Quantum chaos through the lens of quantum channels

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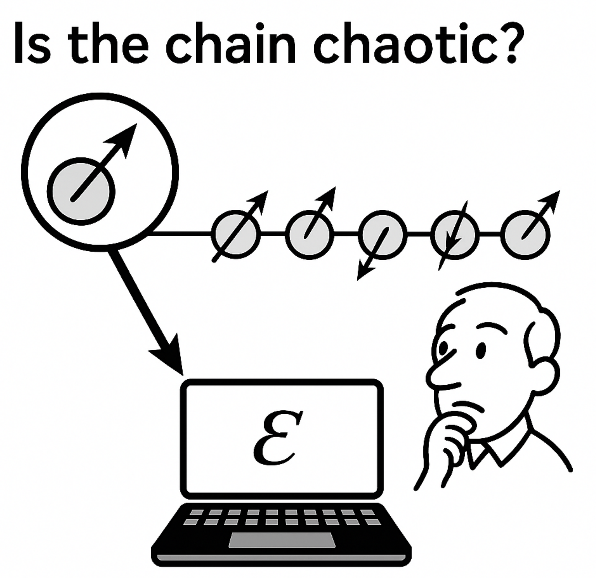
Abstract

Characterizing quantum chaos in many-body systems through standard indicators remains experimentally challenging, as these typically require full-system measurements. We address this limitation by investigating the quantum channel describing a subsystem's reduced dynamics as a diagnostic tool. Specifically, we ask:

