Assignment 3: Edges and Hough transform

Machine perception

2022/2023

General instructions: The assignment is composed of the compulsory tasks and optional tasks (denoted by ★). To approach the assignment defense, you are required to complete at least the compulsory parts. Successfully defending the compulsory part will give you a maximum 75 points out of 100. At the defense, the obligatory tasks **must** be implemented correctly and completely. If your implementation is incomplete or has major errors, you will not be able to successfully defend the assignment. You can choose arbitrarily among the optional tasks to reach the 100 points. The number of points for an optional task is given in the task description. It is possible to obtain more than 100 points on individual assignment. However, the maximum overall points obtained from the 6 assignments is 600 points.

Required formating and submission: Create a folder `assignment3` that you will use during this assignment. Unpack the content of the `assignment3.zip` that you can download from the course webpage to the folder. Save the solutions for the assignments as Python scripts to `assignment3` folder. In order to complete the assignment you have to present your solutions to the teaching assistant. Some assignments contain questions that require sketching, writing or manual calculation. Write these answers down and bring them to the assignment defense as well. The code must be submitted on the e-classroom **before** the defense.

COMMON ERRORS AND DEBUG IDEAS

- check your x and y coordinates
- check your data type: float, uint8
- check your data range: [0,255], [0,1]
- perform simple checks (synthetic data examples)

Introduction

For additional explanation of the theory, required for the following assignment, check the slides from the lectures as well as scientific literature on these topics [1, 4]. A version of the book by Forsyth in Ponce can also be found online [2] – the related theory is located in chapters 8 and 15.

Exercise 1: Image derivatives

The first and the second exercise will deal with the problem of detecting edges in images. A way of detecting edges is by analyzing local changes of grayscale levels. Mathematically this means that we are computing *image derivatives*. The downside of an image derivative at a certain point in the image is that it can be sensitive to image noise. Thus, it is common to soften the image beforehand with a narrow filter $I_b(x, y) = G(x, y) * I(x, y)$ and only then calculate the derivative.

Usually, a Gaussian filter is used to smooth the image. As we will use partial derivatives in the following exercises, we will first look at the decomposition of the partial derivative of the Gaussian kernel. A 2-D Gaussian kernel can be written as a product of two 1-D kernels as:

$$G(x, y) = g(x)g(y), \tag{1}$$

therefore image filtering for image $I(x, y)$ can be formulated as

$$I_b(x, y) = g(x) * g(y) * I(x, y). \tag{2}$$

Taking into account the associative property of the convolution $\frac{d}{dx}(g * f) = (\frac{d}{dx}g) * f$, we can write a partial derivative of the *smoothed* image with respect to x as:

$$I_x(x, y) = \frac{\delta}{\delta x}\Big[g(x) * g(y) * I(x, y)\Big] = \frac{d}{dx}g(x) * \Big[g(y) * I(x, y)\Big]. \tag{3}$$

This means that the input image can be first filtered with a Gaussian kernel with respect to y and then filter the result with the derivative of the Gaussian kernel with respect to x. Similarly, we can define the second partial derivative with respect to x, however, we have to remember to always filter the image before we perform derivation. The second derivative with respect to x is therefore defined as a partial derivative of already derived image:

$$I_{xx}(x, y) = \frac{\delta}{\delta x}\Big[g(x) * g(y) * I_x(x, y)\Big] = \frac{d}{dx}g(x) * \Big[g(y) * I_x(x, y)\Big]. \tag{4}$$

(a) Follow the equations above and derive the equations used to compute first and second derivatives with respect to y: $I_y(x, y)$, $I_{yy}(x, y)$, as well as the mixed derivative $I_{xy}(x, y)$

(b) Implement a function that computes the derivative of a 1-D Gaussian kernel. The formula for the derivative of the Gaussian kernel is:

$$\begin{aligned} \frac{d}{dx}g(x) &= \frac{d}{dx}\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2}) \\ &= -\frac{1}{\sqrt{2\pi}\sigma^3}x\exp(-\frac{x^2}{2\sigma^2}). \end{aligned} \tag{5}$$

Implement the function `gaussdx(sigma)` that works the same as function `gauss` from the previous assignment. Don't forget to normalize the kernel. Be careful as the derivative is an odd function, so a simple sum will not do. Instead normalize the kernel by dividing the values such that the sum of absolute values is 1. Effectively, you have to divide each value by $\sum abs(g_x(x))$.

