $B = \{c, d, e\}$ find:
   (a) (1 point) $|S|$ _____
   (b) (2 points) $\mathcal{P}(S)$ _____
   (c) (1 point) $|\mathcal{P}(S)|$ _____
   (d) (2 points) $A \times B$ _____
   (e) (2 points) $A \cup B$ _____
   (f) (2 points) $A \cap B$ _____
   (g) (2 points) $A \setminus B$ (often seen as $A - B$) _____
   (h) (2 points) $B \setminus A$ (often seen as $B - A$) _____

8. Let:
   $A = \{2, 4, 6\}$
   $B = \{6, 2, 4\}$
   $C = \{1, 2, 3, 4\}$

   Mark each of the following statements as either True or False.
   (a) (1 point) A $\subseteq$ B _____
   (b) (1 point) C $\not\subseteq$ A _____
   (c) (1 point) B $\subseteq$ A _____
   (d) (1 point) C $\not\subseteq$ A _____
   (e) (1 point) A $\subseteq$ C _____
   (f) (1 point) A $\not\subseteq$ B _____
   (g) (1 point) A $\subset$ C _____
   (h) (1 point) B $\not\subseteq$ A _____

Data Representation

9. Convert the following numbers from decimal (base 10) to binary (base 2). Show your work for full credit.

   (a) (1 point) 10

   (b) (2 points) 31

   (c) (2 points) 66

   (d) (2 points) 201

   (e) (3 points) 2679

10. Convert the following binary numbers (base 2) to hexadecimal (base 16). Show your work for full credit.

    (a) (1 point) `1101`

    (b) (1 point) `10011011`

    (c) (2 points) `111100111011`

(d) (2 points) `1010011111000001`

11. Convert the following decimal (base 10) numbers to ternary (base 3). Remember, this question should only use the digits 0, 1, or 2 in the answer and will be based around powers of 3. Show your work for full credit.

(a) (2 points) 27

(b) (3 points) 62

(c) (3 points) 101

12. Compute the following results. Give you answers in hexadecimal format. Show your work for full credit.

(a) (3 points) `0xC7 | 0xA8`

(b) (3 points) `0xC7 & 0xA8`

(c) (2 points) `~0xC7`

Applications

13. You are the manager of a programming team. One of your programmers has changed several if statements. PROVE whether or not the new code is logically equivalent to the old code. (For those who have not programmed before, the code-to-logic mapping is given.)

(a) (8 points) Old code:

```
if (x < 2 || !(1 < x && x < 3))
```

New code:

```
if (x <= 1 || x < 2 || x >= 3)
```

Let:

```
a = x < 2
b = x > 1
c = x < 3
```

Then:
Old code is $a \vee \neg(b \wedge c)$
New code is $\neg b \vee a \vee \neg c$
(Hint: My proof is 3 lines long.)

(b) (8 points) Old code:

```
if (x < y) {
  w = 4;
}
if (x < y) {
        z = 8;
 }
```

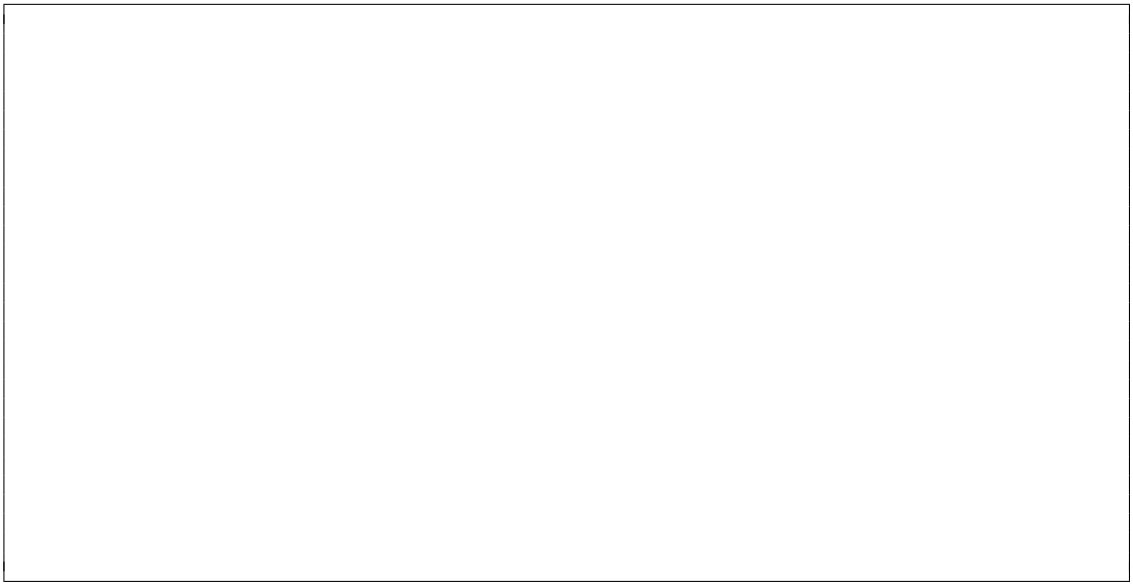
New code:

```
if (x < y) {
  w = 4;
  z = 8;
}
```

Let:

```
a = x < y
b = w = 4
c = z = 8
```

Then:
Old code is $(a \to b) \wedge (a \to c)$
New code is $a \to (b \wedge c)$
(Hint: My proof is 4 lines long.)



14. (10 points) Use logic to find the treasure from the following premises. Show your reasoning. You can assume there is only one treasure in only one building, but you cannot assume the same about the other objects mentioned.

    1. If a signet ring is in Bush, then a shovel is not in Mills.
    2. If a compass is in Bush, then a shovel is in Mills.
    3. Either a signet ring is in Bush, or both a signet ring and compass are in Bush.
    4. If a calculator is in Bush then the treasure is in Mills.
    5. If the treasure is in Olin, then a shovel is also in Olin.
    6. It is not true that both a shovel and signet ring are in Olin.
    7. Either a compass is in Bush or a shovel is in Olin.
    8. If either a signet ring is NOT in Olin or a compass IS in Olin, then both a shovel is in Bush and either a compass or a calculator is also in Bush.