Queueing Simulation (Assignment 3) - GROUP 15

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1 Recapitulation of the model in assignment S-2, now with a battery

a) To simulate the given scenario and determine the minimal battery capacities for the provided alpha values, we have come up with the following solution in Python (python files also provided in the submission):

```
import numpy as np
2 import simpy
3 import matplotlib.pyplot as plt
4 import copy
5 import random
7 import numpy as np
  def generate_exponential_times(starttime, endtime, rate,
     num_samples):
10
11
      values = np.random.exponential(scale=1/rate, size=
     num_samples)
13
      waardes = list(values)
14
      times = []
16
18
      current = 0
      for i in range(0, len(waardes)):
19
          temp = current + waardes[i]
20
          times.append(temp)
21
          current = temp
22
23
24
      times = list(times)
26
      if not all(starttime <= time <= endtime for time in times):</pre>
27
          return generate_exponential_times(starttime, endtime,
     rate, num_samples)
      return times
29
  class Gillespie:
31
32
      def __init__(self, p01, p10):
33
          self.p01 = p01
34
          self.p10 = p10
```

```
def calculate_total_rate(self):
37
           return self.p01 + self.p10
38
39
      def Generate(self):
40
          rando = random.uniform(0, 1)
41
42
          if (rando < self.p01/self.calculate_total_rate()):</pre>
43
               return 1
44
           else:
45
              return 0
46
47
48 class Singleton:
49
      def __init__(self):
50
           self.st = None
51
52
      def singleton(self, list):
53
54
          if self.st == None:
               self.st = list
55
               return self.st
56
57
           else:
               return self.st
59
60 class Process:
61
      def __init__(self, alphas, gamma, 1, p, e, gp = Gillespie
62
      (0.5, 0.5):
          self.alphas = alphas
63
           self.gamma = gamma
64
           self.p = p
65
           self.e = e
66
           self.gp = gp
67
           self.l = 1
           self.total_energy_acquired = 1
69
          self.total_energy_bought = 1
70
          self.B = 0.1
71
           self.results = []
72
           self.amount_of_times = 0
73
           self.output = None
74
          self.jobs = []
75
           self.done = False
76
77
      def getOutput(self):
78
79
          print("
80
          print("Simulation Successfull, running the simulation {
81
      self.amount_of_times} the values are as follows:")
          for alpha in self.alphas:
82
               print(f"Alpha: {alpha}, Average Min Battery
83
      Capacity: {self.results[self.alphas.index(alpha)] / self.
      amount_of_times}")
84
      def simulate(self, max_capacity, lambda_rate, min_capacity
85
     = 1000, amount_of_times = 1, distr_value = 200,
      capacity_increment = 10, time = 100000):
86
           results = []
87
           self.amount_of_times = amount_of_times
88
          for alpha in self.alphas:
```

```
results.append(max_capacity)
90
91
           # Run the simulations amount_of_times to get a more
92
      average result
           for i in range(1, amount_of_times+1):
93
                arrivals = np.random.poisson(lam=lambda_rate)
95
96
                arrival_times = generate_exponential_times(0, time,
97
       self.1, arrivals)
98
                jobs = []
99
                original_wh = []
100
                for j in range(0, len(arrival_times)):
                    job = []
                    tau_k = np.random.exponential(distr_value)
104
105
                    W_k_new = np.random.exponential(distr_value)
                    Wh = W_k_new*tau_k
106
                    original_wh.append(Wh)
107
                    job = [arrival_times[j], tau_k, Wh]
108
109
                    jobs.append(job)
110
                self.jobs = tuple(jobs)
111
112
113
                self.output = []
114
                for i in range(0, len(self.alphas)):
115
                    self.output.append(max_capacity)
116
117
                single = Singleton()
118
119
                # For every alpha value do the simulation
120
                for alpha in self.alphas:
121
122
                    # Find a capacity by iterating over possible
123
      values for it
                    for capacity in range(max_capacity,
      min_capacity, -capacity_increment):
                        taak = single.singleton(copy.deepcopy(jobs)
126
      )
127
                        self.done = False
129
130
                        env = simpy.Environment()
132
                        # Run the simulation
133
                        env.process(self.energy_source_process(env,
134
       alpha, self.gamma, self.p, self.e, taak, capacity, self.
      done))
135
                        # Adjust simulation time accordingly
136
                        env.run(until=time)
137
138
                        if (self.done == True):
139
                             #results[self.alphas.index(alpha)] +=
140
      capacity
```

```
if (self.output[self.alphas.index(alpha
141
      )] > capacity):
                                self.output[self.alphas.index(alpha
142
      )] = capacity
                                print(f"using alpha: {alpha}, job {
143
      self.alphas.index(alpha)} successful with current min: {self
      .output[self.alphas.index(alpha)]}")
                       else:
144
                           print(f"using alpha: {alpha}, job {self
145
      .alphas.index(alpha)} failed when attempting capacity: {
      capacity}, min is {self.output[self.alphas.index(alpha)]}")
                           print("Final", sorted(taak, key=lambda
146
      x: x[2], reverse=True))
                           break
           print("
148
           -----")
          print(f"Simulation Successfull, running the simulation {
      self.amount_of_times} times, the values are as follows:")
           for alpha in self.alphas:
               print(f"Alpha: {alpha}, Average Min Battery
      Capacity: {self.output[self.alphas.index(alpha)] / self.
      amount_of_times}")
153
      def Execute(self, state, job, capacity, alpha, gamma, p, e)
           res = None
156
           finished = False
           left = 0
158
159
           if state == 1:
160
                   total_source = np.random.uniform(0.01, 2)
                   self.total_energy_acquired += total_source
162
                   # Consume energy from the intermittent source
163
                   if job[2] - total_source < 0: # Source is</pre>
164
      enough to satisfy the job
                       remaining_source = total_source - job[2]
165
                       job[2] = 0
166
167
                       if self.B + remaining_source > capacity: #
      Excess source cannot be stored
                           self.B = capacity
168
                           finished = True
169
                           return res, finished, left
170
171
                            self.B += e * remaining_source
172
                           finished = True
173
                           return res, finished, left
174
                   else: # Source is not enough, we need to check
175
      the battery
                       job[2] = job[2] - total_source
176
                       if self.B - job[2] > 0: # Battery is enough
                            self.B = self.B - job[2]
178
                            job[2] = 0
179
                           finished = True
180
                           res = job[1]
                           return res, finished, left
182
                       else: # Battery is not enough
183
                            job[2] = job[2] - self.B
184
                            self.B = 0
```

```
left = job[2]
186
                             res = job[1]
187
                             return res, finished, left
188
                # Check if the energy source is ON
189
190
           else:
                # Energy source is OFF
191
                # Determine whether to reschedule or buy external
192
      energy
                if self.B - job[2] > 0: # Battery is enough
193
                         self.B = self.B - job[2]
194
                         job[2] = 0
195
                         finished = True
196
                         return res, finished, left
197
                else: # Battery is not enough
                    job[2] = job[2] - self.B
199
                    self.B = 0
200
                    left = job[2]
201
                    if ((job[2] + self.total_energy_bought) / self.
202
      total_energy_acquired <= alpha): #Buy external energy
                         cost = p * job[2]
203
                         self.total_energy_acquired += job[2]
204
205
                         self.total_energy_bought += job[2]
                         job[2] = 0
206
                         finished = True
207
                         return res, finished, left
209
                    else:
                         res = gamma
                         return res, finished, left
211
212
213
214
       def energy_source_process(self, env, alpha, gamma, p, e,
215
      tasks, capacity, done):
216
           queue = []
217
           self.total_energy_bought = 1
218
            self.total_energy_acquired = 1
220
           jobs = copy.deepcopy(tasks)
221
222
           while len(jobs) > 0 or len(queue) > 0:
223
224
225
                if (len(queue) > 0):
227
                    job = queue[0]
229
230
                    yield env.timeout(job[1])
231
                    state = self.gp.Generate()
232
                    logic, finished, left = self.Execute(state, job
233
      , capacity, alpha, gamma, p, e)
234
                    if finished:
235
                         queue.pop(0)
236
                    elif (logic == gamma):
237
238
                         rs_time = np.random.exponential(gamma)
                         new_job = [rs_time+job[0], job[1], left]
239
                         queue.pop(0)
240
                         if (rs_time+job[0] > env.now):
```

```
queue.append(new_job)
242
                          else:
243
244
                              jobs.append(new_job)
                              jobs = sorted(jobs, key=lambda x: x[0])
245
                     elif (logic != None):
246
                          yield env.timeout(logic)
247
                     else:
248
                          continue
249
                 else:
250
                     #wait untill time of execution
251
                     if (len(jobs) > 1):
252
                          for j in range(1, len(jobs)):
253
                              if (j \ge len(jobs)):
                                   break
                              if (jobs[j][0] < jobs[0][0]):</pre>
256
                                   queue.append(jobs[j])
257
258
                                   jobs.pop(j)
259
                              else:
                                   break
260
261
                     yield env.timeout(jobs[0][0])
263
                     if (len(jobs) > 1):
264
                          if (j \ge len(jobs)):
265
                              break
                          for j in range(1, len(jobs)):
267
                              if (j \ge len(jobs)):
268
                                   break
269
                              if (jobs[j][0] < jobs[0][0]):</pre>
                                   queue.append(jobs[j])
271
                                   jobs.pop(j)
272
                              else:
273
                                   break
274
                     yield env.timeout(jobs[0][1])
275
276
                     #execute the job logic
278
                     state = self.gp.Generate()
                     logic, finished, left = self.Execute(state,
279
       jobs[0] , capacity, alpha, gamma, p, e)
280
                     if (finished):
281
                          jobs.pop(0)
282
                     elif (logic == gamma):
283
                          rs_time = np.random.exponential(gamma)
                          new_job = [rs_time+jobs[0][0], jobs[0][1],
285
      left]
                          jobs.pop(0)
286
                          jobs.append(new_job)
287
                          jobs = sorted(jobs, key=lambda x: x[0])
288
                     elif (logic != None):
289
                          if (len(jobs) > 1):
290
                              for j in range(1, len(jobs)):
                                   if (j \ge len(jobs)):
292
                                        break
293
                                   if (jobs[j][0] < logic):</pre>
294
                                        queue.append(jobs[j])
295
                                        jobs.pop(j)
296
                                   else:
297
                                        break
298
                          yield env.timeout(logic)
```