(c) The properties of the filter can be analyzed by using an *impulse response function*. This is performed as a convolution of the filter with a Dirac delta function. The discrete version of the Dirac function is constructed as a finite image that has all elements set to 0 except the central element, which is set to a high value (e.g. 1).

```
impulse = np.zeros((50, 50))
impulse[25, 25] = 1
```

Generate a 1-D Gaussian kernel G and a Gaussian derivative kernel D.

What happens if you apply the following operations to the impulse image?

(a) First convolution with G and then convolution with G^T.

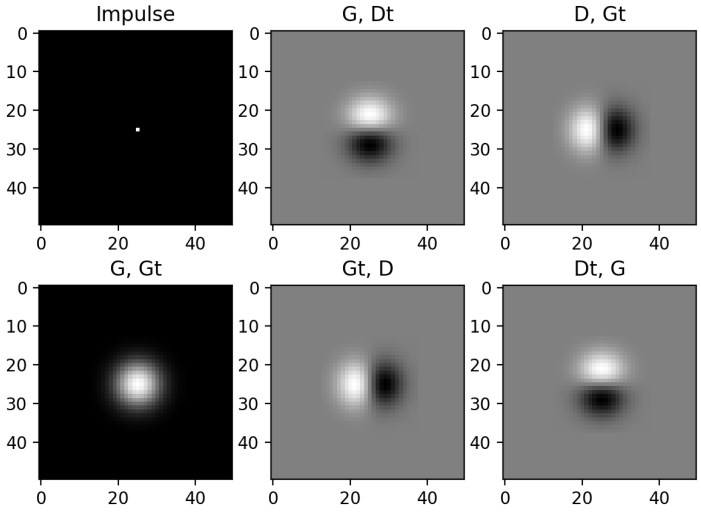
(b) First convolution with G and then convolution with D^T.

(c) First convolution with D and then convolution with G^T.

(d) First convolution with G^T and then convolution with D.

(e) First convolution with D^T and then convolution with G.

Is the order of operations important? Display the images of the impulse responses for different combinations of operations.



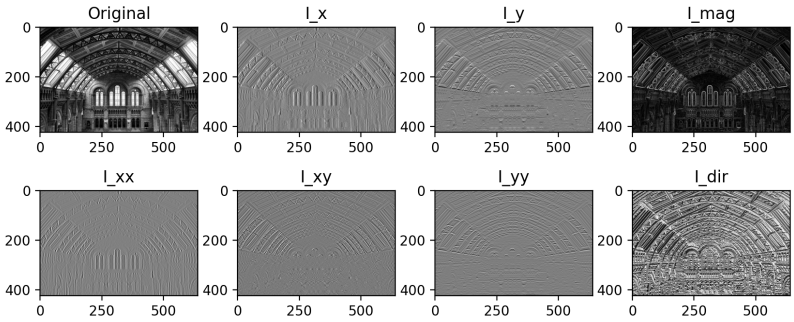
(d) Implement a function that uses functions `gauss` and `gaussdx` to compute both partial derivatives of a given image with respect to x and with respect to y.

Similarly, implement a function that returns partial second order derivatives of a given image.

Additionally, implement the function `gradient_magnitude` that accepts a grayscale image I and returns both derivative magnitudes and derivative angles. Magnitude is calculated as $m(x, y) = \sqrt{(I_x(x, y)^2 + I_y(x, y)^2)}$ and angles are calculated as $\phi(x, y) = \arctan(I_y(x, y)/I_x(x, y))$

Hint: Use function `np.arctan2` to avoid division by zero for calculating the arctangent function.

Use all the implemented functions on the same image and display the results in the same window.



