

# Problem Set 3 Key

## 1 Problem 1

The table below shows the frequency table of the number of referrals offered for expectant mothers who experience socioeconomic barriers for needed healthcare.

Number of Referrals	Frequency
0	90
1	132
2	76
3	10

- What is the probability that a randomly selected participant received at least one referral? [2 pts.]

### Note

We need to calculate the relative frequencies for each category first to calculate the specific probabilities.

```
library(tidyverse)
```

Warning: package 'ggplot2' was built under R version 4.5.1

```
-- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
v dplyr      1.1.4      v readr      2.1.5
v forcats    1.0.0      v stringr    1.5.1
v ggplot2    4.0.0      v tibble     3.2.1
v lubridate  1.9.4      v tidyr      1.3.1
v purrr      1.0.4
-- Conflicts ----- tidyverse_conflicts() --
```

```
x dplyr::filter() masks stats::filter()
x dplyr::lag() masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
```

```
df <- data.frame(Referrals=c(0,1,2,3),
                 Frequency = c(90,132,76,10))

mutate(df,RelFrequency = Frequency/sum(Frequency))
```

	Referrals	Frequency	RelFrequency
1	0	90	0.29220779
2	1	132	0.42857143
3	2	76	0.24675325
4	3	10	0.03246753

OR

```
total <- sum(c(90,132,76,10))

df <- data.frame(Referrals=c(0,1,2,3),
                 Frequency = c(90,132,76,10),
                 Rel.Frequency = c(90,132,76,10)/total)

df
```

	Referrals	Frequency	Rel.Frequency
1	0	90	0.29220779
2	1	132	0.42857143
3	2	76	0.24675325
4	3	10	0.03246753

$$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3)$$

or

$$P(X \geq 1) = 1 - P(X = 0)$$

Hence the probability  $P(X \leq 1)$  is:

```
1-90/sum(c(90,132,76,10))
```

```
[1] 0.7077922
```

```
# OR
```

```
1-0.2922
```

```
[1] 0.7078
```

- What is the expected value of the number of referrals in the sample? [1pt.]

#### Note

The expected number of referrals is the sum of the product of the outcome and their corresponding probabilities.

$$E(X) = 1 * (90/308) + 2 * (132/308) + 3 * (76/308) + 4 * (10/308)$$

```
0*(90/308) + 1*(132/308) + 2*(76/308) + 3*(10/308)
```

```
[1] 1.019481
```

```
# OR
```

```
sum(df$Referrals*df$Rel.Frequency)
```

```
[1] 1.019481
```

## 2 Problem 2

The proportion of individuals from a certain population with an O blood type is 0.44. What is the probability that out of a sample of 30 individuals from this population,

#### Note

We use the binomial distribution!

- a. That exactly half of the individuals in the sample have an O blood type?

```
dbinom(15,30,0.44)
```

```
[1] 0.1162175
```

- b. At most 10 individuals have an O blood type? [1pt]

```
pbinom(10,30,0.44)
```

```
[1] 0.160397
```

### 3 Problem 3

If the mean number of unexcused absences per school year for fifth graders at a public elementary school is 11.8, what is the probability that a randomly selected student will have chronic absenteeism, i.e. at least 18 unexcused absences per school year? [1 pt.]

#### **i** Note

We use the Poisson distribution!