The above code initially generates multiple jobs that arrive at different times according to a Poisson distribution, and here we can apply the common variables method. We can do this by using the same job times (samples) for every configuration of the simulations. During the simulation, τ and W_k values are generated for each job to be completed within the simulation. τ refers to the time each process needs to complete, and W_k refers to the amount of watts the job needs during the completion time. By multiplying these variables we get the amount of Wh (watt-hour) that the job needs so that the system can use a similar unit of measurement. Then the simulation runs multiple times for each alpha value, as well as a wide range of capacity values, each incremented by at each iteration, until a minimal value is found that satisfies the given condition of running autonomously.

We have defined two states, ON (1) and OFF (0), as described in the task. The process switches between these two states using the Gillespie algorithm (line 8). We have tried to use reasonable values such as λ (rate of arrivals), e (Efficiency), and γ (rescheduling variable), which also resulted in reasonable values for the minimal capacities. We can simulate multiple times (line 151) for each scenario and then take the average of those simulations for every alpha value to ensure that the different variables for all the jobs generated do not affect the results significantly, and we can get some clear trends.

Within the function energy_source_process, we have many different steps to ensure that the user can run autonomously. Initially, we start with the job at the top of the queue. If the source is available, then we generate a certain amount of source that could be used to satisfy the requirements of the job, this generation of source uses a uniform distribution. If the job wattage can be satisfied with this amount, then the job is finished and it is removed from the queue. Otherwise, we check our battery storage at that given point and use the energy in the battery. If the battery does not have sufficient energy either, then we check if we can buy external energy, while still staying under the alpha value. If this is also not possible then the job remains in the queue, to see if it could be completed later on by generating more energy.

We assumed the α to be the threshold value that represents the upper limit on the fraction of total energy demands that the system is allowed to satisfy by buying energy from external sources. So, for instance, $\alpha = 0.01$ allows up to 1% of the total energy demands to be met by external purchases. In that sense, we kept track of all the energy that has been bought previously, as well as the total energy demands to see if the energy we are requiring is within the given threshold. If it is, we can safely buy the external energy. Otherwise, we rescheduled the job using the γ variable.

The obtained results after the simulation are shown in Figure 1.

The output in Figure 1 shows that, on average, the minimal capacity decreases as alpha increases.

