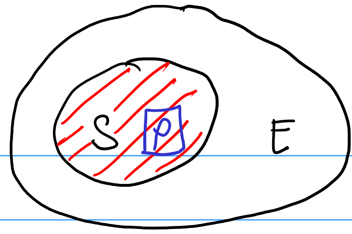
Canales cuánticos

$$A \xrightarrow{\mathcal{E}} B \xrightarrow{\mathcal{E}} C$$



$$\mathcal{E}(\rho) = \mathrm{Tr}_E(U \rho_S \otimes \rho_E U^\dagger) = \underbrace{\sum_j \hat{K}_j \rho \hat{K}_j^\dagger}_{\text{Kraus de } \mathcal{E}} \qquad \hat{K}_j = \langle e_j | U | e_0 \rangle \in \mathcal{B}(\mathcal{H}_S)$$

$\{e_j\}$ base de $\mathcal{H}_E$

$|e_0\rangle$ estado inicial de E

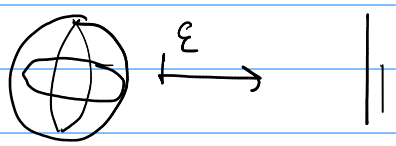
$U \in \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_E)$

$$\Updownarrow$$

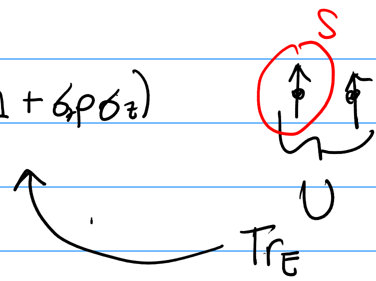
CPTP = canal cuántico $\Longleftrightarrow$ $\mathcal{E}(\rho) = \sum_j K_j \rho K_j^\dagger$

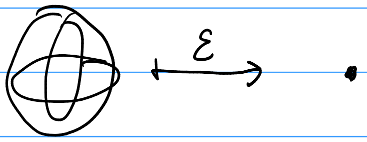
CP: completamente positivo ¿?

TP: preserva la traza ✓ ~ $\langle\psi|\psi\rangle = 1$

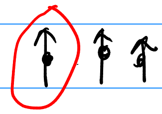


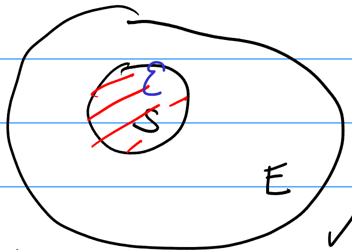
$$\mathcal{E}(\rho) = \frac{1}{2}(\mathbb{1} + \sigma_z \rho \sigma_z)$$





$$\mathcal{E}(\rho) = \frac{1}{4}(\mathbb{1} + \sum_i \sigma_i \rho \sigma_i)$$

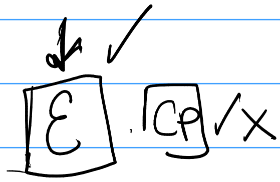




$$\boxed{\rho_{SE} \neq \rho_S \otimes \rho_E}$$

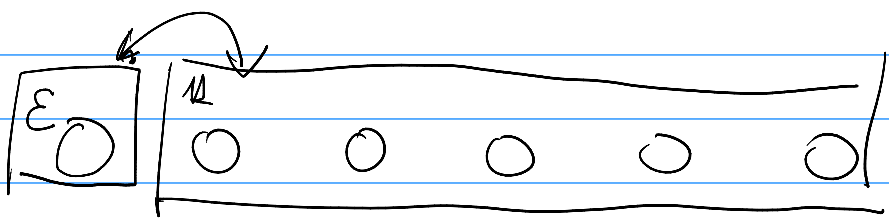
$$(\mathcal{E} \otimes \mathbb{1})(\rho_S \otimes \rho_E) = \mathcal{E}(\rho_S) \otimes \rho_E$$

$$\boxed{(\mathcal{E} \otimes \mathbb{1})\rho_{SE} \geq 0} \rightarrow \text{CP}$$



