

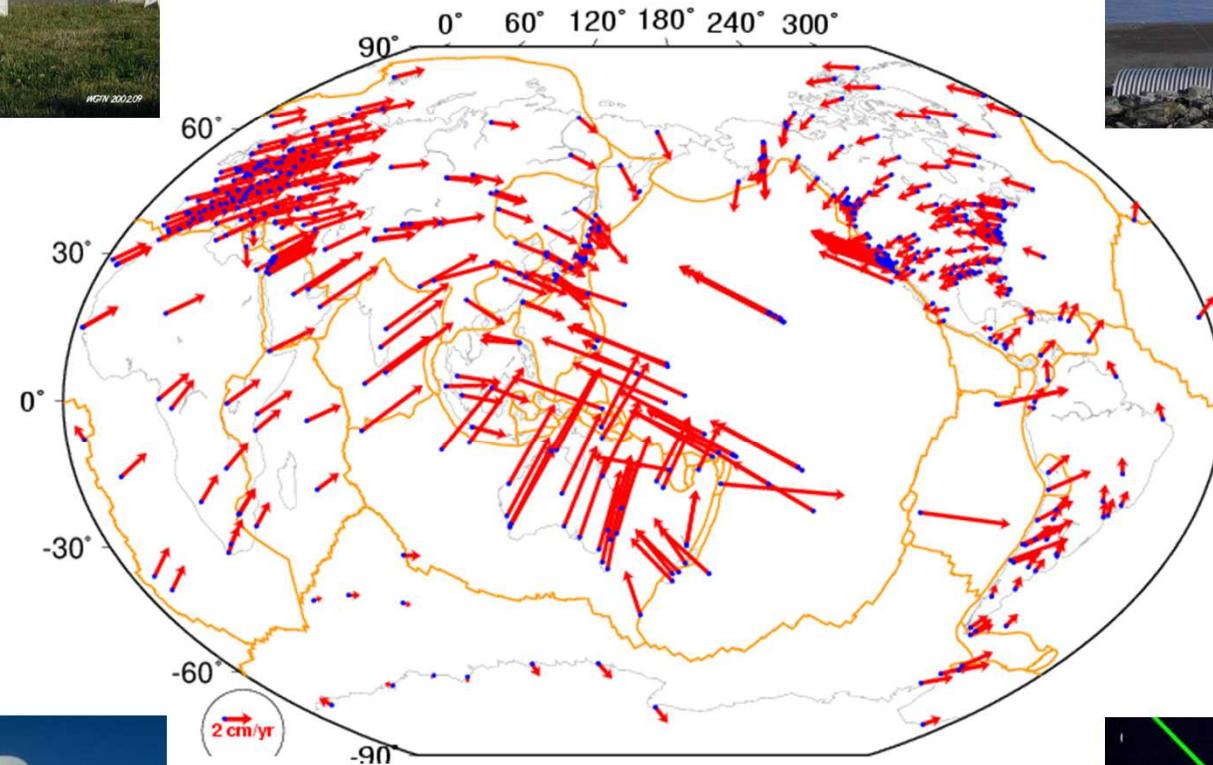
Localisation précise par moyens spatiaux

Global Terrestrial Reference Systems and their realizations

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OUTLINE

- Concept and Definition
- What is a Terrestrial Reference System (TRS), why is it needed and how is it realized ?
- A TRF in practice and motions of the deformable Earth
- Reference frame definition and minimum constraints
- Combination of TRF solutions
- Stacking of TRF time series
- International Terrestrial Reference System (ITRS) and its realization: the International Terrestrial Reference Frame (ITRF)
- Summary of ITRF2008 Geodetic & Geophysical Results
- How to access the ITRF ?
- GNSS associated reference systems and their relationship to ITRF:
 - World Geodetic System (WGS84)
 - Galileo Terrestrial Reference Frame (GTRF)

Defining a Reference System & Frame:

Three main conceptual levels :

- **Ideal Terrestrial Reference System (TRS):**
Ideal, mathematical, theoretical system
- **Terrestrial Reference Frame (TRF):**
Numerical realization of the TRS to which users have access
- **Coordinate System:** mathematical representation of point locations: cartesian (X,Y,Z), geographic (λ , ϕ , h), ...
 - The TRF is a materialization of the TRS inheriting the mathematical properties of the TRS
 - As the TRS, the TRF has an origin, scale & orientation
 - TRF is constructed using space geodesy observations, hence with uncertainties

Ideal Terrestrial Reference System

A tridimensional reference frame (mathematical sense)
Defined in a Euclidian affine space of dimension 3:

Affine Frame (O, E) where:

O : point in space (**Origin**)

E : vector base: orthogonal with the same length:

- vectors co-linear to the base (**Orientation**)
- unit of length (**Scale**)