Figure 1: Output of the Simulation (Single Run)

```
\mathrm{b}) # Code same as part a, above
       def plot_results(self, correlation_values):
 2
           min_battery_capacities = []
 3
 4
           for alpha in self.alphas:
                avg_min_capacity = 0
                for correlation in correlation_values:
                    my\_time = 0
 9
                    job_times = []
10
11
                    while my_time < 20000:</pre>
12
                        new_job_time = np.random.poisson(lam=
13
      lambda_rate) * 100
                        my_time += new_job_time
14
                        if my_time < 20000:</pre>
                             job_times.append(my_time)
17
                    results = []
18
19
                    for capacity in range(0, 2001, 10):
20
                         job_times_copy = copy.copy(job_times)
21
                        env = simpy.Environment()
22
23
                        energy_source = env.process(self.
24
      energy_source_process(env, alpha, self.gamma, self.p, self.
      e, job_times_copy, capacity, correlation))
25
                        env.run(until=20000)
26
27
                        if not job_times_copy:
28
                             results.append(capacity)
29
                             break
30
                        else:
31
                             continue
32
33
                    if results: # Only calculate the average when
      there are results
                        avg_min_capacity += sum(results) / len(
35
      results)
                if avg_min_capacity != 0:
37
                    avg_min_capacity /= len(correlation_values)
38
                min_battery_capacities.append(avg_min_capacity)
39
40
           plt.plot(self.alphas, min_battery_capacities, marker='o
41
           plt.xlabel('Alpha')
42
           plt.ylabel('Average Min Battery Capacity')
43
           plt.title('Influence of Correlation on System
44
      Performance')
           plt.grid(True)
```

```
plt.show()
46
47
     __name__ == "__main__":
48
      lambda_rate = 0.2
49
      gamma = 0.5
50
        = 0.2
          0.8
53
        = Process([0.0, 0.01, 0.05, 0.1], gamma, p, e)
54
      correlation_values = [0.0, 0.2, 0.4, 0.6, 0.8, 1.0]
     Different correlation levels
      p.plot_results(correlation_values)
```

The above code is an extension of part a, it adds a plotting mechanism (def plot_results(self, correlation_values)) to view the correlation factor of the ON/OFF process as the program simulates the environment. The results are plotted to assess how different correlations in the ON-OFF process affect the system's performance under various conditions.

The output of part b can be found in Figure 2. This figure shows the correlation between the increased allowed import of energy and the battery capacity. In the graph, battery capacity decreases as alpha increases. This makes sense because increasing the capacity of the battery translates to being able to hold up more energy that might be needed later. It also makes sense because alpha determines how much external energy can be purchased, which allows the user to run autonomously without requiring as much minimal battery capacity. When additional resources are needed, they can be purchased. Now we rely on another external system to aid the current system in its path towards running autonomously, therefore not requiring as much battery capacity as required when the system is fully autonomous (When alpha = 0).

