Constrained Delaunay Triangulations CDTs

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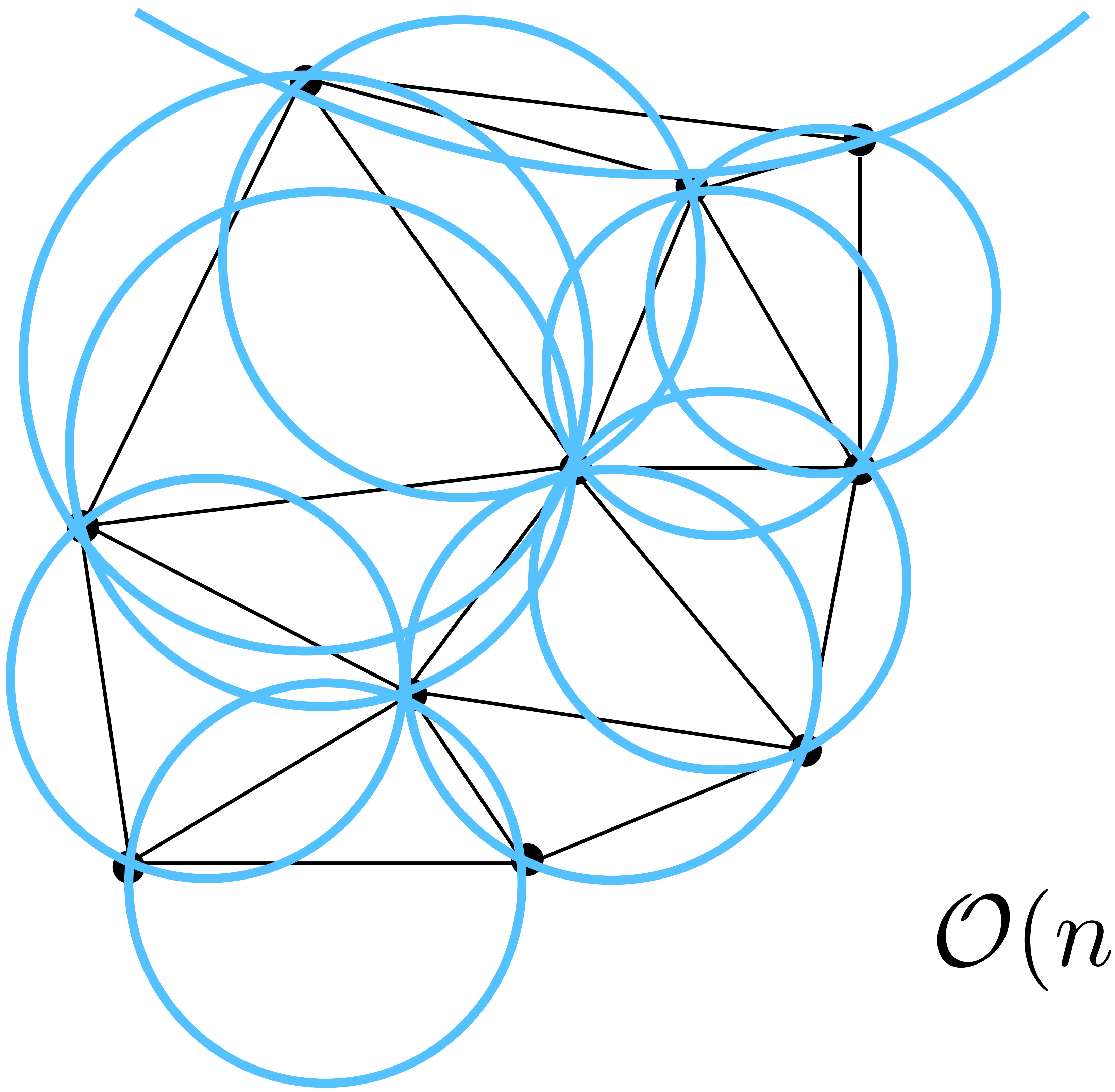
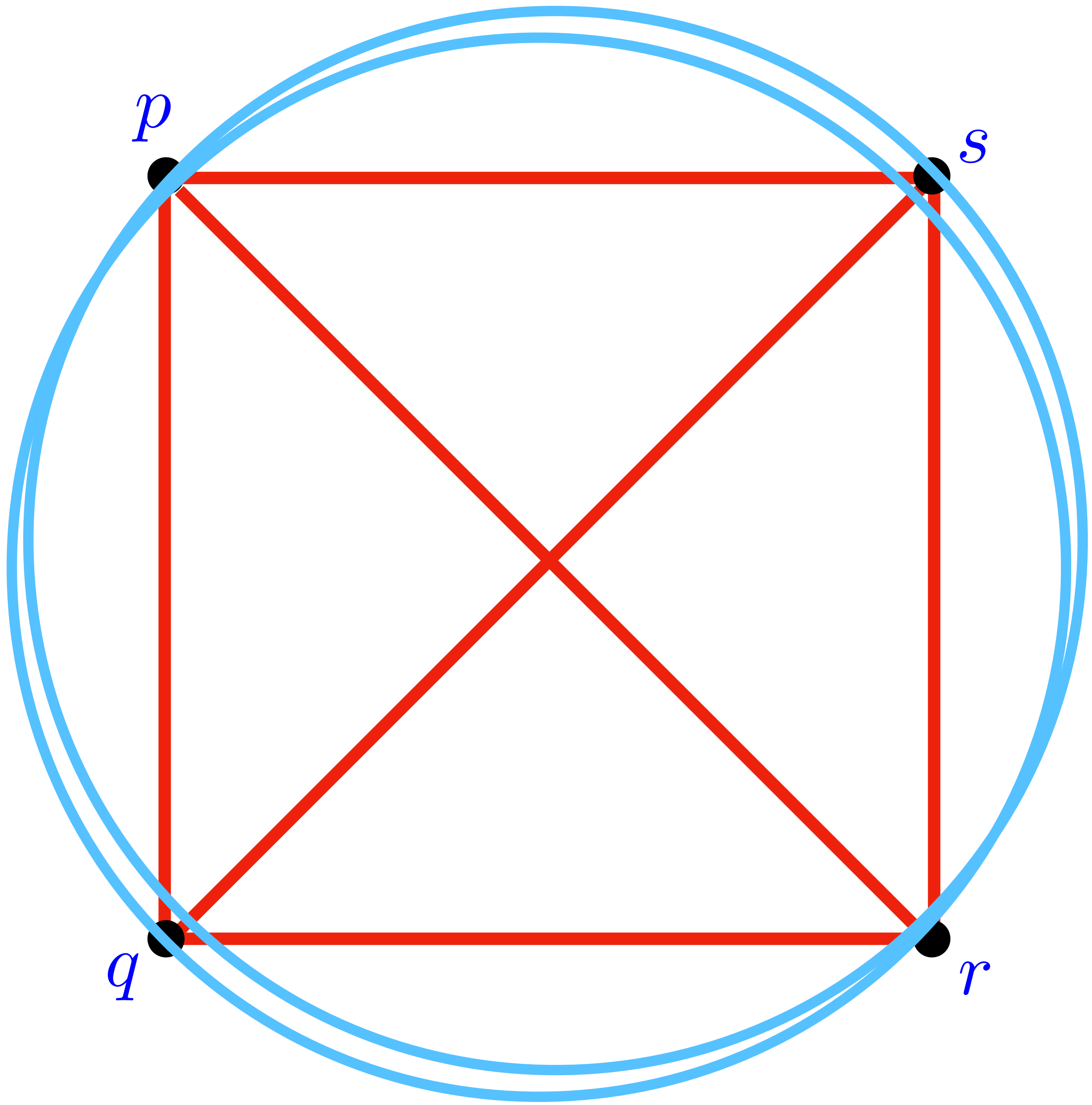
Triangulaciones de Delaunay



