Quantum chaos meets quantum channels

September 6, 2024

1 Model

The spin chain we are interested in studying first is that studied by Mirkin and Wisniacki in Ref. [1]:

eq:H:wisniacki:ising:chain

$$H = \sum_{i=1}^{L} (h_x \sigma_i^x + h_z \sigma_i^z) - \sum_{i=1}^{L-1} J_z \sigma_i^z \sigma_{i+1}^z. \tag{1}$$

2 Mean level spacing ratio

The level spacing ratio $\tilde{r}_n$ is defined as:

eq:level:spacing:ratio

$$\tilde{r}_n = \frac{\min(s_n, s_{n-1})}{\max(s_n, s_{n-1})}, \tag{2}$$

where $s_n = E_{n+1} - E_n$. The mean level spacing ratio $\langle \tilde{r}_n \rangle$ is known to attain the value $\langle \tilde{r}_n \rangle \approx 0.5207$ when the level spacing distribution $P(s)$ is Wigner-Dyson and $\langle \tilde{r}_n \rangle \approx 0.386$ when it is Poisson.

3 Spectral form factor

The spectral form factor $K(t)$ is defined as:

eq:sff

$$K(t) = \frac{1}{2^L} \left\langle \left| \operatorname{Tr} U(t) \right|^2 \right\rangle = \frac{1}{2^L} \left\langle \sum_{i,j} e^{i(E_i - E_j)t} \right\rangle, \tag{3}$$

where $\langle \cdot \rangle$ denotes the ensemble-average over statistically-similar systems.

4 Chaometer's quantum channel

The reduced dyanmics of the chaometer is described by the quantum channel:

$$\mathcal{E}(\rho) = \operatorname{Tr}_E \left(e^{-iHt} \rho \otimes \left| \psi_0^{(E)} \right\rangle \!\! \left\langle \psi_0^{(E)} \right| e^{iHt} \right), \tag{4}$$

where H is that of eq. (1), $\left| \psi_0^{(E)} \right\rangle$ the initial state of all spins except the chaometer, and ρ the initial state of the chaometer.

The chaometer's quantum channel $\mathcal{E}$, in general, is divisible into:

1. A unitary operation rotating the Bloch's sphere.
2. A quantum channel that deforms the Bloch's sphere and translates its origin.

Both operations do not commute.

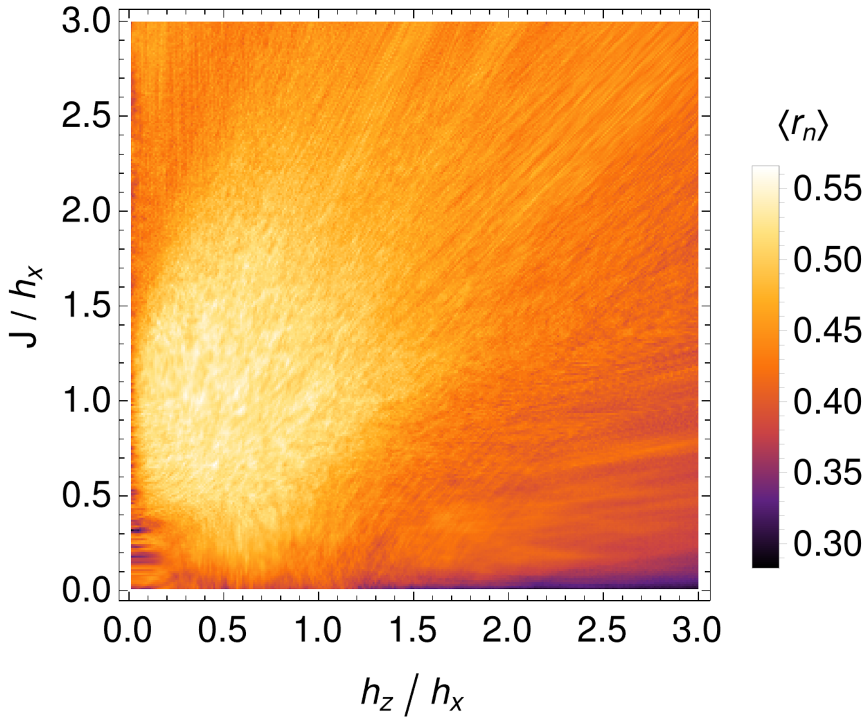

Figure 1: Mean level spacing ratio $\langle \tilde{r}_n \rangle$ [c.f. eq. (2)] of the Ising chain with Hamiltonian (1) as a function of ratios h_z/h_x and J/h_x. We assume $J_z = J \,\forall\, k$.