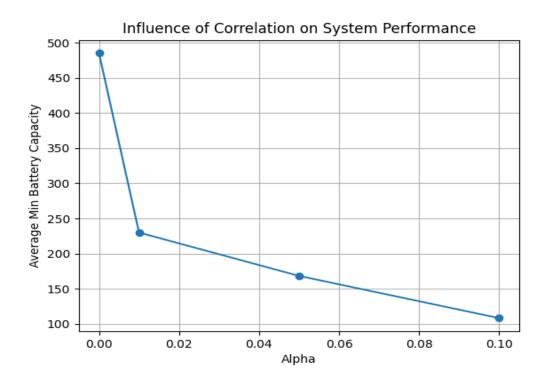


Figure 2: Influence of Correlation on System Performance

2 Scenarios with N users

```
a) import numpy as np
 2 import simpy
 3 import matplotlib.pyplot as plt
 4 import copy
 5 import random
 6 import threading
 8 from concurrent.futures import ThreadPoolExecutor, as_completed
10 class Gillespie(object):
11
12
       def __init__(self, p01, p10):
           self.p01 = p01
13
           self.p10 = p10
14
       def calculate_total_rate(self):
16
           return self.p01 + self.p10
17
18
       def Generate(self):
19
           rando = random.uniform(0, 1)
20
           if (rando < self.p01 / self.calculate_total_rate()):</pre>
21
                return 1
22
23
           else:
               return 0
24
25
       def setP01(self, p):
26
27
           self.p01 = p
28
       def setP10(self, p):
29
           self.p10 = p
30
31
32
   class ExclusiveGillespie(Gillespie):
33
34
       def __init__(self, p01, p10):
35
           super().__init__(p01, p10)
36
           self.first = True
37
           self.lock = threading.Lock()
           self.current = None
39
40
       def Generate(self):
41
           with self.lock:
42
                if (self.first == True):
43
                    self.first = False
44
                    self.current = super().Generate()
45
                return self.current
46
47
       def next(self):
48
           with self.lock:
49
                self.first = True
50
51
       def setP01(self, p):
           with self.lock:
53
                return super().setP01(p)
       def setP10(self, p):
56
           with self.lock:
57
               return super().setP10(p)
58
```

```
59
  class Energy_Collector:
61
       def __init__(self):
62
           self.energy = 0
63
           self.total_additions = 0
64
           self.total_usage = 0
65
           self.lock = threading.Lock()
66
67
       def getEnergy(self):
68
           with self.lock:
69
               return self.energy
70
71
       def addEnergy(self, value):
72
           with self.lock:
73
               self.energy += value
74
               self.total_additions += value
75
76
77
       def useEnergy(self, value):
           with self.lock:
78
                self.energy -= value
                self.total_usage -= value
80
81
       def Check(self):
82
           #debugging
83
           if (round(self.getEnergy(), 5) == round(self.
84
      getTotalAddition() + self.getTotalUsage(), 5)):
               s = 'True, final values are: Remaining Energy: {},
85
      Total Balnce: {}, Total Addition: {}, Total Usage: {}'.
      format(round(self.getEnergy(), 5), round(self.
      getTotalAddition() + self.getTotalUsage(), 5) ,round(self.
      getTotalAddition(), 5), round(self.getTotalUsage(),5))
               print(s)
               return 1
87
           else:
88
               s = 'False, final values are: Remaining Energy: {},
89
       Total Balnce: {}, Total Addition: {}, Total Usage:
      format(round(self.getEnergy(), 5), round(self.
      getTotalAddition() + self.getTotalUsage(), 5) ,round(self.
      getTotalAddition(), 5), round(self.getTotalUsage(),5))
               print(s)
               return 0
91
92
       def getTotalUsage(self):
93
           #debugging
94
           with self.lock:
95
               return self.total_usage
96
97
       def getTotalAddition(self):
98
           #debugging
99
           with self.lock:
100
               return self.total_additions
101
102
   class Simulator:
103
       def __init__(self, max_lambda=1.0, lambda_step=0.1):
104
           self.processes = []
105
           self.Battery = Energy_Collector()
106
           self.max_lambda = max_lambda
           self.lambda_step = lambda_step
108
           self.lambdas = []
```

```
110
       def addUser(self, process):
111
           process.setBattery(self.Battery) # Pass the Battery
112
      object to the process
           self.processes.append(process)
113
       def deleteProcess(self, process):
           self.processes.remove(process)
117
       def readLambdas(self):
118
           for i in self.lambdas:
119
               print(i)
120
121
       def simulate(self, max_capacity=2001, min_capacity = 0,
      amount_of_times = 1, capacity_increment = 10, time = 20000)
           #Function to simulate different processes per user
123
124
           #need to multiproces this function
           #scenario ii
126
           max_lambda = self.max_lambda
127
128
           lambda_step = self.lambda_step
           while max_lambda > 0:
129
               for user in self.processes:
130
                    user.gp.setP01(max_lambda)
                    user.gp.setP10(max_lambda)
132
               with ThreadPoolExecutor(max_workers=4) as executor:
133
                    futures = [executor.submit(user.simulate,
      max_capacity, min_capacity, amount_of_times,
      capacity_increment, time) for user in self.processes]
                    for future in futures:
135
                        future.result()
136
               backlog = False
137
               for user in self.processes:
138
                    if user.getResults()[0] > 0:
139
                        backlog = True
140
141
                        break
               if backlog:
142
                    max_lambda -= lambda_step
143
                    break
144
               else:
145
                    max_lambda += lambda_step
146
                self.Battery.Check()
147
           self.lambdas.append(max_lambda)
148
149
           for process in self.processes:
150
               process.getOutput()
152
153
       def simulate_all_same_on_off_process(self, gp =
154
      ExclusiveGillespie(0.5, 0.5), max_capacity=2001,
      min_capacity = 0, amount_of_times = 1, capacity_increment =
       10, time = 20000):
           #scenario i
155
           for process in self.processes:
156
               process.setGP(gp)
157
           self.simulate(max_capacity, min_capacity,
158
      amount_of_times, capacity_increment, time)
159
```

```
def find_max_lambda(self, max_capacity=2001, min_capacity
160
      =0, amount_of_times=1, capacity_increment=10, time=20000):
           max_lambda = self.max_lambda
161
           lambda_step = self.lambda_step
162
           while max_lambda > 0:
163
                for user in self.processes:
                    user.gp.setP01(max_lambda)
165
                    user.gp.setP10(max_lambda)
166
                with ThreadPoolExecutor(max_workers=len(self.
167
      processes)) as executor:
                    futures = [executor.submit(user.simulate,
168
      max_capacity, min_capacity, amount_of_times,
      capacity_increment, time) for user in self.processes]
                    for future in futures:
                         future.result()
170
                backlog = False
171
                for user in self.processes:
172
173
                    if user.getResults()[0] > 0:
                         backlog = True
174
                         break
175
                if backlog:
176
177
                    max_lambda -= lambda_step
178
                else:
179
                    max_lambda += lambda_step
180
           return max_lambda
181
182
   class Process:
183
184
       def __init__(self, alphas, gamma, p, e, gp = Gillespie(0.5,
185
       0.5)):
           self.alphas = alphas
186
           self.gamma = gamma
187
           self.p = p
188
           self.e = e
189
           self.gp = gp
190
           self.Battery = None
           self.results = None
192
           self.amount_of_times = 0
193
194
       def getResults(self):
195
           if (self.results == None):
196
                return "Simulate the process first"
197
           else:
198
                return self.results
199
200
       def setBattery(self, battery):
201
202
           self.Battery = battery
203
       def setGP(self, gp):
204
           self.gp = gp
205
206
       def energy_source_process(self, env, alpha, gamma, p, e,
207
      job_times, capacity):
208
           state = self.gp.Generate()
           total_energy_demand = 0
210
           B = 0.1
211
           total_energy_bought = 0
212
           jobs = []
```

```
my_time = 0
214
215
            while job_times != []:
216
                # Generate a new job
217
                state = self.gp.Generate()
218
                if job_times and job_times[0] <= my_time:</pre>
220
                     tau_k = np.random.exponential(lambda_rate)
221
                     W_k_new = np.random.exponential(2000) # Random
222
       wattage for the job
                     Wh_new = tau_k * W_k_new
223
                     job_times.pop(0)
224
                     total_energy_demand += Wh_new
225
                     jobs.append(Wh_new)
                my\_time += 10
227
228
                if jobs == []:
229
230
                    continue
                if state == 1:
231
                     total_source = np.random.uniform(0, 10)
232
                     # Consume energy from the intermittent source
233
234
                     if jobs[0] - total_source < 0: # Source is</pre>
      enough to satisfy the job
                         remaining_source = total_source - jobs[0]
235
                         jobs[0] = 0
236
                         jobs.pop(0)
237
                         if B + remaining_source > capacity: #
238
      Excess source cannot be stored
                              self.Battery.addEnergy(remaining_source
239
      )
                              B = capacity
240
241
                         else:
                              B = B + e * remaining_source
242
                     else: # Source is not enough, we need to check
243
      the battery
                         jobs[0] = jobs[0] - total_source
244
                         if B - jobs[0] > 0: # Battery is enough
245
                              B = B - jobs[0]
jobs[0] = 0
246
247
                              jobs.pop(0)
248
                              yield env.timeout(1 / tau_k)
249
250
                         else: # Battery is not enough
251
                              jobs[0] = jobs[0] - B
252
                              B = 0
253
                         yield env.timeout(1 / tau_k)
                # Check if the energy source is ON
255
                else:
256
                     # Energy source is OFF
257
                     # Determine whether to reschedule or buy
258
      external energy
                     if B - jobs[0] > 0: # Battery is enough
                             B = B - jobs[0]jobs[0] = 0
260
261
                              jobs.pop(0)
262
                     else: # Battery is not enough
263
                         jobs[0] = jobs[0] - B
264
                         B = 0
265
                         if (((jobs[0] + total_energy_bought) /
266
      total_energy_demand <= alpha) and (self.Battery.getEnergy()
```

```
> jobs[0])): #Buy external energy
                             cost = p * jobs[0]
267
                             total_energy_bought += jobs[0]
268
                             self.Battery.useEnergy(jobs[0])
269
                             jobs[0] = 0
270
                         else:
                             yield env.timeout(np.random.exponential
272
      (gamma)) # Reschedule
                #print(test)
273
274
275
       def simulate(self, max_capacity= 2000, min_capacity=0,
276
      amount_of_times=1, capacity_increment=10, time=20000):
           results = []
           self.amount_of_times = amount_of_times
278
           for alpha in self.alphas:
279
                results.append(0)
280
281
           for i in range(1, amount_of_times + 1):
282
                for alpha in self.alphas:
283
                    my_time = 0
285
                    job_times = []
286
                    while my_time < time:</pre>
287
                        new_job_time = np.random.poisson(lam=
      lambda_rate) * 100
                        my_time += new_job_time
289
                        if my_time < time:</pre>
290
                             job_times.append(my_time)
291
292
                    for capacity in list(range(min_capacity,
293
      max_capacity, capacity_increment)):
                        job_times_copy = copy.copy(job_times)
294
                        env = simpy.Environment()
295
                        env.process(self.energy_source_process(env,
296
       alpha, self.gamma, self.p, self.e, job_times_copy,
      capacity))
297
                        env.run(until=time) # Adjust simulation
      time accordingly
298
                        if not job_times_copy:
299
                             results[self.alphas.index(alpha)] +=
300
      capacity
                             break # Move on to the next alpha
301
                         else:
302
                             continue
303
                if (isinstance(self.gp, ExclusiveGillespie)):
304
305
                    self.gp.next()
306
           self.results = results
307
           #self.getOutput()
308
309
       def getOutput(self):
310
311
           if (self.Battery.Check()):
312
               print("
313
                              -----")
                print("Simulation Successfull, running the
314
      simulation {self.amount_of_times} the values are as follows
      :")
```

```
for alpha in self.alphas:
315
                   print(f"Alpha: {alpha}, Average Min Battery
316
      Capacity: {self.results[self.alphas.index(alpha)] / self.
      amount_of_times}")
317
  lambda_rate = 0.2 # Poisson stream rate
  gamma = 0.5 # Rescheduling rate
  p = 0.2
           # External energy purchase rate
321 e = 0.8 # Battery charging efficiency
322 B_values = [] # To store minimal battery capacities for
      different alpha values
323
324 # Create User instances
  user1 = Process([0.0, 0.01, 0.05, 0.1], gamma, p, e)
  user2 = Process([0.0, 0.01, 0.05, 0.1], gamma, p, e)
user3 = Process([0.0, 0.01, 0.05, 0.1], gamma, p, e)
329 # Initialize the Simulator
s = Simulator(max_lambda=1.0, lambda_step=0.01)
331
332 # Add User instances to the simulator
  s.addUser(user1)
334 s.addUser(user2)
335 s.addUser(user3)
337 #s.simulate()
s.simulate_all_same_on_off_process(amount_of_times=10)
340 print(s.readLambdas())
```