$$\mathcal{O}(n\,log\,n)$$

Triangulaciones de Delaunay

Unicidad

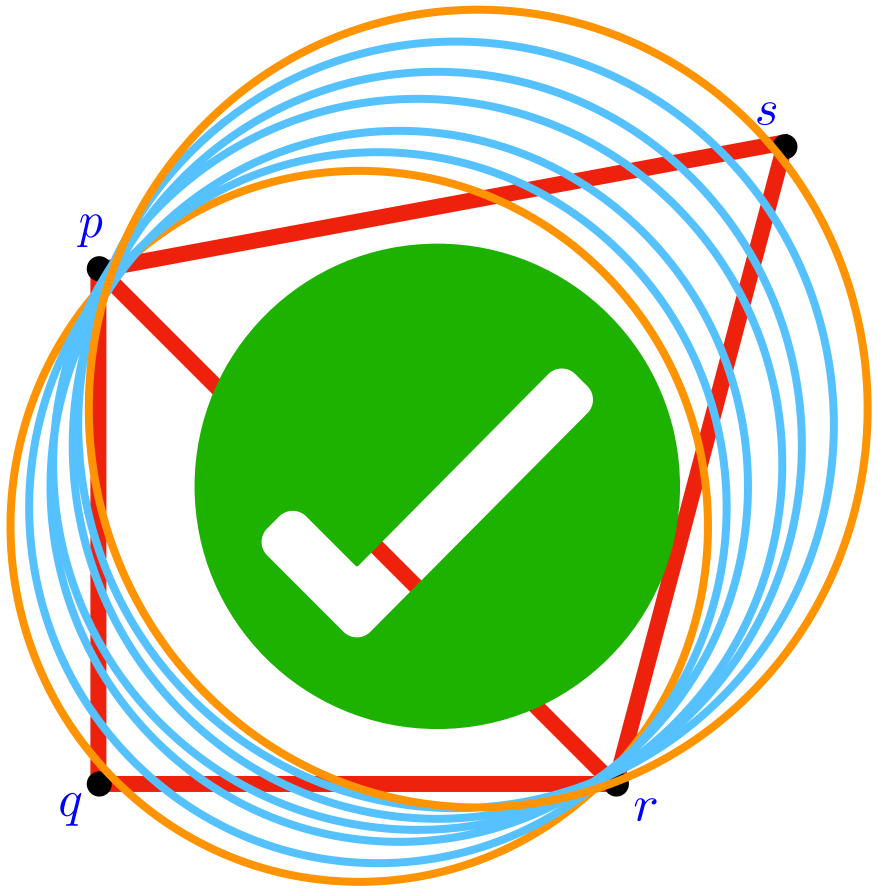


4 vértices cocirculares

Triangulaciones de Delaunay

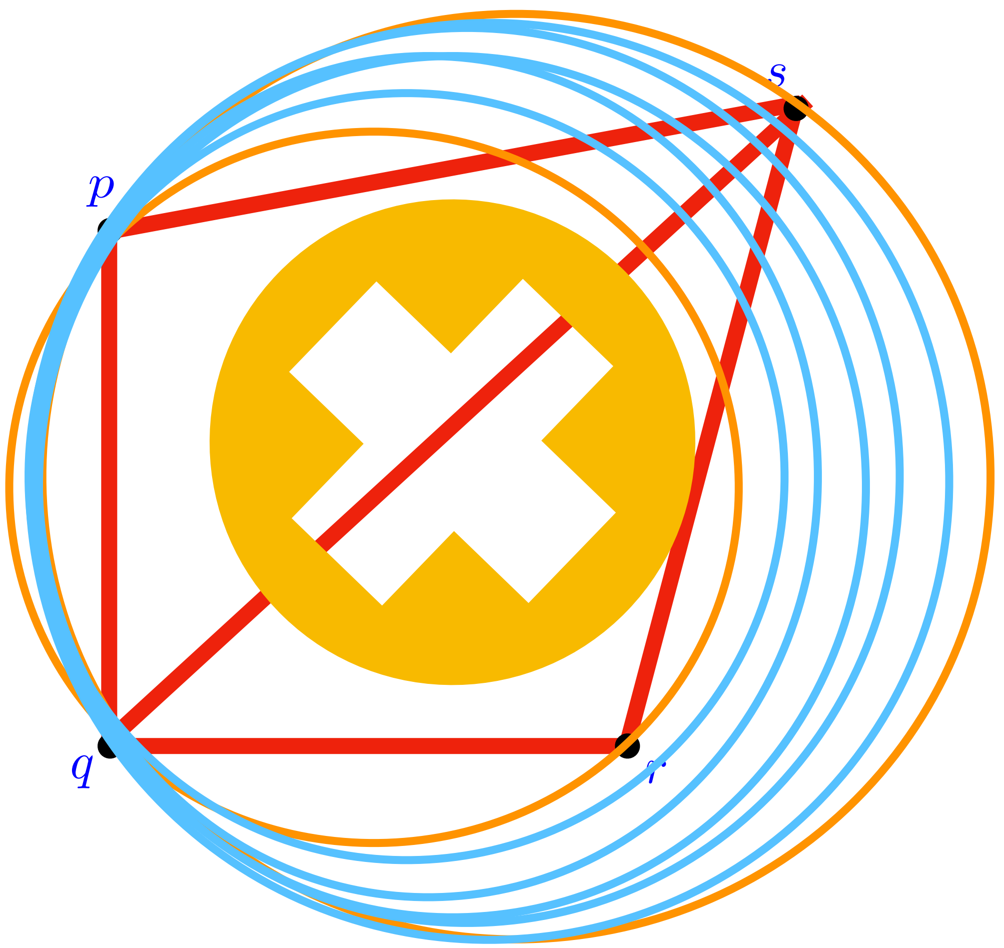
Unicidad

$\mathcal{T}_1$



$\mathcal{C}_1$

$\mathcal{T}_2$



$\mathcal{C}_2$

Diagrama de Voronoi

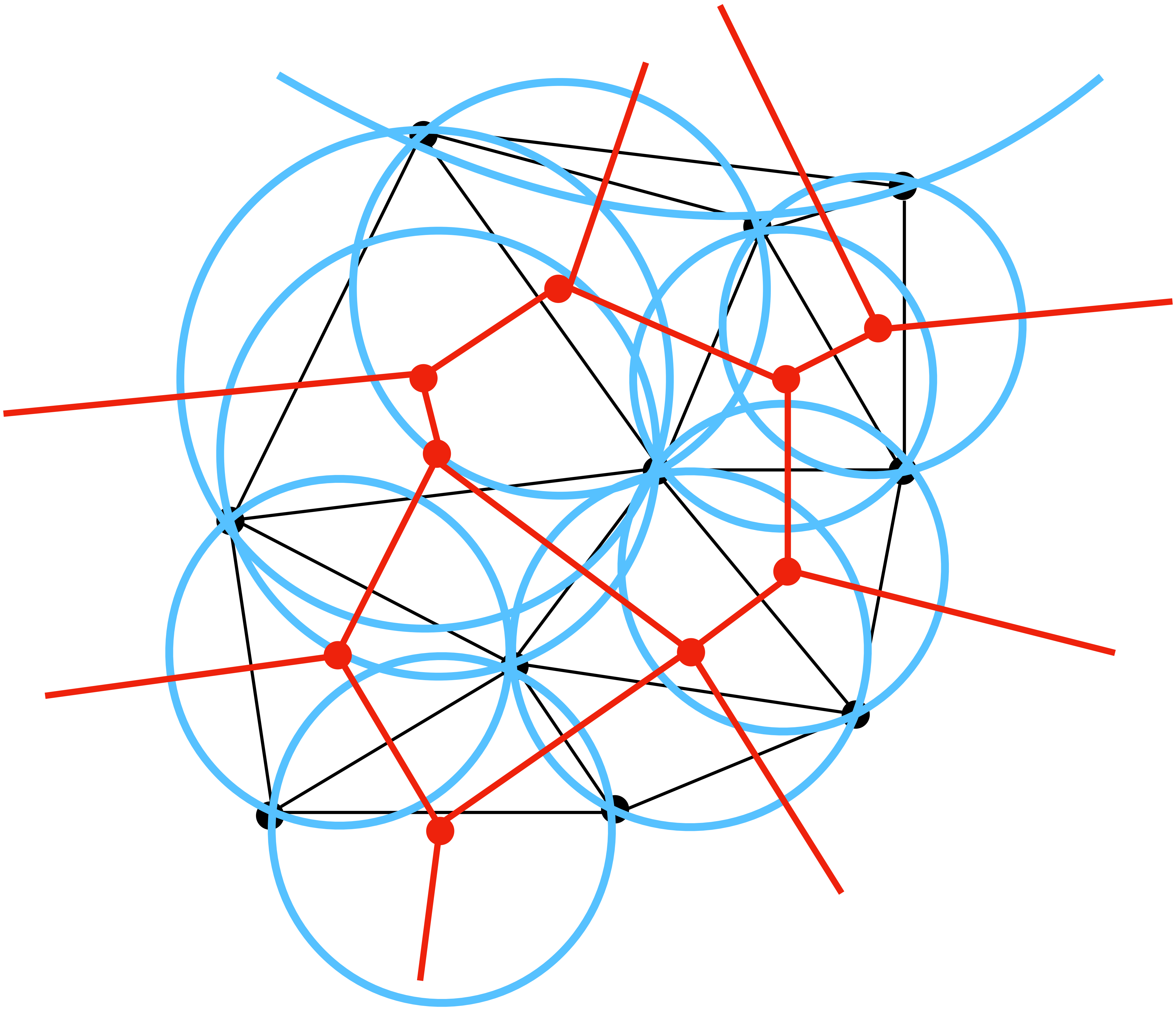
$$P \subset \mathbb{R}^2$$

$$P = \{p_1, p_2, \ldots, p_n\}$$

$$\mathcal{VD}(P)$$

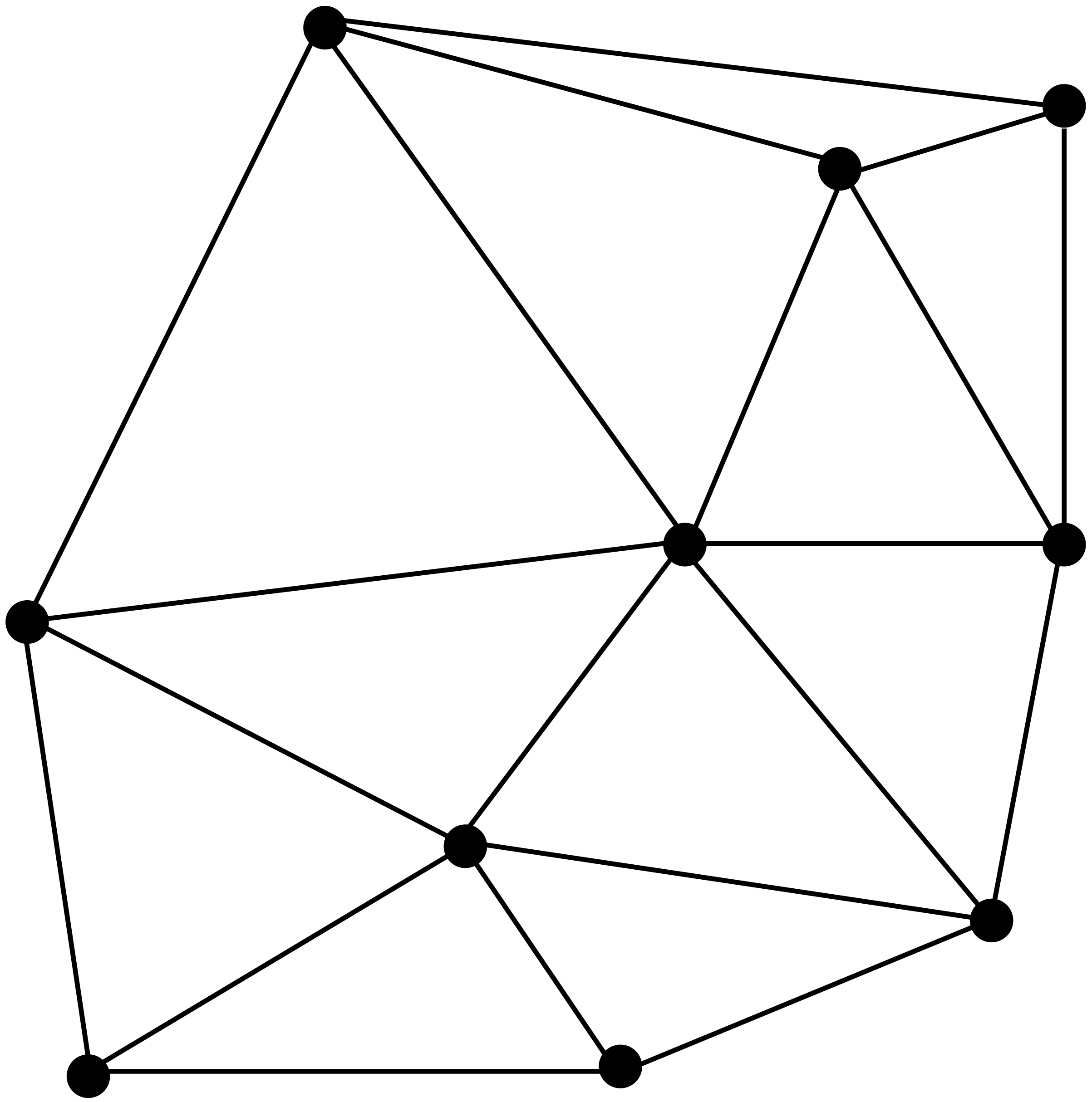
$$q \in \mathcal{V}(p_i) \Leftrightarrow dist(p_i, q) \leq dist(p_j, q) \forall p_j \in P, j \neq i$$

