Guía de ejercicios 1er parcial

MAT 103 - ALGEBRA LINEAL Y TEORIA MATRICIAL

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Ejercicio - Operaciones con matrices

Dadas las matrices $A = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 6 & 3 \\ 4 & -2 & 5 \end{pmatrix}$ y $B = \begin{pmatrix} 8 & 3 & -2 \\ 6 & 1 & 3 \\ -2 & 9 & 2 \end{pmatrix}$, calcular la suma de los elementos de la 2da fila y la suma de elementos de la 3ra columna de X si

$$\frac{1}{2}(X - 3A) = (A^t - 2B)^t + A^t$$

Solución

$$\frac{1}{2}(X - 3A) = (A^t - 2B)^t + A^t$$

$$2\left[\frac{1}{2}(X - 3A)\right] = 2[(A^t - 2B)^t + A^t]$$

$$X - 3A = 2[(A^t - 2B)^t + A^t]$$

$$X - 3A = 2[(A^t)^t - 2B^t + A^t]$$

$$X - 3A = 2[A - 2B^t + A^t]$$

$$X - 3A = 2A - 4B^t + 2A^t$$

$$X = 5A - 4B^t + 2A^t$$

$$A = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 6 & 3 \\ 4 & -2 & 5 \end{pmatrix} \qquad A^t = \begin{pmatrix} 2 & -1 & 4 \\ 3 & 6 & -2 \\ 1 & 3 & 5 \end{pmatrix}$$

$$B = \begin{pmatrix} 8 & 3 & -2 \\ 6 & 1 & 3 \\ -2 & 9 & 2 \end{pmatrix} \qquad B^t = \begin{pmatrix} 8 & 6 & -2 \\ 3 & 1 & 9 \\ -2 & 3 & 2 \end{pmatrix}$$

$$X = 5A - 4B^t + 2A^t \qquad A = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 6 & 3 \\ 4 & -2 & 5 \end{pmatrix} \qquad A^t = \begin{pmatrix} 2 & -1 & 4 \\ 3 & 6 & -2 \\ 1 & 3 & 5 \end{pmatrix} \qquad B^t = \begin{pmatrix} 8 & 6 & -2 \\ 3 & 1 & 9 \\ -2 & 3 & 2 \end{pmatrix}$$

$$X = 5\begin{pmatrix} 2 & 3 & 1 \\ -1 & 6 & 3 \\ 4 & -2 & 5 \end{pmatrix} - 4\begin{pmatrix} 8 & 6 & -2 \\ 3 & 1 & 9 \\ -2 & 3 & 2 \end{pmatrix} + 2\begin{pmatrix} 2 & -1 & 4 \\ 3 & 6 & -2 \\ 1 & 3 & 5 \end{pmatrix}$$

$$X = \begin{pmatrix} 10 & 15 & 5 \\ -5 & 30 & 15 \\ 20 & -10 & 25 \end{pmatrix} - \begin{pmatrix} 32 & 24 & -8 \\ 12 & 4 & 36 \\ -8 & 12 & 8 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 8 \\ 6 & 12 & -4 \\ 2 & 6 & 10 \end{pmatrix}$$

$$X = \begin{pmatrix} -18 & -11 & 21 \\ -11 & 38 & -25 \\ 30 & -16 & 27 \end{pmatrix}$$

$$\boxed{\begin{array}{l} \sum 2da\ fila = -11 + 38 - 25 = 2 \\ \sum 3ra\ columna = 21 - 25 + 27 = 23 \end{array}}$$

Ejercicio - Operaciones con matrices

Dadas las matrices $A_{3\times 3} = \begin{cases} 0 & si\ i > j \\ \pi & si\ i = j \\ -1 & si\ i < j \end{cases}$ y $B_{3\times 3} = \begin{cases} \int_0^{\pi} x \sin x \, dx & si\ i \geq j \\ (i+j)! & si\ i < j \end{cases}$, calcular $3A - 2B$ y $tr[(I + A^t)^t]$

Solución

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad A = \begin{pmatrix} \pi & -1 & -1 \\ 0 & \pi & -1 \\ 0 & 0 & \pi \end{pmatrix}$$

Para B $\quad \displaystyle\int_0^{\pi} x \sin x \, dx = (x \cos x + \sin x) \,|_0^{\pi}$

$$\int_0^{\pi} x \sin x \, dx = \pi$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad B = \begin{pmatrix} \pi & (1+2)! & (1+3)! \\ \pi & \pi & (2+3)! \\ \pi & \pi & \pi \end{pmatrix} \quad B = \begin{pmatrix} \pi & 6 & 24 \\ \pi & \pi & 120 \\ \pi & \pi & \pi \end{pmatrix}$$

$$A = \begin{pmatrix} \pi & -1 & -1 \\ 0 & \pi & -1 \\ 0 & 0 & \pi \end{pmatrix} \qquad B = \begin{pmatrix} \pi & 6 & 24 \\ \pi & \pi & 120 \\ \pi & \pi & \pi \end{pmatrix}$$

$$3A - 2B = 3\begin{pmatrix} \pi & -1 & -1 \\ 0 & \pi & -1 \\ 0 & 0 & \pi \end{pmatrix} - 2\begin{pmatrix} \pi & 6 & 24 \\ \pi & \pi & 120 \\ \pi & \pi & \pi \end{pmatrix}$$

$$3A - 2B = \begin{pmatrix} 3\pi & -3 & -3 \\ 0 & 3\pi & -3 \\ 0 & 0 & 3\pi \end{pmatrix} + \begin{pmatrix} -2\pi & -12 & -48 \\ -2\pi & -2\pi & -240 \\ -2\pi & -2\pi & -2\pi \end{pmatrix}$$

$$\boxed{3A - 2B = \begin{pmatrix} \pi & -15 & -51 \\ -2\pi & \pi & -243 \\ -2\pi & -2\pi & \pi \end{pmatrix}}$$

$$tr[(I + A^t)^t]$$

$$tr[(I + A^t)^t] = tr[I^t + A]$$

$$tr[(I + A^t)^t] = tr[I + A]$$

$$I + A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \pi & -1 & -1 \\ 0 & \pi & -1 \\ 0 & 0 & \pi \end{pmatrix}$$

$$I + A = \begin{pmatrix} \pi + 1 & -1 & -1 \\ 0 & \pi + 1 & -1 \\ 0 & 0 & \pi + 1 \end{pmatrix}$$

$$\boxed{tr[(I + A^t)^t] = 3(\pi + 1)}$$

Ejercicio - Operaciones con matrices

Dadas la matriz $A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}$, calcular A^{37}

Solución

$$A^{37} = \underbrace{A \cdot A \cdot A \cdot \ldots \cdot A \cdot A}_{\text{37 veces}}$$

$$A^2 = A \cdot A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -5 & -6 & -6 \\ 9 & 10 & 9 \\ -4 & -4 & -3 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = A \cdot A^2$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} -5 & -6 & -6 \\ 9 & 10 & 9 \\ -4 & -4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}$$

$$A^3 = A$$

$$A^3 = A$$

$$A^{37} = A^{3\cdot 12+1}$$

$$A^{37} = A^{3\cdot 12} \cdot A$$

$$A^{37} = (A^3)^{12} \cdot A$$

$$A^{37} = (A)^{12} \cdot A$$

$$A^{37} = A^{3\cdot 4} \cdot A$$

$$A^{37} = (A^3)^4 \cdot A$$

$$A^{37} = (A)^4 \cdot A$$

$$A^{37} = A^5 = A^{3+2}$$

$$A^{37} = A^3 \cdot A^2$$

$$A^{37} = A \cdot A^2$$

$$A^{37} = A^3$$

$$A^{37} = A$$

$$\boxed{A^{37} = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}}$$

Ejercicio - Operaciones con matrices

Dada la matriz $N^{\frac{1}{80}} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ -1 & -2 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 2 \end{pmatrix}$, hallar la matriz N

Solución

$$N^{\frac{1}{80}} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}_{3\times 2} \cdot \begin{pmatrix} 1 & 1 & 1 \\ -1 & -2 & 0 \end{pmatrix}_{2\times 3} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 2 \end{pmatrix}_{3\times 3}$$

$$N^{\frac{1}{80}} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}_{3\times 3} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 2 \end{pmatrix}_{3\times 3}$$

$$N^{\frac{1}{80}} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(N^{\frac{1}{80}})^{80} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}^{80}$$

$$N = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}^{80}$$

Sea $X = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$; $N = X^{80}$

$$X = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X^3 = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$X^4 = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X^4 = -X$$

$$X^{80} = X^{4\cdot 20} = (X^4)^{20}$$

$$X^{80} = (-X)^{20}$$

$$X^{80} = X^{20}$$

$$X^{80} = (X^4)^5$$

$$X^{80} = (-X)^5$$

$$X^{80} = -X^5$$

$$X^{80} = -X^4 \cdot X$$

$$X^{80} = -(-X) \cdot X$$

$$X^{80} = X^2$$

$$N = X^{80} = X^2$$

$$\boxed{N = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

Matrices especiales

Matriz nula o matriz cero

$$\Theta = [\theta_{ij}] = \{0 \text{ para todo } i, j$$

$$\Theta = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \Theta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \Theta = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Matriz diagonal

$$D = [d_{ij}] = \begin{cases} d_{ij} & si\ i = j \\ 0 & si\ i \neq j \end{cases} \qquad D = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \qquad D = \begin{pmatrix} 3/5 & 0 & 0 \\ 0 & 2e & 0 \\ 0 & 0 & \pi \end{pmatrix} \qquad D = \begin{pmatrix} 2^6 & 0 & 0 & 0 \\ 0 & 3^6 & 0 & 0 \\ 0 & 0 & 7^6 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matriz escalar

$$K = [k_{ij}] = \begin{cases} c & si\ i = j \\ 0 & si\ i \neq j \end{cases} \qquad K = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \qquad K = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad K = \begin{pmatrix} -14 & 0 & 0 & 0 \\ 0 & -14 & 0 & 0 \\ 0 & 0 & -14 & 0 \\ 0 & 0 & 0 & -14 \end{pmatrix}$$

Matriz identidad

$$I = [i_{ij}] = \begin{cases} 1 & si\ i = j \\ 0 & si\ i \neq j \end{cases} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matriz idempotente $A^2 = A$