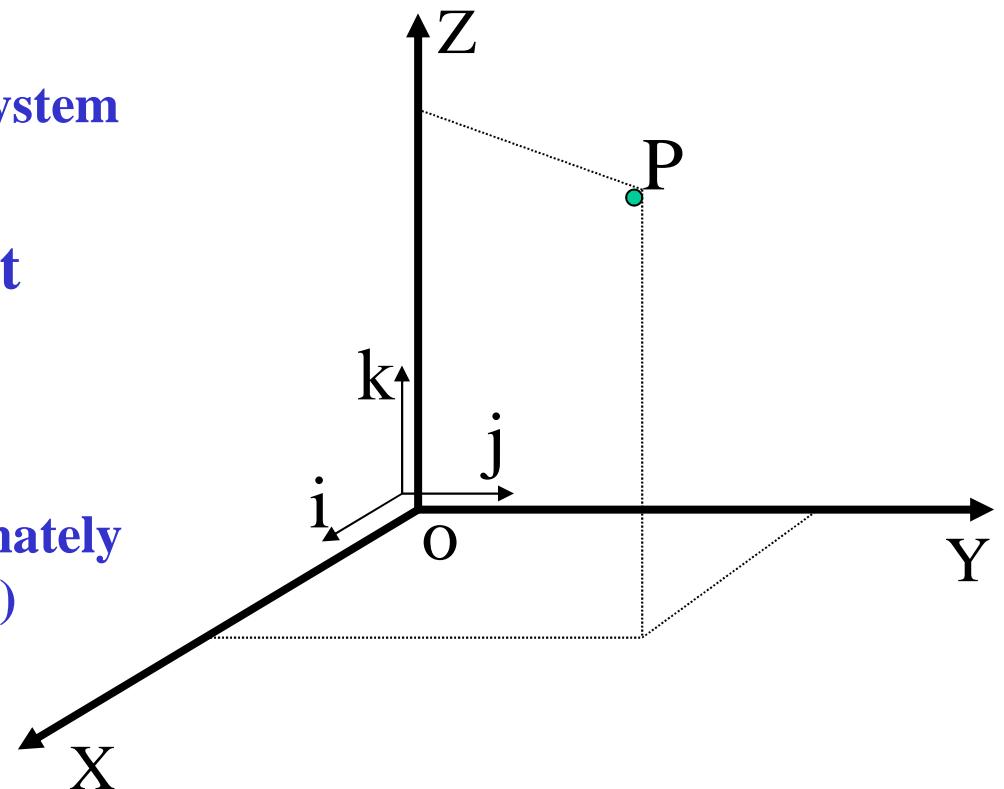
$$\lambda = \left\| \vec{E}_i \right\|_{i=1,2,3}$$