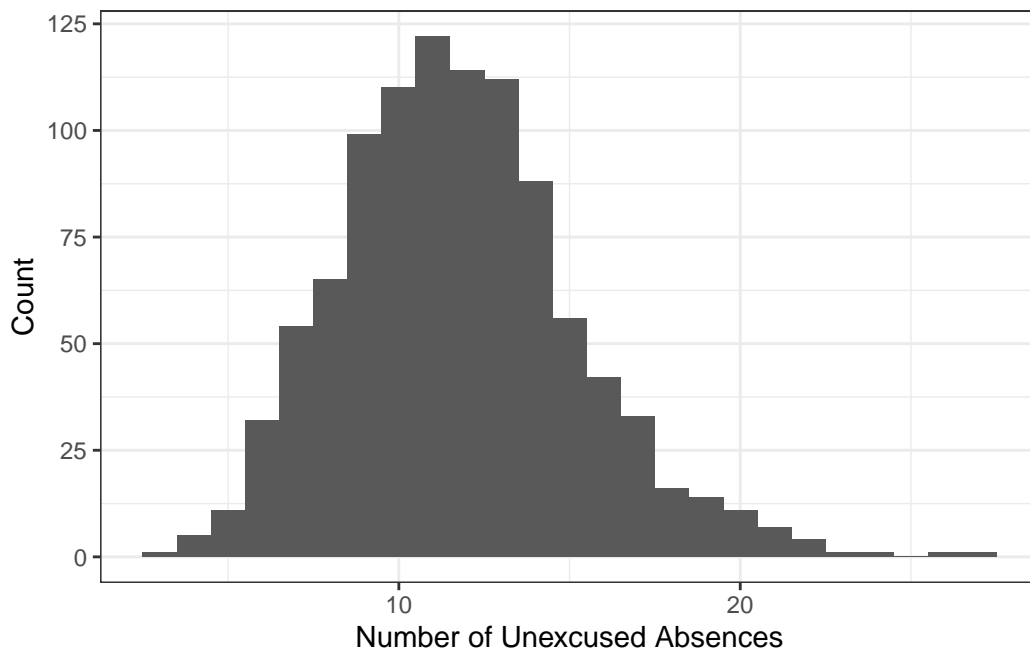
- a. What is the probability that a randomly selected student will have chronic absenteeism, i.e. at least 18 unexcused absences per school year? [1 pt.]

```
1-ppois(17,lambda=11.8)
```

```
[1] 0.05561775
```

- b. Generate 1000 numbers from the associated distribution and create a histogram from these values. [2pts.]

```
library(tidyverse)
Nsim<- 1000 # Simulating 10 times
set.seed(12)
sims <- rpois(Nsim,11.8)
df <- data.frame(simulations=sims)
ggplot(df, aes(x=simulations)) +
  geom_histogram(binwidth=1) +
  theme_bw() +
  labs(x="Number of Unexcused Absences", y="Count")
```



#### 4 Problem 4

A random number generator was used to assign participants to two treatment groups. A random number was generated from a  $UNIF(-1,1)$  distribution. A value between  $-0.3$  and  $0.2$  means the participant is assigned to treatment group A, else they were assigned in B. What is the probability that a participant is assigned in group A? [2pts.]

##### **i** Note

You can use R or manual mathematical calculation of areas. I will be showing the R solution.

We use the CDF to calculate probabilities for continuous distributions such as the normal distribution. We will be using `pnorm()`.

```
pnif(0.2,-1,1) - pnif(-0.3,-1,1)
```

```
[1] 0.25
```

## 5 Problem 5

The mean A1c measurement for residents at a nursing home was 5.6 with a standard deviation of 2.1. Assuming the A1c measurements follow a normal distribution,

### Tip

We use the CDF to calculate probabilities for continuous distributions such as the normal distribution. We will be using `pnorm()`.

- a. What is the probability of randomly selecting a resident who is prediabetic (A1c between 5.7 and 6.4)? [1pt]

```
pnorm(6.4,mean=5.6,sd=2.1) - pnorm(5.7,mean=5.6,sd=2.1)
```

```
[1] 0.1293906
```

- b. What is the probability of randomly selecting a resident who is diabetic (A1c above 6.5)? [1pt]

```
pnorm(6.5,mean=5.6,sd=2.1, lower.tail = F)
```

```
[1] 0.3341176
```

- c. What is the expected number of diabetics out of a sample of 100 residents based on this distribution? (Hint: Randomly selecting a diabetic in this sample is a Bernoulli process.) [2 pts.]

### Tip

Remember that for a Bernoulli process, the expected value  $E(X) = np$  where  $n$  is the sample size and  $p$  is the probability of success.

```
probability_diabetic <- pnorm(6.5,mean=5.6,sd=2.1, lower.tail = F)
sample_size <- 100

sample_size*probability_diabetic
```

```
[1] 33.41176
```

Expected value is `r sample_size*probability_diabetic`. If we're talking about number of diabetics, we **round up** to 34.

d. What is the 85th percentile of the A1c measurements in this nursing home? [1 pt.]

💡 Tip

We can use `qnorm()` to calculate values based on percentiles.

```
qnorm(0.85, mean=5.6, sd=2.1)
```

```
[1] 7.77651
```