Diagrama de Voronoi



CDT

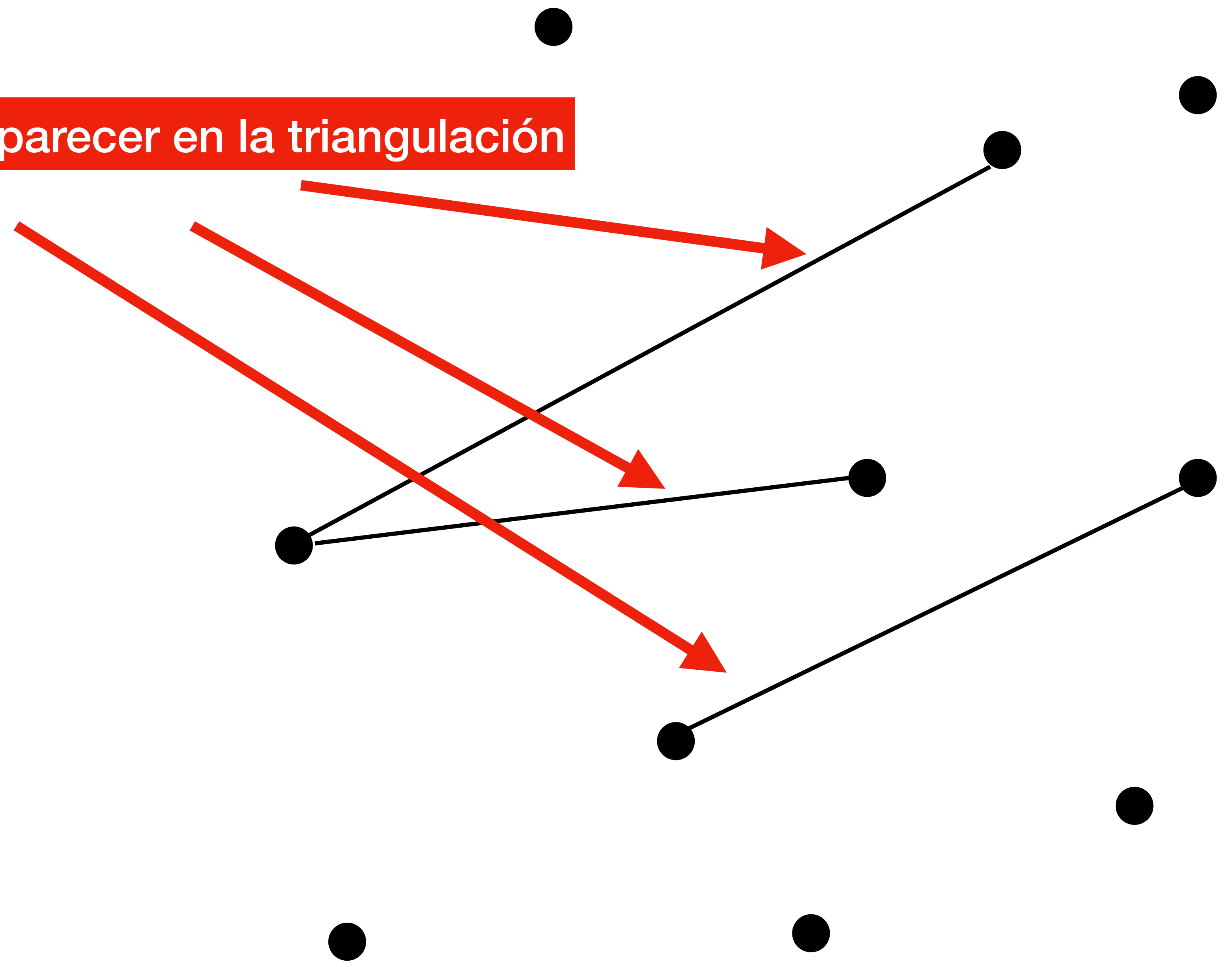
Triangulación de Delaunay



CDT

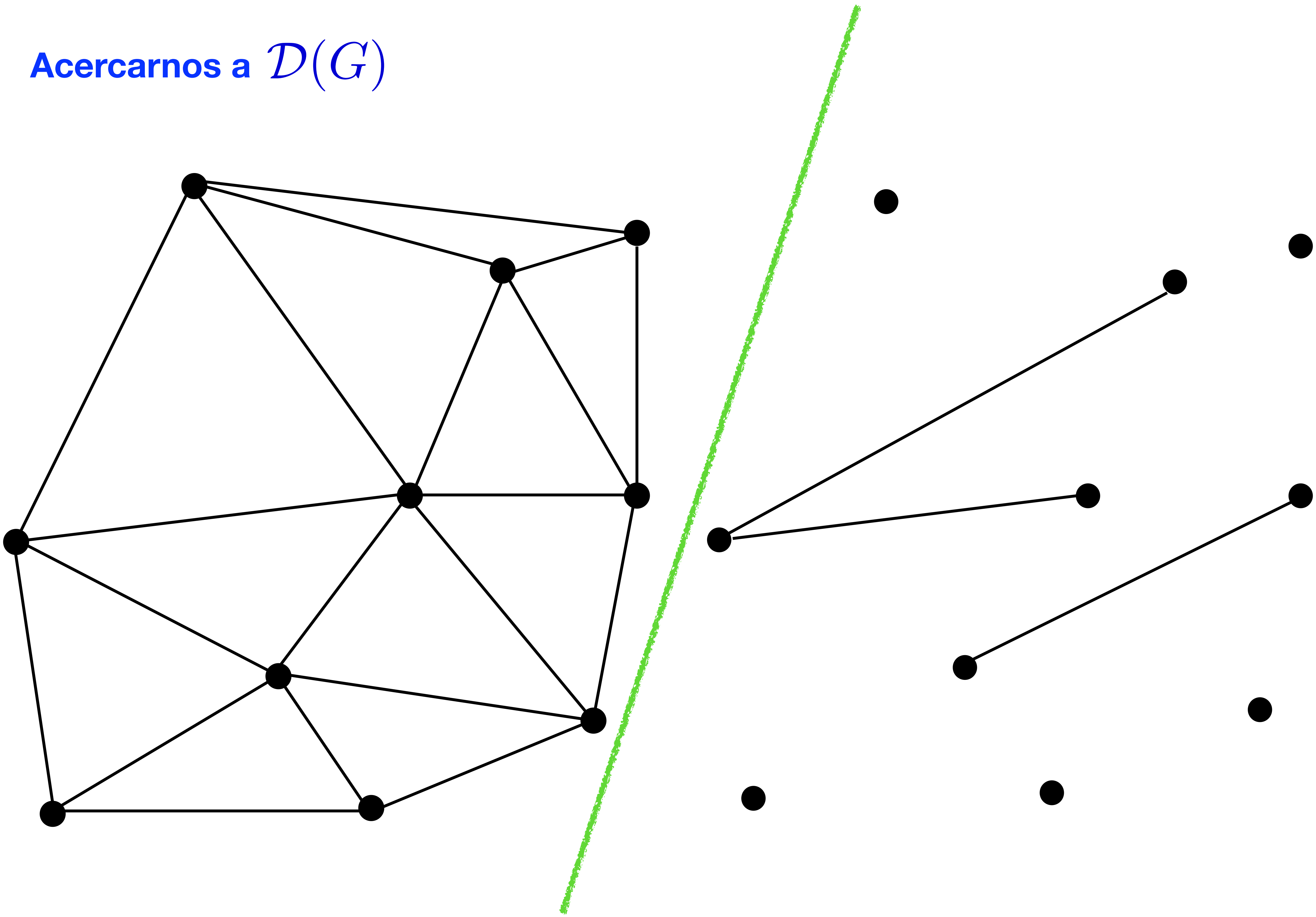
$G = (V, E)$ plana

Deben aparecer en la triangulación



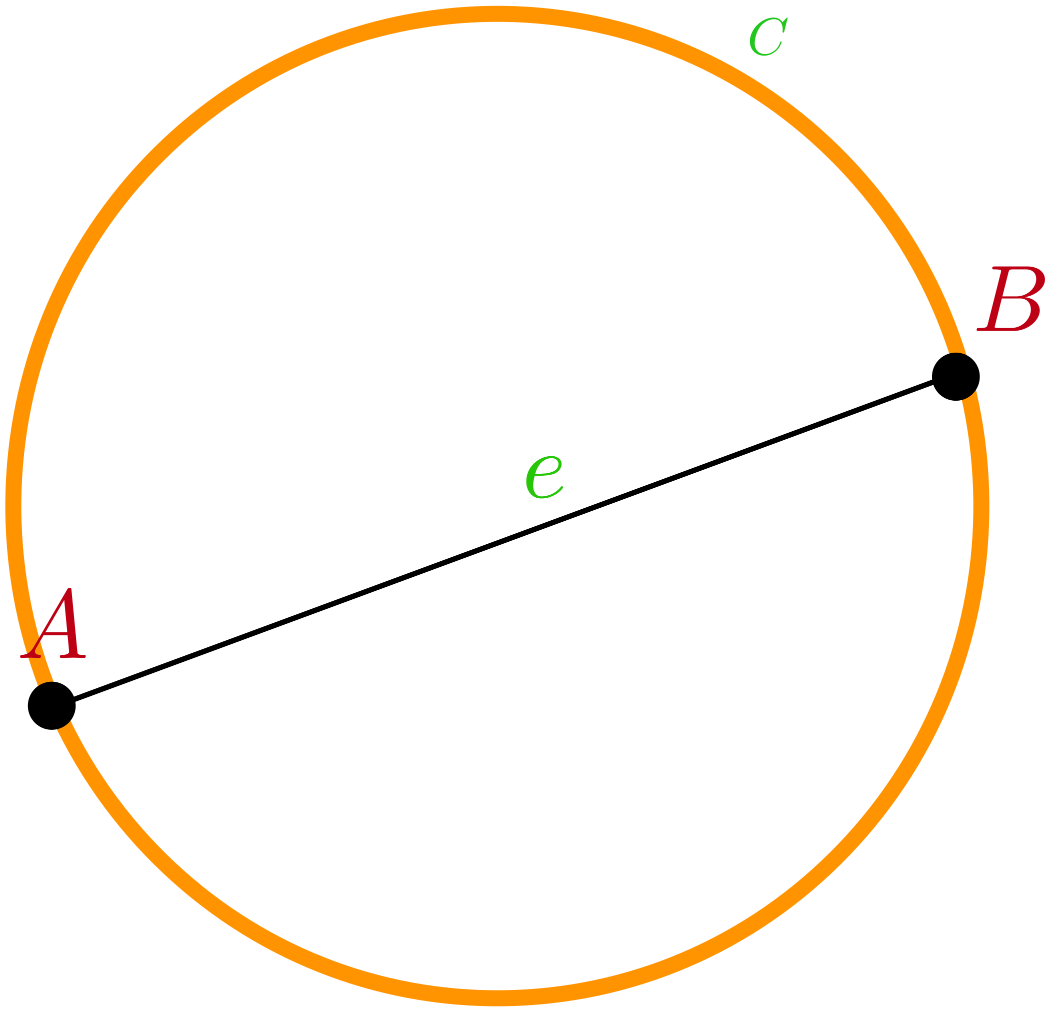
CDT

Acercarnos a $\mathcal{D}(G)$



Definición

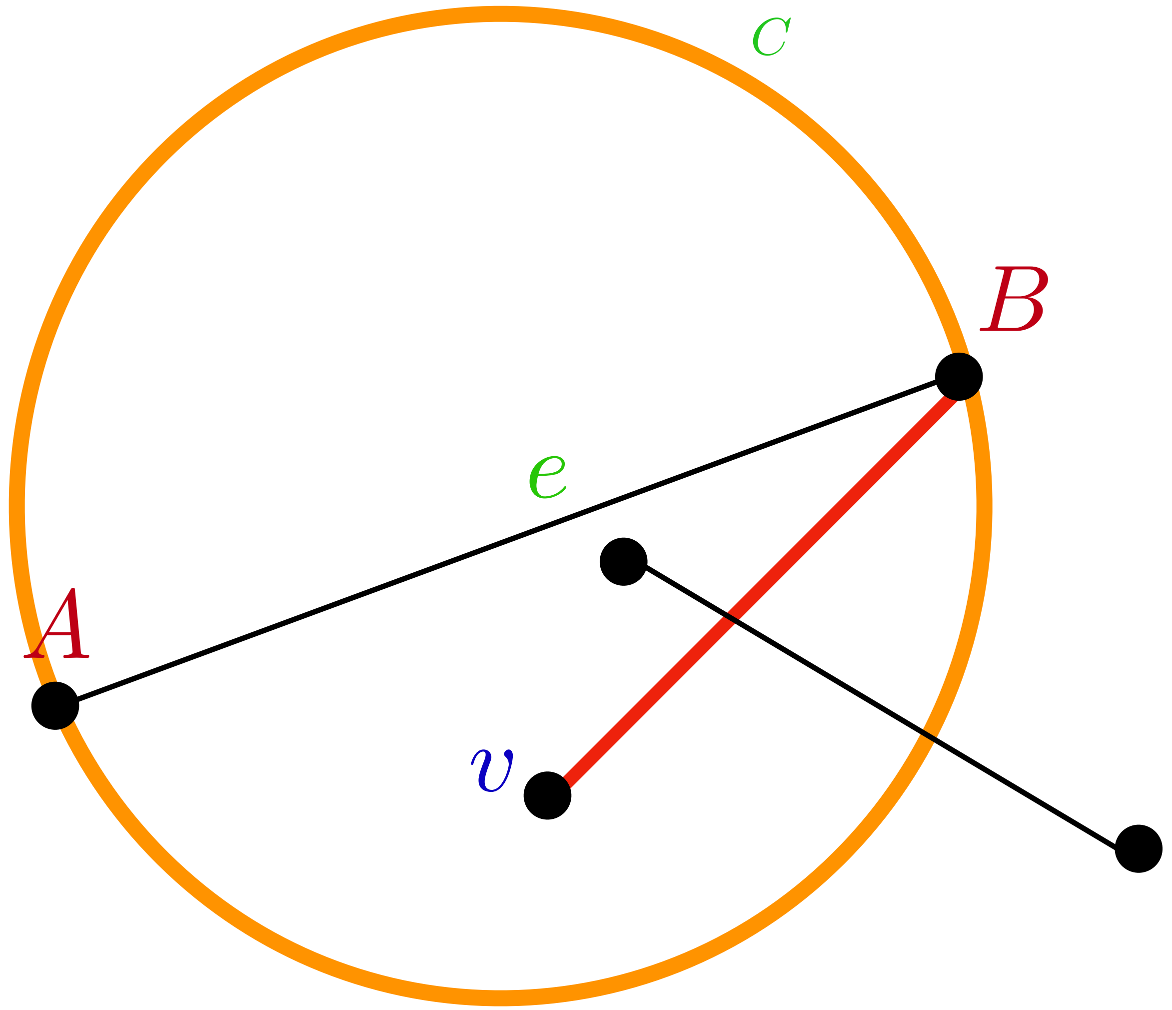
1. $\forall e \in E(T)\ \exists c$



CDT

Definición

2. Si v en c $\Longrightarrow$ 👁 por A ó B



Aplicaciones

$S \subset \mathbb{R}^2$ Conjunto de puntos

$\mathcal{D}(S)$ La triangulación de Delaunay

$\mathcal{E}(S)$ EMST

$$\forall e \in E(\mathcal{E}(S)), e \in E(\mathcal{D}(S))$$

$$\mathcal{E}(S) \subseteq \mathcal{D}(S)$$

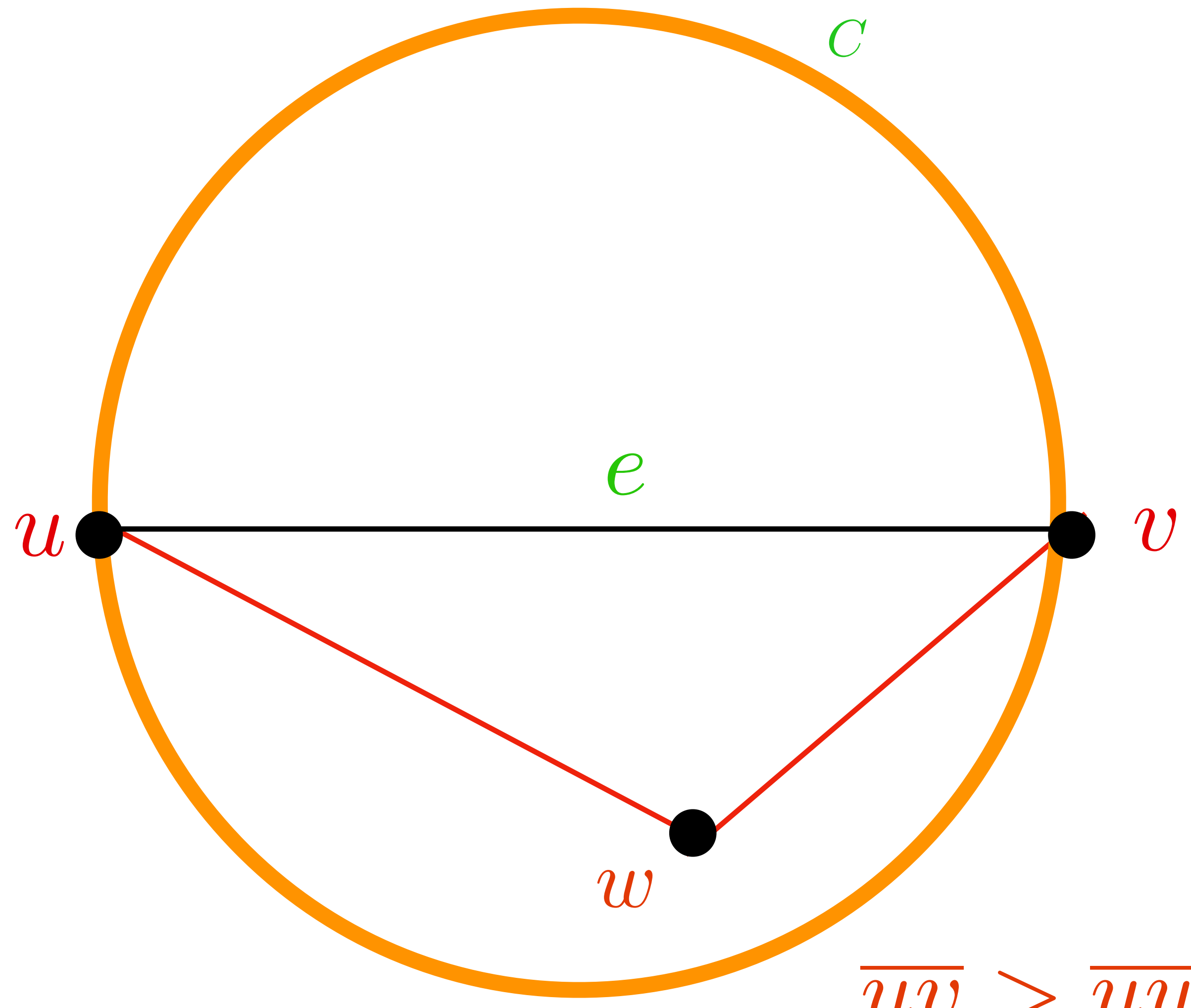
$$e = (u, v) \in E(\mathcal{E}(S)) \ni e \notin E(\mathcal{D}(S))$$

