Quantum chaos meets quantum channels

September 10, 2024

1 Model

The spin chain we are interested in studying first is that studied by Mirkin and Wisniacki in Ref. [1]:

eq:H:wisniacki:ising:chain

$$H = \sum_{i=1}^{L} \left(h_x \sigma_i^x + h_z \sigma_i^z\right) - \sum_{i=1}^{L-1} J_z \sigma_i^z \sigma_{i+1}^z. \tag{1}$$

2 Mean level spacing ratio

The level spacing ratio $\tilde{r}_n$ is defined as:

eq:level:spacing:ratio

$$\tilde{r}_n = \frac{\min(s_n, s_{n-1})}{\max(s_n, s_{n-1})}, \tag{2}$$

where $s_n = E_{n+1} - E_n$. The mean level spacing ratio $\langle \tilde{r}_n \rangle$ is known to attain the value $\langle \tilde{r}_n \rangle \approx 0.5207$ when the level spacing distribution $P(s)$ is Wigner-Dyson and $\langle \tilde{r}_n \rangle \approx 0.386$ when it is Poisson.

3 Spectral form factor

The spectral form factor $K(t)$ is defined as:

eq:sff

$$K(t) = \frac{1}{2^L} \left\langle \left|\mathrm{Tr}\, U(t)\right|^2 \right\rangle = \frac{1}{2^L} \left\langle \sum_{i,j} e^{i(E_i - E_j)t} \right\rangle, \tag{3}$$

where $\langle \cdot \rangle$ denotes the ensemble-average over statistically-similar systems.

4 Chaometer's quantum channel

The reduced dyanmics of the chaometer is described by the quantum channel:

$$\mathcal{E}(\rho) = \mathrm{Tr}_E \left(e^{-iHt} \rho \otimes \left|\psi_0^{(E)}\right\rangle\!\left\langle\psi_0^{(E)}\right| e^{iHt} \right), \tag{4}$$

where H is that of eq. (1), $\left|\psi_0^{(E)}\right\rangle$ the initial state of all spins except the chaometer, and ρ the initial state of the chaometer.

The chaometer's quantum channel $\mathcal{E}$, in general, is divisible into:

1. A unitary operation rotating the Bloch's sphere.
2. A quantum channel that deforms the Bloch's sphere and translates its origin.

Both operations do not commute.

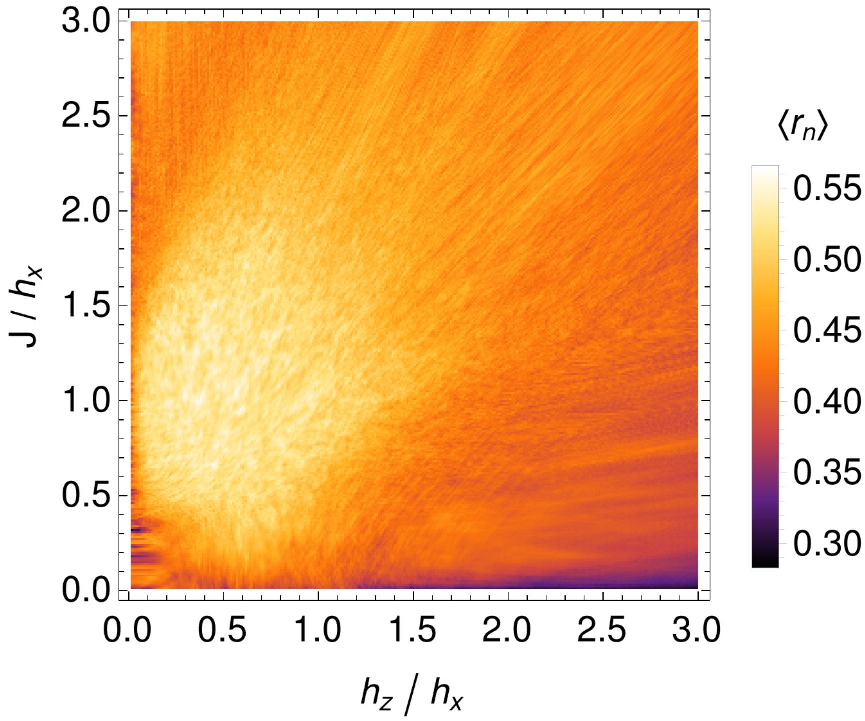

Figure 1: Mean level spacing ratio $\langle \tilde{r}_n \rangle$ [c.f. eq. (2)] of the Ising chain with Hamiltonian (1) as a function of ratios h_z/h_x and J/h_x. We assume $J_z = J \, \forall \, k$.