fig:mean:level:spacing:ratio

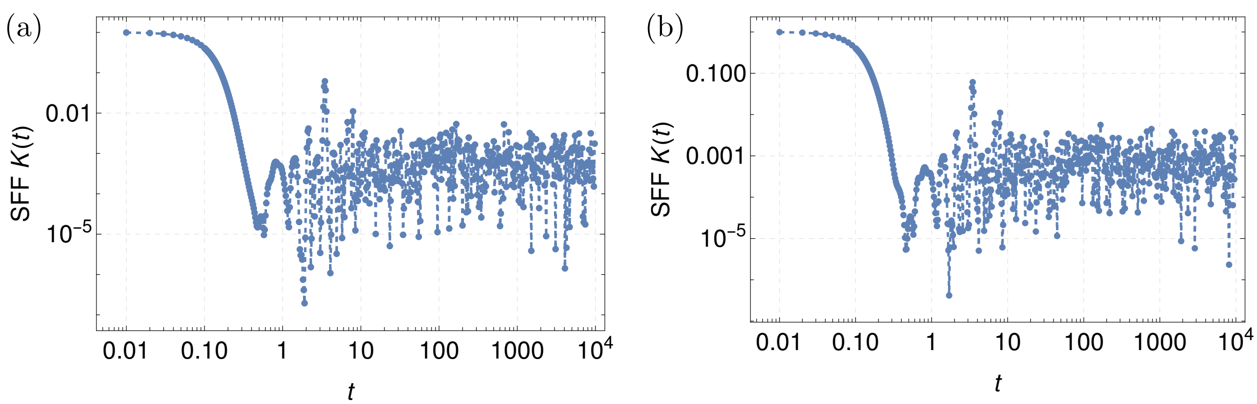

Figure 2: Spectral form factor (SFF) [c.f. (3)] in regular region: $h_z/h_x = 2.5$ and $J/h_x = 1$. **(a)** Whole spectrum. **(b)** Even-parity subspace spectrum.

fig:sff:regular

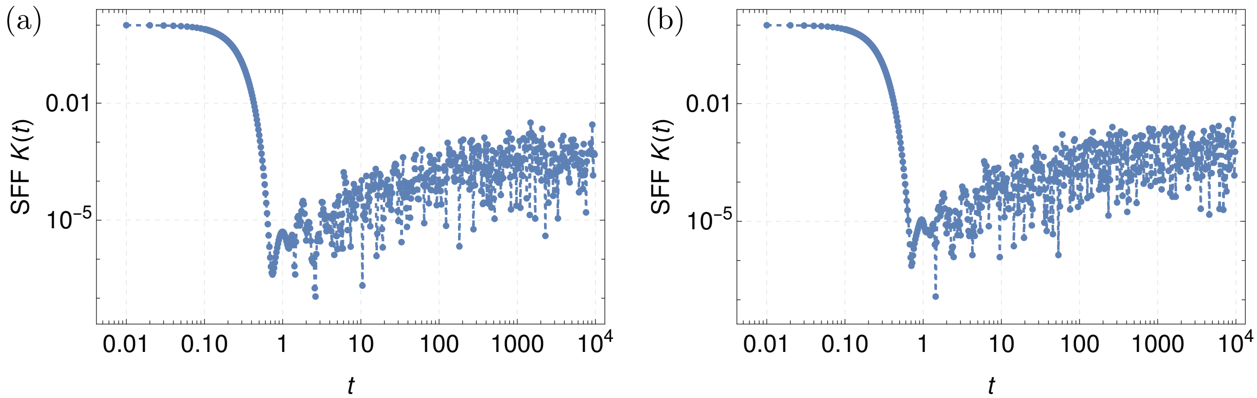

Figure 3: Spectral form factor (SFF) [c.f. (3)] in chaotic region: $h_z/h_x = 0.5$ and $J/h_x = 1$. **(a)** Whole spectrum. **(b)** Even-parity subspace spectrum.

fig:sff:chaotic

References

[1] Nicolás Mirkin and Diego Wisniacki. Quantum chaos, equilibration, and control in extremely short spin chains. *Phys. Rev. E*, 103:L020201, Feb 2021.