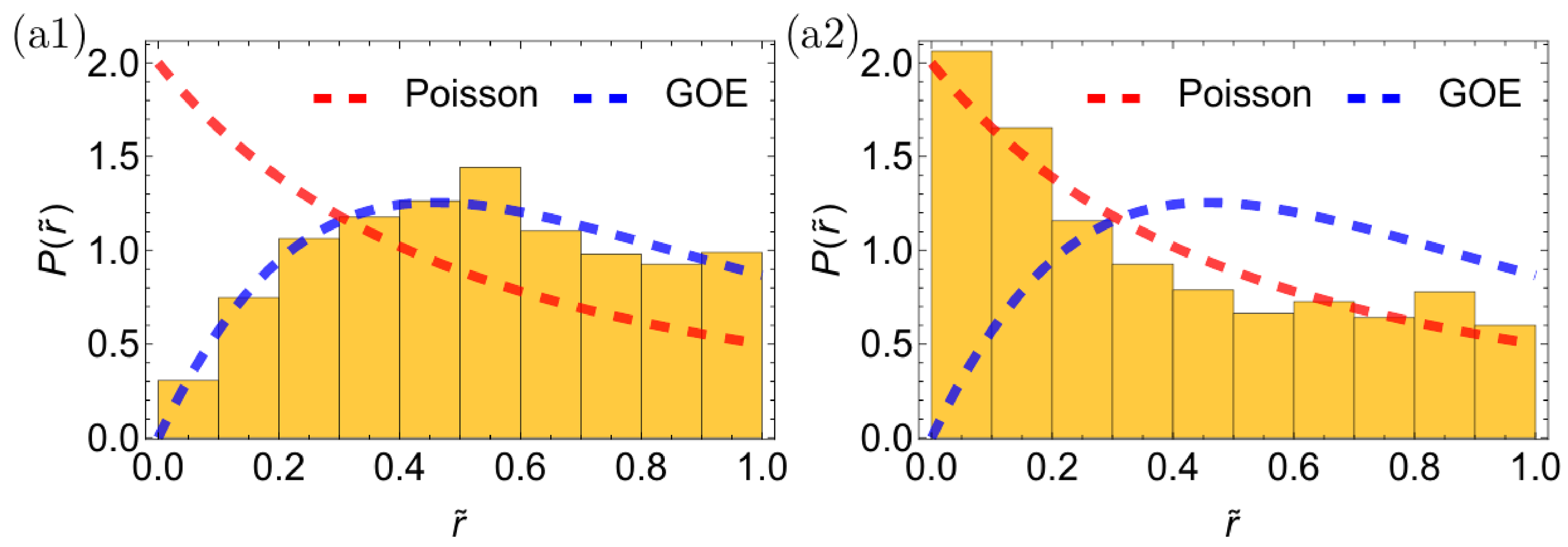


Can the Choi-Jamiołkowski purity serve as an indicator of quantum chaos in many-body systems? We show that it not only can detect chaos-integrability transitions in