CDT

Aplicaciones

Diámetro $\overline{uv}$

$e \notin E(\mathcal{D}(S))$



$\overline{uv} > \overline{uw} \wedge \overline{uv} > wv$

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EMSTs Restringidos

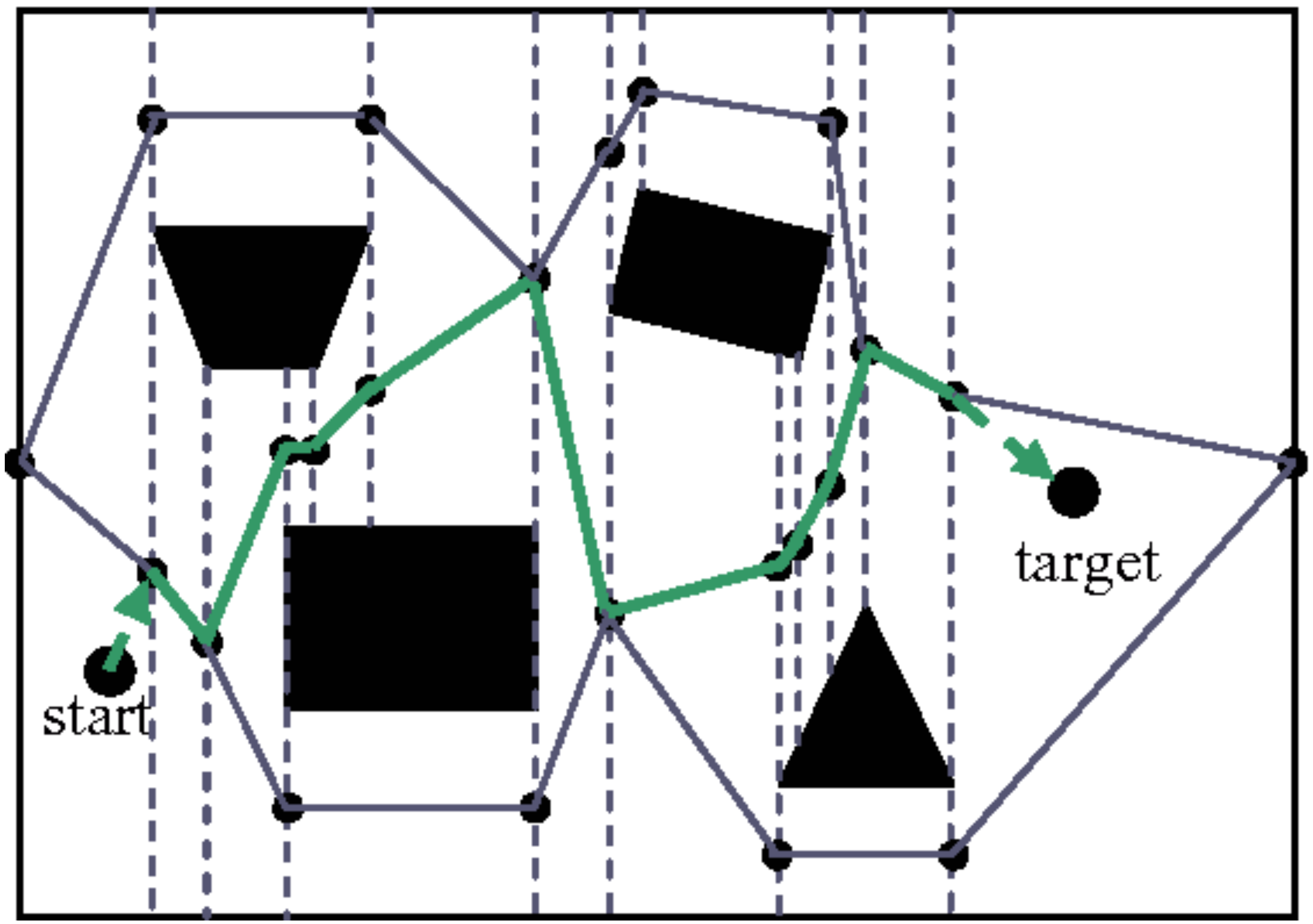
CDT

Aplicaciones

EMSTs Restringidos

CDT

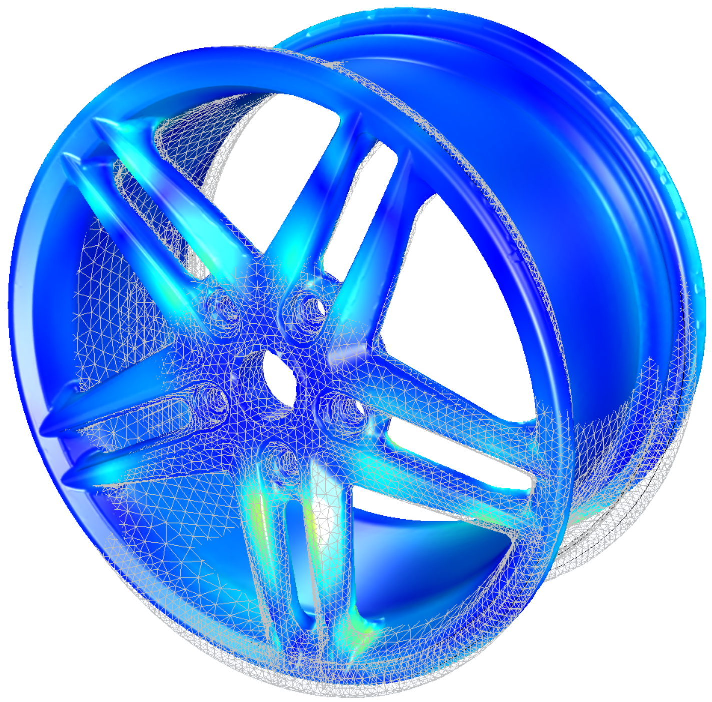
Aplicaciones

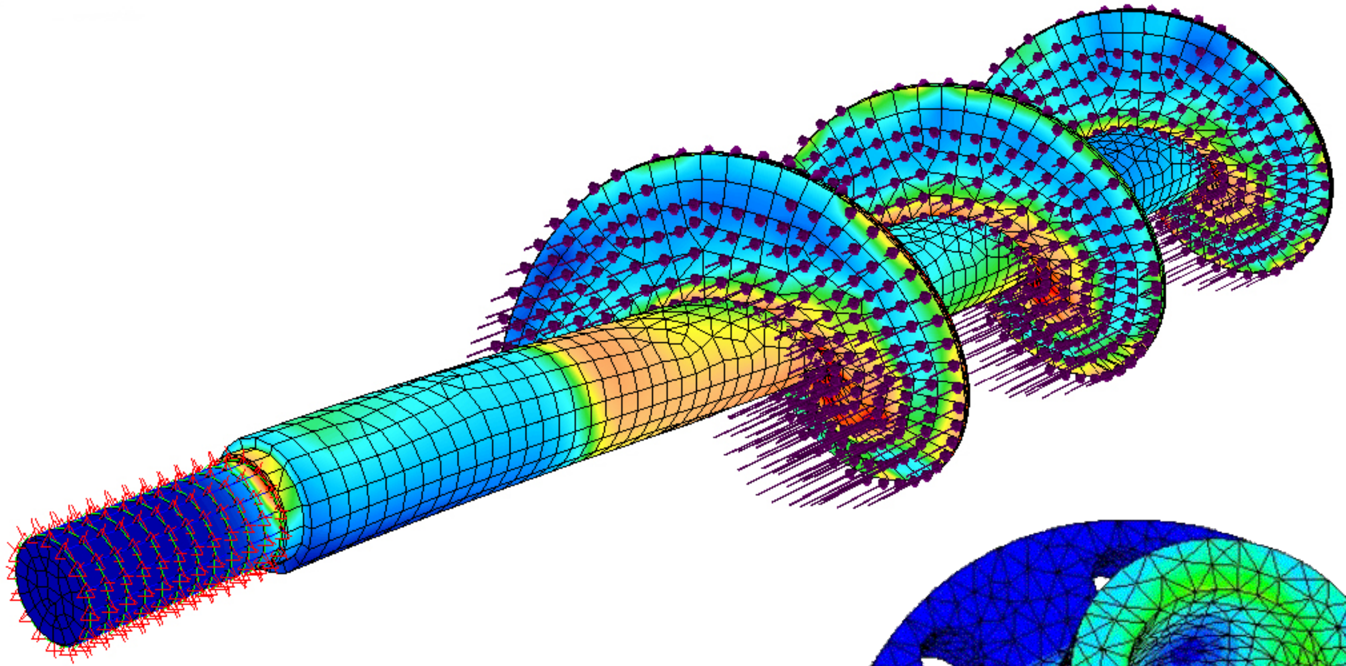


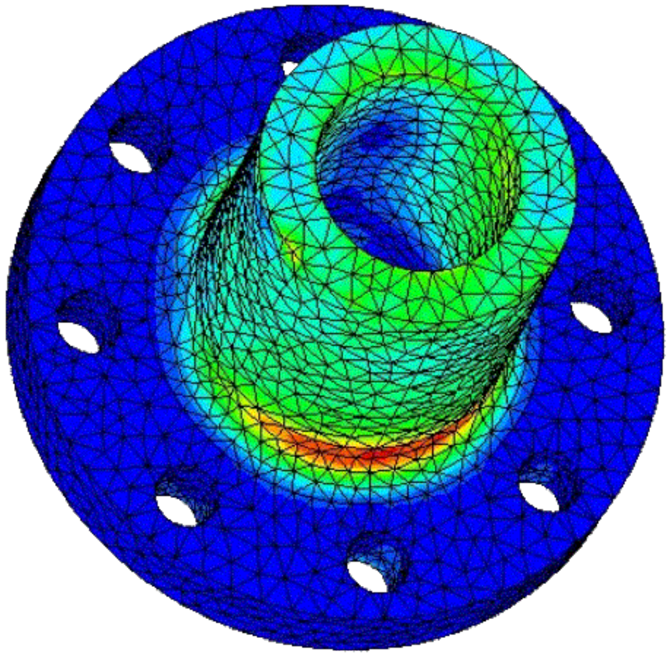
EMSTs Restringidos

Planeación de movimiento

Aplicaciones

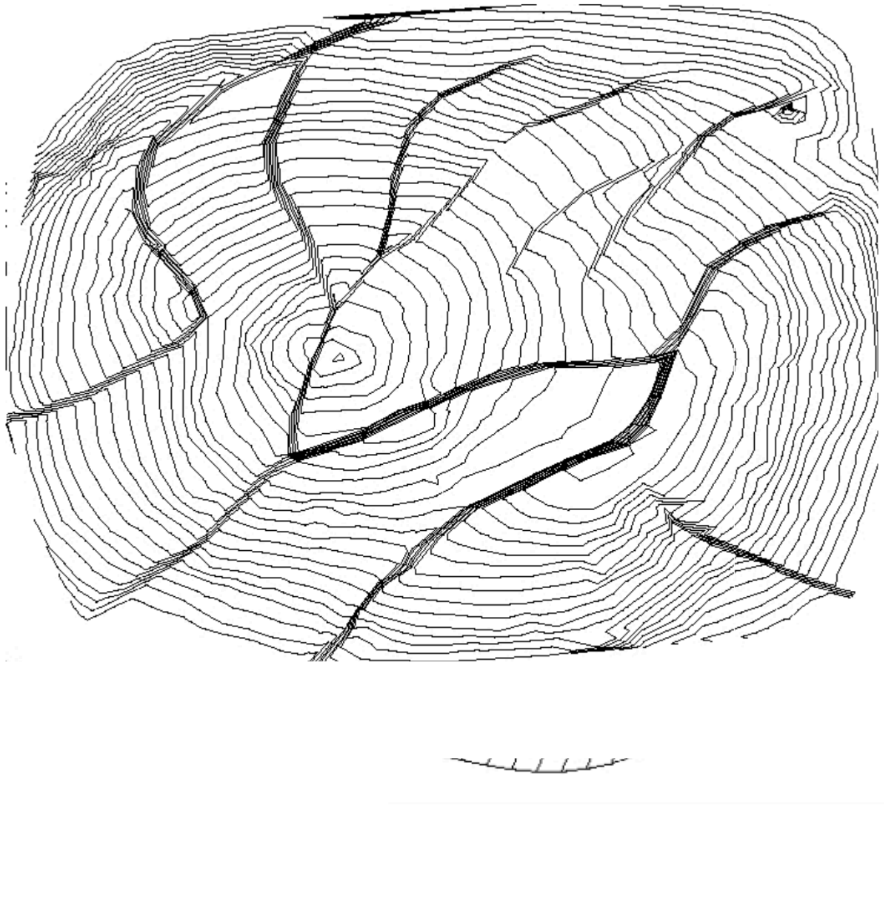


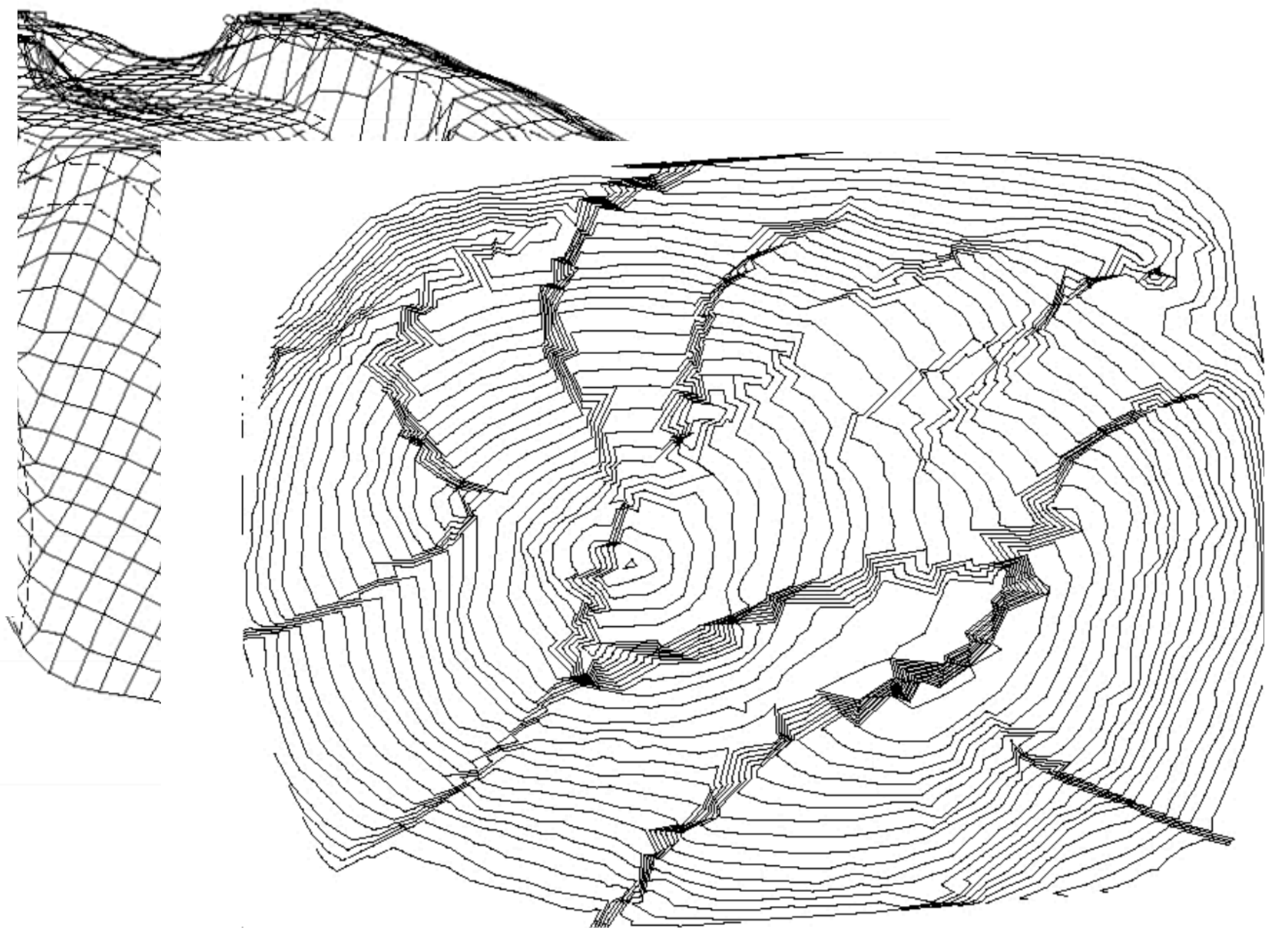




CDT

Aplicaciones

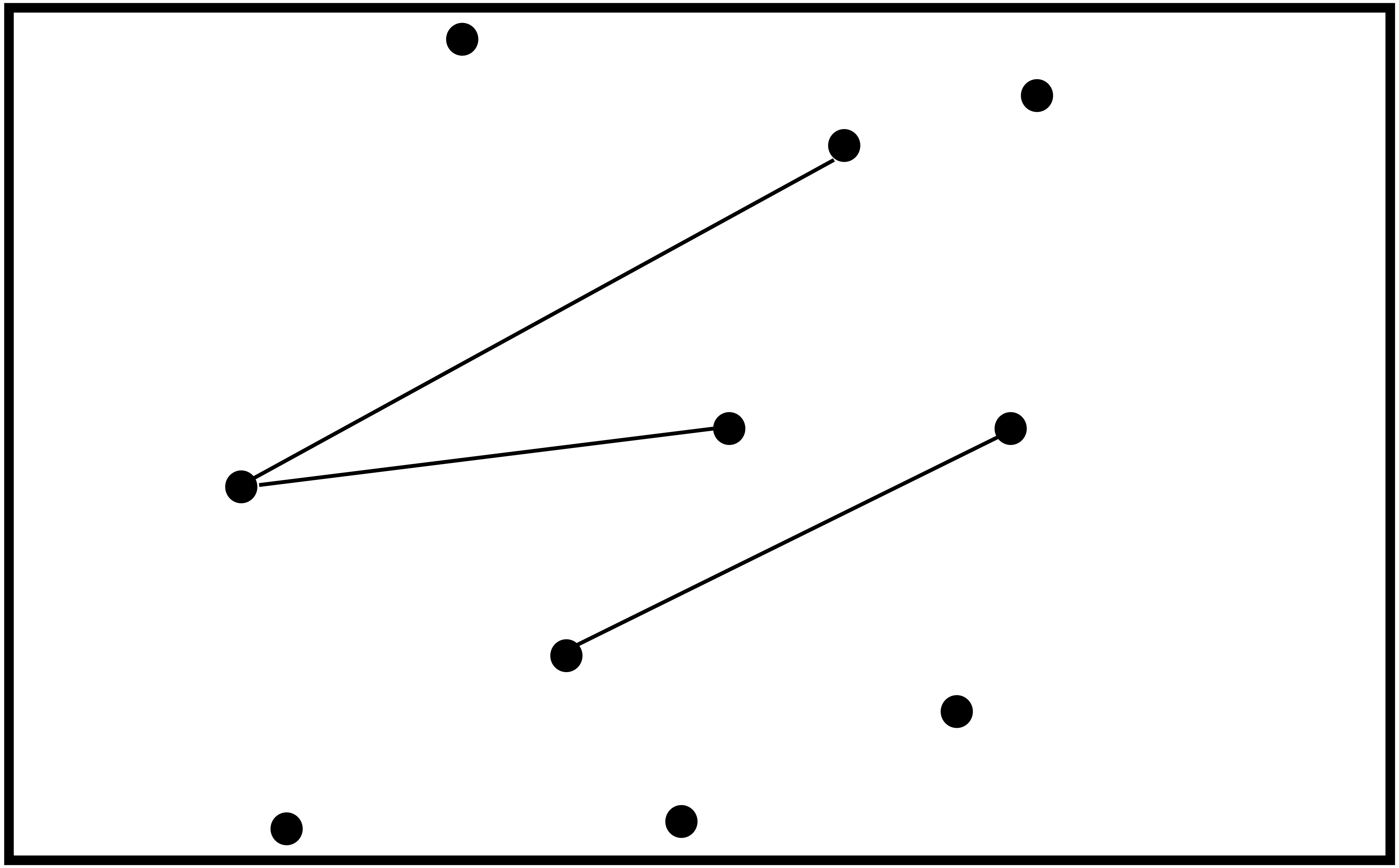