$$\vec{E}_i \cdot \vec{E}_j = \lambda^2 \delta_{ij}$$

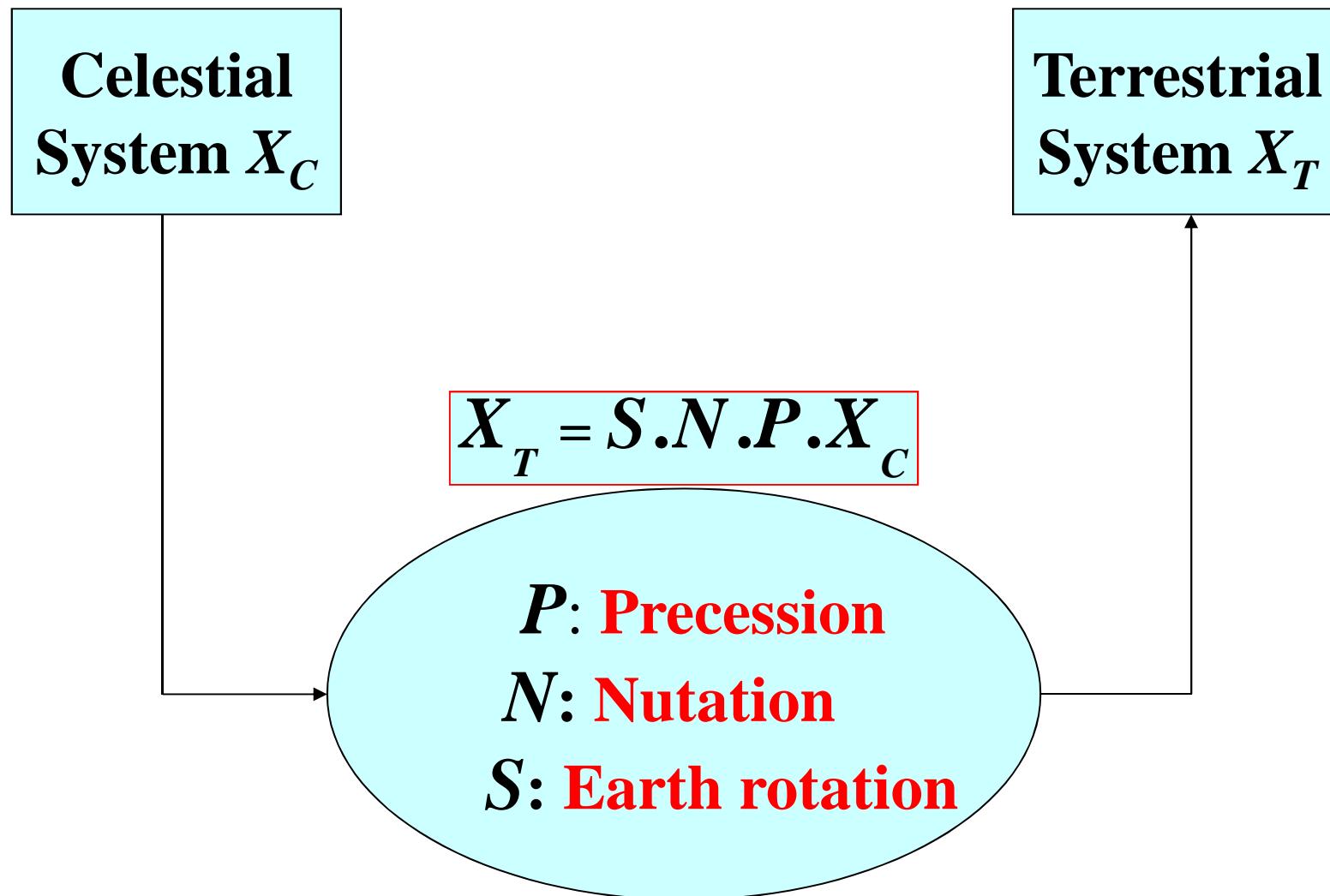
$$(\delta_{ij} = 1, \quad i = j)$$

Terrestrial Reference Frame in the context of space geodesy

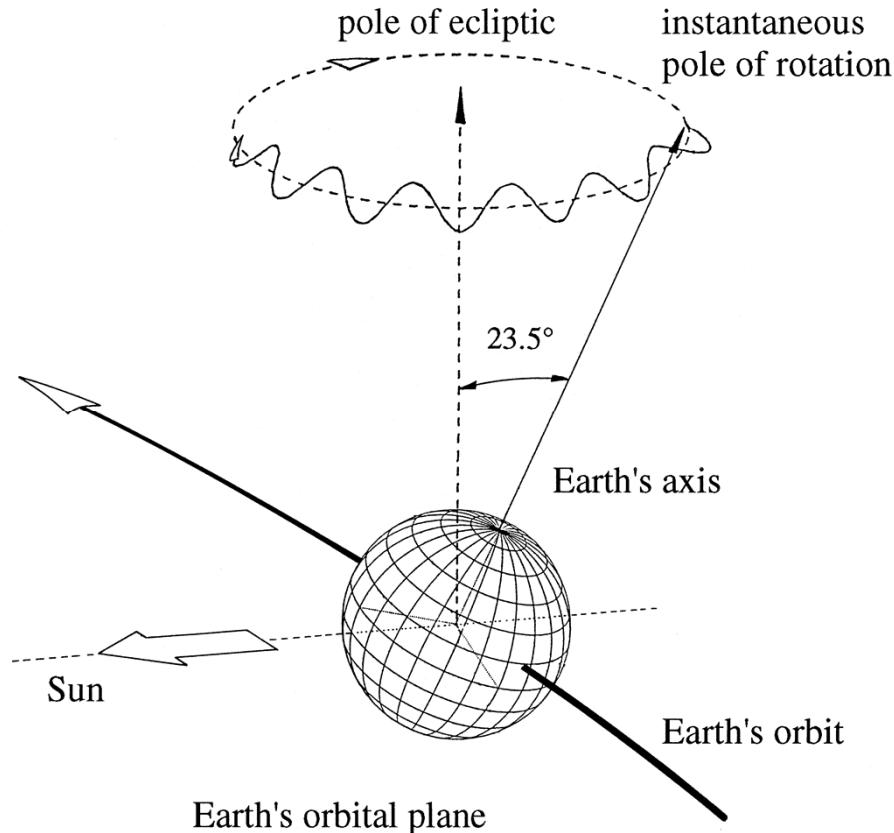
- **Origin:**
 - Center of mass of the Earth System
- **Scale (unit of length):** SI unit
- **Orientation:**
 - Equatorial (Z axis is approximately the direction of the Earth pole)