Matriz involutiva $A^2 = I$

Matriz nilpotente/nulpotente $A^n = \Theta$

Matriz periódica $A^p = I$

p es el periodo $A^{p+1} = A$

Matriz simétrica $A^T = A$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 9 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

Matriz antisimétrica $A^T = -A$

$$A = \begin{pmatrix} 0 & 2 & 3 \\ -2 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix} \qquad A^T = \begin{pmatrix} 0 & -2 & -3 \\ 2 & 0 & -6 \\ 3 & 6 & 0 \end{pmatrix}$$

$$A^T = -\begin{pmatrix} 0 & 2 & 3 \\ -2 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix}$$

Nota:
Las matrices antisimétricas siempre tienen ceros en su diagonal principal

Matriz ortogonal $A^T = A^{-1}$

Ejercicio – Matrices especiales y propiedades

Si $A \cdot B = A$ y $B \cdot A = B$ demostrar que A y B^t son idempotentes

Solución

$$A \cdot B = A$$

$$A \cdot B \cdot A = A \cdot A$$

$$A \cdot B = A^2$$

$$\boxed{A = A^2}$$

$$B \cdot A = B$$

$$B \cdot A \cdot B = B \cdot B$$

$$B \cdot A = B^2 \longrightarrow B \cdot A = B \cdot B$$

$$B = B^2$$

$$(B \cdot A)^t = (B \cdot B)^t$$

$$B^t = B^t \cdot B^t$$

$$\boxed{B^t = (B^t)^2}$$

Ejercicio – Matrices especiales y propiedades

Si A es involuntiva demostrar que $C = \frac{1}{2}(I + A)$ y $D = \frac{1}{2}(I - A)$ son idempotentes,
¿Qué tipo de matriz es $C \cdot D$?

Solución

$A^2 = I$

Demostrar $\quad C^2 = C \quad D^2 = D$

$$C = \frac{1}{2}(I + A)$$

$$C^2 = \left[\frac{1}{2}(I + A)\right]^2$$

$$C^2 = \left[\frac{1}{2}(I + A)\right]^2$$

$$C^2 = \frac{1}{4}(I + A)^2$$

$$C^2 = \frac{1}{4}(I + A)(I + A)$$

$$C^2 = \frac{1}{4}(I^2 + I \cdot A + A \cdot I + A^2)$$

$$C^2 = \frac{1}{4}(I + A + A + I)$$

$$C^2 = \frac{1}{4}(2I + 2A)$$

$$C^2 = \frac{1}{2}(I + A) \qquad \boxed{C^2 = C}$$

$$D = \frac{1}{2}(I - A)$$

$$D^2 = \frac{1}{4}(I - A)(I - A)$$

$$D^2 = \frac{1}{4}(I^2 - I \cdot A - A \cdot I + A^2)$$

$$D^2 = \frac{1}{4}(I - A - A + I)$$

$$D^2 = \frac{1}{2}(I - A) \qquad \boxed{D^2 = D}$$

$$C \cdot D = \frac{1}{2}(I + A) \cdot \frac{1}{2}(I - A)$$

$$C \cdot D = \frac{1}{4}(I + A)(I - A)$$

$$C \cdot D = \frac{1}{4}(I^2 - I \cdot A + A \cdot I - A^2)$$

$$C \cdot D = \frac{1}{4}(I - A + A - I)$$

$$C \cdot D = \frac{1}{4}(\Theta)$$

$$\boxed{C \cdot D = \Theta}$$

Donde Θ es la matriz nula

Operaciones elementales

1, Intercambiar dos filas o dos columnas $f_1 \leftrightarrow f_2$ $c_1 \leftrightarrow c_2$

2, Multiplicar una fila o columna por una constante $cf_4 \to f_4$ $kc_3 \to c_3$

3, Sumar una fila o columna con una fila o columna multiplicada por una constante

$$cf_4 + f_2 \to f_2$$

$$kc_3 + c_4 \to c_4$$

Ejercicio – Operaciones elementales y semejanza

Dada la matriz $A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}$ demostrar el principio de equivalencia o semejanza. Aplicando en A las operaciones elementales $f_1 \leftrightarrow f_2$; $-2f_2 + f_3 \to f_3$; $2C_1 + C_2 \to C_2$; $C_1 + C_3 \to C_3$; $3f_2 + f_1 \to f_1$ llegando a la matriz equivalente B

Solución

Obteniendo la matriz B

$$A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix} \quad f_1 \leftrightarrow f_2$$

$$\begin{pmatrix} -3 & 2 & 9 \\ 1 & -2 & -6 \\ 2 & 0 & -3 \end{pmatrix} \quad -2f_2 + f_3 \to f_3$$

$$\begin{pmatrix} -3 & 2 & 9 \\ 1 & -2 & -6 \\ 0 & 4 & 9 \end{pmatrix}$$

$$2C_1 + C_2 \to C_2$$

$$\begin{pmatrix} -3 & -4 & 9 \\ 1 & 0 & -6 \\ 0 & 4 & 9 \end{pmatrix}$$

$$C_1 + C_3 \to C_3$$

$$\begin{pmatrix} -3 & -4 & 6 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix} \quad 3f_2 + f_1 \to f_1$$

$$B = \begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix}$$

Demostrar $\quad P \cdot A \cdot Q = B$

$$A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}$$

$f_1 \leftrightarrow f_2$; $-2f_2 + f_3 \rightarrow f_3$; $2C_1 + C_2 \rightarrow C_2$; $C_1 + C_3 \rightarrow C_3$; $3f_2 + f_1 \rightarrow f_1$

$$E_3 \cdot E_2 \cdot E_1 \cdot A \cdot K_1 \cdot K_2 = B$$

E_i matrices elementales en fila

K_i matrices elementales en columna

$$P \cdot A \cdot Q = B$$

$$E_3 \cdot E_2 \cdot E_1 = P$$

$$K_1 \cdot K_2 = Q$$

$f_1 \leftrightarrow f_2$ genera E_1

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} f_1 \leftrightarrow f_2$$

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$-2f_2 + f_3 \rightarrow f_3$ genera E_2

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} -2f_2 + f_3 \rightarrow f_3$$

$$E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

$3f_2 + f_1 \rightarrow f_1$ genera E_3

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} 3f_2 + f_1 \rightarrow f_1$$

$$E_3 = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$2C_1 + C_2 \rightarrow C_2$ genera K_1

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$2C_1 + C_2 \rightarrow C_2$$

$$K_1 = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$C_1 + C_3 \rightarrow C_3$ genera K_2

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_1 + C_3 \rightarrow C_3$$

$$K_2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_3 \cdot E_2 \cdot E_1 \cdot A \cdot K_1 \cdot K_2 = B$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix}}_{P} \cdot \underbrace{\begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix}}_{A} \cdot \underbrace{\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{Q} = \underbrace{\begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix}}_{B}$$

$$\boxed{\begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix} = \begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix}}$$

Ejercicio – Operaciones elementales y semejanza

Dada la matriz $A=\begin{pmatrix}1 & -2 & -6\\ -3 & 2 & 9\\ 2 & 0 & -3\end{pmatrix}$; aplicando en A las operaciones elementales $f_1 \leftrightarrow f_2$; $-2f_2+f_3 \rightarrow f_3$; $2C_1+C_2 \rightarrow C_2$; $C_1+C_3 \rightarrow C_3$; $3f_2+f_1 \rightarrow f_1$ se llega a la matriz equivalente B, hallar las matrices P y Q por Gauss

Solución

$$\begin{array}{ccc|ccc} 1 & -2 & -6 & 1 & 0 & 0\\ -3 & 2 & 9 & 0 & 1 & 0\\ 2 & 0 & -3 & 0 & 0 & 1\\ \hline 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \qquad f_1 \leftrightarrow f_2$$

$$\begin{array}{ccc|ccc} -3 & 2 & 9 & 0 & 1 & 0\\ 1 & -2 & -6 & 1 & 0 & 0\\ 2 & 0 & -3 & 0 & 0 & 1\\ \hline 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \qquad -2f_2+f_3 \rightarrow f_3$$

$$\begin{array}{ccc|ccc} -3 & 2 & 9 & 0 & 1 & 0\\ 1 & -2 & -6 & 1 & 0 & 0\\ 0 & 4 & 9 & -2 & 0 & 1\\ \hline 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \qquad 2C_1+C_2 \rightarrow C_2$$

$$\begin{array}{ccc|ccc} -3 & -4 & 9 & 0 & 1 & 0\\ 1 & 0 & -6 & 1 & 0 & 0\\ 0 & 4 & 9 & -2 & 0 & 1\\ \hline 1 & 2 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \qquad C_1+C_3 \rightarrow C_3$$

$$\begin{array}{ccc|ccc} -3 & -4 & 6 & 0 & 1 & 0 \\ 1 & 0 & -5 & 1 & 0 & 0 \\ 0 & 4 & 9 & -2 & 0 & 1 \\ \hline 1 & 2 & 1 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \quad 3f_2 + f_1 \to f_1$$

$$\begin{array}{ccc|ccc} & B & & & P & \\ 0 & -4 & -9 & 3 & 1 & 0 \\ 1 & 0 & -5 & 1 & 0 & 0 \\ 0 & 4 & 9 & -2 & 0 & 1 \\ \hline 1 & 2 & 1 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \\ & Q & & & & \end{array}$$

$$\boxed{\begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -4 & -9 \\ 1 & 0 & -5 \\ 0 & 4 & 9 \end{pmatrix}}$$

Operaciones elementales inversas

1, Intercambiar dos filas o dos columnas $f_1 \leftrightarrow f_2$ $c_1 \leftrightarrow c_2$

Su operación inversa es $f_1 \leftrightarrow f_2$ $c_1 \leftrightarrow c_2$

2, Multiplicar una fila o columna por una constante $cf_4 \rightarrow f_4$ $kc_3 \rightarrow c_3$

Su operación inversa es $\frac{1}{c} f_4 \rightarrow f_4$ $\frac{1}{k} c_3 \rightarrow c_3$

3, Sumar una fila o columna con una fila o columna multiplicada por una constante

$cf_4 + f_2 \rightarrow f_2$ $kc_3 + c_4 \rightarrow c_4$

Su operación inversa es $-cf_4 + f_2 \rightarrow f_2$ $-kc_3 + c_4 \rightarrow c_4$

Ejercicio – Factorización LU y LDU

Dada la matriz $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix}$; hallar la factorización LU y LDU