niento FEM **Interpolación de superficies**

Algoritmo

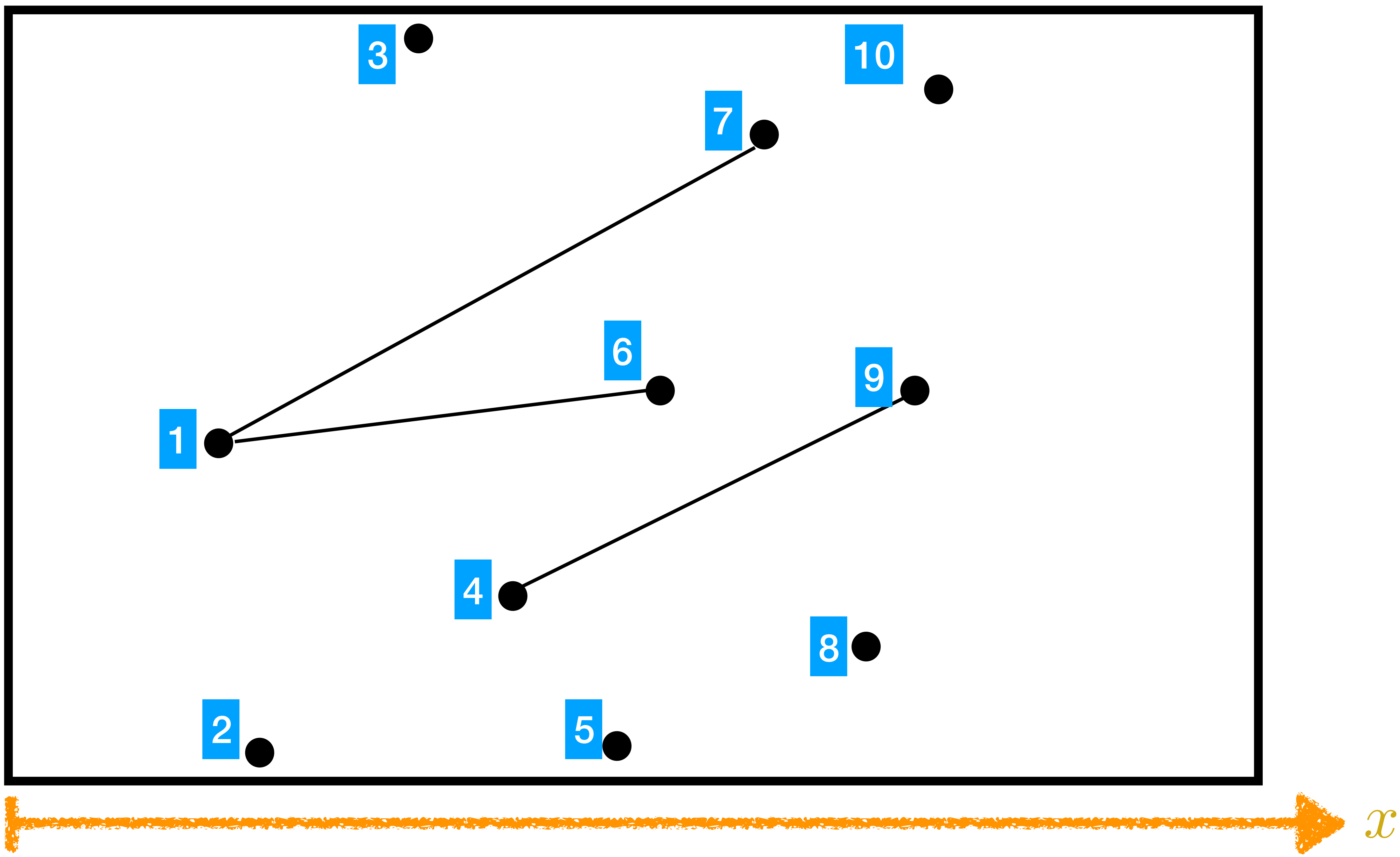
Divide y vencerás



CDT

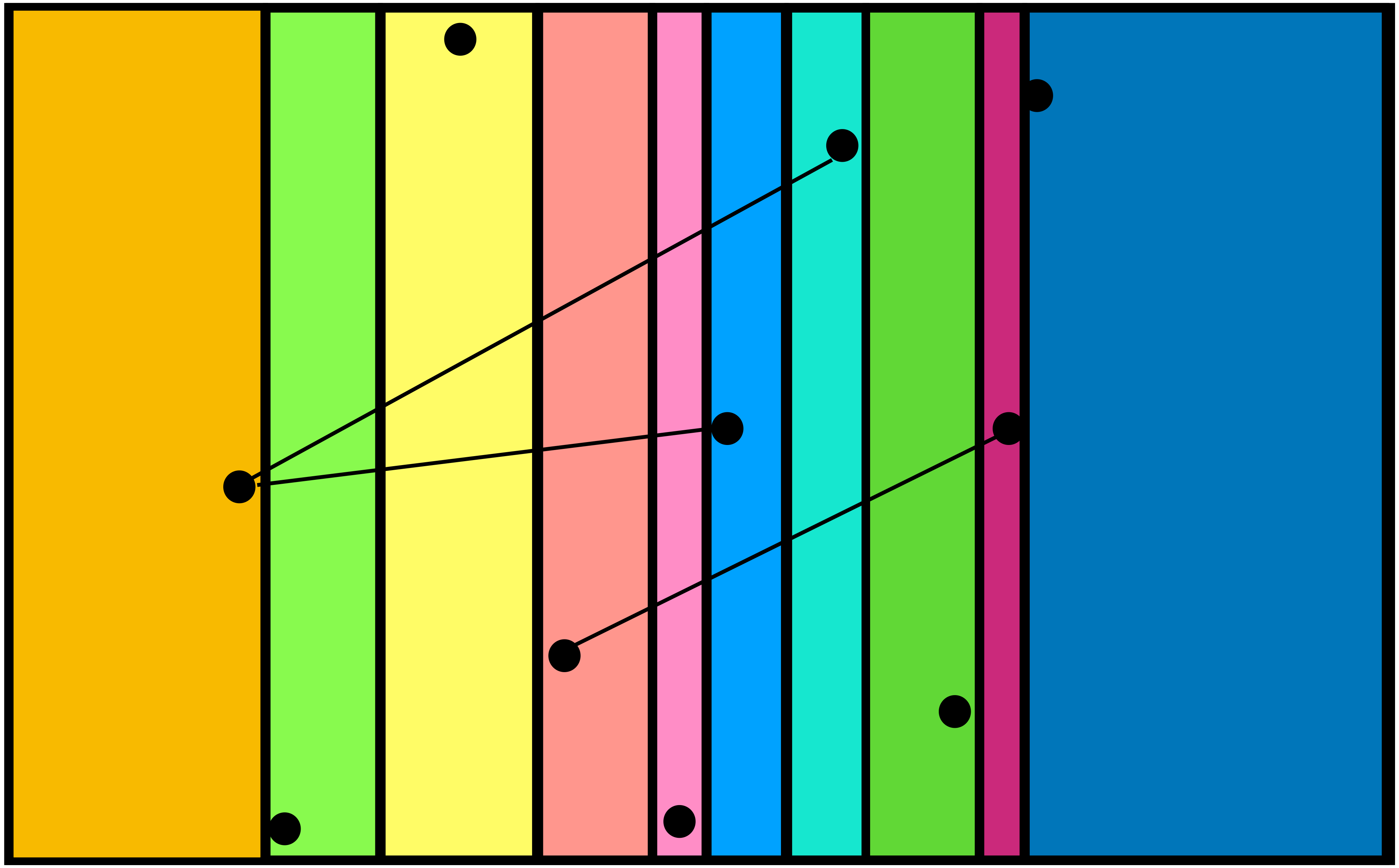
Algoritmo

Divide y vencerás



Algoritmo

Divide y vencerás



CDT

Algoritmo

Divide y vencerás

Calcular la CDT de las franjas

eficientemente

$$\mathcal{O}(n\,log\,n)$$

Combinar franjas contiguas

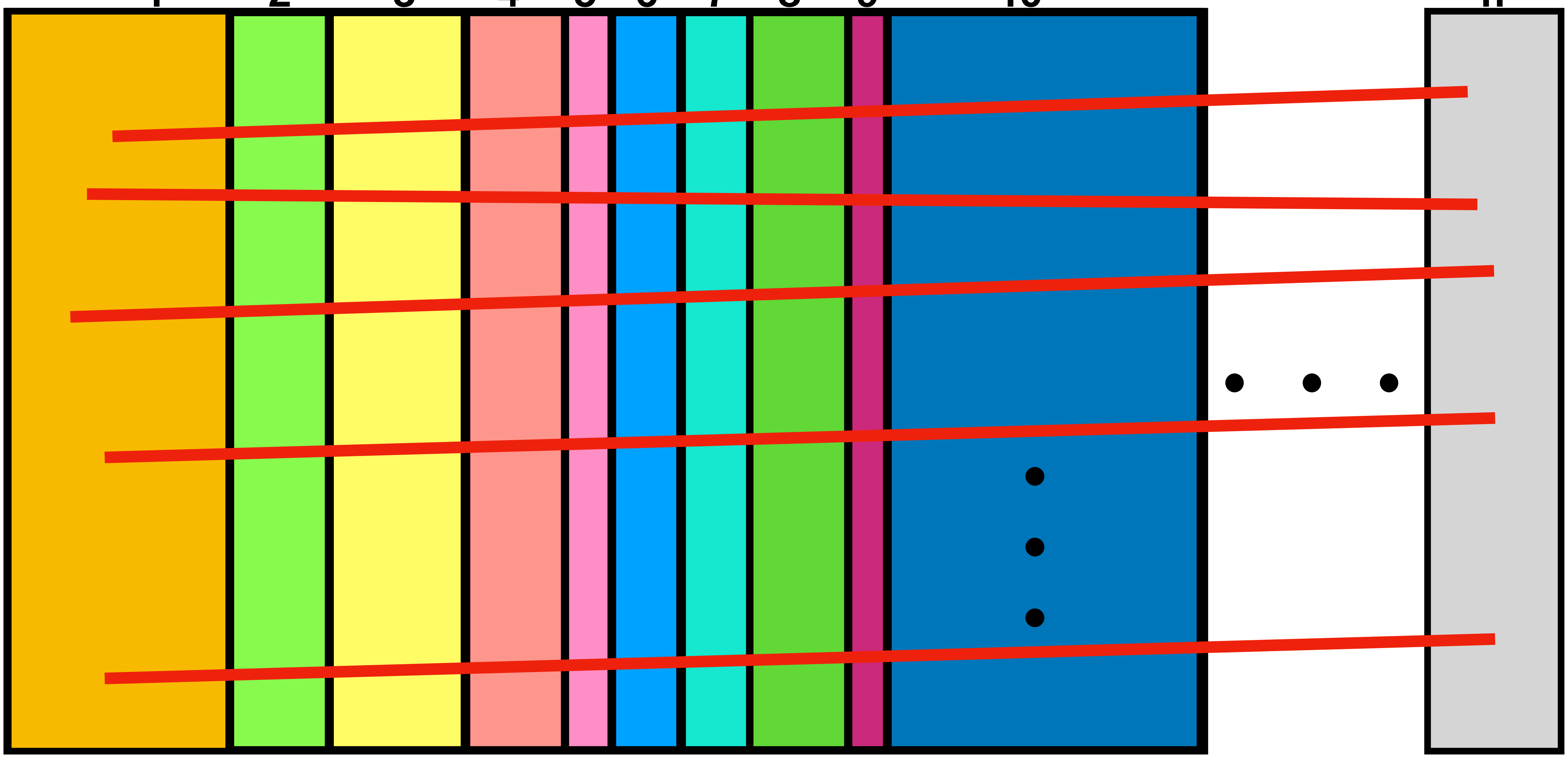
CDT

Algoritmo

Divide y vencerás

$$|V(G)| \leq 3n - 6, \mathcal{O}(n)$$

$$\mathcal{O}(n^2)$$



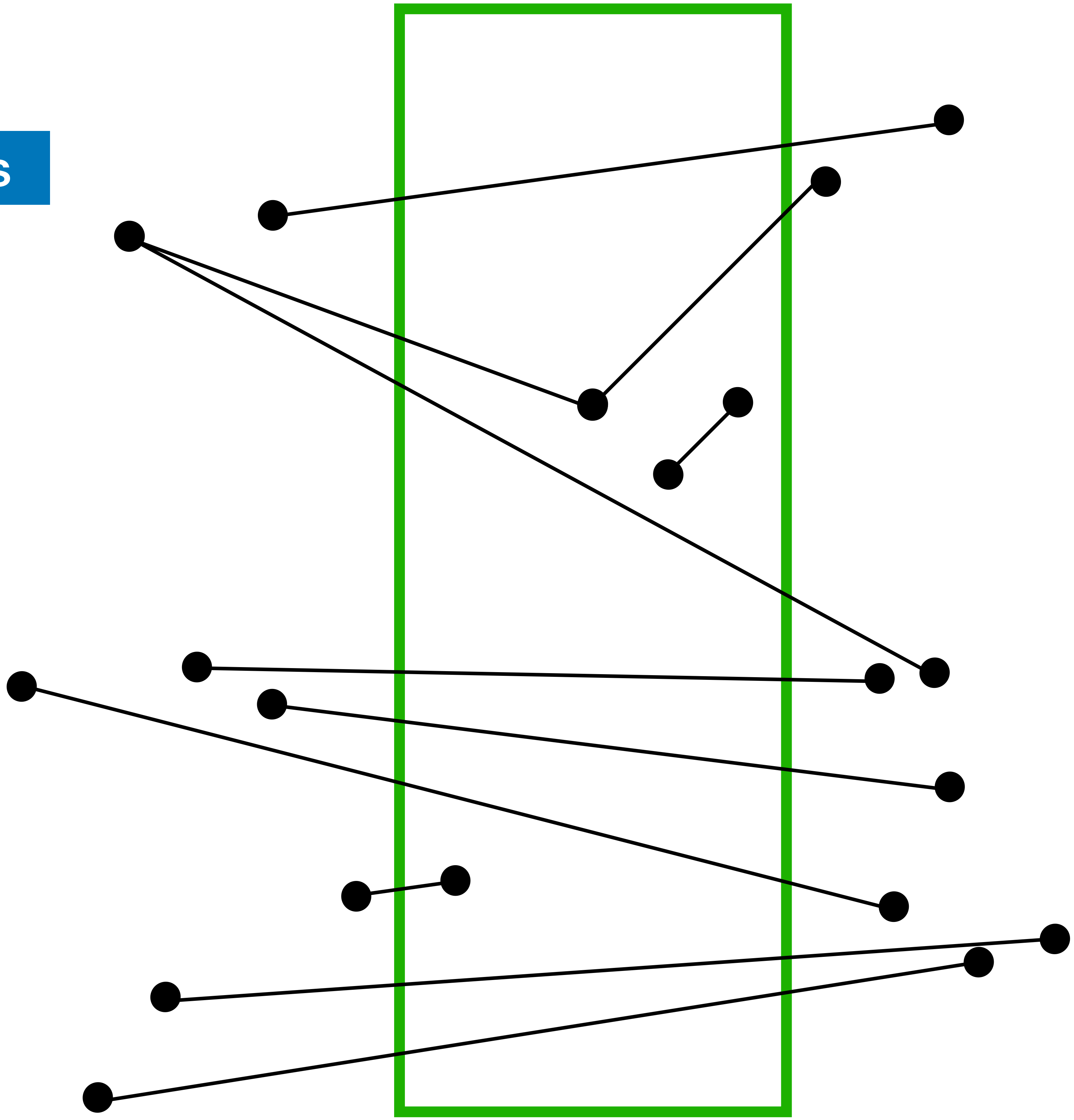
CDT

Algoritmo

Divide y vencerás

Dividir a la franja

en regiones



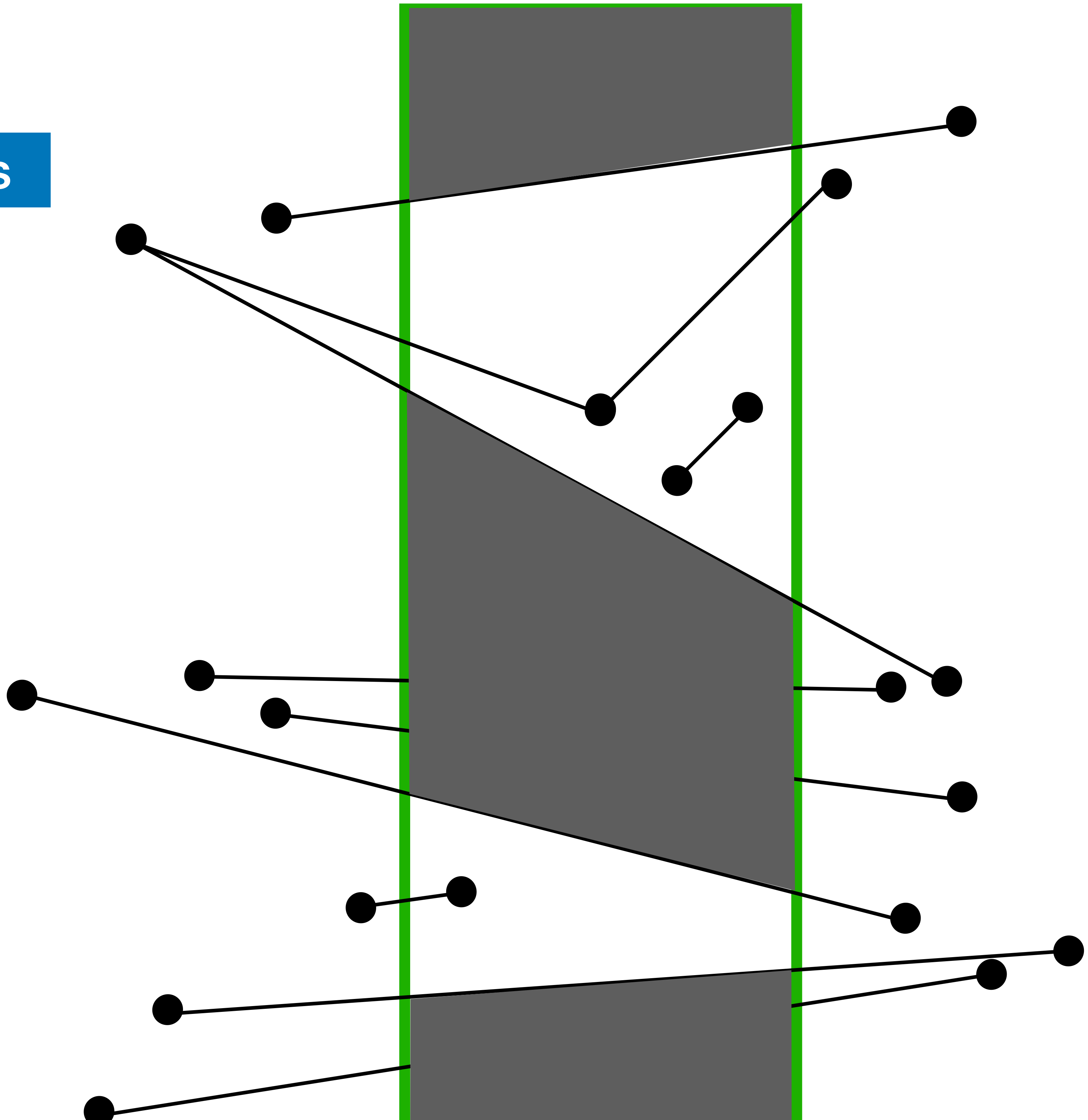
CDT

Algoritmo