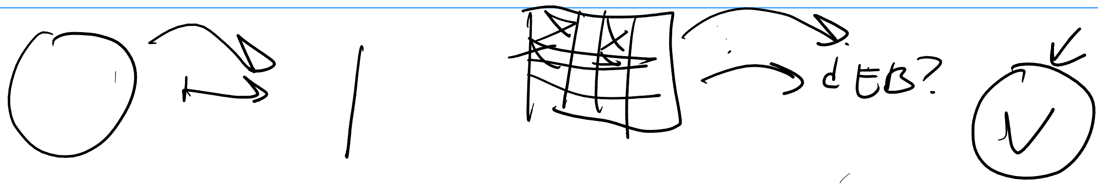
$\dim(S) = d$

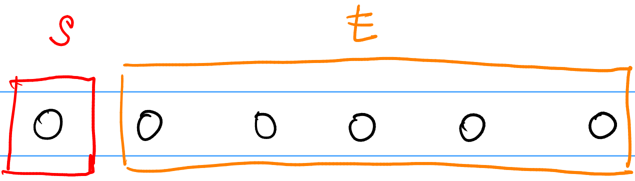
$$\underbrace{(\mathcal{E} \otimes \mathbb{1}_d)|\psi^+\rangle\langle\psi^+| \geq 0}_{\text{Choi}} \Longleftrightarrow \mathcal{E} \text{ es CP} \quad ✓$$



$1/2^N$

$r^{2/2^N!} - 1$





$$H = \sum J \sigma_i^i \sigma_{i+1}^i + \sum h \sigma_i^z$$

$$\to U = e^{-iHt}$$

$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$, $|\psi_0\rangle$: producto aleatorio

$$\mathcal{E}(\rho_S) = \underbrace{Tr_E(U \rho_S \otimes \rho_E U^\dagger)} = \sum_j K_j \rho_S K_j^\dagger, \quad \boxed{K_j = \langle e_j|U|E_0\rangle}$$

$\# K_j = 2^{N_E}$ N_E: # de espines en el entorno

1. Fijamos $|0\rangle\otimes|E_0\rangle$, $|1\rangle\otimes|E_0\rangle$
2. $\mathcal{E}(|0\rangle\langle 0|)$, $\boxed{\mathcal{E}(|0\rangle\langle 1|)}$, $\boxed{\mathcal{E}(|1\rangle\langle 0|)}$, $\mathcal{E}(|1\rangle\langle 1|)$
3. Reconstruir $\mathcal{E}$, $\dim \mathcal{E} = 4\times 4$

$$|0\rangle\langle 0| \mapsto \mathcal{E}(|0\rangle\langle 0|)$$

$$\mathcal{E}\vec{\rho} = \vec{\rho}_f$$

$$\vec{\rho} = \begin{pmatrix}\rho_{00}\\ \rho_{01}\\ \rho_{10}\\ \rho_{11}\end{pmatrix}$$

4. $\to \mathcal{E}(|0\rangle\langle 0|) = Tr_E(e^{-iHt}\, |0\rangle\langle 0| \otimes |E_0\rangle\langle E_0|\, e^{iHt})$

$|0\rangle\langle 0| = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$ $\rho_f = \begin{pmatrix}\cdot & \cdot\\ \cdot & \cdot\end{pmatrix}$

$(\mathcal{E})\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \xmapsto{\mathcal{E}} \begin{pmatrix}\cdot\\ \vdots\end{pmatrix}$ $(0\,0\,0\,1)\begin{pmatrix}\vdots\end{pmatrix}$ $\mathcal{E} = (\cdot \ \cdot \ \cdot \ \cdot$

$\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \mapsto \begin{pmatrix}*\\ *\\ *\\ *\end{pmatrix}$ $\mathcal{E} \mapsto D \mapsto K_j$