This Python code simulates a scenario involving multiple users equipped with batteries and varying energy demands. Each user's behavior is represented as a Markov On-Off process. The different scenarios differ in that if the general state is zero in the first scenario a situation can occur where can able to be bought for a while. This might lead to a higher dependency on chance compared to the second scenario. In the second scenario, every process can generate it's own state, since there are many users the low of averages dictates that there will always be a pool of users contributing to the energy pool. This makes the second scenario more stable and consistent than the first scenario. So we can expect that $\lambda_m ax$ will on average be bigger in scenario 2 compared to scenario 1, our simulation results also supported this assumption. We implemented the logic for the scenario 1 using the $simulate_all_same_on_off_process()$ and for scenario 2, using simulate() functions. We then extracted the values using our function for λ calculations.

b) If we were to use the regeneration method in scenario ii, then let's assume that a regeneration cycle for each user is $X_i(t)$. We can consider the regeneration period to start when the process turns to 'ON' state, and ends when the process returns to 'OFF' state. Then, for each user, we would have a specific duration for the regeneration cycle, t_i . Our expected regeneration period for a single user would then be:

$$E[t_1] = \frac{1}{\lambda} \tag{1}$$

In order to obtain the lower bound, we would have to look at the minimum among all the users. However, the duration of regeneration cycle, t_i is identi-

cally distributed, so the minimum of them would become:

$$E[t_{min}] = min_{i=1}^{N} E[t_i] = E[t_1] = \frac{1}{\lambda}$$
 (2)

As a result, the lower bound of the expected regeneration period does not depend on the number of users N for scenario ii. Regardless of the number of users, the system regenerates at a rate determined by the underlying Poisson stream of jobs. In this case, the regeneration method could be useful in simplifying the analysis, and the number of users will not affect it. So, scaling the process to have more users will not complicate the application of regeneration method in this particular case, since expected regeneration period does not increase with the number of users.

A control variate is a statistical technique used in experimental design and data analysis to reduce the variance (i.e., the spread or variability) of the estimates or outcomes of an experiment or simulation. It is a method for improving the precision and efficiency of statistical estimates or reducing the standard errors of those estimates. We used control variate in this case to create the following code for part a