Divide y vencerás

Dividir a la franja

en regiones

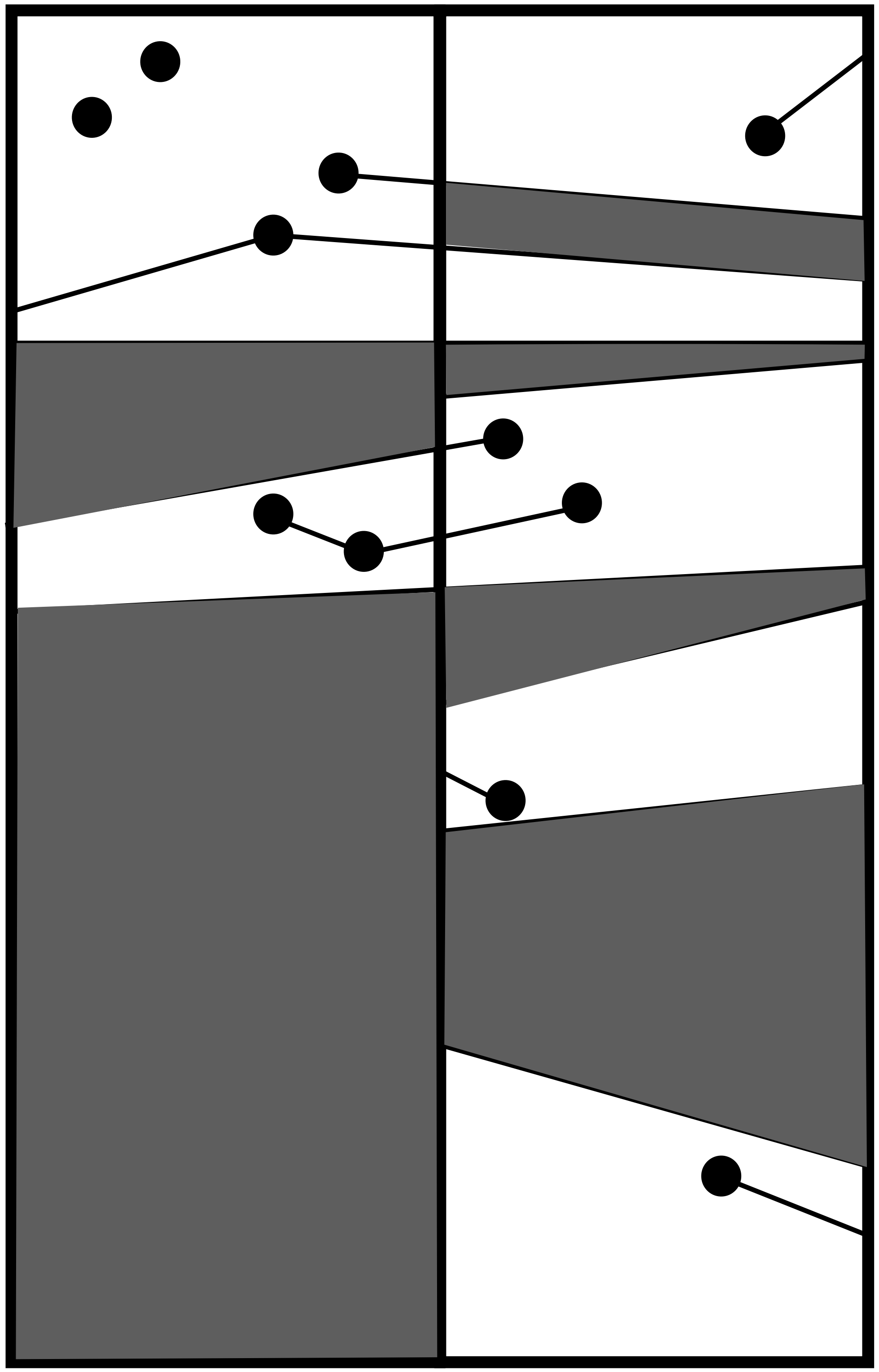


CDT

Algoritmo

Divide y vencerás

Control de las regiones



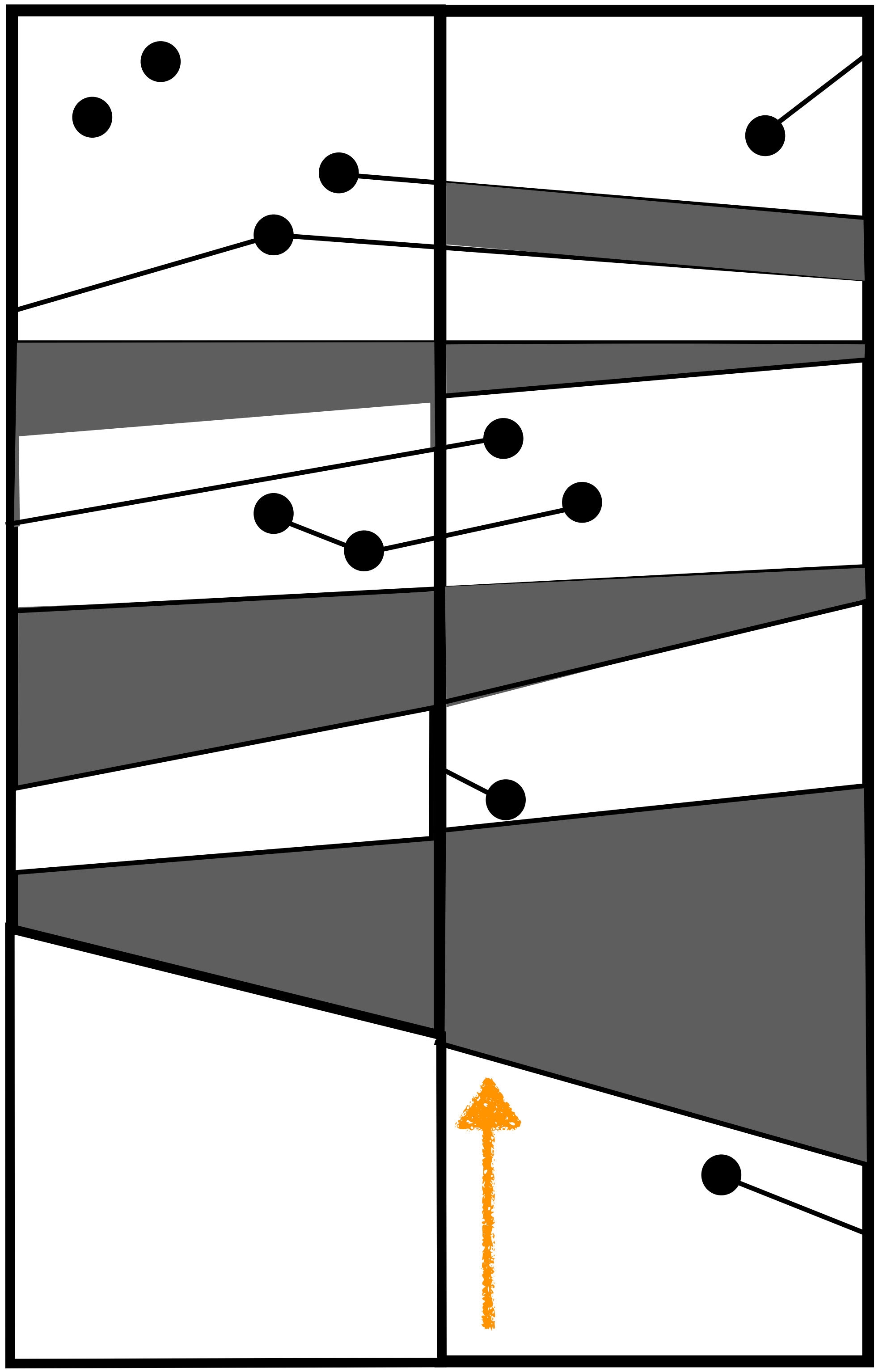
CDT

Algoritmo

Divide y vencerás

Control de las regiones

$$\mathcal{O}(v + h)$$



CDT

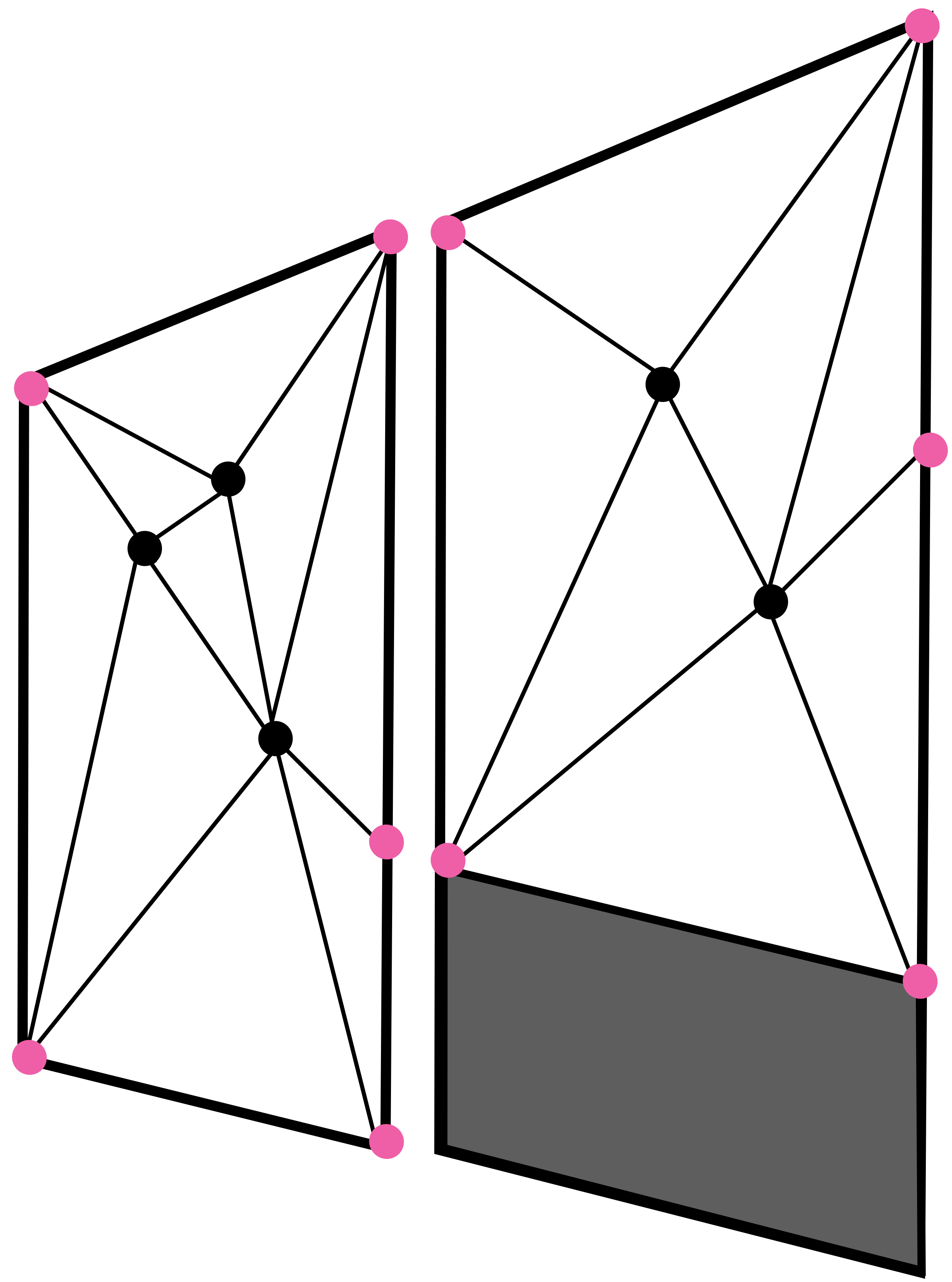
Algoritmo

Divide y vencerás

Mezcla

Introducir vértices falsos

$$\mathcal{O}(v + h)$$



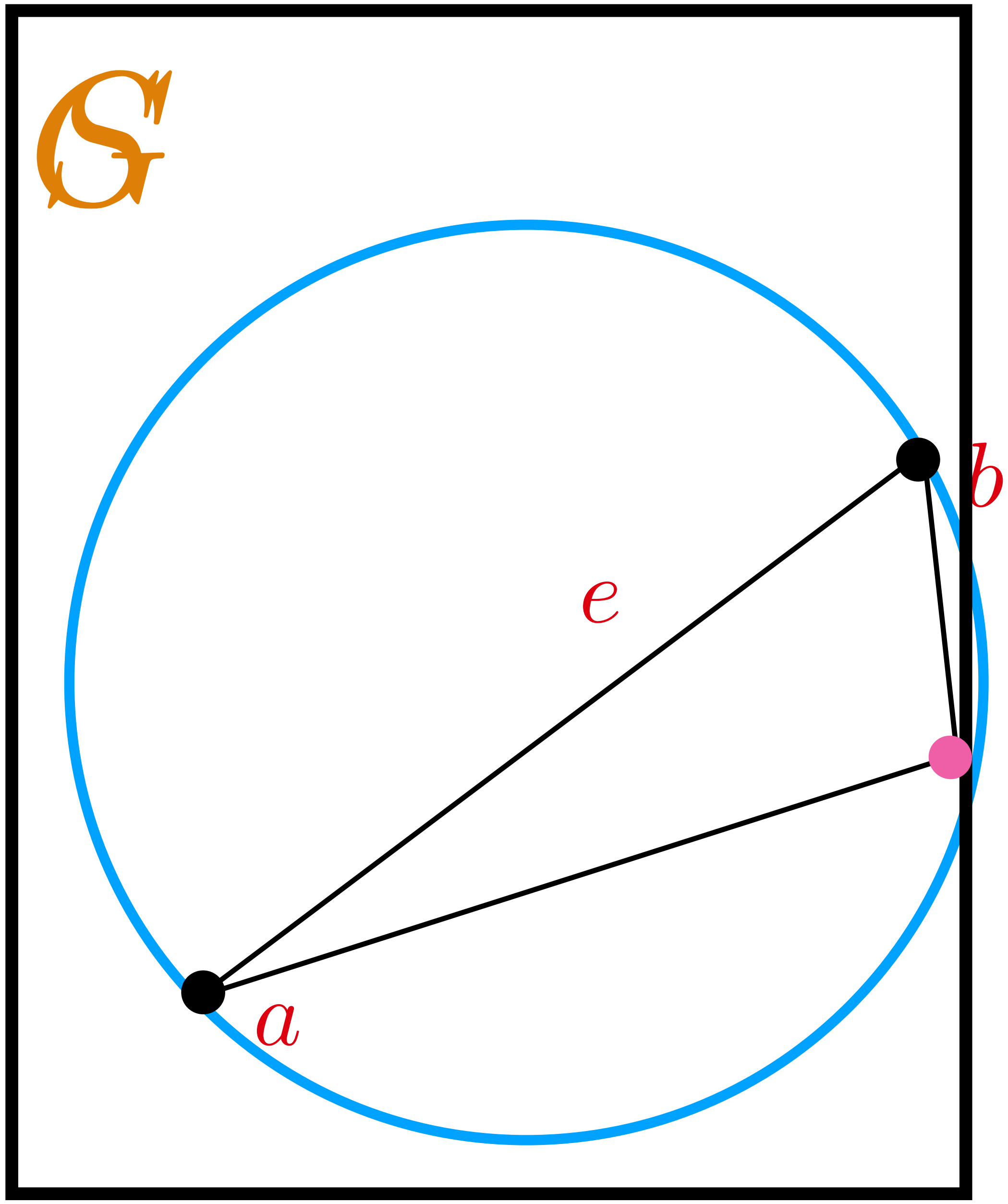
CDT

Algoritmo

Divide y vencerás

Mezcla

Introducir vértices falsos

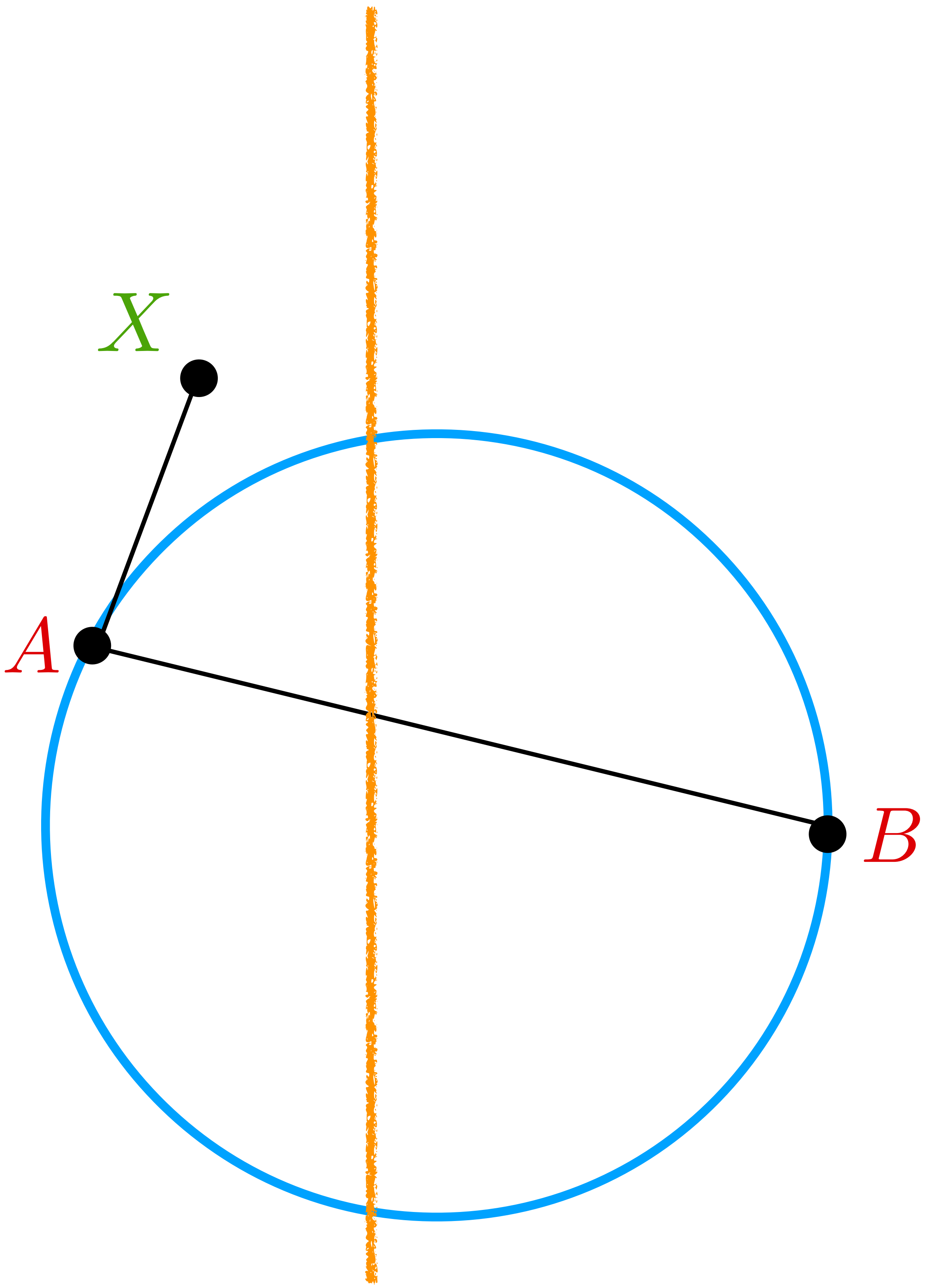


$$e = (a, b) \in E(CTD(G)) \Rightarrow e \in E(CTD(S)), \forall \text{ franja } S$$

