Quantum chaos and quantum channels

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1. Ideas

The evolution of the chaometer qubit can be understood as a quantum channel:

$$\rho_1(t) = \mathcal{E}[\rho_1(0)] = \mathrm{Tr}_E\left(e^{-iHt}\rho_1(0)\otimes\rho_E(0)e^{-iHt}\right), \tag{1}$$

where $\rho_1(0)\otimes\rho_E(0) = |\psi(0)\rangle\langle\psi(0)|$, with $|\psi(0)\rangle$ is a L-qubit random product state; and H the spin chain Hamiltonian.

The quantum channel $\mathcal{E}$ can be written in its Kraus form:

$$\mathcal{E}(\rho_1) = \sum_{i=1}^{r\leq 4} K_i \rho_1 K_j^\dagger, \tag{2}$$

with r is the Kraus rank of $\mathcal{E}$.

The purity $\mathcal{P}$ of the chaometer now reads:

$$\begin{aligned}
\mathcal{P}[\mathcal{E}(\rho_1)] &= \mathrm{Tr}\left\{[\mathcal{E}(\rho_1)]^2\right\} && (3)\\
&= \sum_{i,j=1}^{r\leq 4} \mathrm{Tr}\left(K_i\rho K_i^\dagger K_j \rho K_j^\dagger\right) && (4)\\
&= \sum_{i,j}^{r\leq 4} \mathrm{Tr}\left(K_j^\dagger K_i \rho K_i^\dagger K_j \rho\right). && (5)
\end{aligned}$$

My guess is that the chaotic information of the system has to be encoded in operators $K_i^\dagger K_j$. To further explore this idea I will examine the relationship $\mathcal{P}[\mathcal{E}(\rho_1)] = 1-\eta$.

Integrable:

$$\mathcal{P}[\mathcal{E}(\rho_1)] = \mathrm{mín}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\int_0^T \mathrm{Tr}\big[\rho_i^2(t)\big]\,dt\right)\right] \approx 1 \tag{6}$$

Chaotic:

$$\mathcal{P}[\mathcal{E}(\rho_1)] = \mathrm{máx}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\int_0^T \mathrm{Tr}\big[\rho_i^2(t)\big]\,dt\right)\right] \approx \frac{1}{2} \tag{7}$$