Relation between Terrestrial and Celestial reference Systems

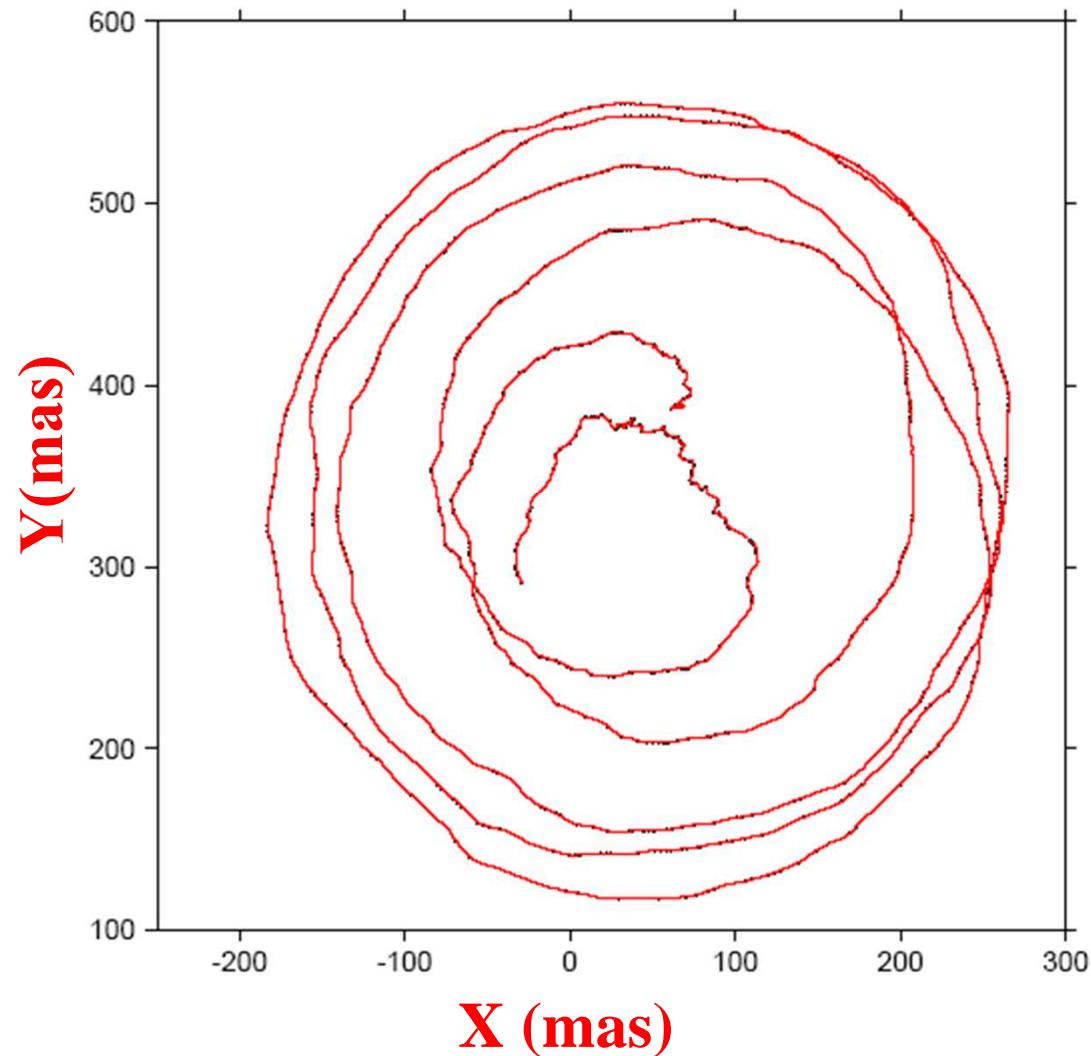


Earth Rotation



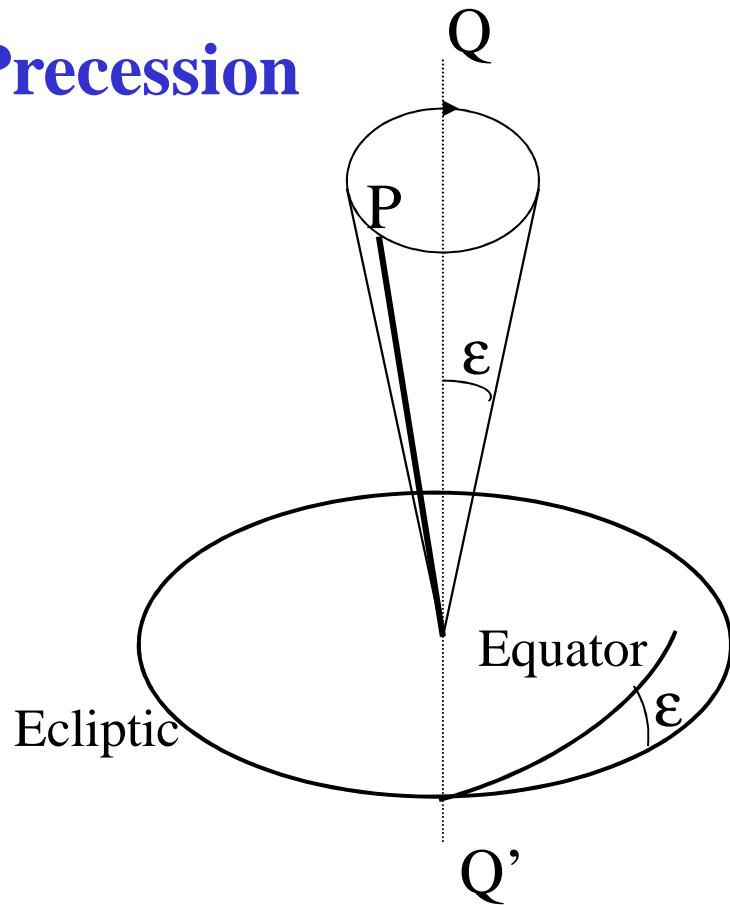
The Earth's rotation axis in inertial space (Beutler, 2004)

Polar Motion



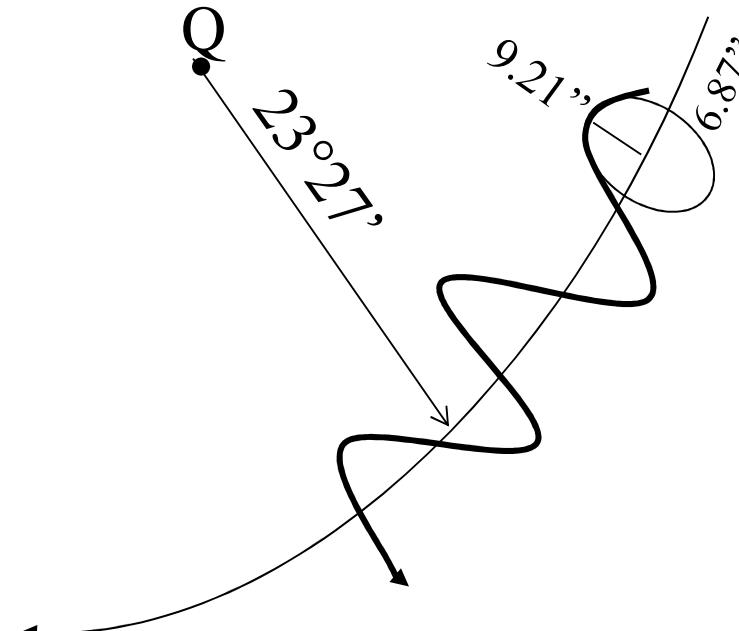
Precession- Nutation

Precession



Displacement in 26000 yrs of the pole axis
along a cone of semi-angle $\varepsilon = 23^{\circ}27'$

Nutation



Elliptic Motion along the precession cone

Transformation between TRS (1/2)

7-parameter similarity:

$$X_2 = T + \lambda \mathcal{R} X_1$$

Translation Vector $T = (T_x, T_y, T_z)^T$

Scale Factor λ

Rotation Matrix $\mathcal{R} = R_x.R_y.R_z$

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos R_1 & \sin R_1 \\ 0 & -\sin R_1 & \cos R_1 \end{pmatrix}$$

$$R_y = \begin{pmatrix} \cos R_2 & 0 & -\sin R_2 \\ 0 & 1 & 0 \\ \sin R_2 & 0 & \cos R_2 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos R_3 & \sin R_3 & 0 \\ -\sin R_3 & \cos R_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation between TRS (2/2)

In space geodesy we use the linearized formula:

$$X_2 = X_1 + T + DX_1 + R.X_1 \quad (1)$$

with: $T = (T_x, T_y, T_z)^T$, $\lambda = (1 + D)$, and $\mathcal{R} = (I + R)$

where $R = \begin{pmatrix} 0 & -R3 & R2 \\ R3 & 0 & -R1 \\ -R2 & R1 & 0 \end{pmatrix}$