(e) ★ (15 points) Gradient information is often used in image recognition. Extend your image retrieval system from the previous assignment to use a simple gradient-based feature instead of color histograms. To calculate this feature, compute gradient magnitudes and angles for the entire image, then divide the image in a 8×8 grid. For each cell of the grid compute a 8 bin histogram of gradient magnitudes with respect to gradient angle (quantize the angles into 8 values, then for each pixel of the cell, add the value of the gradient to the bin specified by the corresponding angle). Combine all the histograms to get a single 1-D feature for every image. Test the new feature on the image database from the previous assignment. Compare the new results to the color histogram based retrieval.

Exercise 2: Edges in images

One of the most widely used edge detector algorithms is Canny edge detector. In this exercise, you will implement parts of Canny's algorithm.

(a) Firstly, create a function `findedges` that accepts an image I, and the parameters *sigma* and *theta*.

The function should create a binary matrix I_e that only keeps pixels higher than threshold *theta*:

$$I_e(x,y) = \begin{cases} 1 & ; I_{mag}(x,y) \geq \vartheta \\ 0 & ; otherwise \end{cases} \tag{6}$$

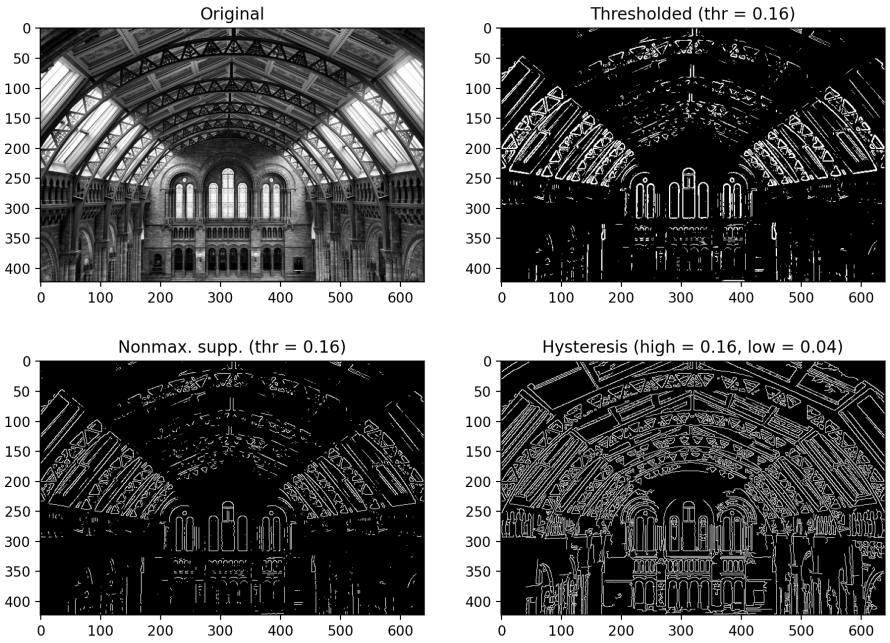
Test the function with the image `museum.png` and display the results for different values of the parameter `theta`. Can you set the parameter so that all the edges in the image are clearly visible?

(b) Using magnitude produces only a first approximation of detected edges. Unfortunately, these are often wide, and we would like to only return edges one pixel wide. Therefore, you will implement non-maxima suppression based on the image derivative magnitudes and angles. Iterate through all the pixels and for each search its 8-neighborhood. Check the neighboring pixels parallel to the gradient direction, and set the current pixel to 0 if it is not the largest in the neighborhood (based on

derivative magnitude). You only need to compute the comparison to actual pixels, interpolating to more accuracy is not required.

(c) ★ (10 points) The final step of Canny's algorithm is edge tracking by hysteresis. Add the final step after performing non-maxima suppression along edges. Hysteresis uses two thresholds $t_{low} < t_{high}$, keeps all pixels above t_{high} and discards all pixels below t_{low}. The pixels between the thresholds are kept only if they are connected to a pixel above t_{high}.

Hint: Since we are looking for connected components containing at least one pixel above t_{high}, you could use something like `cv2.connectedComponentsWithStats` to extract them. Try to avoid explicit `for` loops as much as possible.



Exercise 3: Detecting lines

In this exercise, we will look at the Hough algorithm, in particular the variation of the algorithm that is used to detect lines in an image. For more information about the theory look at the lecture slides as well as the literature [1], and a web applet that demonstrates the Hough transform, e.g. [3].