Solución

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix} \quad -2f_1 + f_2 \to f_2$$

$$\begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ -3 & -2 & 7 \end{pmatrix} \quad 3f_1 + f_3 \to f_3$$

$$\begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 7 & 13 \end{pmatrix} \quad 7f_2 + f_3 \to f_3$$

$$U = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 27 \end{pmatrix}$$

$$E_3E_2E_1A = U$$

$$E_3^{-1}E_3E_2E_1A = E_3^{-1}U$$

$$IE_2E_1A = E_3^{-1}U$$

$$E_2E_1A = E_3^{-1}U$$

$$E_2^{-1}E_2E_1A = E_2^{-1}E_3^{-1}U$$

$$IE_1A = E_2^{-1}E_3^{-1}U$$

$$E_1A = E_2^{-1}E_3^{-1}U$$

$$E_1^{-1}E_1A = E_1^{-1}E_2^{-1}E_3^{-1}U$$

$$IA = E_1^{-1}E_2^{-1}E_3^{-1}U$$

$$A = \underbrace{E_1^{-1}E_2^{-1}E_3^{-1}}_{L}U$$

Matrices elementales

(aplicando las operaciones elementales a la matriz identidad)

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \xrightarrow{-2f_1+f_2\to f_2} E_1=\begin{pmatrix}1&0&0\\-2&1&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \xrightarrow{3f_1+f_3\to f_3} E_2=\begin{pmatrix}1&0&0\\0&1&0\\3&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \xrightarrow{7f_2+f_3\to f_3} E_3=\begin{pmatrix}1&0&0\\0&1&0\\0&7&1\end{pmatrix}$$

Matrices elementales inversas

(aplicando las operaciones elementales inversas a la matriz identidad)

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \xrightarrow{2f_1+f_2\to f_2} E_1^{-1}=\begin{pmatrix}1&0&0\\2&1&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \xrightarrow{-3f_1+f_3\to f_3} E_2^{-1}=\begin{pmatrix}1&0&0\\0&1&0\\-3&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \xrightarrow{-7f_2+f_3\to f_3} E_3^{-1}=\begin{pmatrix}1&0&0\\0&1&0\\0&-7&1\end{pmatrix}$$

$$A = \underbrace{E_1^{-1}E_2^{-1}E_3^{-1}}_{L}U$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 1 \end{pmatrix}U$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 1 \end{pmatrix}U$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7 & 1 \end{pmatrix}U$$

$$\boxed{\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 27 \end{pmatrix}}$$

$$A = LU$$

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 27 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 27 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$A = LDU$$

Nota: D puede ser cualquier matriz diagonal

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7 & 1 \end{pmatrix}\begin{pmatrix} \pi & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{pmatrix}\begin{pmatrix} 1/\pi & 3/\pi & 2/\pi \\ 0/e & -1/e & 2/e \\ 0/i & 0/i & 27/i \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ -3 & -2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7 & 1 \end{pmatrix}\begin{pmatrix} \pi & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{pmatrix}\begin{pmatrix} 1/\pi & 3/\pi & 2/\pi \\ 0 & -1/e & 2/e \\ 0 & 0 & 27/i \end{pmatrix}$$

Ejercicio – Factorización LU y LDU

Expresar la matriz A en la forma LU y LDU

$$A=\begin{pmatrix}2 & -2 & 1 & 2\\ 3 & 3 & 3 & 1\\ 5 & 2 & -3 & 3\\ -1 & 3 & 4 & 2\end{pmatrix}$$

Solución

$$\begin{pmatrix}2 & -2 & 1 & 2\\ 3 & 3 & 3 & 1\\ 5 & 2 & -3 & 3\\ -1 & 3 & 4 & 2\end{pmatrix} -\frac{3}{2} +$$

$$\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 5 & 2 & -3 & 3\\ -1 & 3 & 4 & 2\end{pmatrix} -\frac{5}{2} +$$

$$\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 0 & 7 & -\frac{11}{2} & -2\\ -1 & 3 & 4 & 2\end{pmatrix} \frac{1}{2} +$$

$$\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 0 & 7 & -\frac{11}{2} & -2\\ 0 & 2 & \frac{9}{2} & 3\end{pmatrix} -\frac{7}{6} + \quad -\frac{2}{6} +$$

$$\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 0 & 0 & -\frac{29}{4} & \frac{1}{3}\\ 0 & 0 & 4 & \frac{11}{3}\end{pmatrix} \frac{4}{29} * 4 +$$

$$U=\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 0 & 0 & -\frac{29}{4} & \frac{1}{3}\\ 0 & 0 & 0 & \frac{335}{87}\end{pmatrix}$$

La matriz L se obtiene con los coeficientes cambiados de signo de las operaciones hechas para obtener la matriz U (vea los colores)

$$L=\begin{pmatrix}1 & 0 & 0 & 0\\ \frac{3}{2} & 1 & 0 & 0\\ \frac{5}{2} & \frac{7}{6} & 1 & 0\\ -\frac{1}{2} & \frac{1}{3} & -\frac{16}{29} & 1\end{pmatrix}\qquad U=\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 0 & 0 & -\frac{29}{4} & \frac{1}{3}\\ 0 & 0 & 0 & \frac{335}{87}\end{pmatrix}$$

$$A=\begin{pmatrix}1 & 0 & 0 & 0\\ \frac{3}{2} & 1 & 0 & 0\\ \frac{5}{2} & \frac{7}{6} & 1 & 0\\ -\frac{1}{2} & \frac{1}{3} & -\frac{16}{29} & 1\end{pmatrix}\begin{pmatrix}2 & -2 & 1 & 2\\ 0 & 6 & \frac{3}{2} & -2\\ 0 & 0 & -\frac{29}{4} & \frac{1}{3}\\ 0 & 0 & 0 & \frac{335}{87}\end{pmatrix}$$

$$A=LU$$

$$A=\begin{pmatrix}1 & 0 & 0 & 0\\ \frac{3}{2} & 1 & 0 & 0\\ \frac{5}{2} & \frac{7}{6} & 1 & 0\\ -\frac{1}{2} & \frac{1}{3} & -\frac{16}{29} & 1\end{pmatrix}\begin{pmatrix}2 & 0 & 0 & 0\\ 0 & 6 & 0 & 0\\ 0 & 0 & -\frac{29}{4} & 0\\ 0 & 0 & 0 & \frac{335}{87}\end{pmatrix}\begin{pmatrix}1 & -1 & \frac{1}{2} & 1\\ 0 & 1 & \frac{1}{4} & -\frac{1}{3}\\ 0 & 0 & 1 & -\frac{4}{87}\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$A=LDU$$

Determinantes

Sólo para matrices cuadradas

Matriz A

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Determinante de la Matriz A

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Propiedades

$$|kA| = k^n|A|$$

$$|A \cdot B| = |A| \cdot |B|$$

$$|A^t| = |A|$$

$$|A^m| = |A|^m$$

$$|A^{-1}| = |A|^{-1}$$

Casos usuales para $|A| = 0$

- Si tiene dos filas o dos columnas iguales
- Si tiene una fila o columna con ceros
- Si tiene dos filas o dos columnas proporcionales

Multiplicación de un escalar a un determinante (cf_i ó cC_i)

$$k\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & ka_{12} & a_{13} \\ a_{21} & ka_{22} & a_{23} \\ a_{31} & ka_{32} & a_{33} \end{vmatrix}$$

- La operaciones elementales del tipo $cf_i + f_j \to f_j$ no afectan al valor del determinante

- Al intercambiar dos filas o dos columnas, el determinante cambia de signo ($f_i \leftrightarrow f_j$)

- Para matrices triangulares, el determinante se obtiene multiplicando los elementos de la diagonal principal

Ejercicio – Determinantes

Hallar el determinante de $\begin{pmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{pmatrix}$

Solución

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix}$$

$+$

$$\begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$\boxed{determinante = 0}$$

Ejercicio – Determinantes

Hallar λ si $\det(A) = 0$

$$A = \begin{pmatrix} 1 & \lambda & \lambda & \lambda \\ \lambda & 1 & \lambda & \lambda \\ \lambda & \lambda & 1 & \lambda \\ \lambda & \lambda & \lambda & 1 \end{pmatrix}$$

Solución

$$det(A) = \begin{vmatrix} 1 & \lambda & \lambda & \lambda \\ \lambda & 1 & \lambda & \lambda \\ \lambda & \lambda & 1 & \lambda \\ \lambda & \lambda & \lambda & 1 \end{vmatrix} \; +$$

$$\begin{vmatrix} 1+\lambda & 1+\lambda & 2\lambda & 2\lambda \\ \lambda & 1 & \lambda & \lambda \\ \lambda & \lambda & 1 & \lambda \\ \lambda & \lambda & \lambda & 1 \end{vmatrix} \; +$$

$$\begin{vmatrix} 1+2\lambda & 1+2\lambda & 1+2\lambda & 3\lambda \\ \lambda & 1 & \lambda & \lambda \\ \lambda & \lambda & 1 & \lambda \\ \lambda & \lambda & \lambda & 1 \end{vmatrix} \; +$$

$$\begin{vmatrix} 1+3\lambda & 1+3\lambda & 1+3\lambda & 1+3\lambda \\ \lambda & 1 & \lambda & \lambda \\ \lambda & \lambda & 1 & \lambda \\ \lambda & \lambda & \lambda & 1 \end{vmatrix}$$

$$- \quad - \quad -$$

$$(1+3\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ \lambda & 1 & \lambda & \lambda \\ \lambda & \lambda & 1 & \lambda \\ \lambda & \lambda & \lambda & 1 \end{vmatrix}$$

$$(1+3\lambda)\begin{vmatrix} 1 & 0 & 0 & 0 \\ \lambda & 1-\lambda & 0 & 0 \\ \lambda & 0 & 1-\lambda & 0 \\ \lambda & 0 & 0 & 1-\lambda \end{vmatrix}$$

$$\det(A) = (1+3\lambda)(1-\lambda)^3$$

$$\lambda = -\frac{1}{3}, 1 \quad \rightarrow A\ es\ singular$$

$$\lambda \neq -\frac{1}{3}, 1 \quad \rightarrow A\ no\ es\ singular$$

Ejercicio – Determinantes

Hallar el determinante $A = \begin{pmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{pmatrix}$ sabiendo que es antisimétrica

Solución

$$A = \begin{pmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{pmatrix} \qquad A^t = \begin{pmatrix} a & -b & -c & -d \\ b & a & d & -c \\ c & -d & a & b \\ d & c & -b & a \end{pmatrix}$$

¿ $A = -A^t$?