fig:mean:level:spacing:ratio

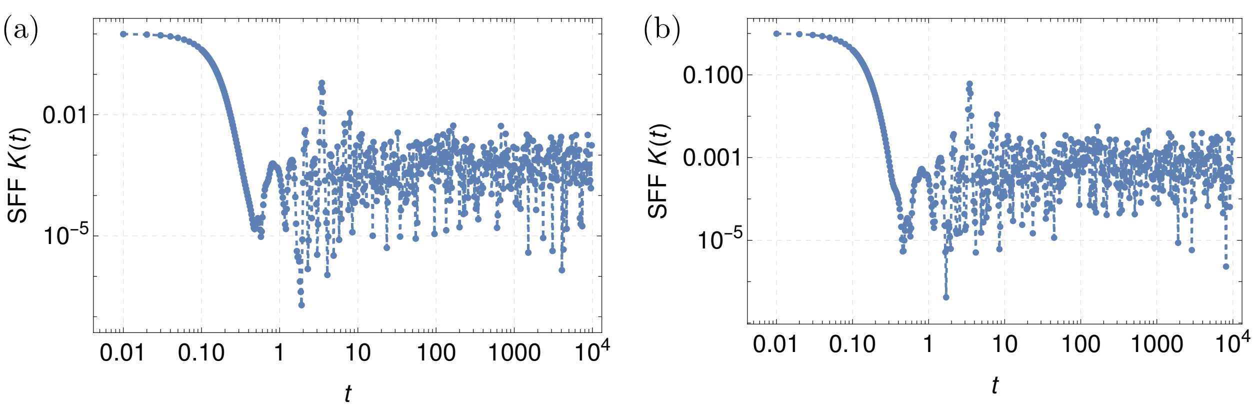

Figure 2: Spectral form factor (SFF) [c.f. (3)] in regular region: $h_z/h_x = 2.5$ and $J/h_x = 1$. **(a)** Whole spectrum. **(b)** Even-parity subspace spectrum.

fig:sff:regular

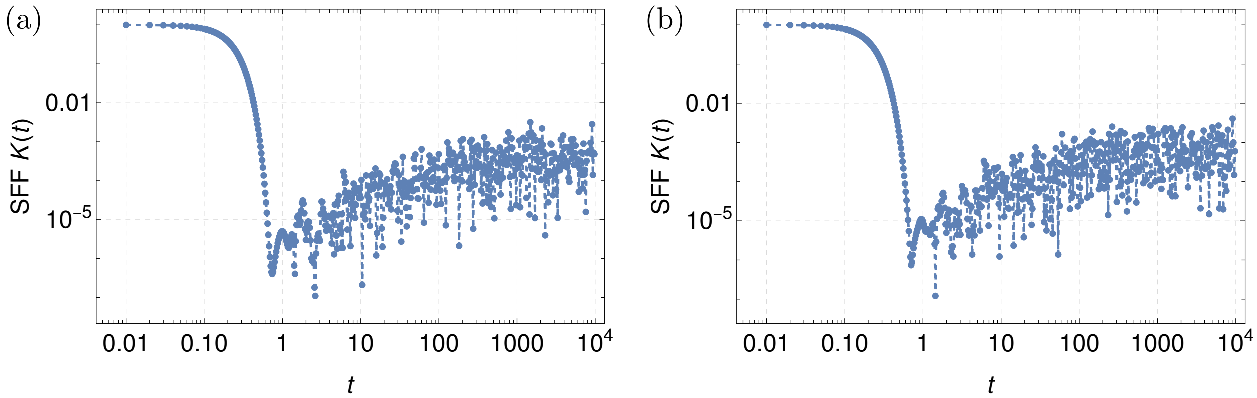

Figure 3: Spectral form factor (SFF) [c.f. (3)] in chaotic region: $h_z/h_x = 0.5$ and $J/h_x = 1$. **(a)** Whole spectrum. **(b)** Even-parity subspace spectrum.

fig:sff:chaotic

5 Purity of the chaometer

Averaged purity $\mathcal{P}$ is defined in Ref. [1] as:

eq:avg:purity

$$\overline{\mathcal{P}} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\int_0^T \mathrm{Tr}\left[\rho_i^2(t)\right]\right) \tag{5}$$

where:

- $\rho_i(t)$: chaometer's density matrix.
- N: number of different random initial states of the whole chain. $N = 50$ in Mirkin and Wisniacki [1].
- T: maximum time. $T = 50$ in Mirkin and Wisniacki [1].

