(9) $y[n]=(n+2)x[n-1]+2x[n-3]$

Teste de linearidade

$T\{x_1[n]\} = (n+2)x_1[n-1]+2x_1[n-3] = y_1[n]$
$T\{x_2[n]\} = (n+2)x_2[n-1]+2x_2[n-3] = y_2[n]$

$T\{ax_1[n]+bx_2[n]\} = (n+2)(ax_1[n-1]+bx_2[n-1])+2(ax_1[n-3]+bx_2[n-3]) =$
$= y_c[n]$

$y_c = ay_1[n]+by_2[n] \Rightarrow$ Linear

Teste de invariância

$T\{x_1[n-k]\} = y_{x[n-k]}[n] = (n+2)(x_1[n-k-1])+2x_1[n-k-3]$

$y_x[n] = y[n-k] = (n+k+2)x[n-k-1]+2x[n-k-3]$

$y_x[n] \neq y_{x[n-k]} \Rightarrow$ Variante no tempo

Causal pois não depende de inputs futuros

—

$y[n]=2x[n-1]-3x[n+4]$ ← não Causal Invariante no tempo

$ay_1[n]+by_2[n] = a(2x_1[n-1]-3x_1[n+4])+b(2x_2[n-1]-3x_2[n+4])$

$y_c[n] = ay_1+by_2 \Rightarrow$ Linear

—

Variante no tempo

$y[n]=2(2n+1)x[n-1]x[n-4]$ Causal

$y_c = 2(2n+1)(ax_1[n-1]+bx_2[n-1])(ax_1[n-4]+bx_2[n-4])$

$ay_1[n]+by_2[n] = 2a(2n+1)x_1[n-1]x[n-4]+2b(2n+1)x_2[n-1]x_2[n-4] \neq$
$\neq y_c[n] \Rightarrow$ Não Linear

(10) $y[n] = 3x[n-1] - x[n-2] + 2x[n-3]$

$$h[n] = 3\delta[n-1] - \delta[n-2] + 2\delta[n-3]$$

$$u[n] * h[n] = \sum_{k=-\infty}^{+\infty} u[k]\, h[n-k] \Leftrightarrow$$

$u[k]$ começa k = a ter significado a partir de $k=0$

$h[n] / u[n] \rightarrow$

k	h	0	1	2	3	4
		1	1	1	1	1
0	0	0	0	0	0	0
1	3	3	3	3		
2	-1	-1	-1	-1		
3	2	2	1	2		
4	0	0	0	0		

$\rightarrow n = 3$

$y[3] = 4$

(OU)

$\Leftrightarrow y[3] = h[3] + h[2] + h[1] = 2 - 1 + 3 = 4$

(11) $h[n] = 3\delta[n-1] - 2\delta[n-2]$

$x[n] = u[n] + 2\delta[n-1] + 3\delta[n-2]$

$$y[2] = \sum_{k=0}^{4} x[k]\, h[2-k] = x[0]h[2] + x[1]h[1] =$$

$$= 1 \cdot (-2) + (1+2)3 = 7$$

(OU)

		0	1	2
		1	3	4
0	0	0	0	0
1	3	3	9	12
2	-2	-2	-6	-8

$\rightarrow 7$

(12) $G(z) = \dfrac{-0{,}3z^{-3} + 1{,}9z^{-4}}{(1-0{,}5z^{-1})(1+0{,}6z^{-1})} \Leftrightarrow, \quad T_s = 0{,}1\,s$

$$\Leftrightarrow \frac{-0{,}3z^{-3} + 1{,}9z^{-4}}{1 + 0{,}6z^{-1} - 0{,}5z^{-1} - 0{,}3z^{-2}} \cdot \frac{z^4}{z^4} = \frac{-0{,}3z + 1{,}9}{z^4 + 0{,}1z^3 - 0{,}3z^2}$$

zeros: $-0{,}3z + 1{,}9 = 0 \Leftrightarrow z = \frac{19}{3}$

polo de ordem 2

polos: $z^4 + 0{,}1z^3 - 0{,}3z^2 = z^2(z^2 + 0{,}1z - 0{,}3) = 0 \Leftrightarrow z = 0 \vee (z^2 + 0{,}1z - 0{,}3) = 0 \Leftrightarrow$

$$\Leftrightarrow z = 0 \vee z = \frac{-0{,}1 \pm \sqrt{0{,}01 + 1{,}2}}{2} = \frac{-0{,}1 \pm \sqrt{1{,}21}}{2} = 0{,}5 \vee -0{,}6$$

polos simples

1 zero 4 polos

Os polos estão contidos na circ. unitária $\Rightarrow$ estável

$$G(z) = \frac{-0{,}3z^{-3} + 1{,}9z^{-4}}{(1-0{,}5z^{-1})(1+0{,}6z^{-1})} = \frac{z^{-3}(-0{,}3 + 1{,}9z^{-1})}{(1-0{,}5z^{-1})(1+0{,}6z^{-1})}$$