spin chains but also outperforms a recently proposed signature.

Quantum chaos

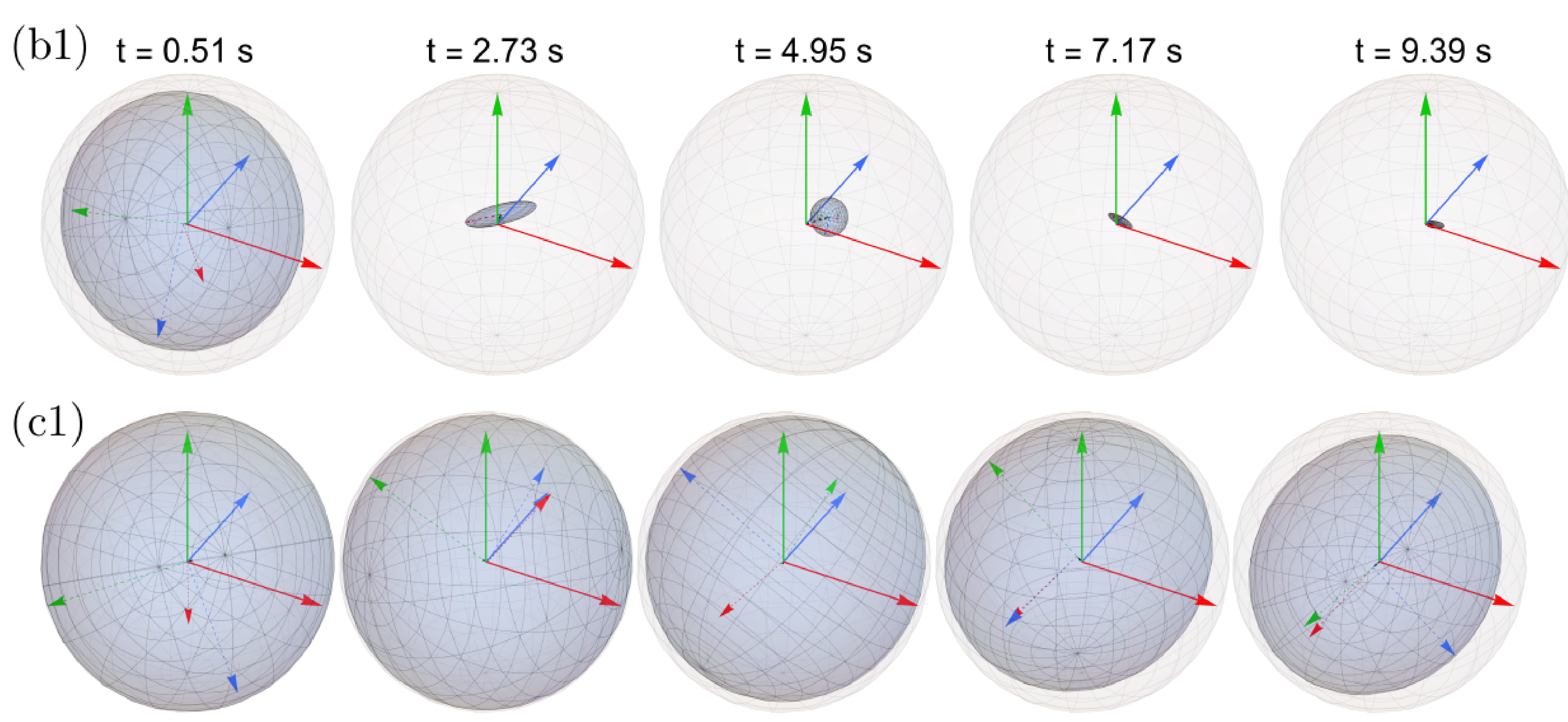
Tipically, quantum chaos is diagnosed via spectral statistics of the energy spectrum E_n,

