Rainbow and Mastumoto-Imai as Signature Schemes

Reporte de Estancia de Investigación

Pérez Ibarra Miguel Esteban **20th October 2020**

Indice

MPKC

- Multivariate Cryptography is the study of PKC's where the public key is given by a set of quadratic polynomials.

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$$P = (p_1(w_1, \ldots, w_n), \ldots, p_m(w_1, \ldots, w_n))$$

- These polynomials are defined on a finite field $\mathbb{K} = \mathbb{F}_q$.

Signature Scheme

Private Key for Rainbow

The maps

$$L_1 : K^{n-v} \mapsto K^{n-v},$$
$$L_2 : K^n \mapsto K^n, \text{ and}$$
$$F : K^n \mapsto K^{n-v}.$$

Signature Scheme

Public Key for Rainbow

The $n - v$ polynomial components of F and the algebraic structure of K

Signing a Document

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$$s^* = S_A(\bar{m}) = L_1^{-1} \circ F^{-1} \circ L_2^{-1}(\bar{m})$$
- define the verification transformation $V_A : M_h S \mapsto \{true, false\}$
$$V_{A,h}(\bar{m}, s^*) = \begin{cases} true & \text{if } L_1 \circ F \circ L_2(s^*) := h(m) \\ false & \text{otherwise,} \end{cases} .$$

Verifying the signature

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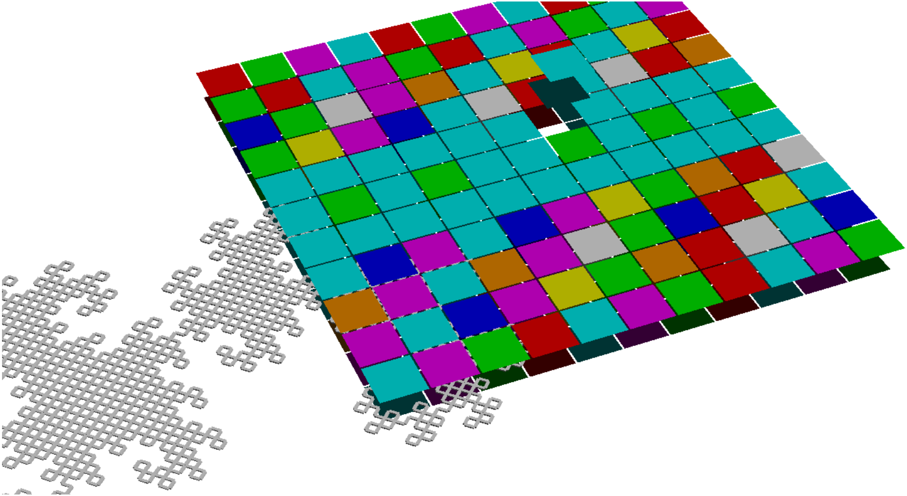
- given $\bar{m}$ and s^*, compute $u = V_{A,h}(\bar{m}, s^*)$

Matsumoto-Imai Signature Scheme



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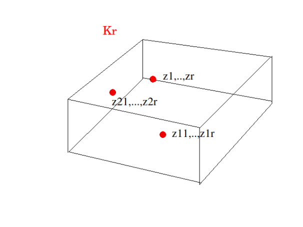
- 

Figure: The set of z's available for entity A

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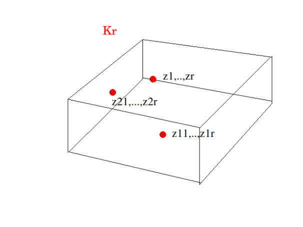
- 

Figure: The set of z's available for entity A

- $z_i(x_1, \ldots, x_n) := \sum_{j=1}^{n} \alpha_{ji} x_j + \beta_i$

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- Entity A must also define a random map $f : K^r \mapsto K^n$ given by
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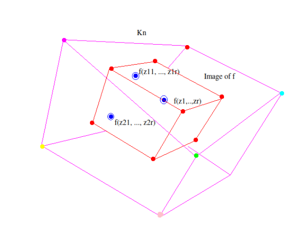
- 

Figure: Image of all the z's in K^r

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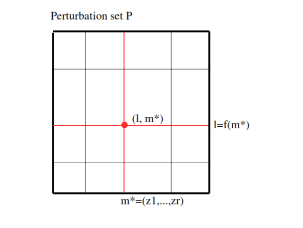

Figure: Perturbation points

Private Key

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The final property is are the linear maps:

$$L_1, L_2 : K^n \mapsto K^n$$

Public key

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- The n multivariate polynomial components of F
- the algebraic structure of K

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- our **digital signature** is then

$$s^* := \hat{F}(\bar{m})$$

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- If $Z(y_{\lambda_0}) := \mu_0$ go to the next step
- Finally if $\bar{m}_0 := L_2^{-1}(y_\lambda)$ is such that $\bar{m}_0 = h(m)$, we are done.

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