$a = 0$

$$A = \begin{pmatrix} 0 & b & c & d \\ -b & 0 & -d & c \\ -c & d & 0 & -b \\ -d & -c & b & 0 \end{pmatrix} \qquad A^t = \begin{pmatrix} 0 & -b & -c & -d \\ b & 0 & d & -c \\ c & -d & 0 & b \\ d & c & -b & 0 \end{pmatrix}$$

$$A * A^t = \begin{pmatrix} 0 & b & c & d \\ -b & 0 & -d & c \\ -c & d & 0 & -b \\ -d & -c & b & 0 \end{pmatrix} \begin{pmatrix} 0 & -b & -c & -d \\ b & 0 & d & -c \\ c & -d & 0 & b \\ d & c & -b & 0 \end{pmatrix} = \begin{pmatrix} b^2+c^2+d^2 & 0 & 0 & 0 \\ 0 & b^2+c^2+d^2 & 0 & 0 \\ 0 & 0 & b^2+c^2+d^2 & 0 \\ 0 & 0 & 0 & b^2+c^2+d^2 \end{pmatrix}$$

$$A * A^t = \begin{pmatrix} b^2+c^2+d^2 & 0 & 0 & 0 \\ 0 & b^2+c^2+d^2 & 0 & 0 \\ 0 & 0 & b^2+c^2+d^2 & 0 \\ 0 & 0 & 0 & b^2+c^2+d^2 \end{pmatrix}$$

$$Propiedad\ |A| = |A^t|$$

$$|A| \cdot |A| = |A| \cdot |A^t|$$

$$|A|^2 = |A| \cdot |A^t|$$

$$Propiedad\ |A \cdot B| = |A| \cdot |B|$$

$$|A|^2 = |A \cdot A^t|$$

$$|A| = \pm\sqrt{|A \cdot A^t|}$$

$$|A \cdot A^t| = \begin{vmatrix} b^2+c^2+d^2 & 0 & 0 & 0 \\ 0 & b^2+c^2+d^2 & 0 & 0 \\ 0 & 0 & b^2+c^2+d^2 & 0 \\ 0 & 0 & 0 & b^2+c^2+d^2 \end{vmatrix}$$

$$|A \cdot A^t| = (b^2+c^2+d^2)^4$$

$$|A| = \pm\sqrt{(b^2+c^2+d^2)^4}$$

$$\boxed{|A| = \pm(b^2+c^2+d^2)^2}$$

Ejercicio –Determinante

Analizar los valores de x para los cuales la matriz F es singular y no singular

$$F=\begin{pmatrix} x & 2 & -2 & 0 \\ -2 & 0 & x & 2 \\ 0 & x & 2 & -2 \\ 2 & -2 & 0 & x \end{pmatrix}$$

Solución

$$|F| = \begin{vmatrix} x & 2 & -2 & 0 \\ -2 & 0 & x & 2 \\ 0 & x & 2 & -2 \\ 2 & -2 & 0 & x \end{vmatrix}$$

$+ \quad -\frac{x}{2} \quad +$

$$|F| = \begin{vmatrix} 0 & 2 & 0 & 0 \\ -2 & 0 & x & 2 \\ -\frac{x^2}{2} & x & x+2 & -2 \\ x+2 & -2 & -2 & x \end{vmatrix}$$

$+ \quad -$

$$|F| = -2\begin{vmatrix} -2 & x & 2 \\ -\frac{x^2}{2} & x+2 & -2 \\ x+2 & -2 & x \end{vmatrix}$$

$+ \quad -\frac{x}{2} \quad +$

$$|F| = -2\begin{vmatrix} -2 & x & 2 \\ -\frac{x^2}{2}-2 & 2x+2 & 0 \\ 2x+2 & -\frac{x^2}{2}-2 & 0 \end{vmatrix}$$

$+ \quad - \quad +$

$$|F| = -4\begin{vmatrix} -\frac{x^2}{2}-2 & 2x+2 \\ 2x+2 & -\frac{x^2}{2}-2 \end{vmatrix}$$

$$|F| = -4\begin{vmatrix} -\frac{x^2}{2} - 2 & 2x + 2 \\ 2x + 2 & -\frac{x^2}{2} - 2 \end{vmatrix}$$

$$|F| = -4\left[\left(-\frac{x^2}{2} - 2\right)^2 - (2x+2)^2\right]$$

$$|F| = -4\left[-\frac{x^2}{2} - 2 - 2x - 2\right]\left[-\frac{x^2}{2} - 2 + 2x + 2\right]$$

$$|F| = -4\left[-\frac{x^2}{2} - 2x - 4\right]\left[-\frac{x^2}{2} + 2x\right]$$

$$|F| = -2\left[-\frac{x^2}{2} - 2x - 4\right]x2\left[-\frac{x}{2} + 2\right]$$

$$|F| = [x^2 + 4x + 8]x[-x + 4]$$

$$|F| = -x[x^2 + 4x + 8][x - 4]$$

Singular $\quad |F| = 0$

$$-x[x^2 + 4x + 8][x - 4] = 0$$

$$\boxed{x = 0 \qquad x = -2 \pm 2i \qquad x = 4}$$

No Singular $\quad |F| \neq 0$

$$\boxed{x \neq 0 \qquad x \neq -2 \pm 2i \qquad x \neq 4}$$

Ejercicio – Determinantes

Hallar el valor de x para que el determinante sea nulo $\begin{vmatrix} x & 0 & -1 & 1 & 0 \\ 1 & x & -1 & 1 & 0 \\ 1 & 0 & x-1 & 0 & 1 \\ 0 & 1 & -1 & x & 1 \\ 0 & 1 & -1 & 0 & x \end{vmatrix}$

Solución

$$\begin{vmatrix} x & 0 & -1 & 1 & 0 \\ 1 & x & -1 & 1 & 0 \\ 1 & 0 & x-1 & 0 & 1 \\ 0 & 1 & -1 & x & 1 \\ 0 & 1 & -1 & 0 & x \end{vmatrix} \begin{array}{l} \\ \\ -1 \quad + \quad -x \quad + \\ \\ \\ \end{array}$$

$$\begin{array}{c} + \\ - \\ + \\ \\ \\ \end{array}\begin{vmatrix} 0 & 0 & -1-x(x-1) & 1 & -x \\ 0 & x & -x & 1 & -1 \\ 1 & 0 & x-1 & 0 & 1 \\ 0 & 1 & -1 & x & 1 \\ 0 & 1 & -1 & 0 & x \end{vmatrix}$$

$$(+1)\begin{vmatrix} 0 & -1-x^2+x & 1 & -x \\ x & -x & 1 & -1 \\ 1 & -1 & x & 1 \\ 1 & -1 & 0 & x \end{vmatrix} \begin{array}{l} \\ \\ \\ -1 \quad + \quad -x \quad + \end{array}$$

$$(+1)\begin{array}{c} + \\ - \\ + \\ - \end{array}\begin{vmatrix} 0 & -1-x^2+x & 1 & -x \\ 0 & 0 & 1 & -1-x^2 \\ 0 & 0 & x & 1-x \\ 1 & -1 & 0 & x \end{vmatrix}$$

$$(+1)(-1)\begin{vmatrix} x-x^2-1 & 1 & -x \\ 0 & 1 & -1-x^2 \\ 0 & x & 1-x \end{vmatrix} \qquad (+1)(-1)(x-x^2-1)\begin{vmatrix} 1 & -1-x^2 \\ x & 1-x \end{vmatrix}$$

$$(+1)(-1)(x-x^2-1)[1(1-x)-x(-1-x^2)]$$

$$(+1)(-1)(x-x^2-1)[1(1-x)-x(-1-x^2)]$$

$$(+1)(-1)(x-x^2-1)[1-x+x+x^3]=0$$

$$-(x-x^2-1)[1+x^3]=0$$

$$(x^2-x+1)[1+x^3]=0$$

$$(x^2-x+1)[1+x](1-x+x^2)=0$$

$$(x+1)(x^2-x+1)^2=0$$

$$\boxed{x=-1 \qquad\qquad x=\frac{1\pm\sqrt{3}i}{2}}$$

Ejercicio – Cofactores y adjunta

Hallar la matriz de cofactores y matriz adjunta de $A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & -1 & 6 \\ -5 & 1 & 2 \end{pmatrix}$

Solución

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & -1 & 6 \\ -5 & 1 & 2 \end{pmatrix}$$

$$cofact(A) = \begin{pmatrix} +\begin{vmatrix} -1 & 6 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 6 \\ -5 & 2 \end{vmatrix} & +\begin{vmatrix} 4 & -1 \\ -5 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 3 & -1 \\ -5 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ -5 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & -1 \\ -1 & 6 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 4 & 6 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} \end{pmatrix}$$

$$cofact(A) = \begin{pmatrix} +(-2-6) & -(8+30) & +(4-5) \\ -(4+1) & +(6-5) & -(3+10) \\ +(12-1) & -(18+4) & +(-3-8) \end{pmatrix}$$

$$\boxed{cofact(A) = \begin{pmatrix} -8 & -38 & -1 \\ -5 & 1 & -13 \\ 11 & -22 & -11 \end{pmatrix}}$$

$$adj(A) = [cofact(A)]^t$$

$$\boxed{adj(A) = \begin{pmatrix} -8 & -5 & 11 \\ -38 & 1 & -22 \\ -1 & -13 & -11 \end{pmatrix}}$$

