On my honor, I have not given, nor received, nor witnessed any unauthorized assistance on this work.

Print name and sign: ____

Question:	1	2	3	4	5	6	Total
Points:	6	5	8	4	5	2	30
Score:							

1. Given the statement “If there is a hurricane, then it is raining” find the following:

   (a) (2 points) Give the original statement in formal propositional notation (i.e. define p, q, and the appropriate logical connectives).

   Solution: Let p be the statement “there is a hurricane” and q be the statement “it is raining”. Then $p \to q$.

   (b) (2 points) The converse of the statement in propositional logic and English

   Solution: $q \to p$
   If it is raining, then there is a hurricane.

   (c) (2 points) The contrapositive of the statement in propositional logic and English.

   Solution: $\neg q \to \neg p$
   If it is not raining, then there is no hurricane.

2. (5 points) Show (via truth tables) that $(p \leftrightarrow q) \to (p \to q)$ is a tautology.

Solution:

p	q	$p \leftrightarrow q$	$p \to q$	$(p \leftrightarrow q) \to (p \to q)$
0	0	1	1	1
0	1	0	1	1
1	0	0	0	1
1	1	1	1	1

For full credit, you MUST show the two intermediate column in the truth table. Showing only the final result (all trues) is worth 1 point of credit only.

3. Let $A = \{w, x, y, z\}$ and $B = \{a, y\}$.
   (a) (1 point) Is $x \in A$? yes
   (b) (1 point) Is $\{a\} \in B$? no
   (c) (1 point) Is $y \subseteq B$? no
   (d) (1 point) Is $\{z\} \subseteq A$? yes
   (e) (2 points) Give the power set of B.

   Solution: $\{\emptyset, \{a\}, \{y\}, \{a, y\}\}$

   (f) (2 points) Show $A \times B$.

   Solution: $\{(w, a), (w, y), (x, a), (x, y), (y, a), (y, y), (z, a), (z, y)\}$

4. Conversions:
   (a) (2 points) Convert 42 (base 10) to 8-bit binary (base 2).

   Solution: `00101010`

   (b) (2 points) Convert 10011101 (8-bit binary, base 2) to hexadecimal (base 16)

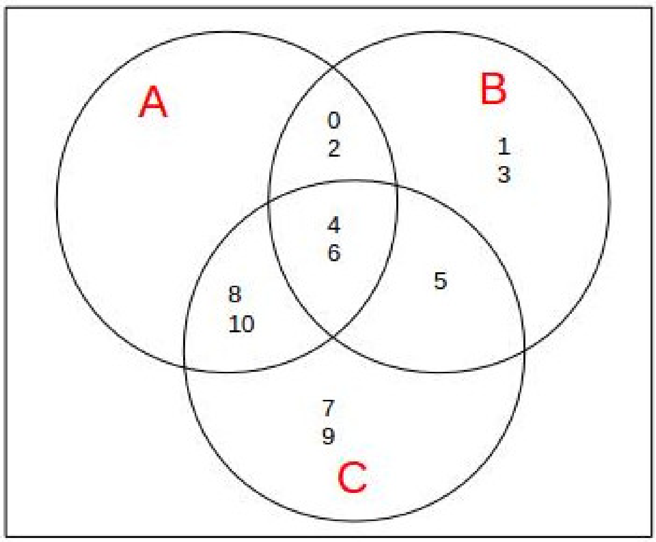
   Solution: `0x9D`

5. Let $A = \{0, 2, 4, 6, 8, 10\} B = \{0, 1, 2, 3, 4, 5, 6\}$ and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find:
   (a) (2 points) $(A \cup B) \cap C$

Solution:
$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 8, 10\}$
$(A \cup B) \cap C = \{0, 1, 2, 3, 4, 5, 6, 8, 10\} \cap C$
$= \{0, 1, 2, 3, 4, 5, 6, 8, 10\} \cap \{4, 5, 6, 7, 8, 9, 10\}$
$= \{4, 5, 6, 8, 10\}$

(b) (3 points) Draw a Venn diagram representing the three sets.

Solution:



6. (2 points) You are given the code:

```
if(x > 4) {
    x = 0;
}
```

Represent this code as formal propositional logic – that is, define propositions and the connectives to represent the above code.

Solution: Let p be the proposition `x > 4` and q be the proposition `x = 0`. Then our representation is $p \to q$

Standard Sets:

- $\mathbb{R}$: set of real numbers
- $\mathbb{Q}$: set of rational numbers $\{a/b : a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ and $b \neq 0\}$
- $\mathbb{N}$: set of natural numbers $\{0, 1, 2, 3\}$
- $\mathbb{Z}$: set of all integers $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- $\mathbb{Z}+$: set of positive integers $\{1, 2, 3, ...\}$

Set Symbols:

- $\subseteq$: "is a subset of"
- $\subset$: "is a proper subset of"
- $\in$: "is an element of"

Identity Laws:	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination Laws:	$p \vee T \equiv T$ $p \wedge F \equiv F$
Idempotent Laws:	$p \vee p \equiv p$ $p \wedge p \equiv p$
Double negation Law:	$\neg(\neg p) \equiv p$
Commutative Laws:	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative Laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive Laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
DeMorgan's Laws:	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Absorption Laws:	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
Negation Laws:	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$