3 A spatial, time-evolving process

a) The following is our attempt at Part 3:

```
# Import additional libraries
2 from scipy.spatial.distance import pdist, squareform
4 import numpy as np
5 import simpy
6 import matplotlib.pyplot as plt
7 import copy
8 import random
9 import threading
11 from concurrent.futures import ThreadPoolExecutor, as_completed
12
class Gillespie(object):
14
      def __init__(self, p01, p10):
15
          self.p01 = p01
16
          self.p10 = p10
17
18
      def calculate_total_rate(self):
19
           return self.p01 + self.p10
20
21
      def Generate(self):
22
          rando = random.uniform(0, 1)
23
           if (rando < self.p01 / self.calculate_total_rate()):</pre>
24
               return 1
          else:
26
               return 0
27
28
      def setP01(self, p):
29
           self.p01 = p
30
31
      def setP10(self, p):
32
           self.p10 = p
```

```
34
  class ExclusiveGillespie(Gillespie):
37
      def __init__(self, p01, p10):
38
           super().__init__(p01, p10)
39
           self.first = True
40
           self.lock = threading.Lock()
41
          self.current = None
42
43
      def Generate(self):
44
          with self.lock:
45
               if (self.first == True):
46
                   self.first = False
47
                   self.current = super().Generate()
48
               return self.current
49
50
51
      def next(self):
           with self.lock:
               self.first = True
53
      def setP01(self, p):
          with self.lock:
56
               return super().setP01(p)
57
      def setP10(self, p):
59
          with self.lock:
60
               return super().setP10(p)
61
  class Energy_Collector:
63
64
      def __init__(self):
65
          self.energy = 0
          self.total_additions = 0
67
           self.total_usage = 0
68
          self.lock = threading.Lock()
69
70
      def getEnergy(self):
71
          with self.lock:
72
               return self.energy
73
74
      def addEnergy(self, value):
75
          with self.lock:
76
               self.energy += value
               self.total_additions += value
78
79
      def useEnergy(self, value):
80
          with self.lock:
81
               self.energy -= value
82
               self.total_usage -= value
83
      def Check(self):
          #debugging
86
          if (round(self.getEnergy(), 5) == round(self.
87
     getTotalAddition() + self.getTotalUsage(), 5)):
               s = 'True, final values are: Remaining Energy: {},
     Total Balnce: {}, Total Addition: {}, Total Usage: {}'.
     format(round(self.getEnergy(), 5), round(self.
     getTotalAddition() + self.getTotalUsage(), 5) ,round(self.
     getTotalAddition(), 5), round(self.getTotalUsage(),5))
```

```
print(s)
89
               return 1
90
           else:
91
               s = 'False, final values are: Remaining Energy: {},
92
       Total Balnce: {}, Total Addition: {}, Total Usage: {}'.
      format(round(self.getEnergy(), 5), round(self.
      getTotalAddition() + self.getTotalUsage(), 5) ,round(self.
      getTotalAddition(), 5), round(self.getTotalUsage(),5))
               print(s)
93
               return 0
94
95
       def getTotalUsage(self):
96
           #debugging
97
           with self.lock:
               return self.total_usage
99
100
       def getTotalAddition(self):
101
           #debugging
102
           with self.lock:
               return self.total_additions
106
   class Simulator:
       def __init__(self, max_lambda=1.0, lambda_step=0.1):
           self.processes = []
108
           self.Battery = Energy_Collector()
           self.max_lambda = max_lambda
           self.lambda_step = lambda_step
111
           self.lambdas = []
112
113
       def addUser(self, process):
114
           process.setBattery(self.Battery) # Pass the Battery
115
      object to the process
           self.processes.append(process)
116
117
       def deleteProcess(self, process):
118
           self.processes.remove(process)
119
120
       def readLambdas(self):
121
           for i in self.lambdas:
               print(i)
123
      def simulate(self, max_capacity=2001, min_capacity = 0,
      amount_of_times = 1, capacity_increment = 10, time = 20000)
           #Function to simulate different processes per user
126
           #need to multiproces this function
           #scenario ii
128
129
           max_lambda = self.max_lambda
130
           lambda_step = self.lambda_step
131
           while max_lambda > 0:
132
               for user in self.processes:
133
                    user.gp.setP01(max_lambda)
134
                    user.gp.setP10(max_lambda)
135
               with ThreadPoolExecutor(max_workers=4) as executor:
136
                   futures = [executor.submit(user.simulate,
137
      max_capacity, min_capacity, amount_of_times,
      capacity_increment, time) for user in self.processes]
                    for future in futures:
138
                        future.result()
```

```
backlog = False
140
                for user in self.processes:
141
                    if user.getResults()[0] > 0:
142
                        backlog = True
143
                        break
144
                if backlog:
                    max_lambda -= lambda_step
146
                    break
147
                else:
148
                    max_lambda += lambda_step
149
                self.Battery.Check()
150
           self.lambdas.append(max_lambda)
152
           for process in self.processes:
                process.getOutput()
154
156
157
       def simulate_all_same_on_off_process(self, gp =
      ExclusiveGillespie(0.5, 0.5), max_capacity=2001,
      min_capacity = 0, amount_of_times = 1, capacity_increment =
       10, time = 20000):
158
           #scenario i
           for process in self.processes:
159
               process.setGP(gp)
160
           self.simulate(max_capacity, min_capacity,
161
      amount_of_times, capacity_increment, time)
162
       def find_max_lambda(self, max_capacity=2001, min_capacity
163
      =0, amount_of_times=1, capacity_increment=10, time=20000):
           max_lambda = self.max_lambda
164
           lambda_step = self.lambda_step
165
           while max_lambda > 0:
166
               for user in self.processes:
                    user.gp.setP01(max_lambda)
168
                    user.gp.setP10(max_lambda)
169
                with ThreadPoolExecutor(max_workers=len(self.
170
      processes)) as executor:
                    futures = [executor.submit(user.simulate,
171
      max_capacity, min_capacity, amount_of_times,
      capacity_increment, time) for user in self.processes]
                    for future in futures:
172
                        future.result()
173
                backlog = False
174
                for user in self.processes:
                    if user.getResults()[0] > 0:
                        backlog = True
                        break
178
                if backlog:
179
                    max_lambda -= lambda_step
180
181
182
                    max_lambda += lambda_step
183
           return max_lambda
184
185
   class Process:
186
187
       def __init__(self, alphas, gamma, p, e, gp = Gillespie(0.5,
188
       0.5)):
189
           self.alphas = alphas
           self.gamma = gamma
```

```
self.p = p
191
            self.e = e
192
            self.gp = gp
193
            self.Battery = None
194
           self.results = None
195
            self.amount_of_times = 0
197
       def getResults(self):
198
            if (self.results == None):
199
                return "Simulate the process first"
200
201
                return self.results
202
203
       def setBattery(self, battery):
            self.Battery = battery
205
206
       def setGP(self, gp):
207
208
            self.gp = gp
209
       def energy_source_process(self, env, alpha, gamma, p, e,
210
      job_times, capacity):
211
           state = self.gp.Generate()
212
           total_energy_demand = 0
213
           B = 0.1
214
           total_energy_bought = 0
215
           jobs = []
           my\_time = 0
217
218
            while job_times != []:
219
                # Generate a new job
220
                state = self.gp.Generate()
221
222
                if job_times and job_times[0] <= my_time:</pre>
223
                    tau_k = np.random.exponential(lambda_rate)
224
                    W_k_new = np.random.exponential(2000) # Random
225
       wattage for the job
                    Wh_new = tau_k * W_k_new
226
                    job_times.pop(0)
227