Propiedades de la matriz adjunta

$$adj(I_n) = I_n \qquad n \ es \ el \ orden \ de \ la \ matriz$$

$$adj(A^{-1}) = \left(adj(A)\right)^{-1}$$

$$adj(A^{t}) = \left(adj(A)\right)^{t} = cofact(A)$$

$$adj(A \cdot B) = adj(A) \cdot adj(B)$$

$$adj(kA) = k^{n-1} adj(A)$$

$$|adj(A)| = |A|^{n-1}$$

$$|adj(adj(A))| = |A|^{(n-1)^2}$$

Recordar:

$$|kA| = k^{n}|A|$$

$$|A^{t}| = |A|$$

$$|A^{m}| = |A|^{m}$$

$$|A^{-1}| = |A|^{-1}$$

Ejercicio – Cofactores y adjunta

Demostrar que $\left|adj\left(adj(A)\right)\right| = |A|^{(n-1)^2}$

Solución

$$A^{-1} = \frac{1}{|A|}adj(A)$$

$$|A| \cdot A^{-1} = adj(A)$$

$$adj(|A| \cdot A^{-1}) = adj(adj(A))$$

$$adj(adj(A)) = adj(|A| \cdot A^{-1})$$

$$adj(kB) = k^{n-1}adj(B)$$

$$adj\left(adj(A)\right) = |A|^{n-1}adj(A^{-1})$$

$$adj(B^{-1}) = \left(adj(B)\right)^{-1}$$

$$adj\left(adj(A)\right) = |A|^{n-1}\left(adj(A)\right)^{-1}$$

$$\left|adj\left(adj(A)\right)\right| = \left||A|^{n-1}\left(adj(A)\right)^{-1}\right|$$

$$|kB| = k^n|B|$$

$$\left|adj\left(adj(A)\right)\right| = (|A|^{n-1})^n\left|\left(adj(A)\right)^{-1}\right|$$

$$|B^{-1}| = |B|^{-1}$$

$$\left|adj\left(adj(A)\right)\right| = (|A|^{n-1})^n|adj(A)|^{-1}$$

$$|adj(A)| = |A|^{n-1}$$

$$\left|adj\left(adj(A)\right)\right| = (|A|^{n-1})^n(|A|^{n-1})^{-1}$$

$$\left|adj\left(adj(A)\right)\right| = (|A|^{n^2-n})(|A|^{1-n})$$

$$\left|adj\left(adj(A)\right)\right| = |A|^{n^2-2n+1}$$

$$\boxed{\left|adj\left(adj(A)\right)\right| = |A|^{(n-1)^2}}$$

Ejercicio – Determinante y adjunta

10. Hallar $|adj(adj(A))|$

$$A = \begin{pmatrix} x^2 & (x+1)^2 & (x+2)^2 \\ y^2 & (y+1)^2 & (y+2)^2 \\ z^2 & (z+1)^2 & (z+2)^2 \end{pmatrix}$$

Solución

$$\left|adj\left(adj(A)\right)\right| = |A|^{(n-1)^2}$$

$$|A| = \begin{vmatrix} x^2 & (x+1)^2 & (x+2)^2 \\ y^2 & (y+1)^2 & (y+2)^2 \\ z^2 & (z+1)^2 & (z+2)^2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} x^2 & x^2+2x+1 & x^2+4x+4 \\ y^2 & y^2+2y+1 & y^2+4y+4 \\ z^2 & z^2+2z+1 & z^2+4z+4 \end{vmatrix}$$

$-$ $-$

$$|A| = \begin{vmatrix} x^2 & 2x+1 & 4x+4 \\ y^2 & 2y+1 & 4y+4 \\ z^2 & 2z+1 & 4z+4 \end{vmatrix}$$

-2 $+$

$$|A| = \begin{vmatrix} x^2 & 2x+1 & 2 \\ y^2 & 2y+1 & 2 \\ z^2 & 2z+1 & 2 \end{vmatrix}$$

$-$ $-$

$$|A| = \begin{vmatrix} x^2 & 2x+1 & 2 \\ y^2-x^2 & 2y-2x & 0 \\ z^2-x^2 & 2z-2x & 0 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} y^2-x^2 & 2y-2x \\ z^2-x^2 & 2z-2x \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} y^2-x^2 & 2(y-x) \\ z^2-x^2 & 2(z-x) \end{vmatrix}$$

$$|A| = 4\begin{vmatrix} (y-x)(y+x) & y-x \\ (z-x)(z+x) & z-x \end{vmatrix}$$

$$|A| = 4\begin{vmatrix} (y-x)(y+x) & y-x \\ (z-x)(z+x) & z-x \end{vmatrix}$$

$$|A| = 4[(y-x)(y+x)(z-x) - (z-x)(z+x)(y-x)]$$

$$|A| = 4(y-x)(z-x)[(y+x) - (z+x)]$$

$$|A| = 4(y-x)(z-x)(y-z)$$

$$|A| = 4(x-y)(x-z)(y-z)$$

$$\left|adj\left(adj(A)\right)\right| = |A|^{(n-1)^2}$$

$$\left|adj\left(adj(A)\right)\right| = [4(x-y)(x-z)(y-z)\ ]^{(3-1)^2}$$

$$\boxed{\left|adj\left(adj(A)\right)\right| = [4(x-y)(x-z)(y-z)\ ]^{4}}$$

Propiedades de la inversa

$$A^{-1} = \frac{1}{|A|} Adj(A)$$

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

$$(A^{-1})^{-1} = A$$

$$(kA)^{-1} = \frac{1}{k} A^{-1}$$

$$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$$

$$(A^{-1})^{t} = (A^{t})^{-1}$$

$$(A^{-1})^{m} = (A^{m})^{-1}$$

Ejercicio – Inversa

Hallar la matriz X de la ecuación $(AX^{-1}B)^t = AB$

Si $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$ y $B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$

Solución

$$(AX^{-1}B)^t = AB$$

$$[(AX^{-1}B)^t]^t = [AB]^t$$

$$(AX^{-1}B) = [AB]^t$$

$$A^{-1} \cdot AX^{-1}B = A^{-1} \cdot [AB]^t$$

$$I \cdot X^{-1}B = A^{-1} \cdot [AB]^t$$

$$X^{-1}B \cdot B^{-1} = A^{-1} \cdot [AB]^t \cdot B^{-1}$$

$$X^{-1} \cdot I = A^{-1} \cdot [AB]^t \cdot B^{-1}$$

$$X^{-1} = A^{-1} \cdot [AB]^t \cdot B^{-1}$$

$$(X^{-1})^{-1} = (A^{-1} \cdot [AB]^t \cdot B^{-1})^{-1}$$

$$X = (A^{-1} \cdot [AB]^t \cdot B^{-1})^{-1}$$

$$(P \cdot Q)^{-1} = Q^{-1} \cdot P^{-1}$$

$$X = (B^{-1})^{-1} \cdot ([AB]^t)^{-1} \cdot (A^{-1})^{-1}$$

$$(P^{-1})^{-1} = P$$

$$X = B \cdot ([AB]^t)^{-1} \cdot A$$

$$(A^{-1})^t = (A^t)^{-1}$$

$$X = B \cdot ([AB]^{-1})^t \cdot A$$

$$X = B \cdot ([AB]^{-1})^t \cdot A \qquad A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 0 \\ 1 & 2 & 3 \end{pmatrix}$$

$$D^{-1} = \frac{1}{|D|} adj(D)$$

$$D \mid I \longrightarrow I \mid D^{-1}$$

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 1 \\ 3 & 4 & 0 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} -3 \quad -2 \\ + \\ + \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 1 \\ 0 & -2 & -9 & 0 & 1 & -3 \\ 0 & -1 & -5 & 1 & 0 & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 1 \\ 0 & -1 & -5 & 1 & 0 & -2 \\ 0 & -2 & -9 & 0 & 1 & -3 \end{array}\right) \begin{array}{l} \\ -2 \;\; + \;\; (-1) \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 1 \\ 0 & 1 & 5 & -1 & 0 & 2 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} + \\ -2 \\ -5 \;\; + \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -7 & 2 & 0 & -3 \\ 0 & 1 & 0 & 9 & -5 & -3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} \\ + \\ 7 \end{array}$$

$$I \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -12 & 7 & 4 \\ 0 & 1 & 0 & 9 & -5 & -3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) D^{-1}$$

$$X = B \cdot ([AB]^{-1})^t \cdot A \qquad A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$[AB]^{-1} = \begin{pmatrix} -12 & 7 & 4 \\ 9 & -5 & -3 \\ -2 & 1 & 1 \end{pmatrix} \qquad ([AB]^{-1})^t = \begin{pmatrix} -12 & 9 & -2 \\ 7 & -5 & 1 \\ 4 & -3 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -12 & 9 & -2 \\ 7 & -5 & 1 \\ 4 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} 7 & 2 & -2 \\ 2 & 1 & 0 \\ -8 & -2 & 3 \end{pmatrix}}$$

Ejercicio –Determinante adjunta, inversa

Si se sabe que la adjunta de la matriz A es

$$\text{adj(A)}=\begin{pmatrix} -3 & a & -3 \\ -14 & 21 & 7 \\ 6 & 9 & 6 \end{pmatrix}$$

Y que el determinante de A^{-1} es: $\det(A^{-1}) = -\frac{1}{63}$, halle el valor de a y la matriz A

Solución

$$|A^{-1}| = -\frac{1}{63}$$

$$|A|^{-1} = -\frac{1}{63}$$

$$\frac{1}{|A|} = -\frac{1}{63}$$

$$|A| = -63$$

$$|adj(A)| = \begin{vmatrix} -3 & a & -3 \\ -14 & 21 & 7 \\ 6 & 9 & 6 \end{vmatrix}$$

$-$

$$|adj(A)| = \begin{vmatrix} -3 & a & 0 \\ -14 & 21 & 21 \\ 6 & 9 & 0 \end{vmatrix}$$

$$|adj(A)| = -21\begin{vmatrix} -3 & a \\ 6 & 9 \end{vmatrix}$$

$$|adj(A)| = -21(-27 - 6a)$$

$$|adj(A)| = +21 \cdot 3(2a + 9)$$

$$|adj(A)| = +63(2a + 9)$$

$$|adj(A)| = |A|^{n-1}$$

$$63(2a + 9) = (-63)^{3-1}$$

$$63(2a + 9) = 63^2$$

$$2a + 9 = 63$$

$$2a = 54$$

$$\boxed{a = 27}$$

$$adj(A) = \begin{pmatrix} -3 & 27 & -3 \\ -14 & 21 & 7 \\ 6 & 9 & 6 \end{pmatrix}$$

$$|A| = -63$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$(A^{-1})^{-1} = \left(\frac{1}{|A|} adj(A)\right)^{-1}$$