$T_a = 3 \cdot T_s = 0{,}3\,s$

Atraso puro

$G(1) = \frac{-0,3+1,9}{(1-0,5)(1+0,6)} = \frac{1,6}{0,8} = 2 \leftarrow$ Ganho em regime estacionário

(13) $G(z) = \frac{0,4\,z^{-3}}{1-0,8z^{-1}} = H(z)\Big|_{c.i.\ nulas} = \mathcal{Z}\{h[n]\Big|_{c.i.\ nulas}\} \Rightarrow$ ← Atraso

$\Rightarrow h[n] = \mathcal{Z}^{-1}\{G(z)\} = \underline{0,4 \cdot 0,8^{n-3}\, u[n-3]}$

$a^n u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1-az^{-1}}$

(14) $y[n] = 0,5x[n-1] + 0,3x[n-3] + 1,1y[n-1] - 0,3y[n-2]$

$x[n] = 5u[n-2] - 2\delta[n-5]$

Começamos por descobrir resposta em frequência:

$Y(z) = 0,5X(z)z^{-1} + 0,3X(z)z^{-3} + 1,1Y(z)z^{-1} - 0,3Y(z)z^{-2} \Leftrightarrow$

$\Leftrightarrow Y(z)(1-1,1z^{-1}+0,3z^{-2}) = X(z)(0,5z^{-1}+0,3z^{-3}) \Leftrightarrow$

$\Leftrightarrow \frac{Y(z)}{X(z)} = \frac{0,5z^{-1}+0,3z^{-3}}{1-1,1z^{-1}+0,3z^{-2}} = H(z)\Big|_{c.i.\ nulas}$

Como $U(z) = \frac{1}{1-z^{-1}}$ e $\mathcal{Z}\{\delta\} = 1$:

$X(z) = \left(\frac{5}{1-z^{-1}}\right)z^{-2} - 2z^{-5}$

} Obter $\mathcal{Z}\{x[n]\}$

Obter $Y(z)$:

$Y(z) = H(z)\Big|_{c.i.\ nulas} \cdot X(z) = \left(\frac{0,5z^{-1}+0,3z^{-3}}{1-1,1z^{-1}+0,3z^{-2}}\right)\left(\frac{5z^{-2}}{1-z^{-1}} - 2z^{-5}\right)$

Aplicar teorema do valor final

$\lim_{n\to+\infty} y[n] = \lim_{z\to1}(1-z^{-1})Y(z) = \lim_{z\to1}\left(\frac{\cancel{(1-z^{-1})}5z^{-2}}{\cancel{1-z^{-1}}} - \cancel{(1-z^{-1})2z^{-5}}\right)\left(\frac{0,5z^{-1}+0,3z^{-3}}{1-1,1z^{-1}+0,3z^{-2}}\right) =$

$(1-z^{-1}) \to 0$

$= 5\frac{0,5+0,3}{1-1,1+0,3} = 5 \cdot \frac{0,8}{1-0,8} = 5 \cdot 4 = 20$

$3e^{j\boxed{\frac{\pi}{2}}}$

(15) $\Omega = 3\,rad \quad H(3) = 3j; \quad x[n] = 2\sin[3n]$

$y[n] = |H(3)| \cdot 2\sin\left[3n + \frac{\pi}{2}\right] = 6\sin\left[3n + \frac{\pi}{2}\right]$

(16) $x(t) = 4(\sin(3t+1))^2 = 4\,\frac{1}{2}\left(1 - \cos(6t+2)\right) = \underset{m=0}{2} - 2\underset{m=1}{\cos(6t+2)} \quad w_0 = 6$

$x(t) = 2\cos(5t) + \sin(5t-1) = 2\underset{m=1}{\cos(5t)} + \underset{m=1}{\cos\left(5t - 1 + \frac{\pi}{2}\right)}$