                    total_energy_demand += Wh_new
228
                    jobs.append(Wh_new)
229
                my\_time += 10
230
231
                if jobs == []:
                    continue
233
                if state == 1:
234
                    total_source = np.random.uniform(0, 10)
235
                    # Consume energy from the intermittent source
236
                    if jobs[0] - total_source < 0: # Source is</pre>
237
      enough to satisfy the job
                         remaining_source = total_source - jobs[0]
238
                         jobs[0] = 0
239
                         jobs.pop(0)
240
                         if B + remaining_source > capacity: #
241
      Excess source cannot be stored
                             self.Battery.addEnergy(remaining_source
      )
                             B = capacity
                         else:
244
                              B = B + e * remaining_source
```

```
else: # Source is not enough, we need to check
246
      the battery
                         jobs[0] = jobs[0] - total_source
247
                         if B - jobs[0] > 0: # Battery is enough
248
                             B = B - jobs[0]
249
                             jobs[0] = 0
                             jobs.pop(0)
251
                             yield env.timeout(1 / tau_k)
252
253
                         else: # Battery is not enough
254
                             jobs[0] = jobs[0] - B
255
                             B = 0
256
                         yield env.timeout(1 / tau_k)
257
                # Check if the energy source is ON
259
                    # Energy source is OFF
260
                    # Determine whether to reschedule or buy
261
      external energy
                    if B - jobs[0] > 0: # Battery is enough
262
                             B = B - jobs[0]
263
                             jobs[0] = 0
264
                             jobs.pop(0)
265
                    else: # Battery is not enough
266
                         jobs[0] = jobs[0] - B
267
                         B = 0
268
                         if (((jobs[0] + total_energy_bought) /
269
      total_energy_demand <= alpha) and (self.Battery.getEnergy()
       > jobs[0])): #Buy external energy
                             cost = p * jobs[0]
270
                             total_energy_bought += jobs[0]
271
                             self.Battery.useEnergy(jobs[0])
272
                             jobs[0] = 0
273
                         else:
274
                             yield env.timeout(np.random.exponential
275
      (gamma)) # Reschedule
                #print(test)
276
278
       def simulate(self, max_capacity= 2000, min_capacity=0,
279
      amount_of_times=1, capacity_increment=10, time=20000):
           results = []
280
           self.amount_of_times = amount_of_times
281
           for alpha in self.alphas:
282
                results.append(0)
284
           for i in range(1, amount_of_times + 1):
285
                for alpha in self.alphas:
286
                    my\_time = 0
287
                    job_times = []
288
289
                    while my_time < time:</pre>
290
                         new_job_time = np.random.poisson(lam=
291
      lambda_rate) * 100
                         my_time += new_job_time
292
                         if my_time < time:</pre>
293
                             job_times.append(my_time)
294
295
                    for capacity in list(range(min_capacity,
296
      max_capacity, capacity_increment)):
                         job_times_copy = copy.copy(job_times)
```

```
env = simpy.Environment()
298
                        env.process(self.energy_source_process(env,
299
       alpha, self.gamma, self.p, self.e, job_times_copy,
      capacity))
                        env.run(until=time) # Adjust simulation
300
      time accordingly
301
                        if not job_times_copy:
302
                            results[self.alphas.index(alpha)] +=
303
      capacity
                            break # Move on to the next alpha
304
                        else:
305
                             continue
306
                if (isinstance(self.gp, ExclusiveGillespie)):
                    self.gp.next()
308
309
           self.results = results
310
311
           #self.getOutput()
312
       def getOutput(self):
313
314
315
           if (self.Battery.Check()):
               print("
316
               print("Simulation Successfull, running the
      simulation {self.amount_of_times} the values are as follows
      :")
               for alpha in self.alphas:
318
                    print(f"Alpha: {alpha}, Average Min Battery
319
      Capacity: {self.results[self.alphas.index(alpha)] / self.
      amount_of_times}")
320
321
  class RingGraphSimulator(Simulator):
322
       def __init__(self, num_users, ring_rate, alphas=None,
323
      max_lambda=1.0, lambda_step=0.1, **kwargs):
           super().__init__(max_lambda, lambda_step)
324
           if alphas is None:
325
               alphas = []
                            # Set a default value or handle it as
326
      needed
           self.alphas = alphas
           self.num_users = num_users
328
           self.ring_rate = ring_rate
329
           self.distance_matrix = self.
      generate_ring_distance_matrix()
331
           self.processes = []
332
333
           self.Battery = Energy_Collector()
           self.gamma = gamma
334
           self.p = p
335
           self.e = e
336
           self.lambdas = []
337
338
       def generate_ring_distance_matrix(self):
339
           positions = np.linspace(0, 2 * np.pi, self.num_users,
340
      endpoint=False)
           positions = np.column_stack([np.cos(positions), np.sin(
341
      positions)])
           return squareform(pdist(positions))
342
```

```
def generate_ring_correlated_states(self, env, user, alpha,
344
       gamma, p, e, job_times, capacity):
           state = user.gp.Generate()
345
           total_energy_demand = 0
346
           B = 0.1
347
           total_energy_bought = 0
           jobs = []
349
           my\_time = 0
350
351
           while job_times:
                # Generate a new job
353
                state = user.gp.Generate()
354
                if job_times and job_times[0] <= my_time:</pre>
                    tau_k = np.random.exponential(lambda_rate)
357
                    W_k_{new} = np.random.exponential(2000)
                                                              # Random
358
       wattage for the job
359
                    Wh_new = tau_k * W_k_new
                    job_times.pop(0)
360
                    total_energy_demand += Wh_new
361
                    jobs.append(Wh_new)
                my_time += 10
363
364
                if not jobs:
365
                    continue
367
                if state == 1:
368
                    total_source = np.random.uniform(0, 10)
369
                    # Consume energy from the intermittent source
                    if jobs[0] - total_source < 0: # Source is</pre>
371
      enough to satisfy the job
                        remaining_source = total_source - jobs[0]
372
                        jobs[0] = 0
373
                        jobs.pop(0)
374
                        if B + remaining_source > capacity:
375
      Excess source cannot be stored
376
                             user.Battery.addEnergy(remaining_source
                             B = capacity
377
                         else:
378
                             B = B + e * remaining_source
379
                    else: # Source is not enough, we need to check
380
       the battery
                         jobs[0] = jobs[0] - total_source
                         if B - jobs[0] > 0:
                                              # Battery is enough
382
                             B = B - jobs[0]
383
                             jobs[0] = 0
384
                             jobs.pop(0)
385
                             yield env.timeout(1 / tau_k)
386
387
                         else: # Battery is not enough
388
                             jobs[0] = jobs[0] - B
                             B = 0
390
                         yield env.timeout(1 / tau_k)
391
                else:
392
                    # Energy source is OFF
393
                    # Determine whether to reschedule or buy
394
      external energy
                    if B - jobs[0] > 0: # Battery is enough
395
                        B = B - jobs[0]
```

```
jobs[0] = 0
397
                        jobs.pop(0)
