Quantum chaos and quantum channels

19 de agosto de 2024

1. Ideas

$$\mathcal{E}(\rho_1) = \mathrm{Tr}_E \left(e^{-iHt} \rho_1(0) \otimes \rho_E(0) e^{-iHt} \right) \tag{1}$$

where

$$\rho_1 \otimes \rho_E = |\psi\rangle\langle\psi| \tag{2}$$

where $|\psi\rangle$ is a random pure state of N qubits.

The quantum channel may be written as

$$\mathcal{E}(\rho_1) = \sum_i K_i \rho_1 K_j^\dagger \tag{3}$$

The purity of the chaometer can be written as

$$\mathcal{P}[\mathcal{E}(\rho)] = \mathrm{Tr}\left[\mathcal{E}^2(\rho)\right] \tag{4}$$

$$= \sum_{i,j} \mathrm{Tr}\left(K_i \rho K_i^\dagger K_j \rho K_j^\dagger \right) \tag{5}$$

$$= \sum_{i,j} \mathrm{Tr}\left(K_j^\dagger K_i \rho K_i^\dagger K_j \rho \right). \tag{6}$$

Therefore, I conjecture that the chaotic information of the system has to be encoded in operators $K_i^\dagger K_j$. To further explore this idea I will explore the relationship $\mathcal{P}[\mathcal{E}(\rho)] = \eta$.