We have a point in the image $p_0 = (x_0, y_0)$. If we know that the equation of a line is $y = mx + c$, which are all the lines that are running through the point p_0? The answer is simple: all the lines whose parameters m and c correspond to the equation $y_0 = mx_0 + c$. If we fix the values (x_0, y_0), then the variable parameters again describe a line, however, this time the line is in the (m, c) space that we also call the parameter space. If we consider a new point $p_1 = (x_1, y_1)$, this new point also has a line in the (m, c) space. This line crosses the p_0 line at a point (m', c'). The point (m', c') then defines a line in (x, y) space that connects the points p_0 and p_1.

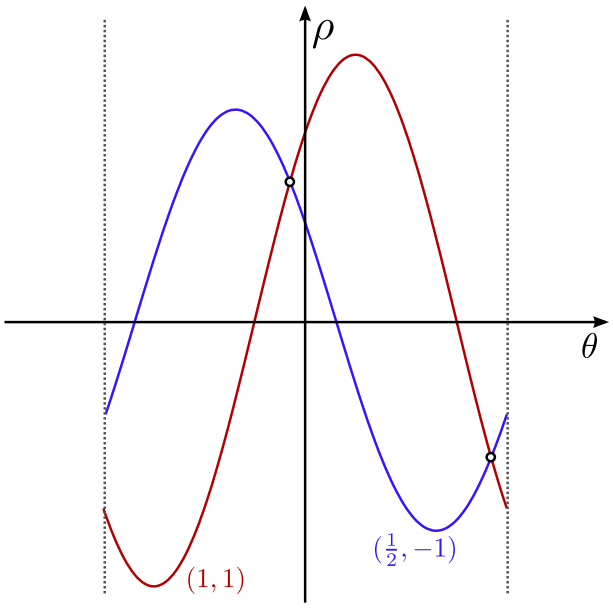
Question: Analytically solve the problem by using Hough transform: In 2D space you are given four points $(0,0)$, $(1,1)$, $(1,0)$, $(2,2)$. Define the equations of the lines that run through at least two of these points.

If we want to find all the lines in an image using the Hough approach in a program, we have to proceed as described. The parameter space (m,c) is first quantized as an *accumulator* matrix. For each edge pixel, we *draw* a corresponding line in the (m,c) space in an additive manner (increase the value by 1). All the image elements that lie on the same line in the input image will generate lines in the (m,c) space that will intersect at the same point and therefore increase the value of the same accumulator cell. This means that local maxima in the (m,c) space define the lines in the input image that contain a lot of the detected edge pixels.

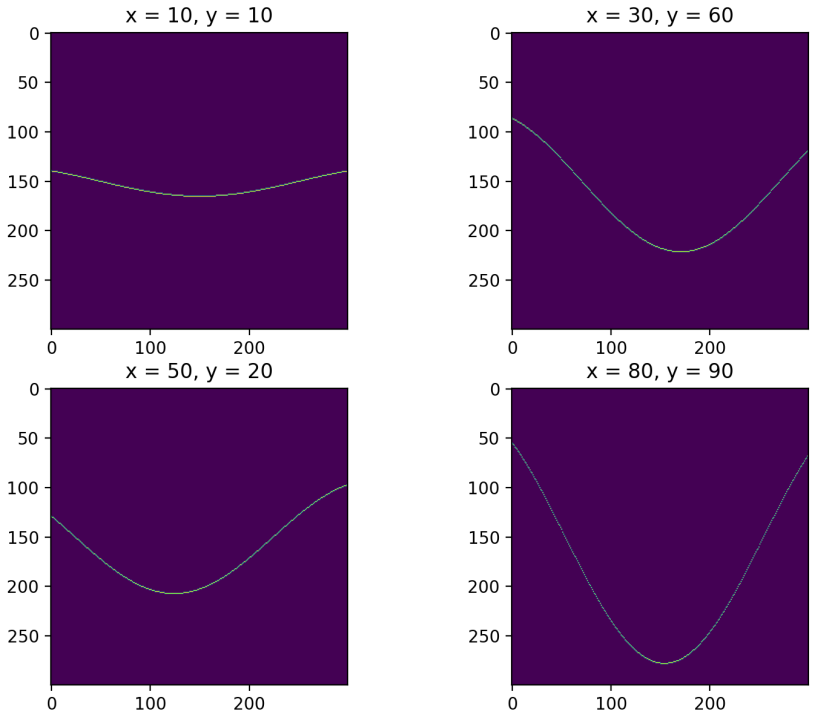
In real scenarios, the $y = mx + c$ formulation of a line is inefficient. This is especially apparent in the case of vertical lines, there m becomes infinite. This problem can be avoided by a different line parametrization, for example using *polar coordinates*. In this case, the equation of a line looks like