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Also, the normalized averaged purity is defined as

eq:avg:norm:purity

$$\overline{P}_{Norm} = \frac{\overline{P} - \min(\overline{P})}{\max(\overline{P}) - \min(\overline{P})}, \tag{6}$$

where $\max(\overline{P})$ and $\min(\overline{P})$ are the minimum and maximum value obtained when sweeping the parameter range (h_z in their case).

In Fig. 4 we plot one realization of the dynamics of purity of the chaometer. We compare in Fig. 5 our results and those of Mirkin and Wisniacki [1].

6 Purity of Choi matrix

We investigate the purity of Choi matrix of the quantum channel $\mathcal{E}(t)$ of the chaometer in Fig. 6.

$$\mathrm{Tr}\left\{[\mathcal{E}^R(t)/2]^2\right\} \tag{7}$$

JA: Comentar el problema de la normalización de Mirkin y Wisniacki y que parece que se resuelve aquí

7 Non complete positiveness of $\Lambda(t, s)$

Any quantum channel $\mathcal{E}(t)$ can composed as

eq:Lambda

$$\mathcal{E}(t) = \Lambda(t, s) \circ \mathcal{E}(s, 0), \tag{8}$$

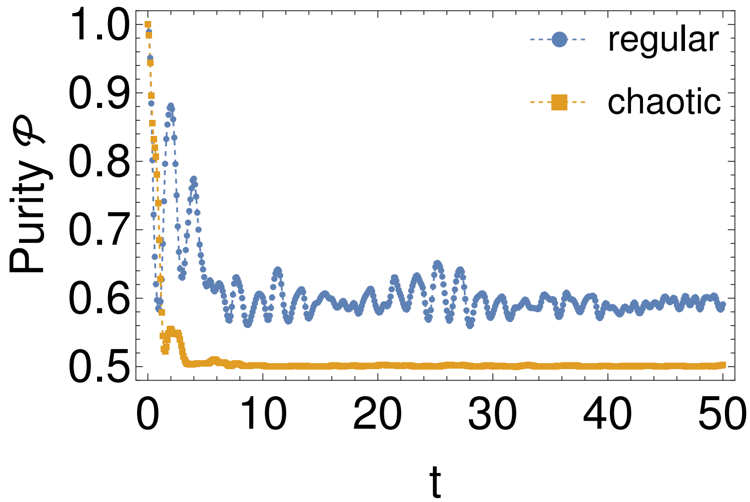

Figure 4: Dynamics of chaometer's purity of a single realization with the same initial state of the environment of the first chaometer's quantum channel of the videos. *fig:purity:one:realization*

nonetheless, $\Lambda(t, s)$ is not in general completely positive. A way to quantify how far is $\Lambda(t, s)$ from being completely positive is through $\tilde{\lambda}$: *eq:lambda:tilde*

$$\tilde{\lambda} = |\min(0, \lambda_{\text{smallest}})|, \tag{9}$$

where $\lambda_{\text{smallest}}$ is the smallest eigenvalue of $\Lambda^R(t, s)/2$. We have added the factor $1/2$ just so $\mathrm{Tr}\left[\Lambda^R(t, s)/2\right] = 1$.

Let us fix $s = 0.1$ and investigate the complete positiviness of $\Lambda(t, s)$, see Fig. 7.

References

[1] Nicolás Mirkin and Diego Wisniacki. Quantum chaos, equilibration, and control in extremely short spin chains. *Phys. Rev. E*, 103:L020201, Feb 2021.