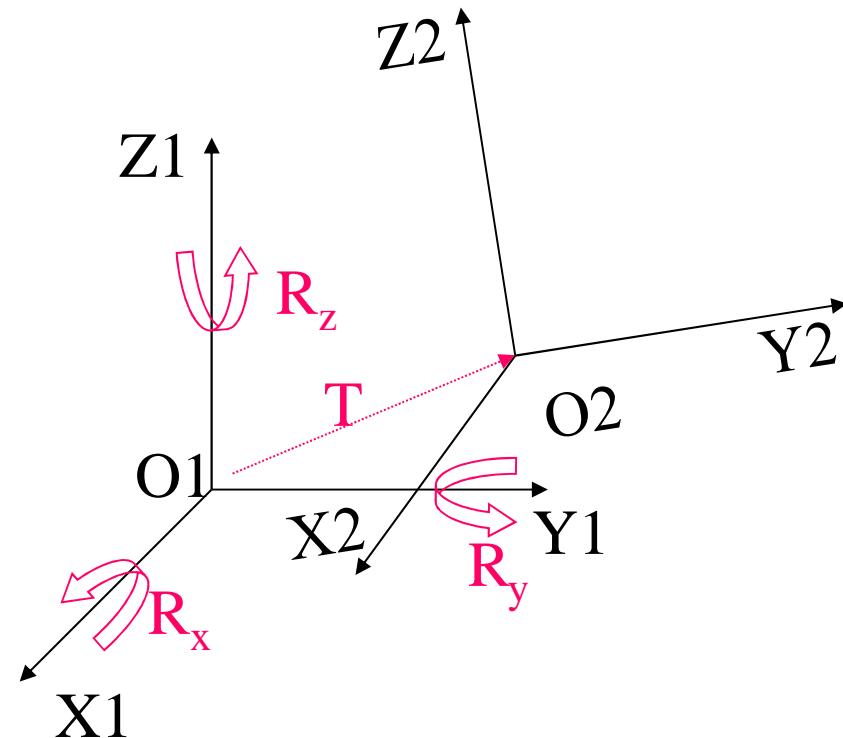
since T is less than 100 meters, D & R less than 10^{-5}

The terms of 2nd ordre are neglected: less than $10^{-10} \approx 0.6$ mm.

Differentiating equation 1 with respect to time, we have:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \underbrace{\dot{D}\dot{X}_1}_{\approx 0} + \dot{DX}_1 + \underbrace{\dot{R}\dot{X}_1}_{\approx 0} + \dot{RX}_1$$

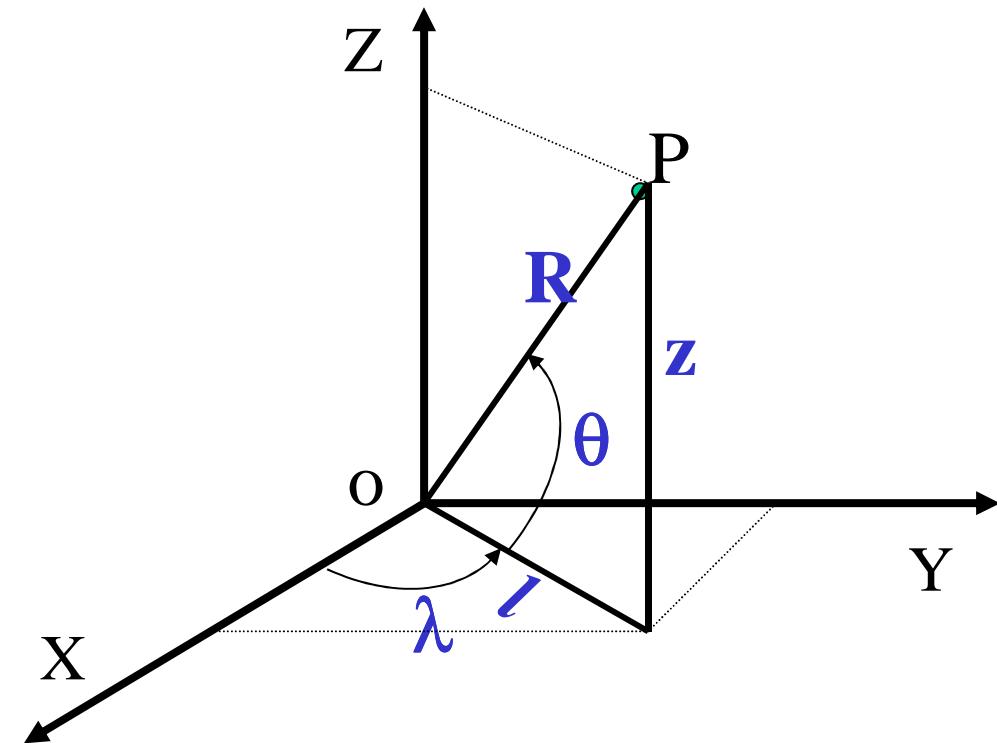
From one RF to another ?



$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_2 = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} + \begin{pmatrix} D - R_z & R_y \\ R_z & D - R_x \\ -R_y & R_x & D \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1$$

Coordinate Systems

- Cartesian: X, Y, Z
- Ellipsoidal: λ , ϕ , h
- Mapping: E, N, h
- Spherical: R, θ , λ
- Cylindrical: l , λ , Z



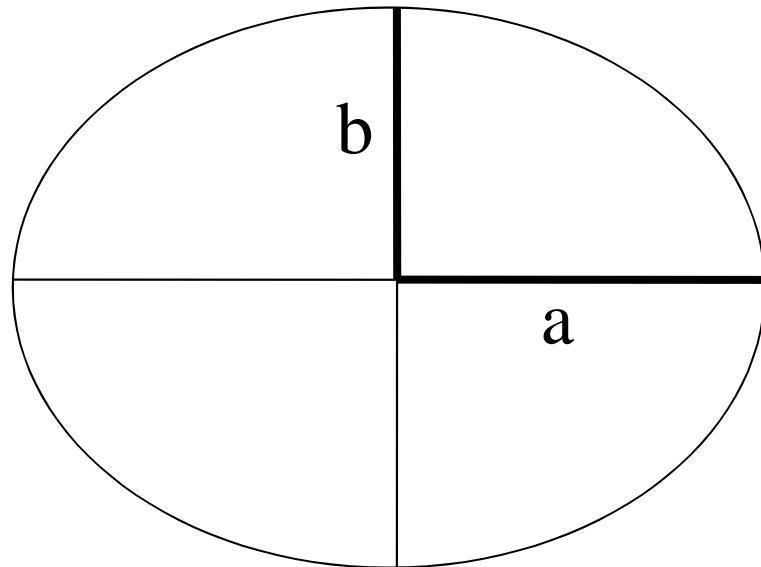
$$\begin{array}{|c} l \cos\lambda \\ OP \\ l \sin\lambda \\ z \end{array}$$