$$A = \left(\frac{1}{|A|}\right)^{-1} \left(adj(A)\right)^{-1}$$

$$A = |A| \left(adj(A)\right)^{-1}$$

$$\begin{array}{ccc|ccc l}
-3 & 27 & -3 & 1 & 0 & 0 & -1/3 \\
-14 & 21 & 7 & 0 & 1 & 0 & -1/7 \\
6 & 9 & 6 & 0 & 0 & 1 & 1/3 \\
\hline
1 & -9 & 1 & -1/3 & 0 & 0 & -2 \\
2 & -3 & -1 & 0 & -1/7 & 0 & + \\
2 & 3 & 2 & 0 & 0 & 1/3 & + \\
\hline
1 & -9 & 1 & -1/3 & 0 & 0 & \\
0 & 15 & -3 & 2/3 & -1/7 & 0 & 1/3 \\
0 & 21 & 0 & 2/3 & 0 & 1/3 & 1/21 \\
\hline
1 & -9 & 1 & -1/3 & 0 & 0 & \\
0 & 5 & -1 & 2/9 & -1/21 & 0 & \\
0 & 1 & 0 & 2/63 & 0 & 1/63 & \\
\hline
1 & -9 & 1 & -1/3 & 0 & 0 & \\
0 & 1 & 0 & 2/63 & 0 & 1/63 & \\
0 & 5 & -1 & 2/9 & -1/21 & 0 &
\end{array}$$

$$\left.\begin{array}{ccc|ccc}1 & -9 & 1 & -1/3 & 0 & 0\\ 0 & 1 & 0 & 2/63 & 0 & 1/63\\ 0 & 5 & -1 & 2/9 & -1/21 & 0\end{array}\right. \quad \begin{array}{l} 9 \quad + \\ -5 \\ + \end{array}$$

$$\left.\begin{array}{ccc|ccc}1 & 0 & 1 & -1/21 & 0 & 9/63\\ 0 & 1 & 0 & 2/63 & 0 & 1/63\\ 0 & 0 & -1 & 4/63 & -1/21 & -5/63\end{array}\right. \quad \begin{array}{l} + \\ (-1) \end{array}$$

$$\left.\begin{array}{ccc|ccc}1 & 0 & 0 & 1/63 & -1/21 & 4/63\\ 0 & 1 & 0 & 2/63 & 0 & 1/63\\ 0 & 0 & 1 & -4/63 & 1/21 & 5/63\end{array}\right.$$

$$\left(adj(A)\right)^{-1} = \begin{pmatrix}\frac{1}{63} & \frac{-1}{21} & \frac{4}{63}\\ \frac{2}{63} & 0 & \frac{1}{63}\\ \frac{-4}{63} & \frac{1}{21} & \frac{5}{63}\end{pmatrix}$$

$$A = |A|\left(adj(A)\right)^{-1}$$

$$|A| = -63$$

$$A = -63\begin{pmatrix}\frac{1}{63} & \frac{-1}{21} & \frac{4}{63}\\ \frac{2}{63} & 0 & \frac{1}{63}\\ \frac{-4}{63} & \frac{1}{21} & \frac{5}{63}\end{pmatrix}$$

$$\boxed{A = \begin{pmatrix}-1 & 3 & -4\\ -2 & 0 & -1\\ 4 & -3 & -5\end{pmatrix}}$$

Ejercicio – Determinante, Matriz inversa

Calcular el determinante y la inversa por medio de operaciones elementales para la matriz A, incluir todo el procedimiento y operaciones elementales:

$$A=\begin{pmatrix}1 & -1 & 1 & -1\\ 2 & 2 & 2 & 2\\ 1 & 3 & 1 & 2\\ 4 & 4 & 3 & 4\end{pmatrix}$$

Solución

$$|A|=\begin{vmatrix}1 & -1 & 1 & -1\\ 2 & 2 & 2 & 2\\ 1 & 3 & 1 & 2\\ 4 & 4 & 3 & 4\end{vmatrix}\quad - \quad + \quad +$$

$$|A|=\begin{vmatrix}1 & 0 & 0 & 0\\ 2 & 4 & 0 & 4\\ 1 & 4 & 0 & 3\\ 4 & 8 & -1 & 8\end{vmatrix}$$

$$|A|=\begin{vmatrix}4 & 0 & 4\\ 4 & 0 & 3\\ 8 & -1 & 8\end{vmatrix}$$

$$|A|=-(-1)\begin{vmatrix}4 & 4\\ 4 & 3\end{vmatrix}$$

$$|A|=12-16$$

$$\boxed{|A|=-4}$$

$$\left(\begin{array}{cccc|cccc}1 & -1 & 1 & -1 & 1 & 0 & 0 & 0\\ 2 & 2 & 2 & 2 & 0 & 1 & 0 & 0\\ 1 & 3 & 1 & 2 & 0 & 0 & 1 & 0\\ 4 & 4 & 3 & 4 & 0 & 0 & 0 & 1\end{array}\right)\quad -2\ +\quad -1\ +\quad -4\ +$$

$$\left(\begin{array}{cccc|cccc}1 & -1 & 1 & -1 & 1 & 0 & 0 & 0\\ 0 & 4 & 0 & 4 & -2 & 1 & 0 & 0\\ 0 & 4 & 0 & 3 & -1 & 0 & 1 & 0\\ 0 & 8 & -1 & 8 & -4 & 0 & 0 & 1\end{array}\right)\quad -1\ +$$

$$\left(\begin{array}{cccc|cccc}1 & -1 & 1 & -1 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0\\ 0 & 4 & 0 & 3 & -1 & 0 & 1 & 0\\ 0 & 8 & -1 & 8 & -4 & 0 & 0 & 1\end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 4 & 0 & 3 & -1 & 0 & 1 & 0 \\ 0 & 8 & -1 & 8 & -4 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} + \\ -3 \quad -8 \\ + \\ + \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 4 & 0 & 0 & 2 & -3 & 4 & 0 \\ 0 & 8 & -1 & 0 & 4 & -8 & 8 & 1 \end{array}\right) \begin{array}{l} \\ \\ -2 \\ + \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 4 & 0 & 0 & 2 & -3 & 4 & 0 \\ 0 & 0 & -1 & 0 & 0 & -2 & 0 & 1 \end{array}\right) \begin{array}{l} + \\ \\ 1/4 \\ -1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & -3/4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & -1 \end{array}\right) \begin{array}{l} + \\ \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/2 & -7/4 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & -3/4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & -1 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/2 & -7/4 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1/2 & -3/4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \end{array}\right)$$

$$\boxed{A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{7}{4} & 0 & 1 \\ \frac{1}{2} & -\frac{3}{4} & 1 & 0 \\ 0 & 2 & 0 & -1 \\ -1 & 1 & -1 & 0 \end{pmatrix}}$$

Ejercicio – Operaciones con matrices, determinante, inversa

Encontrar a tal que la matriz G sea no singular donde:

$$BG = \begin{pmatrix} a-1 & 1 & -1 \\ 0 & -1 & a \\ -1 & a & 1 \end{pmatrix}; B = ED^k; E = [e_{ij}]_{3\times 3} = \begin{cases} i \cdot j^{-k} & si\ i \leq j \\ 0 & si\ i > j \end{cases}\ y\ D = [d_{ij}]_{3\times 3} = \begin{cases} i & si\ i = j \\ 0 & si\ i \neq j \end{cases}$$

Solución

$$E = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}$$

$$E = \begin{pmatrix} 1 \cdot 1^{-k} & 1 \cdot 2^{-k} & 1 \cdot 3^{-k} \\ 0 & 2 \cdot 2^{-k} & 2 \cdot 3^{-k} \\ 0 & 0 & 3 \cdot 3^{-k} \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 2^{-k} & 3^{-k} \\ 0 & 2 \cdot 2^{-k} & 2 \cdot 3^{-k} \\ 0 & 0 & 3 \cdot 3^{-k} \end{pmatrix}$$

$$D = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$D^k = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^k & 0 \\ 0 & 0 & 3^k \end{pmatrix}$$

$$B = ED^K$$

$$B = \begin{pmatrix} 1 & 2^{-k} & 3^{-k} \\ 0 & 2 \cdot 2^{-k} & 2 \cdot 3^{-k} \\ 0 & 0 & 3 \cdot 3^{-k} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^k & 0 \\ 0 & 0 & 3^k \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$G = B^{-1} \begin{pmatrix} a-1 & 1 & -1 \\ 0 & -1 & a \\ -1 & a & 1 \end{pmatrix}$$

$$B=\begin{pmatrix}1&1&1\\0&2&2\\0&0&3\end{pmatrix}\qquad G=B^{-1}\begin{pmatrix}a-1&1&-1\\0&-1&a\\-1&a&1\end{pmatrix}$$

$$B^{-1}=\begin{pmatrix}1&-\frac{1}{2}&0\\0&\frac{1}{2}&-\frac{1}{3}\\0&0&\frac{1}{3}\end{pmatrix}$$

$$\left.\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&2&2&0&1&0\\0&0&3&0&0&1\end{array}\right.\quad\begin{array}{c}\\1/2\\1/3\end{array}$$

$$\left.\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&1&0&1/2&0\\0&0&1&0&0&1/3\end{array}\right.\quad\begin{array}{c}\\\\-1\quad+\end{array}$$

$$\left.\begin{array}{ccc|ccc}1&1&0&1&0&-1/3\\0&1&0&0&1/2&-1/3\\0&0&1&0&0&1/3\end{array}\right.\quad\begin{array}{c}\\-1\quad+\\\end{array}$$

$$\left.\begin{array}{ccc|ccc}1&0&0&1&-1/2&0\\0&1&0&0&1/2&-1/3\\0&0&1&0&0&1/3\end{array}\right.$$

$$G=\begin{pmatrix}1&-\frac{1}{2}&0\\0&\frac{1}{2}&-\frac{1}{3}\\0&0&\frac{1}{3}\end{pmatrix}\begin{pmatrix}a-1&1&-1\\0&-1&a\\-1&a&1\end{pmatrix}$$

$$G=\begin{pmatrix}a-1&\frac{3}{2}&-\frac{a}{2}+1\\\frac{1}{3}&-\frac{a}{3}-\frac{1}{2}&\frac{a}{2}-\frac{1}{3}\\-\frac{1}{3}&\frac{a}{3}&\frac{1}{3}\end{pmatrix}$$

$$|G| = \begin{vmatrix} a-1 & \frac{3}{2} & -\frac{a}{2}+1 \\ \frac{1}{3} & -\frac{a}{3}-\frac{1}{2} & \frac{a}{2}-\frac{1}{3} \\ -\frac{1}{3} & \frac{a}{3} & \frac{1}{3} \end{vmatrix} +$$

$$|G| = \begin{vmatrix} a-1 & \frac{3}{2} & -\frac{a}{2}+1 \\ 0 & -\frac{1}{2} & \frac{a}{2} \\ -\frac{1}{3} & \frac{a}{3} & \frac{1}{3} \end{vmatrix} +$$

$$|G| = \begin{vmatrix} a-1 & 1 & 1 \\ 0 & -\frac{1}{2} & \frac{a}{2} \\ -\frac{1}{3} & \frac{a}{3} & \frac{1}{3} \end{vmatrix}$$