398
                    else: # Battery is not enough
399
                        jobs[0] = jobs[0] - B
400
                        B = 0
401
                        if ((jobs[0] + total_energy_bought) /
      total_energy_demand <= alpha) and (
                                 user.Battery.getEnergy() > jobs[0])
403
         # Buy external energy
                             cost = p * jobs[0]
404
                             total_energy_bought += jobs[0]
405
                             user.Battery.useEnergy(jobs[0])
406
                             jobs[0] = 0
407
                         else:
                             yield env.timeout(np.random.exponential
409
      (gamma))
                # Reschedule
410
411
       def simulate_ring_graph(self, gp=ExclusiveGillespie(0.5,
      0.5), max_capacity=2001, min_capacity=0,
                                 amount_of_times=1,
412
      capacity_increment=10, time=20000):
413
           for process in self.processes:
                process.setGP(gp)
414
415
           for user in self.processes:
416
                env = simpy.Environment()
417
                user_processes = [self.
418
      generate_ring_correlated_states(env, user, user.gp.p01,
      self.gamma, self.p, self.e,
419
            [], max_capacity) for _ in range(amount_of_times)]
                env.process(self.ring_process(env, user_processes,
420
      max_capacity, min_capacity, amount_of_times,
                                                 capacity_increment,
421
      time))
                env.run(until=time)
422
423
       def ring_process(self, env, user_processes, max_capacity,
424
      min_capacity, amount_of_times, capacity_increment, time):
           for alpha in self.alphas:
425
                results = []
426
427
                for i in range(amount_of_times):
428
                    user_jobs = [list(process) for process in
      user_processes]
430
                    my\_time = 0
431
432
                    job_times = []
433
                    while my_time < time:</pre>
434
                        my_time += 10
435
                        yield env.timeout(1)
436
                        new_job_time = np.random.poisson(lam=
437
      lambda_rate) * 100
                        my_time += new_job_time
438
                        if my_time < time:</pre>
                             job_times.append(my_time)
440
441
                    for capacity in range(min_capacity,
442
      max_capacity, capacity_increment):
```

```
job_times_copy = copy.copy(job_times)
443
444
                        user_jobs = [list(self.
      generate_ring_correlated_states(env, self.processes[i],
      alpha, self.gamma, self.p, self.e, job_times_copy, capacity
      )) for i in range(len(self.processes))]
                        env.run(until=time) # Adjust simulation
446
      time accordingly
447
                        if not job_times_copy:
448
                            results.append(sum([self.processes[i].
449
      Battery.getEnergy() for i in range(len(self.processes))]))
                            break # Move on to the next alpha
450
                        else:
                            continue
452
453
               average_result = sum(results) / len(results)
454
455
               print(f"Alpha: {alpha}, Average Min Battery
      Capacity: {average_result}")
456
457
       def calculate_temporal_correlation(self, user_processes):
           # Flatten the generator and convert it to a list
459
           flat_processes = [item for sublist in user_processes
460
      for item in sublist]
461
           # Check if the flattened list is not empty
462
           if flat_processes:
463
               # Calculate temporal correlation for a single user
464
               return np.corrcoef(flat_processes)
           else:
466
               print("User processes are empty.")
467
               return np.empty((0, 0))
468
469
       def calculate_spatial_correlation(self, user_results):
470
           # Check if user_results is empty
471
           if not user_results or not user_results[0]:
               print("Spatial correlation data is empty.")
473
               return np.empty((0, 0))
474
475
           # Calculate spatial correlation between different users
           return np.corrcoef(user_results)
477
478
       def plot_temporal_correlation(self,
479
      temporal_correlation_matrix):
           # Flatten the matrix to a 1D array
480
           if temporal_correlation_matrix.size > 0:
481
               plt.imshow(temporal_correlation_matrix, cmap='
482
      viridis', origin='lower', interpolation='none')
               plt.colorbar()
483
               plt.title('Temporal Correlation Matrix')
484
               plt.show()
           else:
486
               print("Temporal correlation matrix is empty.")
487
488
       def plot_spatial_correlation(self,
      spatial_correlation_matrix):
           # Ensure that the matrix is not empty
490
           if spatial_correlation_matrix.size > 0:
491
```

```
plt.imshow(spatial_correlation_matrix, cmap='
492
      viridis', origin='lower', interpolation='none')
               plt.colorbar()
493
               plt.title('Temporal Correlation Matrix')
494
               plt.show()
495
           else:
               print("Temporal correlation matrix is empty.")
497
498
499
      def analyze_correlations(self, gp=ExclusiveGillespie(0.5,
500
      0.5), max_capacity=2001, min_capacity=0,
                             amount_of_times=1, capacity_increment
501
      =10, time =20000):
           for process in self.processes:
               process.setGP(gp)
503
504
           user_processes = []
505
506
507
           for user in self.processes:
               env = simpy.Environment()
508
               processes = [self.generate_ring_correlated_states(
509
      env, user, user.gp.p01, self.gamma, self.p, self.e,
510
      [], max_capacity) for _ in range(amount_of_times)]
               env.process(self.ring_process(env, processes,
511
      max_capacity, min_capacity, amount_of_times,
                                               capacity_increment,
512
      time))
               env.run(until=time)
513
               user_processes.append(processes)
514
515
           # Analyze temporal correlation
516
           temporal_correlation_matrix = self.
517
      calculate_temporal_correlation(user_processes[0][0])
           self.plot_temporal_correlation(
518
      temporal_correlation_matrix)
519
           # Analyze spatial correlation
520
           user_results = [
521
           [sum([self.processes[i].Battery.getEnergy() for i in
522
      range(len(self.processes))]) for _ in range(amount_of_times
      )]
           for _ in range(len(self.processes))]
523
           spatial_correlation_matrix = self.
      calculate_spatial_correlation(user_results)
           self.plot_spatial_correlation(
      spatial_correlation_matrix)
526 # . .
10 lambda_rate = 0.2 # Poisson stream rate
529 gamma = 0.5 # Rescheduling rate
p = 0.2 # External energy purchase rate
e = 0.8 # Battery charging efficiency
532 B_values = [] # To store minimal battery capacities for
     different alpha values
534 # Initialize the RingGraphSimulator
alphas_ring = [0.0, 0.01, 0.05, 0.1]
```

```
user1 = Process(alphas=[0.0, 0.01, 0.05, 0.1], gamma=0.5, p
     =0.2, e=0.8, gp=Gillespie(0.5, 0.5))
538 user2 = Process(alphas=[0.0, 0.01, 0.05, 0.1], gamma=0.5, p
     =0.2, e=0.8, gp=Gillespie(0.5, 0.5))
_{539} user3 = Process(alphas=[0.0, 0.01, 0.05, 0.1], gamma=0.5, p
      =0.2, e=0.8, gp=Gillespie(0.5, 0.5))
540
541 # Initialize the RingGraphSimulator
ring_simulator = RingGraphSimulator(num_users=3, alphas=
     alphas_ring, max_lambda=1.0, lambda_step=0.01, ring_rate
      =0.1, gamma=0.5, p=0.2, e=0.8)
543
^{544} # Add User instances to the simulator
545 ring_simulator.addUser(user1)
546 ring_simulator.addUser(user2)
547 ring_simulator.addUser(user3)
549 # Simulate the ring graph scenario
ring_simulator.analyze_correlations(gp=ExclusiveGillespie(0.5,
     0.5), amount_of_times=10)
552 # Print lambdas
553 ring_simulator.readLambdas()
```

This code defines a simulation framework for studying energy usage and battery capacity within a network of users connected in a ring graph. It comprises several key components, each serving a specific purpose. We tried generating a ring graph of N users each having a background state and a sequence of probabilities depending on the distance. However, this algorithm does not output accurate and uniform results each run.

b)