Cylindrical

$$\begin{array}{|c} R\cos\theta \cos\lambda \\ OP \\ R\cos\theta \sin\lambda \\ R\sin\theta \end{array}$$

Spherical

Ellipsoidal and Cartesian Coordinates: Ellipsoid definition



a: semi major axis
b: semi minor axis
f: flattening
e: eccentricity

$$e^2 = \frac{a^2 - b^2}{a^2}, \quad f = \frac{a - b}{a}$$

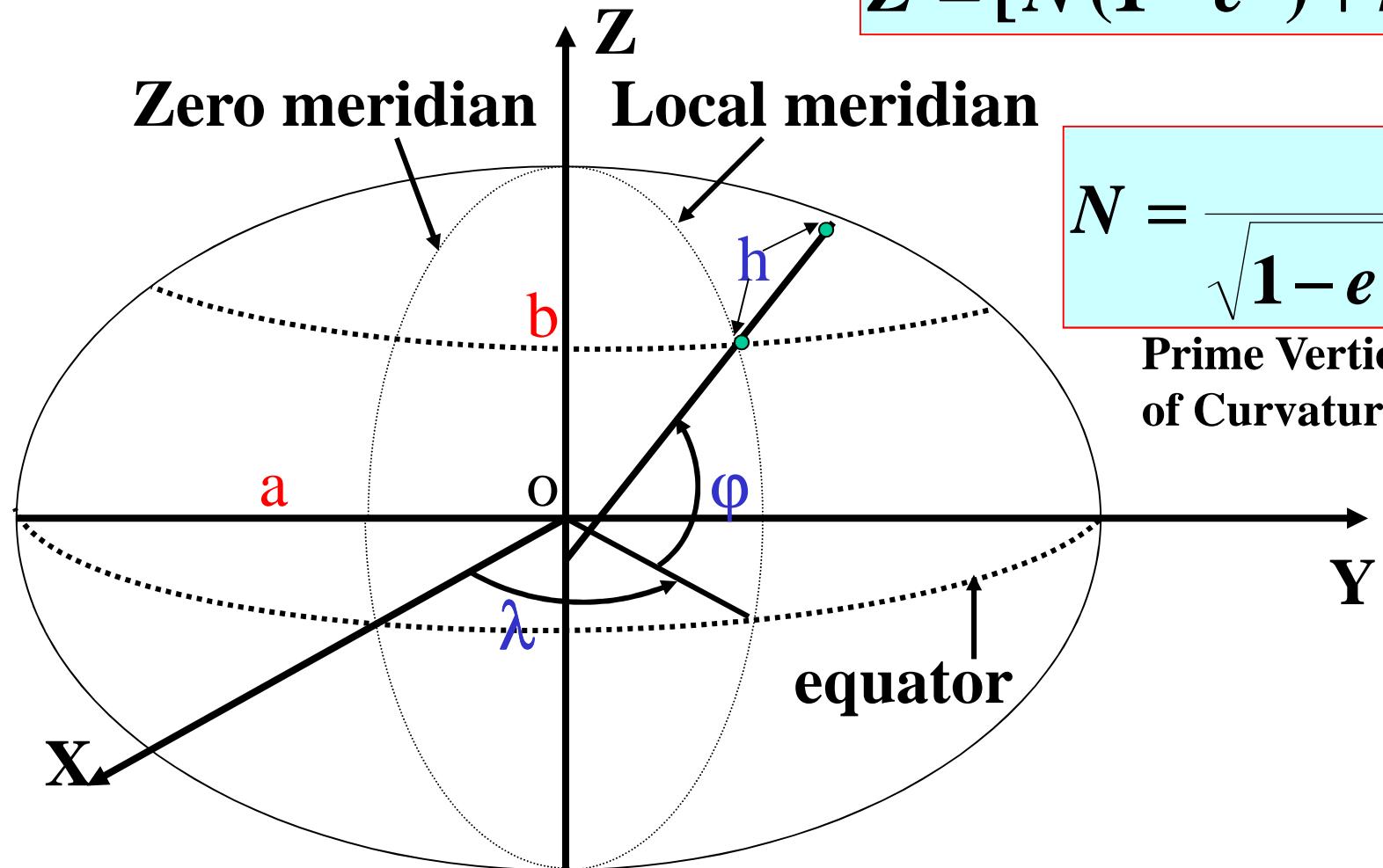
**(a,b), (a,f), or (a,e²) define entirely and
geometrically the ellipsoid**

Ellipsoidal and Cartesian Coordinates

$$X = (N + h) \cos \lambda \cos \varphi$$

$$Y = (N + h) \sin \lambda \cos \varphi$$

$$Z = [N(1 - e^2) + h] \sin \varphi$$



$(X, Y, Z) \Rightarrow (\lambda, \phi, h)$

$$f = 1 - \sqrt{1 - e^2}$$

$$R = \sqrt{X^2 + Y^2 + Z^2}$$

$$\lambda = \arctg\left(\frac{Y}{X}\right)$$

$$\mu = \arctg\left[\frac{Z}{\sqrt{X^2 + Y^2}} \left((1-f) + \left(\frac{e^2 a}{R}\right) \right)\right]$$

$$\phi = \arctg\left[\frac{Z(1-f) + e^2 a \sin^3 \mu}{(1-f)[X^2 + Y^2 - e^2 a \cos^3 \mu]}\right]$$

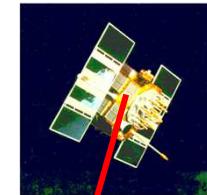
$$h = \sqrt{X^2 + Y^2} [\cos \phi + Z \sin \phi] - a \sqrt{1 - e^2 \sin^2 \phi}$$

Why is a Reference System/Frame needed?