$$\tilde{r}_n = \min\left(r_n, \frac{1}{r_n}\right), \quad r_n = \frac{E_{n+1} - E_n}{E_n - E_{n-1}}. \tag{1}$$



Quantum channels

Quantum channels are completely positive and trace-preserving maps. System-environment representation: $\mathcal{E}(\rho) = \mathrm{Tr}_E\left(e^{-iHt}\rho \otimes |\psi_E\rangle\langle\psi_E|\, e^{iHt}\right)$.



Choi-Jamiolkowsky (CJ) matrix $\mathcal{D}$

If $\mathcal{E}$ acts over a d-dimensional system, then:

$$\mathcal{D} = (\mathcal{E} \otimes \mathbb{1}_d)[|\mathrm{Bell}\rangle\langle\mathrm{Bell}|]. \tag{2}$$

Let λ_i and v_i be the eigenvalues and eigenvectors reshaped as matrices of $\mathcal{D}$, then:

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger, \quad K_i = \sqrt{\lambda_i} v_i \tag{3}$$

The purity of $\mathcal{D}$ has been termed the *unitarity* of $\mathcal{E}$, and $\frac{1}{d^2} \leq \mathrm{Tr}(\mathcal{D}^2) = \sum_i \lambda_i^2 \leq 1$.

Operational interpretation of $\mathrm{Tr}(\mathcal{D}^2)$

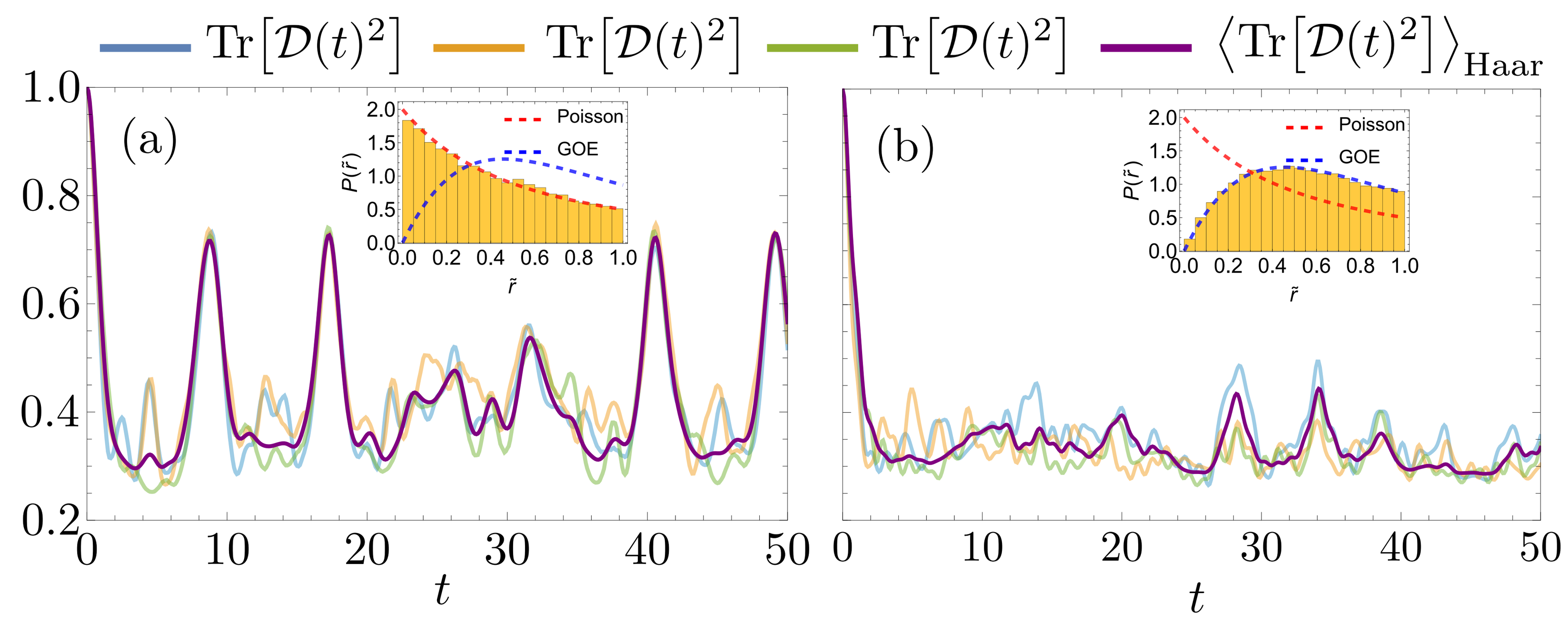
We show that $\mathrm{Tr}(\mathcal{D}^2)$ can be expressed as:

$$\mathrm{Tr}(\mathcal{D}^2) = \left\langle \psi \middle| \mathrm{Tr}_S\left\{ U^\dagger \Lambda\left[U \left(\frac{\mathbb{1}_2}{2} \otimes |\psi\rangle\langle\psi|\right) U^\dagger \right] U \right\} \middle| \psi \right\rangle. \tag{4}$$

Thus, it can interpreted as the probability of recovering the environment in its initial state after:

1. evolving forward the joint system, initially in the state $\mathbb{1}_S/2 \otimes |\psi\rangle\langle\psi|$, with U
2. applying the channel Λ
3. evolving backward with $U^\dagger$.

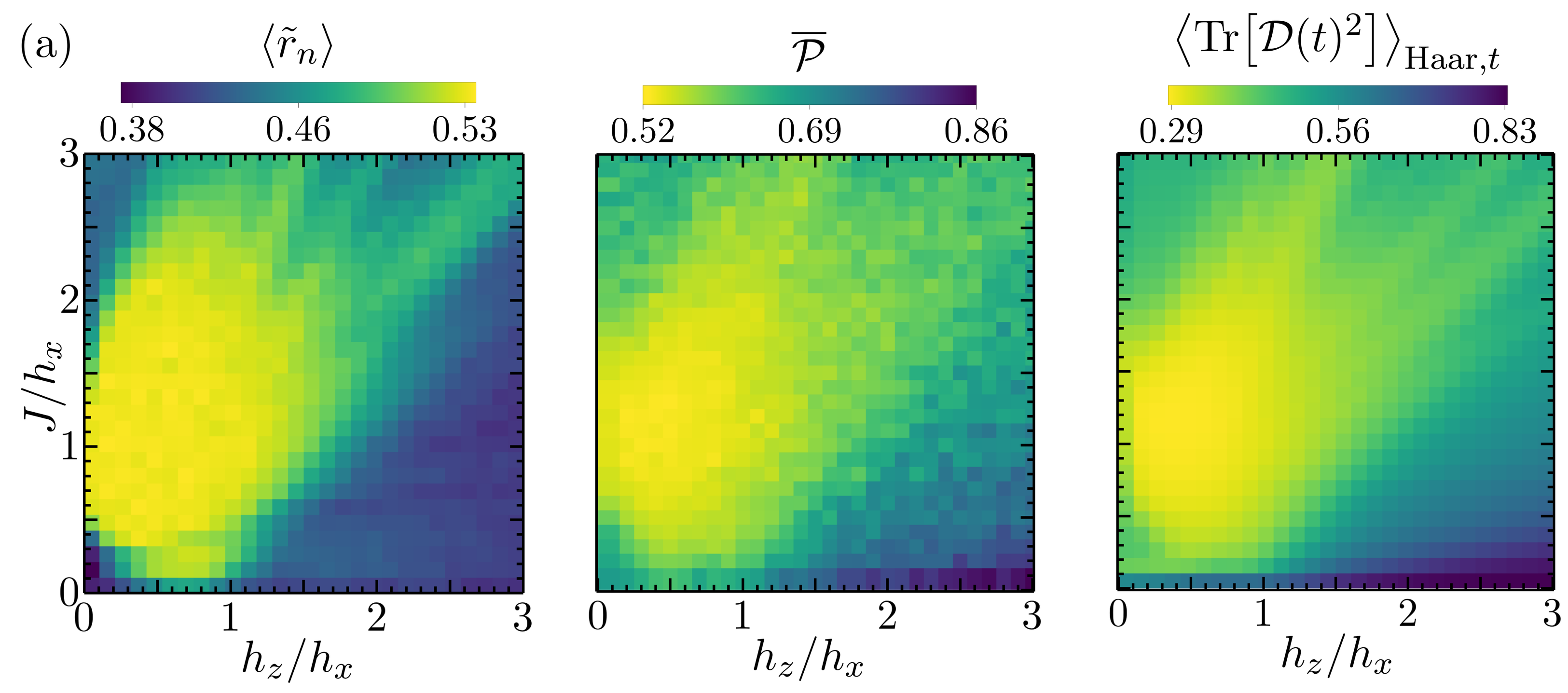