$+$

$$|G| = \begin{vmatrix} a-1 & 1 & a \\ 0 & -\frac{1}{2} & \frac{a}{2} \\ -\frac{1}{3} & \frac{a}{3} & 0 \end{vmatrix} -2 \quad +$$

$$|G| = \begin{vmatrix} a-1 & 2 & 0 \\ 0 & -\frac{1}{2} & \frac{a}{2} \\ -\frac{1}{3} & \frac{a}{3} & 0 \end{vmatrix}$$

$$|G| = -\frac{a}{2}\begin{vmatrix} a-1 & 2 \\ -\frac{1}{3} & \frac{a}{3} \end{vmatrix}$$

$$|G| = -\frac{a}{2}\left[\frac{a}{3}(a-1)+\frac{2}{3}\right]$$

$$|G| = -\frac{a}{6}[a^2-a+2]$$

G No Singular

$$|G| \neq 0$$

$$\boxed{a \neq 0 \qquad a \neq \frac{1 \pm \sqrt{7}}{2}}$$

Ejercicio – Sistemas de ecuaciones

En el siguiente sistema encuentre los valores de los ángulos α, β, y γ por métodos matriciales

$$2\sin\alpha - \cos\beta + 3\tan\gamma = 3$$
$$-2\tan\gamma + 4\sin\alpha + 2\cos\beta = 2$$
$$-3\cos\beta + \tan\gamma + 6\sin\alpha = 9$$

Solución

$$\begin{pmatrix} 2 & -1 & 3 \\ 4 & 2 & -2 \\ 6 & -3 & 1 \end{pmatrix}\begin{pmatrix} \sin\alpha \\ \cos\beta \\ \tan\gamma \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 3 & \vdots & 3 \\ 4 & 2 & -2 & \vdots & 2 \\ 6 & -3 & 1 & \vdots & 9 \end{pmatrix} \begin{matrix} -2 \; + \quad -3 \; + \\ \\ \\ \end{matrix}$$

$$\begin{pmatrix} 2 & -1 & 3 & \vdots & 3 \\ 0 & 4 & -8 & \vdots & -4 \\ 0 & 0 & -8 & \vdots & 0 \end{pmatrix} \begin{matrix} \\ 1/4 \\ -1/8 \end{matrix}$$

$$\begin{pmatrix} 2 & -1 & 3 & \vdots & 3 \\ 0 & 1 & -2 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 0 \end{pmatrix} \begin{matrix} \\ \\ 2 \; + \quad -3 \; + \end{matrix}$$

$$\begin{pmatrix} 2 & -1 & 0 & \vdots & 3 \\ 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 0 \end{pmatrix} \begin{matrix} \\ + \\ \\ \end{matrix}$$

$$\begin{pmatrix} 2 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 0 \end{pmatrix} \begin{matrix} 1/2 \\ \\ \\ \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} \sin\alpha \\ \cos\beta \\ \tan\gamma \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\sin\alpha = 1 \qquad \boxed{\alpha = \frac{\pi}{2}}$$

$$\cos\beta = -1 \qquad \boxed{\beta = \pi}$$

$$\tan\gamma = 0 \qquad \boxed{\gamma = 0}$$

Ejercicio – Sistemas de ecuaciones

En el sistema
$$\begin{aligned} ax + y + z &= 1 \\ x + ay + z &= b \\ x + y + az &= 1 \end{aligned}$$

Determinar los valores de a y b de manera que:

a) los planos se intercepten en el punto
b) los planos se intercepten en muchos puntos
c) los planos sean paralelos

Solución

$$\begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} \quad + \quad +$$

$$|A| = \begin{vmatrix} a+2 & a+2 & a+2 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix}$$

$$|A| = (a+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} \quad - \quad -$$

$$|A| = (a+2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & 0 \\ 0 & 0 & a-1 \end{vmatrix}$$

$$|A| = (a+2)(a-1)^2$$

$a = -2 \qquad a = 1$

$a \neq -2$ $a \neq 1$ $todo\ b$	Se interceptan en un punto (única solución)

$$\begin{pmatrix} a & 1 & 1 & \vdots & 1 \\ 1 & a & 1 & \vdots & b \\ 1 & 1 & a & \vdots & 1 \end{pmatrix}$$

$Si\ a = -2$

$$\begin{pmatrix} -2 & 1 & 1 & \vdots & 1 \\ 1 & -2 & 1 & \vdots & b \\ 1 & 1 & -2 & \vdots & 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 1 & 1 & \vdots & 1 \\ 1 & -2 & 1 & \vdots & b \\ 1 & 1 & -2 & \vdots & 1 \end{pmatrix} \begin{array}{l} \\ \\ -1 \quad + \quad 2 \quad + \end{array}$$

$$\begin{pmatrix} 0 & 3 & -3 & \vdots & 3 \\ 0 & -3 & 3 & \vdots & b-1 \\ 1 & 1 & -2 & \vdots & 1 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 3 & -3 & \vdots & 3 \\ 0 & 0 & 0 & \vdots & b-2 \\ 1 & 1 & -2 & \vdots & 1 \end{pmatrix}$$

$Si\ a = -2$	$b = 2$	Se interceptan en muchos puntos (infinitas soluciones)

$Si\ a = -2$	$b \neq 2$	Planos paralelos (sin solución)

$$\begin{pmatrix} a & 1 & 1 & \vdots & 1 \\ 1 & a & 1 & \vdots & b \\ 1 & 1 & a & \vdots & 1 \end{pmatrix}$$

$Si\ a = 1$

$$\begin{pmatrix} 1 & 1 & 1 & \vdots & 1 \\ 1 & 1 & 1 & \vdots & b \\ 1 & 1 & 1 & \vdots & 1 \end{pmatrix} \begin{array}{l} -1 \quad + \\ \\ \quad + \end{array}$$

$$\begin{pmatrix} 1 & 1 & 1 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & b-1 \\ 0 & 0 & 0 & \vdots & 0 \end{pmatrix}$$

$Si\ a = 1$	$b = 1$	Se interceptan en muchos puntos (infinitas soluciones)

$Si\ a = 1$	$b \neq 1$	Planos paralelos (sin solución)

34. Dado el sistema hallar los valores de "a" para que el sistema sea consistente determinado, consistente indeterminado, inconsistente :

$$3ax + (3a-7)y + (a-5)z = (a-1)$$
$$(2a-1)x + (4a-1)y + 2az = a+1$$
$$4ax + (5a-7)y + (2a-5)z = 0$$

Solución

$$\begin{pmatrix} 3a & 3a-7 & a-5 \\ 2a-1 & 4a-1 & 2a \\ 4a & 5a-7 & 2a-5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a-1 \\ a+1 \\ 0 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3a & 3a-7 & a-5 \\ 2a-1 & 4a-1 & 2a \\ 4a & 5a-7 & 2a-5 \end{vmatrix}$$

$-$

$$|A| = \begin{vmatrix} 3a & -7 & a-5 \\ 2a-1 & 2a & 2a \\ 4a & a-7 & 2a-5 \end{vmatrix}$$

$-$

$$|A| = \begin{vmatrix} 3a & -7 & a+2 \\ 2a-1 & 2a & 0 \\ 4a & a-7 & a+2 \end{vmatrix}$$

$-$

$$|A| = \begin{vmatrix} 3a & -7 & a+2 \\ 2a-1 & 2a & 0 \\ a & a & 0 \end{vmatrix}$$

$$|A| = (a+2)\begin{vmatrix} 2a-1 & 2a \\ a & a \end{vmatrix}$$

$$|A| = (a+2)a(2a-1-2a)$$

$$|A| = -a(a+2)$$

$a = 0 \qquad a = -2$

Consistente determinado

$a \neq 0 \qquad a \neq -2$

Si $a = 0$

$$\left(\begin{array}{ccc:c} 3a & 3a-7 & a-5 & a-1 \\ 2a-1 & 4a-1 & 2a & a+1 \\ 4a & 5a-7 & 2a-5 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc:c} 0 & -7 & -5 & -1 \\ -1 & -1 & 0 & 1 \\ 0 & -7 & -5 & 0 \end{array}\right) \; -$$

$$\left(\begin{array}{ccc:c} 0 & -7 & -5 & -1 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right) \quad ¿\,0 = 1?$$

Inconsistente

Si $a = -2$

$$\left(\begin{array}{ccc:c} -6 & -13 & -7 & -3 \\ -5 & -9 & -4 & -1 \\ -8 & -17 & -9 & 0 \end{array}\right) \begin{array}{l} -1 \\ + \\ + \end{array}$$

$$\left(\begin{array}{ccc:c} -6 & -13 & -7 & -3 \\ 1 & 4 & 3 & 2 \\ -2 & -4 & -2 & 3 \end{array}\right) \begin{array}{l} \\ 2 \\ + \end{array} \quad \begin{array}{l} 6 \; + \\ \\ \\ \end{array}$$

$$\left(\begin{array}{ccc:c} 0 & 11 & 11 & 9 \\ 1 & 4 & 3 & 2 \\ 0 & 4 & 4 & 7 \end{array}\right) \begin{array}{l} -4/11 \\ \\ + \end{array}$$

$$\left(\begin{array}{ccc:c} 0 & 11 & 11 & 9 \\ 1 & 4 & 3 & 2 \\ 0 & 0 & 0 & 41/11 \end{array}\right) \quad ¿\,0 = 41/11?$$

Inconsistente

Consistente determinado	$a \neq 0$	$a \neq -2$
Inconsistente	$a = 0$	$a = -2$
Consistente indeterminado	ningún a	