$$x\cos(\vartheta) + y\sin(\vartheta) = \rho. \tag{7}$$

The algorithm remains more or less the same, the only difference is that a point in a (x,y) space generates a sinusoid in the (ϑ,ρ) space. For points $(1,1)$ and $(\frac{1}{2},-1)$ the corresponding curves in the parameter space are shown in the figure below.



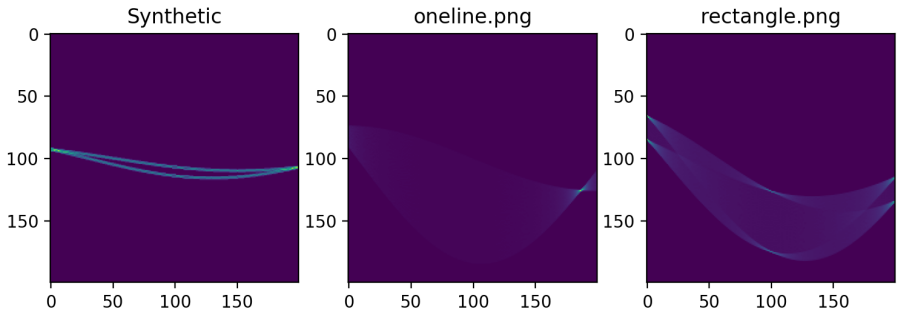
(a) Create an accumulator array defined by the resolution on ρ and ϑ values. Calculate the sinusoid that represents all the lines that pass through some nonzero point. Increment the corresponding cells in the accumulator array. Experiment with different positions of the nonzero point to see how the sinusoid changes. You can set the number of accumulator bins on each axis to 300 to begin with.



(b) Implement the function `hough_find_lines` that accepts a binary image, the number of bins for ϑ and ρ (allow the possibility of them being different) and a threshold.

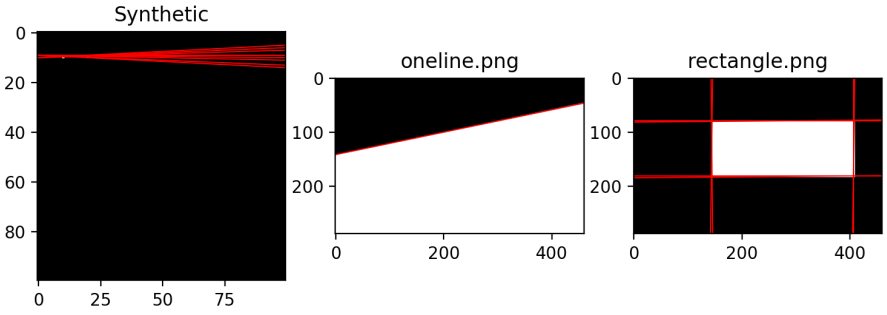
Create an accumulator matrix `A` for the parameter space (ρ, ϑ). Parameter ϑ is defined in the interval from $-\pi/2$ to $\pi/2$, ρ is defined on the interval from $-D$ to D, where D is the length of the image diagonal. For each nonzero pixel in the image, generate a curve in the (ρ, ϑ) space by using the equation (7) for all possible values of ϑ and increase the corresponding cells in `A`. Display the accumulator matrix. Test the method on your own synthetic images ((e.g. 100×100 black image, with two white pixels at $(10, 10)$ and $(10, 20)$).

Finally, test your function on two synthetic images `oneline.png` and `rectangle.png`. First, you should obtain an edge map for each image using either your function `findedges` or some inbuilt function. Run your implementation of the Hough algorithm on the resulting edge maps.

