Grupo II

5. $\begin{vmatrix} 0 & -1 & -1 & 2 \\ 1 & -2 & 0 & 1 \\ -1 & k & 1 & -1 \\ -1 & 1 & 0 & 1 \end{vmatrix} \xrightarrow[\substack{L_3+L_1 \\ L_4+L_1}]{L_1 \leftrightarrow L_2} - \begin{vmatrix} 1 & -2 & 0 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & k-1 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} = -1 \times \begin{vmatrix} -1 & -1 & 2 \\ k-1 & 1 & 0 \\ -1 & 0 & 2 \end{vmatrix} = -1 \times \left(-1 \times \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} + 2 \times \begin{vmatrix} -1 & -1 \\ k-1 & 1 \end{vmatrix} \right) =$

$= -1 \times (-1 \times (-2) + 2 \times (-1 + k - 1)) =$

$= -1 \times (2 + 2k - 4) =$

$= -2k + 2$

Para serem linearmente dependentes,

$\det(A) = 0 \Leftrightarrow$

$\Leftrightarrow -2k + 2 = 0$

$\Leftrightarrow k = 1$

6. $v = (1, 0, 3)$ $\quad t = (0, 1, 1)$ $\quad w = (1, -1, 2)$

a) $\begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 3 & 2 \end{bmatrix} \xrightarrow[L_3 - 3L_1]{} \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} \xrightarrow[\substack{L_1 - L_2 \\ L_3 + L_2}]{-L_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

car = 2, logo não geram $\mathbb{R}^3$, seria necessário sempre um terceiro vetor. Verdadeiro

b) $v = at + bw$

$(1, 0, 3) = a(0, 1, 1) + b(1, -1, 2)$

$1 = b \wedge 0 = a - b \wedge 3 = a + 2b$

$b = 1 \wedge a = 1 \wedge 3 = 1 + 2.1$ P.V.

Logo, v pode ser obtido como combinação linear dos vetores t e w. Verdadeiro

c) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix} \xrightarrow[L_3 - 3L_1]{} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow[L_3 - L_2]{} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

Nem todos os vetores são linearmente independentes, então não formam uma base. Falso

Grupo III

7. a) $x + 2y - z - w = 0 \wedge y - z - w = 0$

$x = -2y + z + w \wedge y = z + w$

$x = -2z - 2w + z + w \wedge y = z + w$

$x = -z - w \wedge y = z + w$

$(-z - w, \ z + w, \ z, \ w)$

$z(-1, 1, 1, 0) + w(-1, 1, 0, 1)$

$\begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow[\substack{L_2 - L_1 \\ L_3 - L_1}]{-L_1} \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \xrightarrow[\substack{L_1 - L_3 \\ L_4 - L_3}]{-L_3} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Os vetores são linearmente independentes, logo formam base de G.

Base de $G = ((-1, 1, 1, 0), (-1, 1, 0, 1))$

b) $F + G = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -2 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[\substack{L_2 + L_1 \\ L_3 + 2L_1 \\ L_4 - L_1}]{} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow[L_3 + 2L_2]{} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow[\substack{L_1 + L_3 \\ L_4 - L_3}]{-\frac{1}{3}L_3} \begin{bmatrix} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & 2/3 \end{bmatrix}$

$$\xrightarrow[\substack{L_1-\frac{1}{3}L_4 \\ L_3-\frac{4}{3}L_4}]{\frac{3}{2}L_4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Base de $F+G = ((1,-1,-2,1), (0,1,-2,0), (-1,1,1,0), (-1,1,0,1))$

$\dim(F+G) = 4$

$\dim(F+G) = \dim F + \dim G - \dim F\cap G \Leftrightarrow$

$\Leftrightarrow 4 = 2+2-\dim F\cap G$

$\Leftrightarrow \dim F\cap G = 0$

8. a) $u = (x,y,z)$

$\varphi(u) + \varphi(u') = (x+y-z,\ x+3y+z) + (x'+y'-z',\ x'+3y'+z')$

$= ((x+y-z)+(x'+y'-z'),\ (x+3y+z)+(x'+3y'+z'))$

$\varphi(u+u') = ((x+y-z)+(x'+y'-z'),\ (x+3y+z)+(x'+3y'+z'))$

$\varphi(\alpha u) = (\alpha(x+y-z),\ \alpha(x+3y+z))$

$\alpha(\varphi(u)) = \alpha(x+y-z,\ x+3y+z)$

$\varphi(u)+\varphi(u') = \varphi(u+u')$ e $\alpha\varphi(u) = \varphi(\alpha u)$

Logo, φ é aplicação linear.

b) $\text{Nuc } f \Rightarrow \varphi(x,y,z) = 0_E \Leftrightarrow$

$\Leftrightarrow (x+y-z,\ x+3y+z) = (0,0)$

$\Leftrightarrow x+y-z=0 \wedge x+3y+z=0$

Apenas $(0,0,0)$ é solução do sistema.

Então, $\text{Nuc } f = ((0,0,0))$

$\text{Im } f = (x+y-z,\ x+3y+z)$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 3 & 1 \end{bmatrix} \xrightarrow[L_2-L_1]{} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \xrightarrow[L_1-L_2]{\frac{1}{2}L_2} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \end{bmatrix}$$

Base $\text{Im } f = ((1,1), (1,3))$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & \frac{2}{3} \end{bmatrix}$$

9. $M(\varphi, \text{b.c.}\mathbb{R}^3, \text{b.c.}\mathbb{R}^2)$ $\quad A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \end{bmatrix}$

$B_{\mathbb{R}^3} = ((1,2,0), (0,1,0), (0,1,1))$

$B_{\mathbb{R}^2} = ((1,1), (-1,1))$

$$\varphi(1,2,0) = \underbrace{\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \end{bmatrix}}_{2\times 3} \times \underbrace{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}_{3\times 1} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\varphi(0,1,0) = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\varphi(0,1,1) = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$(5,8) = a(1,1) + b(-1,1)$ $\quad a-b=5 \wedge a+b=8 \Leftrightarrow a=\frac{13}{2} \wedge b=\frac{3}{2}$

$(2,3) = a(1,1) + b(-1,1)$ $\quad a-b=2 \wedge a+b=3 \Leftrightarrow a=\frac{5}{2} \wedge b=\frac{1}{2}$

$(1,1) = a(1,1) + b(-1,1)$ $\quad a-b=1 \wedge a+b=1 \Leftrightarrow a=1 \wedge b=0$

$$\mathfrak{M}(e, B, B') = \begin{bmatrix} 13/2 & 5/2 & 1 \\ 3/2 & 1/2 & 0 \end{bmatrix}$$

10. a) $\det(B - \alpha I) = \begin{vmatrix} 2-\alpha & 1 & 1 \\ 1 & 1-\alpha & 0 \\ 1 & 0 & 1-\alpha \end{vmatrix} = (2-\alpha) \times \begin{vmatrix} 1-\alpha & 0 \\ 0 & 1-\alpha \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 0 & 1-\alpha \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 1 \\ 1-\alpha & 0 \end{vmatrix} =$

$= (2-\alpha)(1-\alpha)^2 - (1-\alpha) - (1-\alpha) =$