- Precise Orbit Determination for:
 - GNSS: Global Navigation Satellite Systems
 - Other satellite missions: Altimetry, Oceanography, Gravity
- Earth Sciences Applications
 - Tectonic motion and crustal deformation
 - Mean sea level variations
 - Earth rotation
 - ...
- Geo-referencing applications
 - Navigation: Aviation, Terrestrial, Maritime
 - National geodetic systems
 - Cartography & Positioning

What is a Reference Frame in practice?

- Earth fixed/centred RF: allows determination of station location/position **as a function of time**

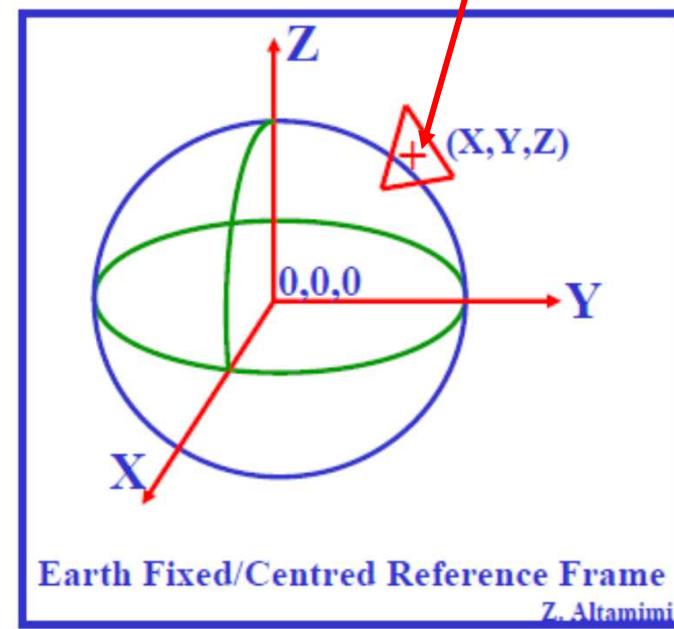


- It seems so simple, but ... we have to deal with:

- Relativity theory
- Forces acting on the satellite
- The atmosphere
- Earth rotation
- Solid Earth and ocean tides
- Tectonic motion
- ...

- Station positions and velocities are now determined with mm and mm/yr precision

Origin, Scale & Orientation



"Motions" of the deformable Earth

- **Nearly linear motion:**
 - Tectonic motion: horizontal
 - Post-Galcial Rebound: Vertical & Horizontal
- **Non-Linear motion:**
 - Seasonal: Annual, Semi & Inter-Annual caused by loading effects
 - Rupture, transient: uneven motion caused by Earthquakes, Volcano Eruptions, etc.

Crust-based TRF

The instantaneous position of a point on Earth Crust at epoch t could be written as :

$$X(t) = X_0 + \dot{X} \cdot (t - t_0) + \sum_i \Delta X_i(t)$$

- X_0 : point position at a reference epoch t_0
 \dot{X} : point linear velocity
 $\Delta X_i(t)$: high frequency time variations:
- Solid Earth, Ocean & Pole tides
- Loading effects: atmosphere, ocean, hydrology,
Post-glacial-Rebound
- ... Earthquakes

Reference Frame Representations

- "Quasi-Instantaneous" Frame: mean station positions at "short" interval:
 - One hour, 6-h, 12-h, one day, one week

==> Non-linear motion embedded in time series of quasi-instantaneous frames
- Long-Term Secular Frame: mean station positions at a reference epoch (t_0) and station velocities: $X(t) = X_0 + V^*(t - t_0)$

Implementation of a TRF

- Definition at a given epoch, by selecting 7 parameters, tending to satisfy the theoretical definition of the corresponding TRS
- A law of time evolution, by selecting 7 rates of the 7 parameters,
assuming linear station motion!
- ==> 14 parameters are needed to define a TRF

How to define the 14 parameters ?

« TRF definition »

- Origin & rate: CoM (Satellite Techniques)
 - Scale & rate: depends on physical parameters
 - Orientation: conventional
 - Orient. Rate: conventional: Geophysical meaning
(Tectonic Plate Motion)
-
- ==> Lack of information for some parameters:
 - Orientation & rate (all techniques)
 - Origin & rate in case of VLBI
 - ==> Rank Deficiency in terms of Normal Eq. System

Implementation of a TRF in practice

The normal equation constructed upon observations of space techniques is written in the form of:

$$N.(\Delta X) = K \quad (1)$$

where $\Delta X = X_{est} - X_{apr}$ are the linearized unknowns

Eq. (1) is a singular system: has a rank deficiency equal to the number of TRF parameters not given by the observations.

Additional constraints are needed:

- Tight constraints $(\sigma \leq 10^{-10}) \text{ m}$
 - Removable constraints $(\sigma \approx 10^{-5}) \text{ m}$
 - Loose constraints $(\sigma \geq 1) \text{ m}$
- Applied over station coordinates
$$(X_{est} - X_{apr}) = 0 \quad (\sigma)$$
-
- Minimum constraints (applied over the TRF parameters, see next)

TRF definition using minimum constraints (1/3)

The standard relation linking two TRFs 1 and 2 is:

$$X_2 = X_1 + A\theta$$

$$X_i = (x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i)^T$$

$$\theta = (T1, T2, T3, D, R1, R2, R3, \dot{T}1, \dot{T}2, \dot{T}3, \dot{D}, \dot{R}1, \dot{R}2, \dot{R}3)^T$$