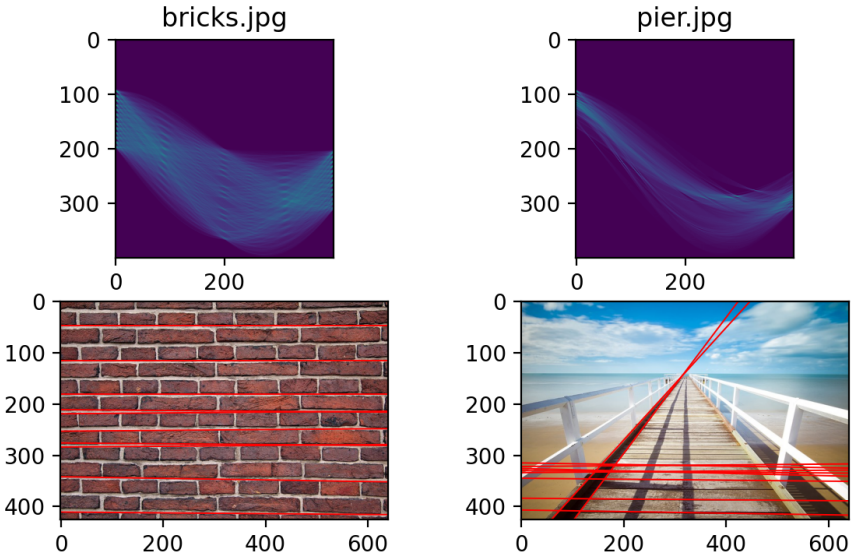


(c) The sinusoids don't usually intersect in only one point, resulting in more than one detected line. Implement a function named `nonmaxima_suppression_box` that checks the neighborhood of each pixel and set it to 0 if it is not the maximum value in the neighborhood (only consider 8-neighborhood).

(d) Search the parameter space and extract all the parameter pairs (ρ, ϑ) whose corresponding accumulator cell value is greater than a specified threshold `threshold`. Draw the lines that correspond to the parameter pairs using the `draw_line` function that you can find in the supplementary material.