Algoritmo

Divide y vencerás

Mezcla

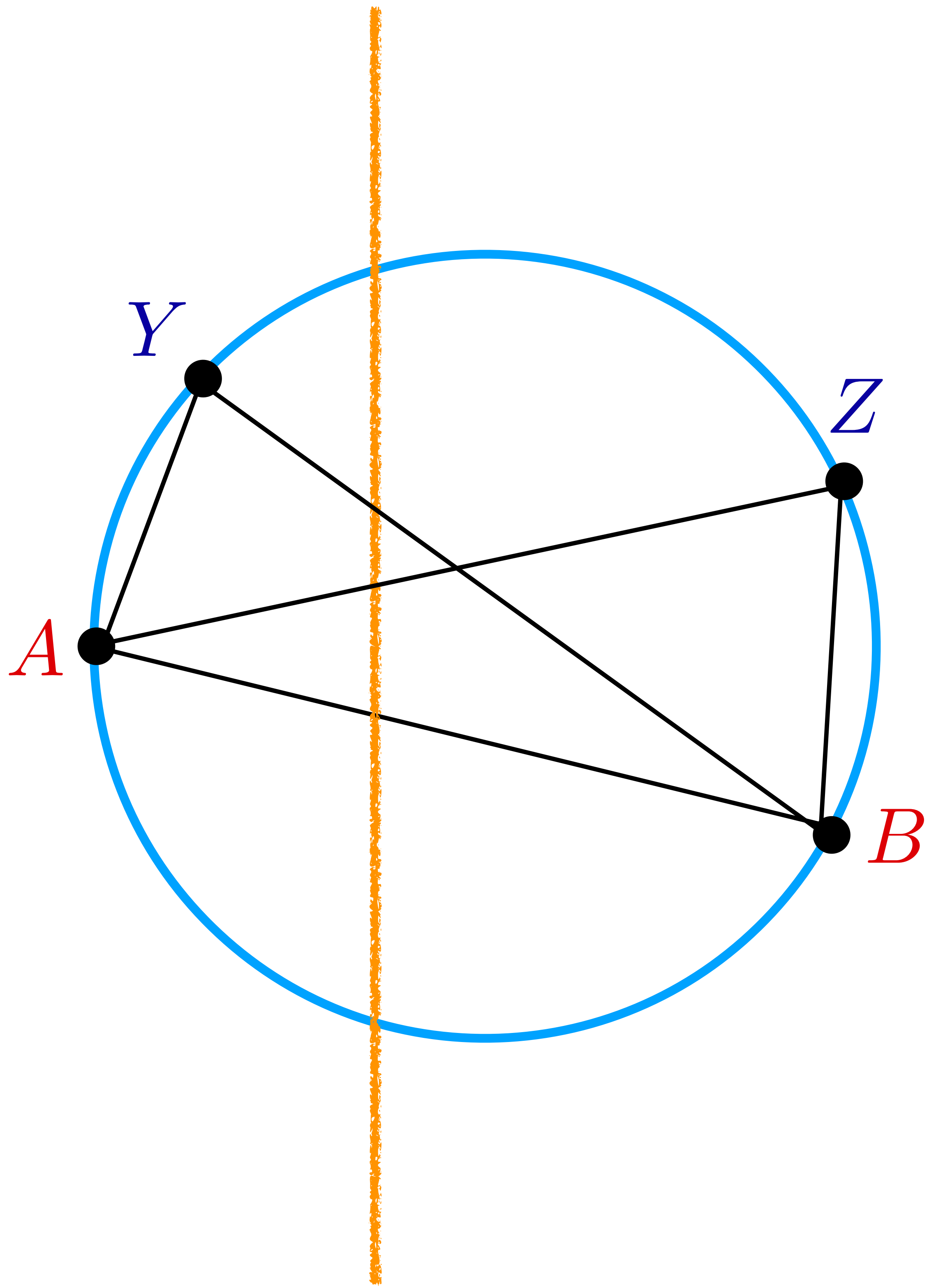


CDT

Algoritmo

Divide y vencerás

Mezcla



CDT

Algoritmo

Divide y vencerás

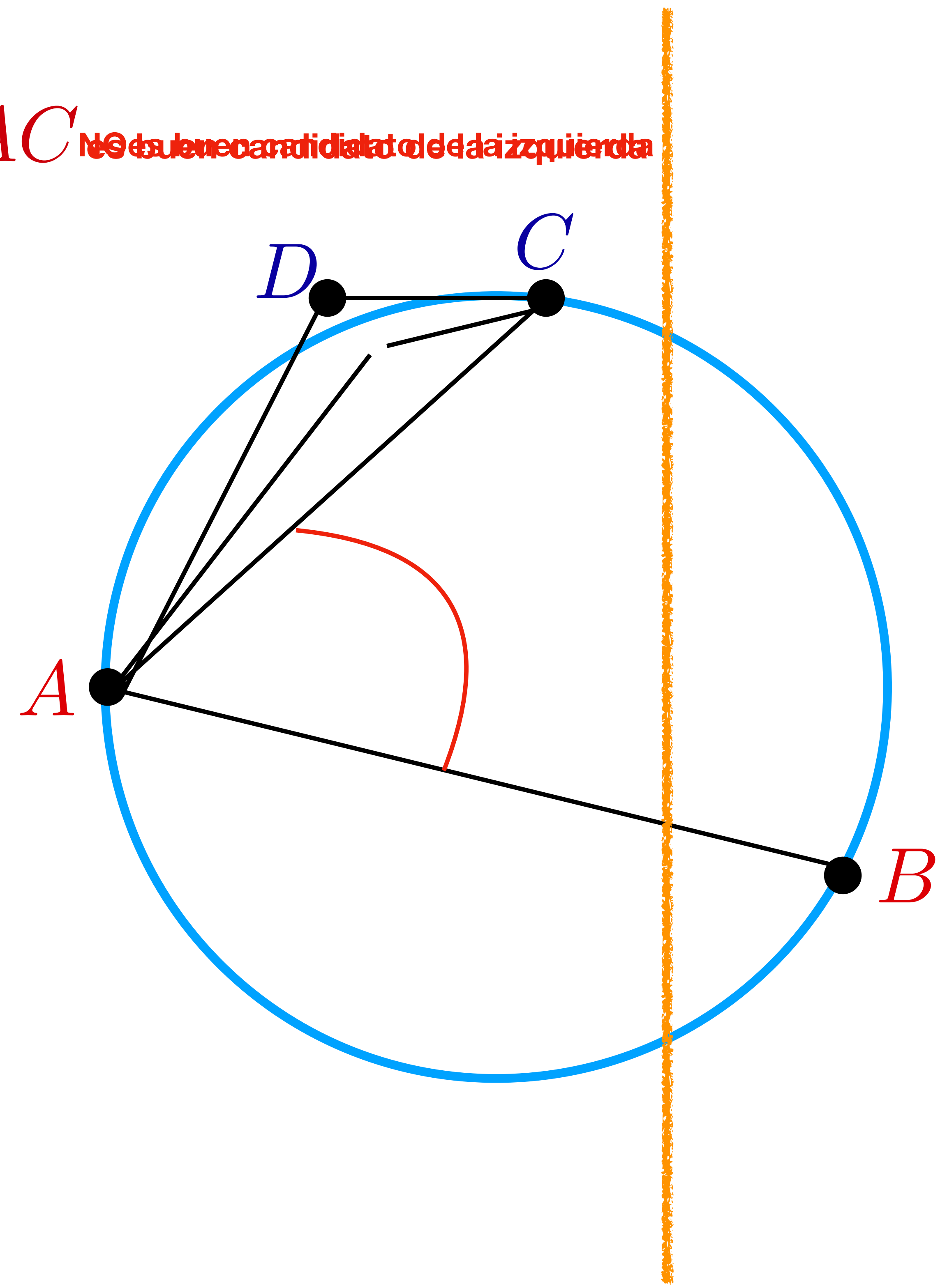
Mezcla

Encontrar al mejor

candidato de cada lado

AC NO es buen candidato de la izquierda

AC es buen candidato de la izquierda



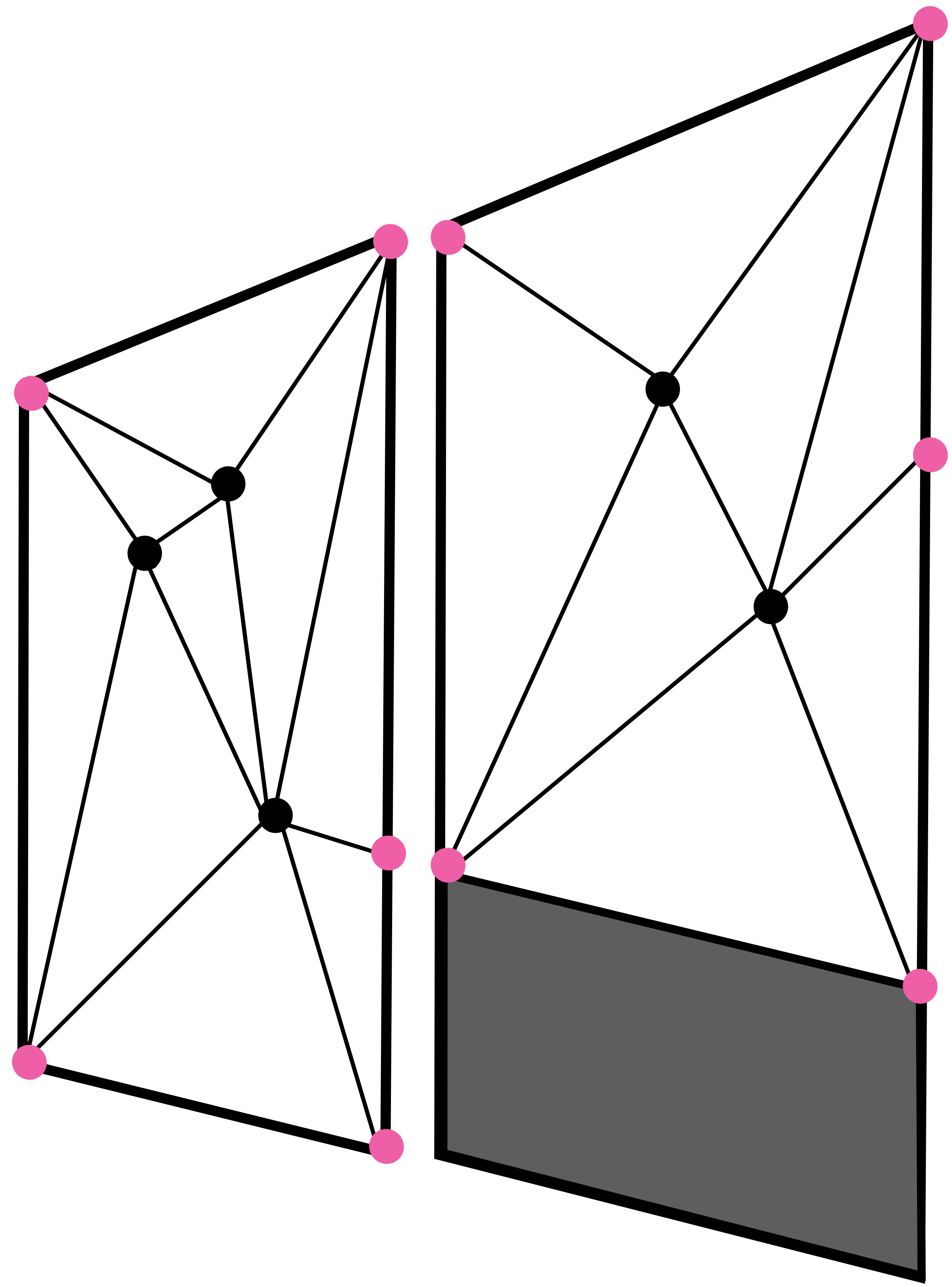
CDT

Algoritmo

Divide y vencerás

Mezcla

1. Quitar vértices falsos.



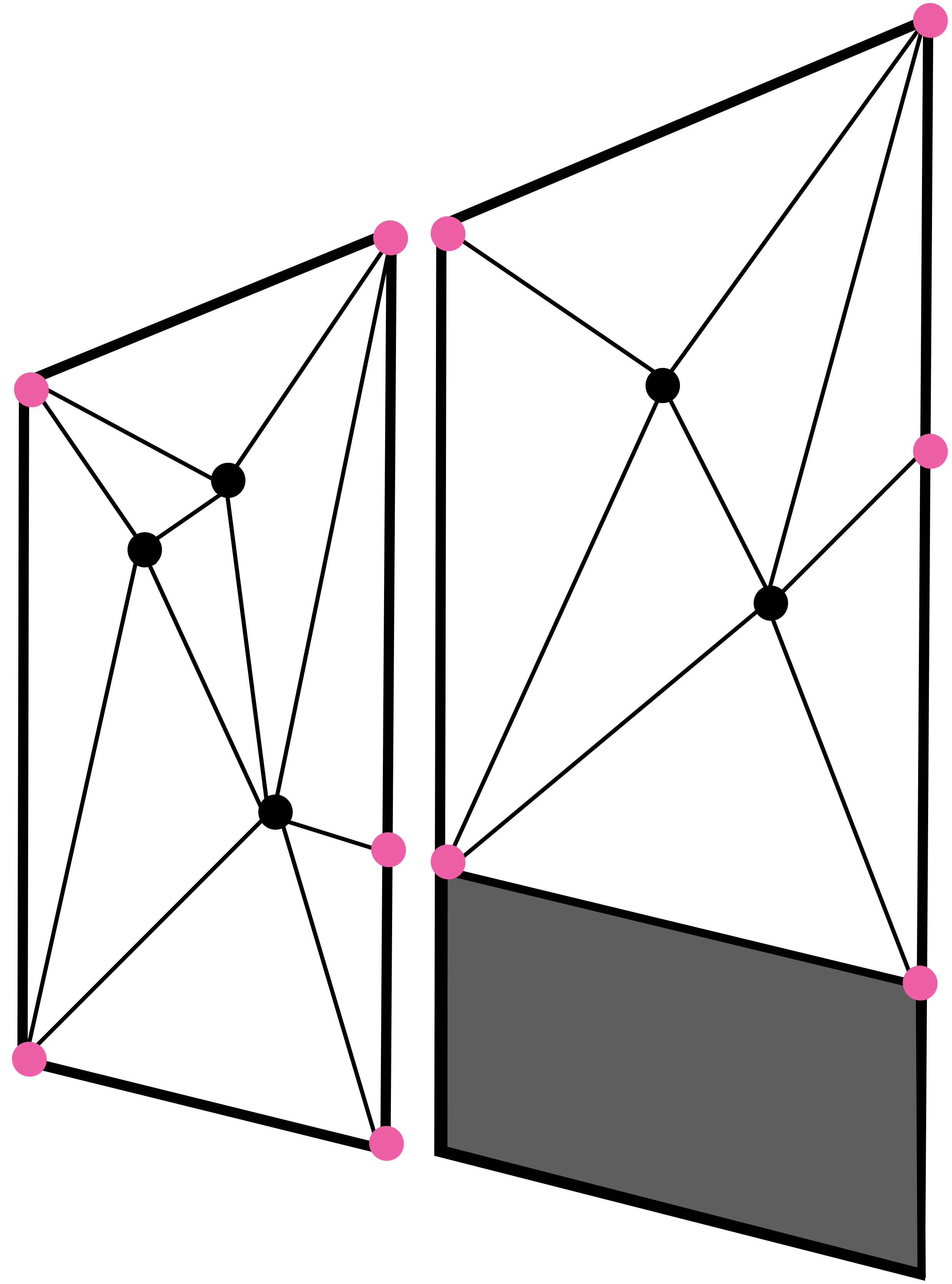
CDT

Algoritmo

Divide y vencerás

Mezcla

1. Quitar vértices falsos.



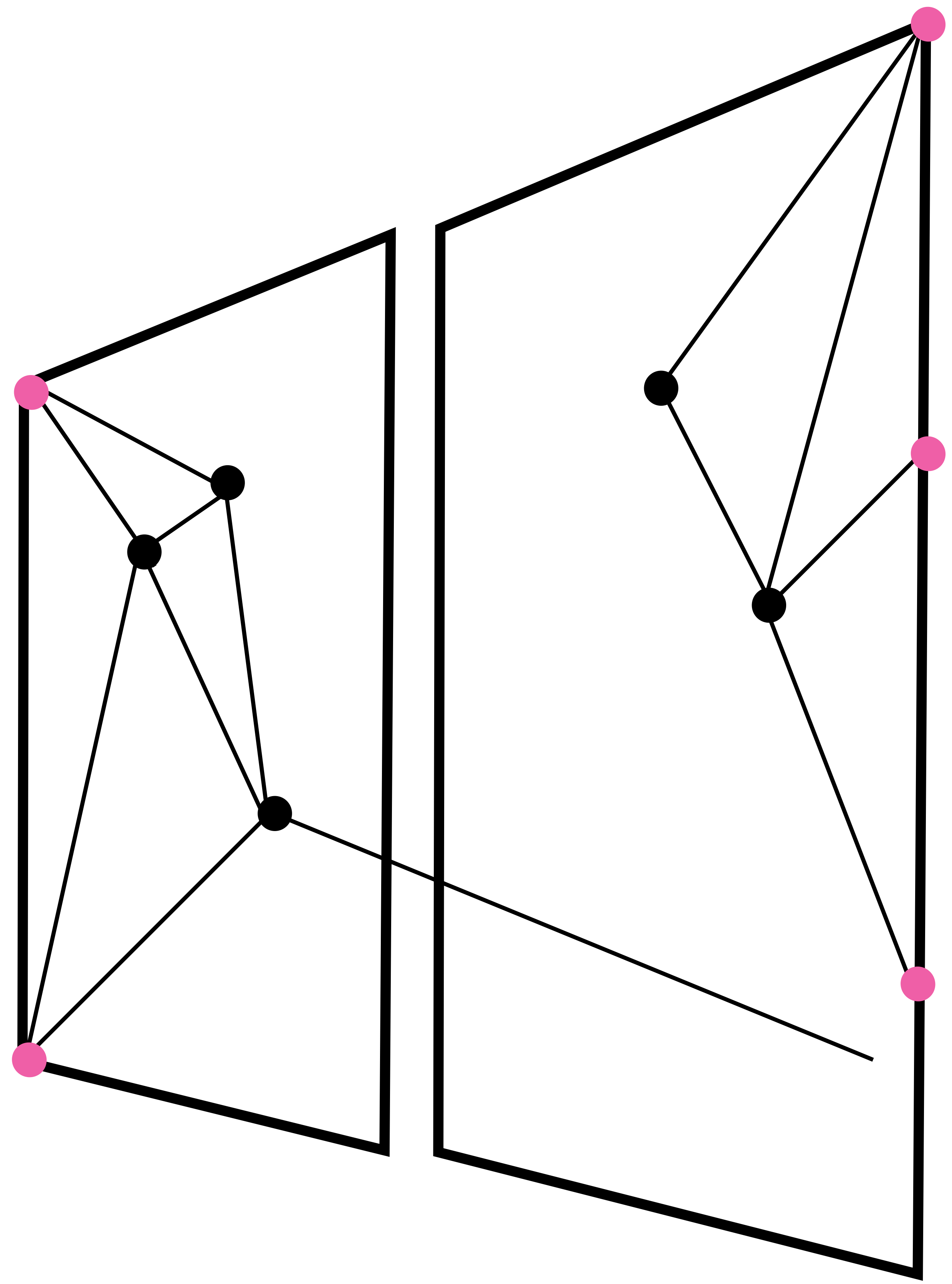
CDT

Algoritmo

Divide y vencerás

Mezcla

1. Quitar vértices falsos.



CDT

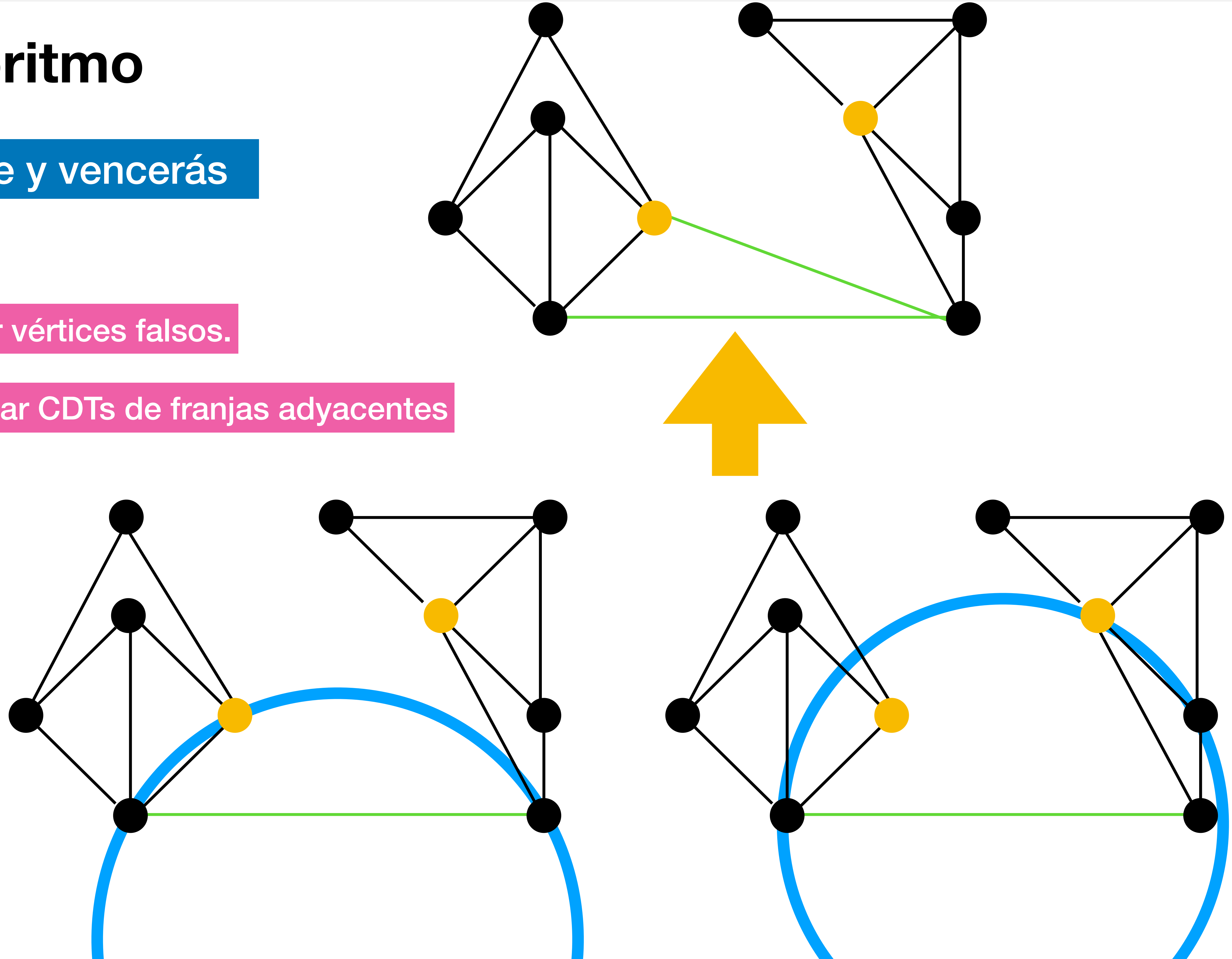