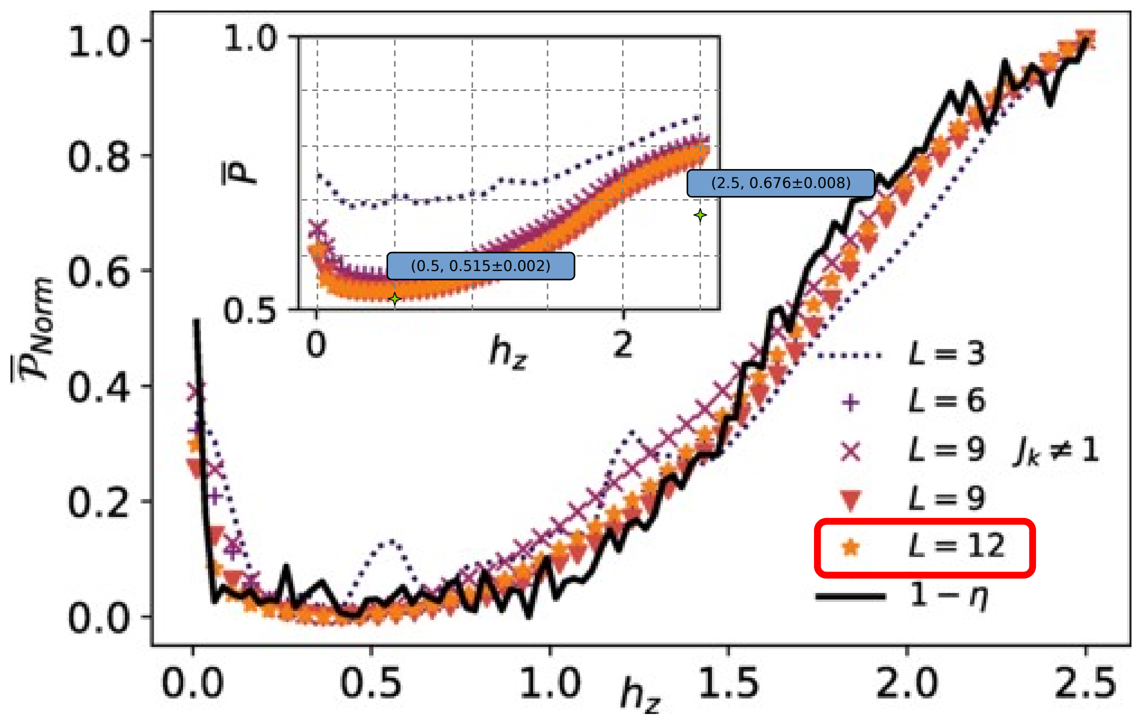

Figure 5: Averaged $\overline{P}$ and averaged normalized purities $\overline{P}_{Norm}$ [c.f. eqs. (5) and (6).] for $N = 50$ random initial states of the chaometer for each quantum channel showed in the videos. JA: Tendría más sentido sacar la pureza del canal. Lo pienso Taken and modified from Ref. [1]. *fig:mirkin2021:fig2:modified*

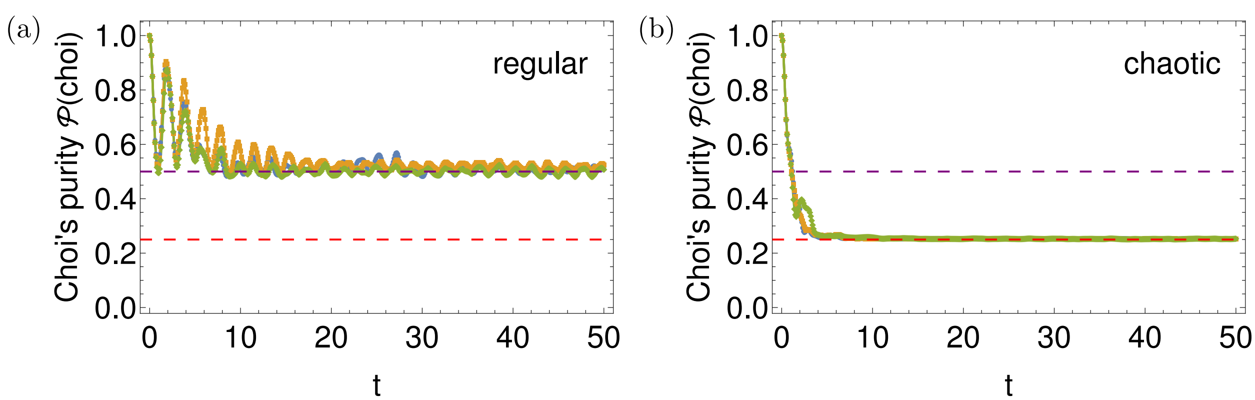

Figure 6: Purity of Choi matrix in (a) regular ($h_z = 2.5$) and (b) chaotic ($h_z = 0.5$) for the three random initial states showed in the video. *fig:choi:purity*