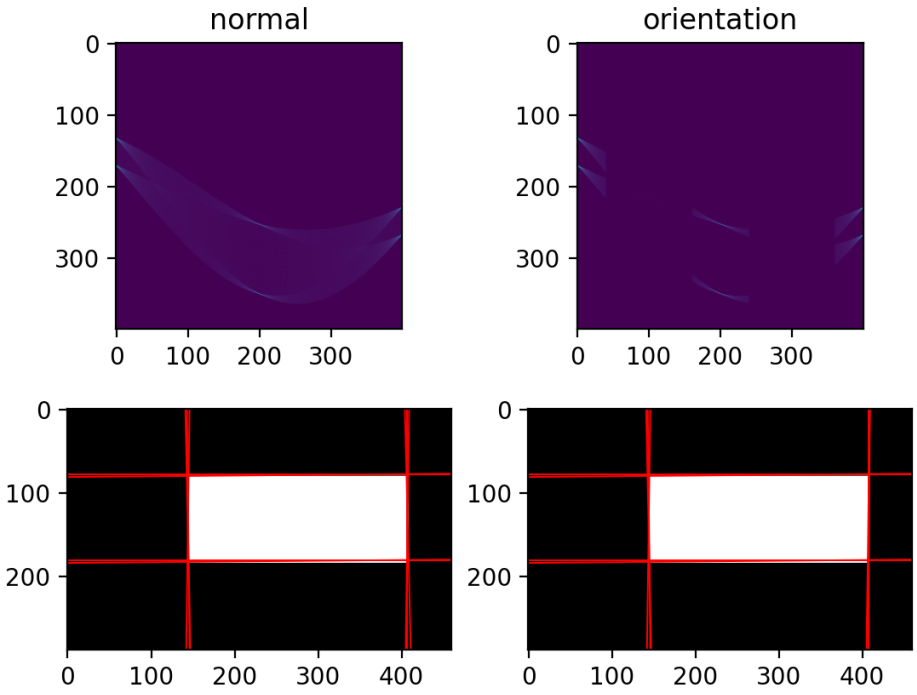


(e) Read the image from files `bricks.jpg` and `pier.jpg`. Change the image to grayscale and detect edges. Then detect lines using your algorithm. As the results will likely depend on the number of pixels that vote for a specific cell and this depends on the size of the image and the resolution of the accumulator, try sorting the pairs by their corresponding cell values in descending order and only select the top $n = 10$ lines. Display the results and experiment with parameters of Hough algorithm as well as the edge detection algorithm, e.g. try changing the number of cells in the accumulator or σ parameter in edge detection to obtain results that are similar or better to the ones shown on the image below.

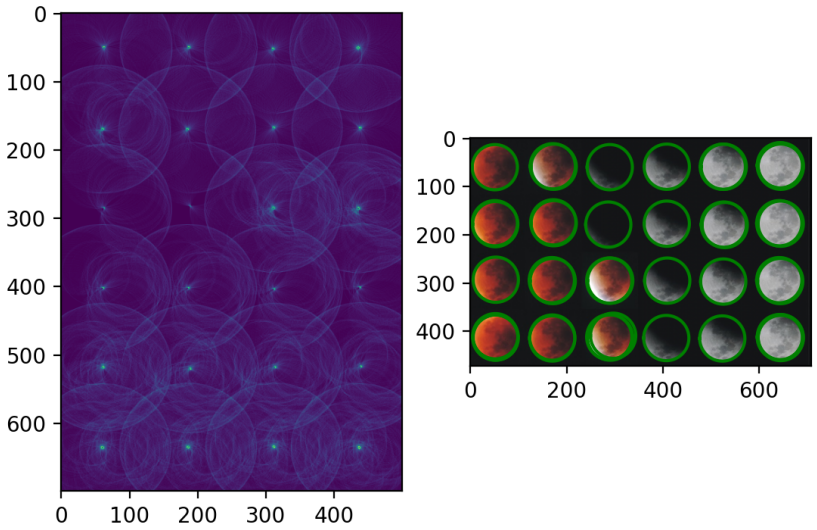


(f) ★ (5 points) A problem of the Hough transform is that we need a new dimension for each additional parameter in the model, which makes the execution slow for more complex models. We can avoid such parameters if we can reduce the parameter space, e.g. by introducing domain knowledge. Recall from the previous exercise that we can get the local gradient angle besides its magnitude. This angle is perpendicular to the edge and can be used to limit the scope of the parameter ϑ for a specific edge point. We therefore do not have to increase the values of the cells of the entire range of ϑ (calculate multiple values of ρ), but can use the local angle and only work with a single (ρ, ϑ) pair for each edge point.

Copy your implementation of the line detector to a new function and modify the algorithm so that it also accepts the matrix of edge angles. Note that the angle values were probably calculated using the `np.arctan2(dy, dx)` function that returns the values between wider range. You have to adjust the angles so that they are within the $[-\pi/2, \pi/2]$ interval. Test the modified function on several images and compare the results with the original implementation.



(g) ★ (5 points) Implement a Hough transform that detects circles of a fixed radius. You can test the algorithm on image `eclipse.jpg`. Try using a radius somewhere between 45 and 50 pixels.



(h) ★ (15 points) Not all lines can accumulate the same number of votes, e.g. if the image is not square, candidates along the longer dimension are in better position.

Extend your algorithm so that it normalizes the number of votes according to the maximum number of votes possible for a given line (how many pixels does a line cover along its crossing of the image). Demonstrate the difference on some non-rectangular images where the difference can be shown clearly.

References

[1] D. A. Forsyth and J. Ponce. *Computer Vision: A Modern Approach.* Prentice Hall, 2002.

[2] D. A. Forsyth and J. Ponce. Computer vision: A modern approach (online version). http://www.cs.washington.edu/education/courses/cse455/02wi/readings/book-7-revised-a-indx.pdf, 2003.

[3] Simon Hohl. Interactive hough transform. http://dersmon.github.io/HoughTransformationDemo/.

[4] R. E. Woods, R. C. Gonzalez, and P. A. Wintz. *Digital Image Processing, 3 ed.* Pearson Education, 2010.