$w_0 = 5$

$x(t) = 4\sin(6t)\cos(9t-6) = \frac{4}{2}\left(\sin(6t+9t-6) + \sin(6t-9t+6)\right) =$

$= 2\left(\sin(15t-6) + \sin(-3t+6)\right) =$

$= 2\left(\cos\left(\frac{\pi}{2} - (15t-6)\right) + \cos\left(\frac{\pi}{2} + 3t - 6\right)\right) \quad mdc(15,3) = 3 = w_0$

$m = 5 \wedge m = 1$

$x(t) = \underset{m=0}{1} + \underset{m=1}{\cos(5t-1)} \quad w_0 = 5$

(17) $T_0 = 2\pi \quad |c_3| = \frac{C_3}{2} \quad |c_4| = \frac{C_4}{2} \quad |c_0| = C_0 \quad \theta_3 = -\angle c_{-3}$

(18). O maior m é 5 $\Rightarrow w_0 \cdot 5 = 100\pi \Rightarrow w_0 = 20\pi \Rightarrow f_0 = 10\,Hz$

$w_2 = 2 \cdot 10 = 20\,Hz \quad w_5 = 50\,z$

(19) V

ATD Modelo 2024

① $x(t) = 2 + 3\sin(t)\cos(2t) + 5\sin(4t) - \cos(3t)$

$\quad 0 \qquad mdc(1,2)=1 \qquad 4 \qquad 3 \qquad w \in \{1, 4, 3\}$

② $w \in \{8\pi, 16\pi, 20\pi\}$ $\quad w_0 = mdc(8\pi, 16\pi, 20\pi) = 4\pi \Rightarrow f_0 = 2 \Rightarrow T_0 = \frac{1}{2}$ s

③ $x(t) = 4\cos^2(2t) = 4\,\frac{1}{2}\left(1 + \cos(2 \cdot 2t)\right) = 2\left(1 + \cos(4t)\right) = 2 + 2\cos(4t)$

④ $E = \int_{-2}^{2} |x(t)|^2\,dt < \infty \Rightarrow$ É sinal de energia $\Rightarrow$ A potência média é nula

⑤ $x(-t) = 2 + 4\sin(-3t) = 2 - 4\sin(3t) \neq -x(t) \neq x(t)$

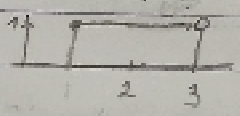
$x(-t) = 4\sin^2(-3t) = 4\,\frac{1}{2}\left(1 - \cos(-6t)\right) = 2 - 2\cos(6t) = x(t)$

$x(-t) = 2 + \cos(-2t) = 2 + \cos(2t) = x(t)$

$x(-t) = 4\sin(-3t)\cos(-2t) = -4\sin(3t)\cos(2t) = -x(t)$

⑥ $x[n] = 2n\left(\underbrace{u[n-1] - u[n-3]} + \delta[n-4]\right) \ominus$

Diferença entre degraus $\rightarrow$



$u[n-1] - u[n-3] =$

$= \left(\delta[n-1] + \delta[n-2] + \delta[n-3] + \dots\right) - \left(\delta[n-3] + \delta[n-4] + \dots\right) =$

$= \delta[n-1] + \delta[n-2]$

$\ominus\, 2n\left(\delta[n-1] + \delta[n-2] + \delta[n-4]\right) \Rightarrow E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \underset{n=1}{2^2} + \underset{n=2}{4^2} + \underset{n=4}{8^2} = 84$ J

⑦ $x(t) = t^2 - 4$ $\quad [-4, 4]$ com 4 subintervalos $\Rightarrow \Delta t = \frac{|-4| + |4|}{4} = 2$ $\quad [-4, -2, 0, 2, 4]$

$E = \int_{-4=t_1}^{4=t_n} |t^2 - 4|^2 \simeq \frac{\Delta t}{3}\left(x(t_1) + x(t_n) + 4 \cdot \sum_{i\,par} x(t_i) + 2 \cdot \sum_{i\,impar} x(t_i)\right) =$

$= \frac{2}{3}\left[|x(-4)|^2 + |x(4)|^2 + 4\left(|x(-2)|^2 + |x(2)|^2\right) + 2\left(|x(0)|^2\right)\right] =$

$= \frac{640}{3}$ J

⑧ $x[n] = 2n\left(u[n+1] - u[n-6]\right)$ $\quad$ transformação linear: $n \Rightarrow an - b = 3n + 2$

$y[n] = 2(3n+2)\left(u[3n+3] - u[3n-4]\right)$