Algoritmo

Divide y vencerás

Mezcla

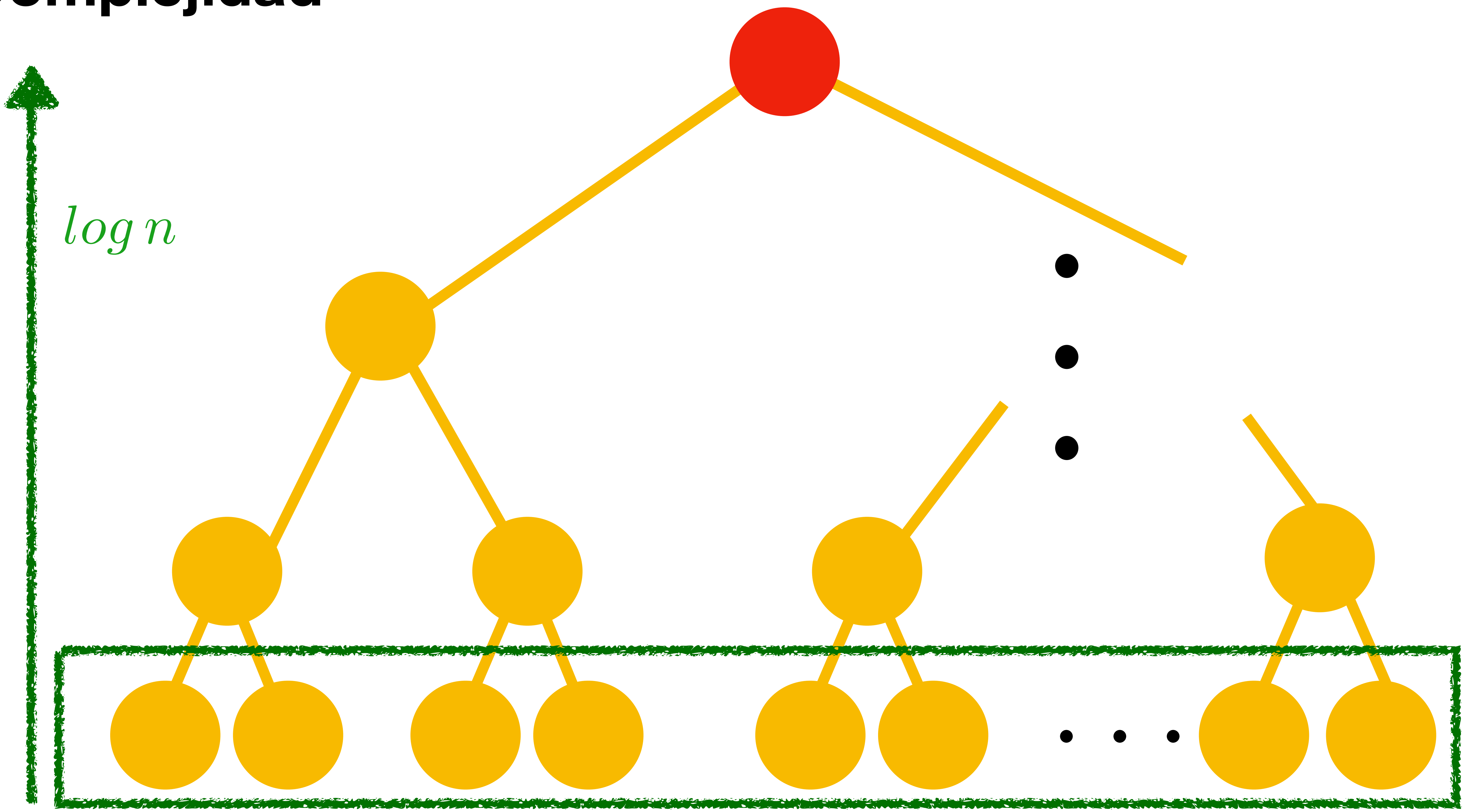
1. Quitar vértices falsos.

2. Mezclar CDTs de franjas adyacentes



CDT

Complejidad



$$\mathcal{O}(|V| + |E|) = \mathcal{O}(n + n) = \mathcal{O}(n)$$

Complejidad

$$\mathcal{O}(n\, log\, n)$$

Complejidad

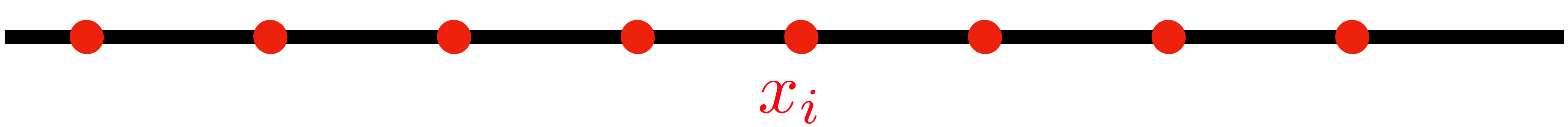
$$\Theta(n\,log\,n)$$

CDT

Complejidad

Ordenamiento $\propto$ Triangulación de Delaunay

$$x_1, x_2, \ldots, x_n \in \mathbb{R}^2$$



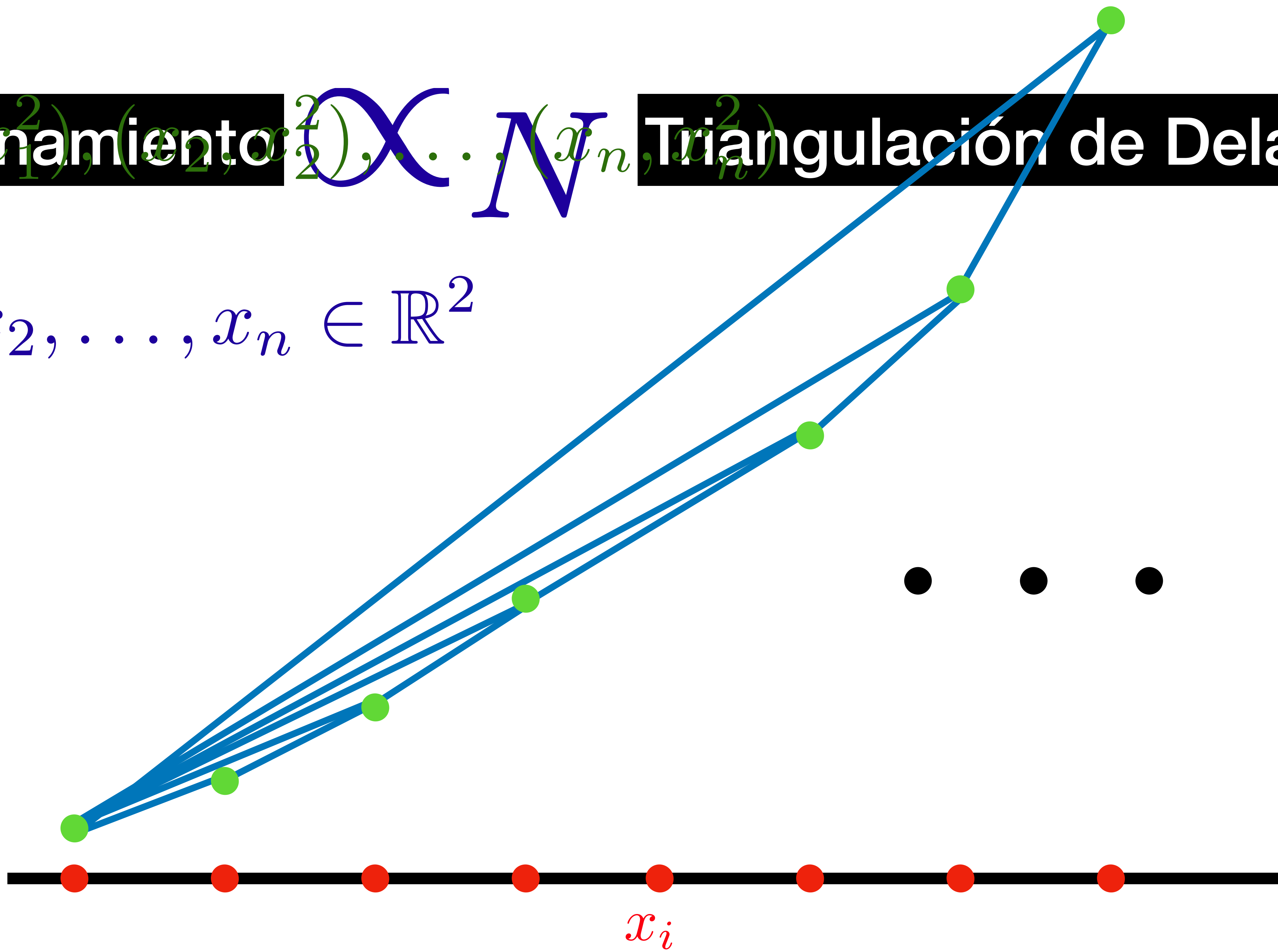
CDT

Complejidad

Ordenamiento $\propto N$ Triangulación de Delaunay

$(x_1, x_1^2), (x_2, x_2^2), \ldots, (x_n, x_n^2)$

$x_1, x_2, \ldots, x_n \in \mathbb{R}^2$



Complejidad

$$\Omega(n\,log\,n)$$