Exercise list nº 1: Python

1. From the following list of attribution instructions, which are correct and which aren't? If they are correct, please provide the corresponding result. Otherwise, change the statements to make them correct.
   (a) `x = 2 + 5`
   (b) `y - 1 = 6`
   (c) `x - y = 3`
   (d) `x = x - y`
   Please assume all used variables have been previously declared.
2. What's the result of the execution of the following code:

```
x = 4
y = 1
z = 1

y = x + z
x = x * 2
z = y - x
```

3. Write a code that calculates the volume of a cylinder, given its radius of the base and height.
4. Write a program that reads a score (nonnegative integer) of a student on a scale from 0 to 20, and converts it to a qualitative classification according to the table:

Score	Grade
0-9	Insufficient
10-13	Average
14-16	Good
17-20	Excellent

   Suggestion: Use the instruction

```
score = input("Please introduce the score of the student: ")
score = int(score)
```

   to store in variable `score` the input obtained from the console and convert it to an integer.
5. Let $x \in \mathbb{R}$. Write a program that evaluates the following sums:
   (a) $\sum_{n=0}^{N} n$;
   (b) $\sum_{n=0}^{N} x^n$;
   (c) $\sum_{n=0}^{N} \frac{x^n}{n!}$;
   (d) $\sum_{n=0}^{N} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

6. Create a list with all multiples, between 0 and 100, of a nonnegative integer $n \in [0, 100]$.

7. Write a code that outputs to the console the first $n \in \mathbb{N}$ elements of the Fibonacci sequence.

8. Write a program that, given $n, m \in \mathbb{N}$, calculates the sum of all even numbers between n and m.

9. Create a program that calculates the maximum, minimum, sum and average of a set of 3 integers values, provided by the user on the console.

10. Build a program that takes $n \in \mathbb{N}$ and verifies if its digits are in ascending or descending order or not ordered at all.

11. Write a program that organizes the return of change at a payment station. The program must ask for the amount to be paid and the amount entered into the machine. Then it must calculate the change, indicating the number of coins and bills to be returned, in order to minimize their number. Consider all coins and bills up to 50 euros.

    Suggestion: Create a list of all existing bills and coins in descending order and make all calculations with integer values (in cents), to avoid rounding errors.

12. Write a program that sums all the natural numbers starting with 1 and ending when the sum exceeds a limit specified by the user. For example, if the user has entered 7 as the limit, it would add the numbers 1, 2 and 3, as their sum is 6 while $1 + 2 + 3 + 4 = 10$. In the end, the code must return the last added number.

13. A possible formula to approximate the value of $\frac{\pi}{2}$ is:

$$p(n) = \frac{2 \cdot 2}{1 \cdot 3}\frac{4 \cdot 4}{3 \cdot 5}\frac{6 \cdot 6}{5 \cdot 7} \cdots \frac{2n \cdot 2n}{(2n-1) \cdot (2n+1)} \approx \frac{\pi}{2},\ n \in \mathbb{N}$$

    Write a program that approximates $\frac{\pi}{2}$ such that

$$\left|\frac{\pi}{2} - p(n)\right| < 0.0001,$$

    for some n.

14. A quick numerical method to calculate the square root of a number $x\mathbb{R}^+$ is to use the following iterative process:

$$y_{n+1} = \frac{1}{2}\left(y_n + \frac{x}{y_n}\right), \quad n \geq 0$$

    where y_0 is an initial approximation for the value of the square root (for example, $\frac{x}{2}$).

    At any given iteration, the associated error is estimated by the absolute value of the difference between two consecutive calculated values, that is, by $|y_{n+1} - y_n|$. Write a program that approximates the square root of a number by this method. The program should read the number whose square root is to be approximated and the threshold value for $|y_{n+1} - y_n|$.

15. Given a list of integers, implement an algorithm that determines whether an integer provided by the user is in the list. If the number is in the list, the algorithm must return the index of the first position in which this element appears.

16. Write a program that checks whether a given natural number is prime.

17. Use the previous code to write a program that lists the first N prime numbers.

18. Given a list of integers, implement the following sorting algorithm (ascending):

    - go through all elements in the list and compare each element with the next in the list; if they are out of order, reverse their order;
    - the process of going through the previous list must be repeated until the entire list is sorted in ascending order.

19. Ulam's conjecture states that, for any natural number, the iterative repetition of the following operation

    - if the number is even, divide it by 2
    - if the number is odd, multiply it by 3 and add 1,

    will eventually reach the value 1 in a finite number of steps. Write a program that checks the validity of Ulam's conjecture for all natural numbers smaller than N.

    For each number tested, the program must calculate the number of iterations needed for the sequence to reach 1. At the end, it should print the maximum number of iterations in the set of all numbers tested.

20. Goldbach's conjecture states that all even numbers greater than 2 are equal to the sum of two prime numbers. Write a program that, given an even number greater than 2, finds two prime numbers that verify this conjecture. For example, the number 18 is decomposed into the sum $5 + 13$.