CJ purity as quantum chaos indicator

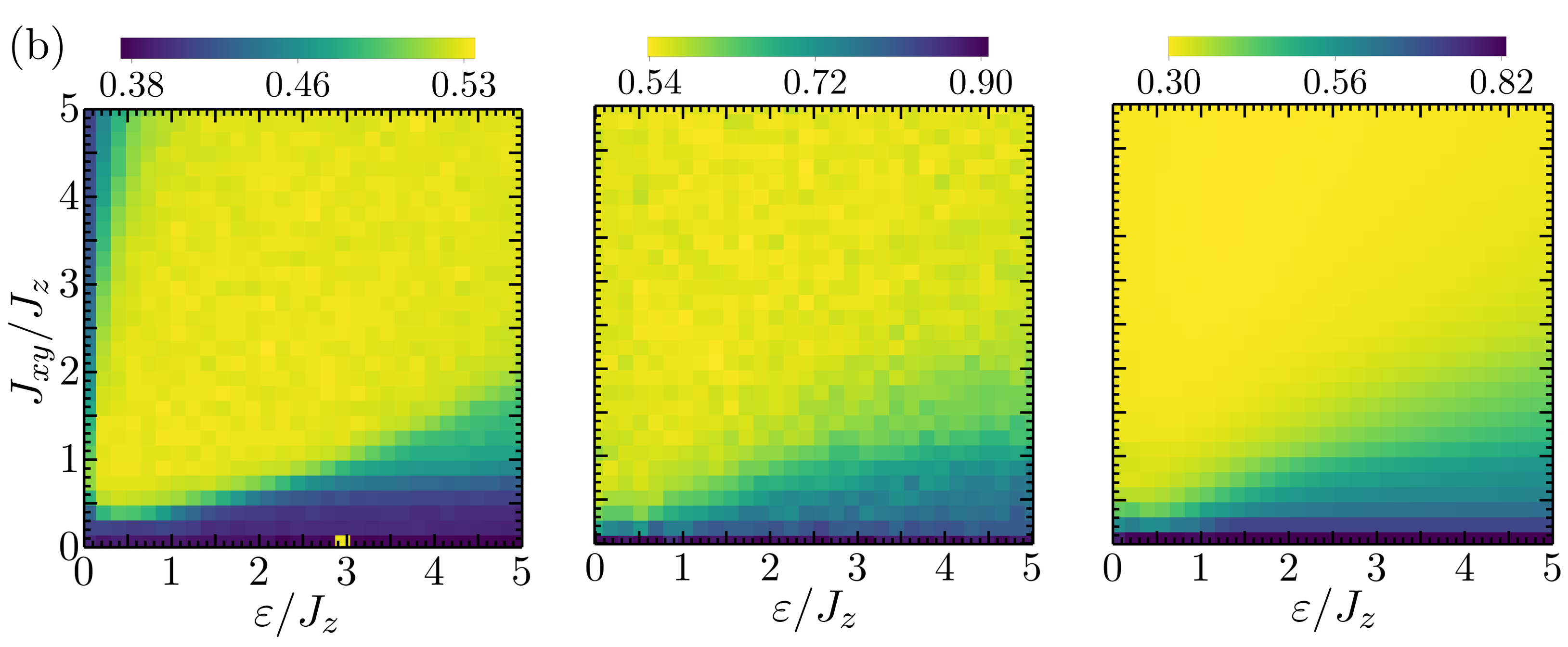


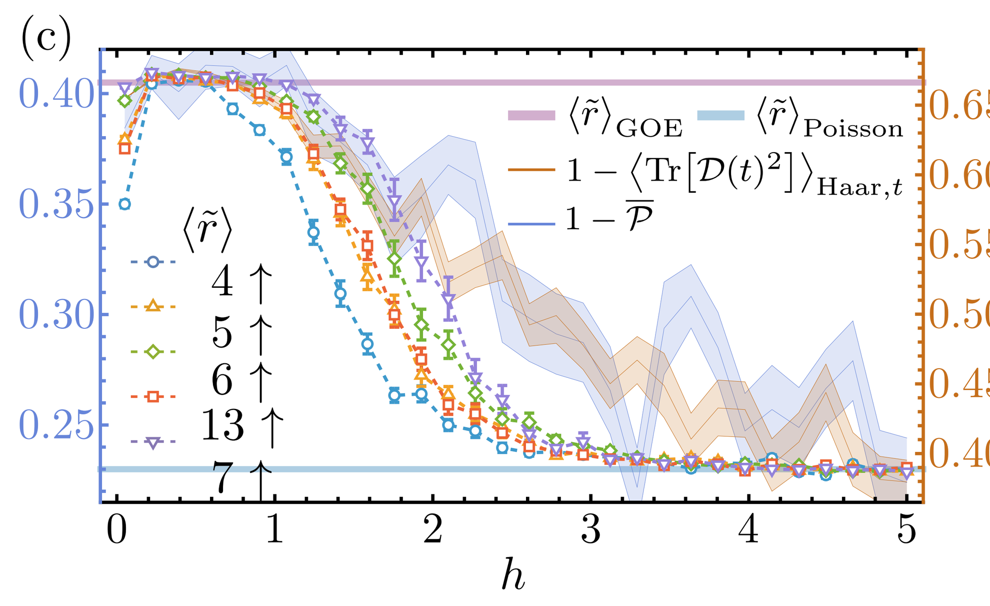
(a) $H = \sum_{i=1}^{L} (h_x \sigma_i^x + h_z \sigma_i^z) - J \sum_{i=1}^{L-1} \sigma_i^z \sigma_{i+1}^z$

(b) $H = \frac{1}{4} \sum_{i=1}^{L-1} \left[J_{xy}(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y) + J_z \sigma_i^z \sigma_{i+1}^z \right] + \frac{1}{2} \varepsilon \sigma_d^z$

(c) $H = \frac{1}{4} \sum_{i=1}^{L-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z) + \frac{1}{2} \sum_{i=1}^{L} h_i \sigma_i^z$







Support from project UNAM-PAPIIT IG101324