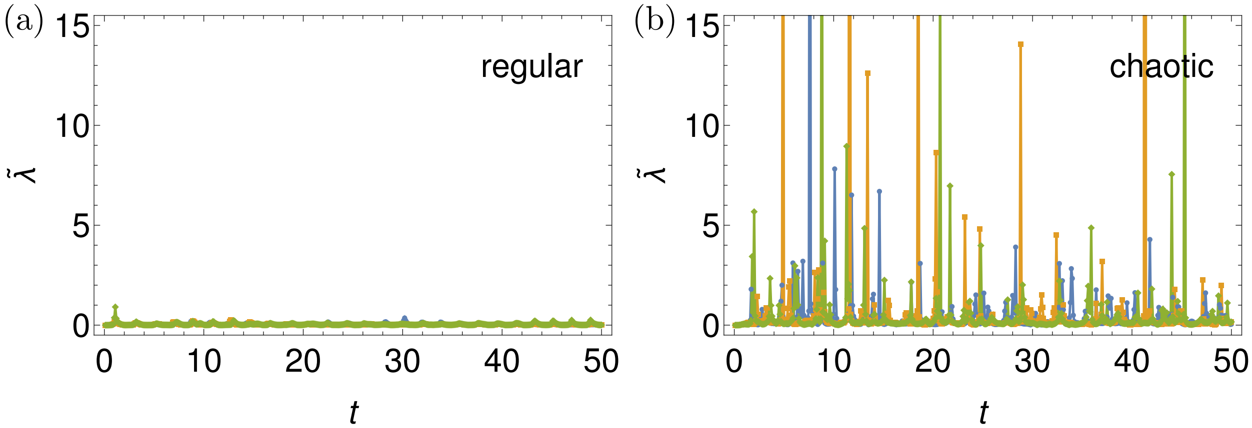

Figure 7: Most negative eigenvalue $\tilde{\lambda}$ of map $\Lambda(t,s)/2$, with $s = 0.1$ [c.f. eqs. (8) and (9)]. *fig:lambda:tilde:choi*

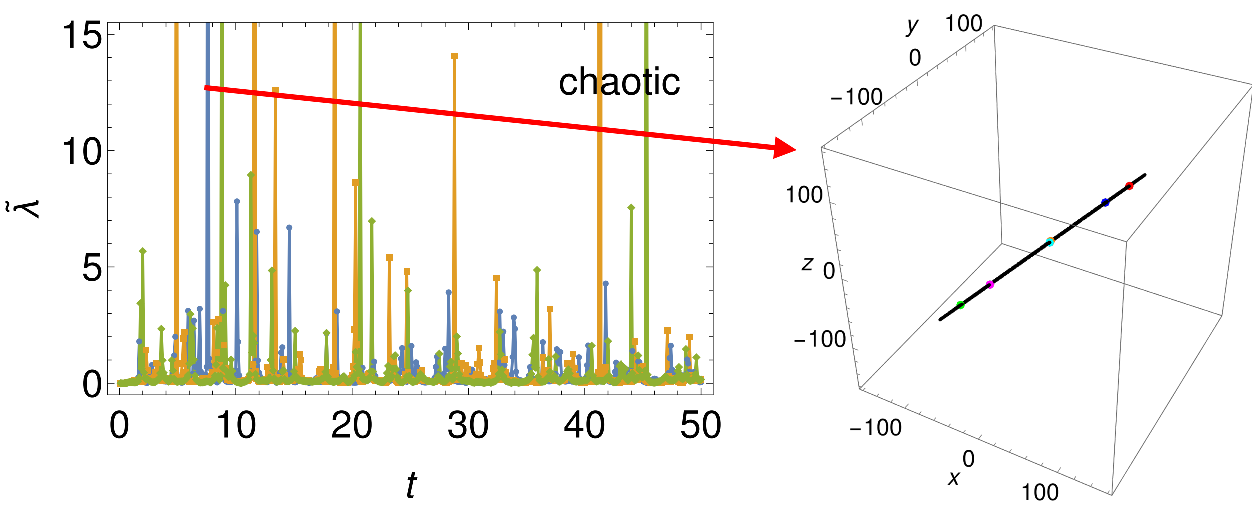

Figure 8: Burst of the Bloch sphere at $t = 0.5$ s. *fig:burst*