Ejercicio – Sistemas de ecuaciones

En el siguiente sistema de ecuaciones, se pide hallar el valor de “x” para que se tenga a) solución única b) infinitas soluciones c) no tenga solución

$$(2x-3)x+3y+3z=x$$
$$3x+(2x-3)y+3z=3$$
$$3x+3y+(2x-3)z=-x+3$$

Solución

Cambiamos la notación de x a k ctte

$$(2k-3)x+3y+3z=k$$
$$3x+(2k-3)y+3z=3$$
$$3x+3y+(2k-3)z=-k+3$$

$$A_{3\times3}X_{3\times1}=B_{3\times1}$$

$$\begin{pmatrix}2k-3 & 3 & 3\\ 3 & 2k-3 & 3\\ 3 & 3 & 2k-3\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}k\\ -3\\ -k+3\end{pmatrix}$$

$$X_{3\times1}=A^{-1}_{3\times3}B_{3\times1}$$

$$A^{-1}=\frac{1}{|A|}adj(A)$$

$$|A|=\begin{vmatrix}2k-3 & 3 & 3\\ 3 & 2k-3 & 3\\ 3 & 3 & 2k-3\end{vmatrix}\begin{matrix}+\\ \\ \end{matrix}\ \begin{matrix}+\\ \\ \end{matrix}$$

$$|A|=\begin{vmatrix}2k+3 & 2k+3 & 2k+3\\ 3 & 2k-3 & 3\\ 3 & 3 & 2k-3\end{vmatrix}$$

$$|A|=(2k+3)\begin{vmatrix}1 & 1 & 1\\ 3 & 2k-3 & 3\\ 3 & 3 & 2k-3\end{vmatrix}\begin{matrix}-3\ +\\ \\ +\end{matrix}$$

$$|A|=(2k+3)\begin{vmatrix}1 & 1 & 1\\ 0 & 2k-6 & 0\\ 0 & 0 & 2k-6\end{vmatrix}$$

$$|A| = (2k+3)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2k-6 & 0 \\ 0 & 0 & 2k-6 \end{vmatrix}$$

$$|A| = (2k+3)\begin{vmatrix} 2k-6 & 0 \\ 0 & 2k-6 \end{vmatrix}$$

$$|A| = (2k+3)(2k-6)^2$$

Si |A|=0

$$(2k+3)(2k-6)^2 = 0$$

$$k = -\frac{3}{2} \quad \text{ó} \quad k = 3$$

En ese caso se tienen infinitas soluciones o no hay solución

Entonces si |A|≠ 0 se tiene solución única

$$\boxed{k \neq -\frac{3}{2} \quad \text{ó} \quad k \neq 3 \quad \text{solución única}}$$

$$\begin{pmatrix} 2k-3 & 3 & 3 \\ 3 & 2k-3 & 3 \\ 3 & 3 & 2k-3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} k \\ -3 \\ -k+3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2k-3 & 3 & 3 & k \\ 3 & 2k-3 & 3 & -3 \\ 3 & 3 & 2k-3 & -k+3 \end{array}\right)$$

$$Si\ k = -\frac{3}{2}$$

$$\left(\begin{array}{ccc|c} -3-3 & 3 & 3 & -\frac{3}{2} \\ 3 & -3-3 & 3 & -3 \\ 3 & 3 & -3-3 & \frac{3}{2}+3 \end{array}\right)$$

$$\left(\begin{array}{ccccc} -6 & 3 & 3 & \vdots & -\frac{3}{2} \\ 3 & -6 & 3 & \vdots & -3 \\ 3 & 3 & -6 & \vdots & \frac{9}{2} \end{array}\right) \quad -1 \; + \quad 2 \; +$$

$$\left(\begin{array}{ccccc} 0 & 9 & -9 & \vdots & \frac{15}{2} \\ 0 & -9 & 9 & \vdots & -\frac{15}{2} \\ 3 & 3 & -6 & \vdots & \frac{9}{2} \end{array}\right) \quad +1 \; +$$

$$\left(\begin{array}{ccccc} 0 & 0 & 0 & \vdots & 0 \\ 0 & -9 & 9 & \vdots & -\frac{15}{2} \\ 3 & 3 & -6 & \vdots & \frac{9}{2} \end{array}\right)$$

$$\left(\begin{array}{ccccc} 3 & 3 & -6 & \vdots & \frac{9}{2} \\ 0 & -9 & 9 & \vdots & -\frac{15}{2} \\ 0 & 0 & 0 & \vdots & 0 \end{array}\right)$$

$$\boxed{k = -\frac{3}{2} \qquad \text{Infinitas soluciones}}$$

$$\left(\begin{array}{ccccc} 2k-3 & 3 & 3 & \vdots & k \\ 3 & 2k-3 & 3 & \vdots & -3 \\ 3 & 3 & 2k-3 & \vdots & -k+3 \end{array}\right)$$

$Si\ k = 3$

$$\left(\begin{array}{ccccc} 3 & 3 & 3 & \vdots & 3 \\ 3 & 3 & 3 & \vdots & -3 \\ 3 & 3 & 3 & \vdots & 0 \end{array}\right)$$

$$\begin{pmatrix} 3 & 3 & 3 & \vdots & 3 \\ 3 & 3 & 3 & \vdots & -3 \\ 3 & 3 & 3 & \vdots & 0 \end{pmatrix} \begin{matrix} -1 \\ + \\ + \end{matrix}$$

$$\begin{pmatrix} 3 & 3 & 3 & \vdots & 3 \\ 0 & 0 & 0 & \vdots & -6 \\ 0 & 0 & 0 & \vdots & -3 \end{pmatrix}$$

$$\boxed{k = 3 \qquad \text{No hay solución}}$$

Ejercicio – Sistemas de ecuaciones

Hallar los valores de "p" y "q" tal que el sistema de ecuaciones: $A_{3\times3}X_{3\times1} + 3pX_{3\times1} = -4X_{3\times1} + B^t$ sea:

a) Consistente determinado.

b) Consistente indeterminado.

c) Inconsistente.

$$A = \begin{pmatrix} 2p & -3 & -3 \\ -3 & 2p & -3 \\ 3 & -3 & 2p \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad B = (2q+2 \quad 3q-1 \quad 0)$$

Solución

$$A_{3\times3}X_{3\times1} + 3pX_{3\times1} = -4X_{3\times1} + B^t$$

$$A_{3\times3}X_{3\times1} + 3pX_{3\times1} + 4X_{3\times1} = B^t$$

$$(A_{3\times3} + 3pI + 4I)X_{3\times1} = B^t$$

$$\left[\begin{pmatrix} 2p & -3 & -3 \\ -3 & 2p & -3 \\ 3 & -3 & 2p \end{pmatrix} + 3p\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 4\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2q+2 \\ 3q-1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 5p+4 & -3 & -3 \\ -3 & 5p+4 & -3 \\ 3 & -3 & 5p+4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2q+2 \\ 3q-1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 5p+4 & -3 & -3 \\ -3 & 5p+4 & -3 \\ 3 & -3 & 5p+4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2q+2 \\ 3q-1 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 5p+4 & -3 & -3 \\ -3 & 5p+4 & -3 \\ 3 & -3 & 5p+4 \end{vmatrix} +$$

$$\begin{vmatrix} 5p+4 & -3 & -3 \\ -3 & 5p+4 & -3 \\ 0 & 5p+1 & 5p+1 \end{vmatrix}$$

$$(5p+1)\begin{vmatrix} 5p+4 & -3 & -3 \\ -3 & 5p+4 & -3 \\ 0 & 1 & 1 \end{vmatrix} \begin{matrix} + \\ + \\ 3 \end{matrix}$$

$$(5p+1)\begin{vmatrix} 5p+4 & 0 & 0 \\ -3 & 5p+7 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$(5p+1)\begin{vmatrix} 5p+4 & 0 \\ -3 & 5p+7 \end{vmatrix}$$

$$(5p+1)(5p+4)(5p+7) = 0$$

$$p = -\frac{1}{5} \qquad p = -\frac{4}{5} \qquad p = -\frac{7}{5}$$

$$\left(\begin{array}{ccc|c} 5p+4 & -3 & -3 & 2q+2 \\ -3 & 5p+4 & -3 & 3q-1 \\ 3 & -3 & 5p+4 & 0 \end{array}\right)$$

$$Si\ p = -\frac{1}{5} \qquad \left(\begin{array}{ccc|c} 3 & -3 & -3 & 2q+2 \\ -3 & 3 & -3 & 3q-1 \\ 3 & -3 & 3 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 0 & 0 & -6 & 5q+1 \\ -3 & 3 & -3 & 3q-1 \\ 0 & 0 & 0 & 3q-1 \end{array}\right)$$

$$q = \frac{1}{3} \quad consistente\ indeterminado$$

$$q \neq \frac{1}{3} \quad inconsistente$$

$$Si\ p=-\frac{4}{5} \qquad \begin{pmatrix} 0 & -3 & -3 & \vdots & 2q+2 \\ -3 & 0 & -3 & \vdots & 3q-1 \\ 3 & -3 & 0 & \vdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & -3 & \vdots & 2q+2 \\ -3 & 0 & -3 & \vdots & 3q-1 \\ 0 & -3 & -3 & \vdots & 3q-1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & -3 & \vdots & 2q+2 \\ -3 & 0 & -3 & \vdots & 3q-1 \\ 0 & 0 & 0 & \vdots & q-3 \end{pmatrix}$$

$q=3\ \ consistente\ indeterminado$

$q\neq 3\ inconsistente$

$\wedge$

$$Si\ p=-\frac{7}{5} \qquad \begin{pmatrix} -3 & -3 & -3 & \vdots & 2q+2 \\ -3 & -3 & -3 & \vdots & 3q-1 \\ 3 & -3 & -3 & \vdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -6 & -6 & \vdots & 2q+2 \\ 0 & -6 & -6 & \vdots & 3q-1 \\ 3 & -3 & -3 & \vdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & \vdots & -q+3 \\ 0 & -6 & -6 & \vdots & 3q-1 \\ 3 & -3 & -3 & \vdots & 0 \end{pmatrix}$$

$q=3\ \ consistente\ indeterminado$

$q\neq 3\ inconsistente$

Consistente determinado	$p\neq-\frac{1}{5}\ \wedge\ p\neq-\frac{4}{5}\ \wedge\ p\neq-\frac{7}{5}\ \wedge\ cualquier\ q$
Inconsistente	$p=-\frac{1}{5}\ \wedge\ q\neq\frac{1}{3}\quad \vee\quad p=-\frac{4}{5}\ \wedge\ q\neq 3\quad \vee\quad p=-\frac{7}{5}\ \wedge\ q\neq 3$
Consistente indeterminado	$p=-\frac{1}{5}\ \wedge\ q=\frac{1}{3}\quad \vee\quad p=-\frac{4}{5}\ \wedge\ q=3\quad \vee\quad p=-\frac{7}{5}\ \wedge\ q=3$