θ is the vector of the 7 (14) transformation parameters

Least squares adjustment gives for θ :

$$\theta = \underbrace{(A^T A)^{-1}}_{\mathbf{B}} A^T (X_2 - X_1)$$

A : design matrix of partial derivatives given in the next slide

The Design matrix A

14 parameters

7 parameters

$$A = \begin{pmatrix} \dots & \dots \\ 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 & & & & & & \\ 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 & & & & & & 0 \\ 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 & & & & & & \\ & & & & & & & 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\ & & & \approx 0 & & & & 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\ & & & & & & & 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \\ & & & & & & & \dots \end{pmatrix}$$

Note: A could be reduced to specific parameters. E.g. if only rotations and rotation rates are needed, then the first 4 columns of the two parts of A are deleted

TRF definition using minimum constraints (2/3)

- The equation of minimum constraints is written as:

$$B(X_2 - X_1) = 0 \quad (\Sigma_\theta)$$

It nullifies the 7 (14) transformation parameters between TRF 1 and TRF 2 at the Σ_θ level

- The normal equation form is written as:

$$B^T \Sigma_\theta^{-1} B (X_2 - X_1) = 0$$

Σ_θ is a diagonal matrix containing small variances of the 7(14) parameters, usually at the level of 0.1 mm

TRF definition using minimum constraints (3/3)

Considering the normal equation of space geodesy:

$$N_{nc}(\Delta X) = K \quad (1)$$

where $\Delta X = X_{est} - X_{apr}$ are the linearized unknowns

Selecting a reference solution X_R , the equation of minimal constraints is given by:

$$B^T \Sigma_\theta^{-1} B(\Delta X) = B^T \Sigma_\theta^{-1} B(X_R - X_{apr}) \quad (2)$$

Accumulating (1) and (2), we have:

$$(N_{nc} + B^T \Sigma_\theta^{-1} B)(\Delta X) = K + B^T \Sigma_\theta^{-1} B(X_R - X_{apr})$$

Note: if $X_R = X_{apr}$, the 2nd term of the right-hand side vanishes

Combination of daily or weekly TRF solutions (1/3)

The basic combination model is written as:

$$X_s^i = X_c^i + T_s + D_s X_c^i + R_s X_c^i$$

Inputs: X_s^i , coordinates of point i of individual solution s .

Outputs (unknowns): combined coordinates X_c^i and transformation parameters T_s, D_s, R_s from TRF s to TRF c .

Note that the translation vector T_s and the rotation matrix R_s have each three components around the three axes X, Y, Z .

The unknown parameters are linearized around their approximate values: x_0^i, y_0^i, z_0^i , so that $x_c^i = x_0^i + \delta x^i$ (respectively y_c^i, z_c^i).

Note: this combination model is valid at a give epoch, t_s , for both the input and output station coordinates

Combination of daily or weekly TRF solutions (2/3)

The observation equation system is written as:

$$(I \quad A_s) \begin{pmatrix} \delta\chi_s \\ \delta T_s \end{pmatrix} + B_s = V_s$$

and the normal equation is:

$$\begin{pmatrix} P_s & P_s A_s \\ A_s^T P_s & A_s^T P_s A_s \end{pmatrix} \begin{pmatrix} \delta\chi_s \\ \delta T_s \end{pmatrix} + \begin{pmatrix} P_s B_s \\ A_s^T P_s B_s \end{pmatrix} = 0$$

where I is the identity matrix, A_s is the design matrix related to solution s , $\delta\chi_s$ and δT_s are the linearized unknowns of station coordinates and transformation parameters, respectively. B_s are the (observed - computed) values and V_s are the residuals. P_s : weight matrix = Σ_s^{-1} : inverse of variance-covariance matrix.

Combination of daily or weekly TRF solutions (3/3)

The design matrix A_s has the following form:

$$A_s = \begin{pmatrix} \cdot & \cdot \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i & \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i & \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 & \\ \cdot & \cdot \end{pmatrix}$$

Definition of the combined TRF

- The normal equation system described in the previous slides is singular and has a rank deficiency of 7 parameters.
- The 7 parameters are the defining parameters of the combined TRF c : origin (3 components), scale (1 component) and orientation (3 components).
- The combined TRF c , could be defined by, e.g.:
 - Fixing to given values 7 parameters among those to be estimated
 - Using minimum constraint equation over a selected set of stations of a reference TRF solution X_R . Refer to slide 24 for more details...

Combination of long-term TRF solutions

The basic combination model is extended to include station velocities and is written as:

$$\begin{cases} X_s^i = X_c^i + T_s + D_s X_c^i + R_s X_c^i \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_s + \dot{D}_s X_c^i + \dot{R}_s X_c^i \end{cases}$$

where the dotted parameters are their time derivatives.

Inputs: X_s^i , position of point i , at epoch t_s and velocities, \dot{X}_s^i , of individual solution s .

Outputs: combined positions X_c^i , at epoch t_s , velocities and transformation parameters T_s, D_s, R_s , at epoch t_s , from TRF s to TRF c .

In the same way as for daily or weekly TRF combination, observation and normal equations could easily be derived.

Note: this combination model is only valid at a give epoch, both for the input and output station coordinates

Stacking of TRF time series

The basic combination model is written as:

$$X_s^i = X_c^i(t_0) + (t_s - t_0)\dot{X} + T_s + D_s X_c^i + R_s X_c^i$$

Inputs: Time series of station positions, X_s^i , at different epochs t_s .

Outputs: combined positions X_c^i at epoch t_0 , velocities and transformation parameters T_s, D_s, R_s from TRF s to TRF c .

Here also, observation and normal equations are constructed and solved by least squares adjustment.

Space Geodesy Techniques

- Very Long Baseline Interferometry (**VLBI**)
 - Lunar Laser Ranging (**LLR**)
 - Satellite Laser Ranging (**SLR**)
 - DORIS
 - GNSS: **GPS, GLONASS, GALILEO, COMPASS,**
...
-
- Local tie vectors at **co-location sites**

Complex of Space Geodesy instruments



SLR/LLR



VLBI



GPS



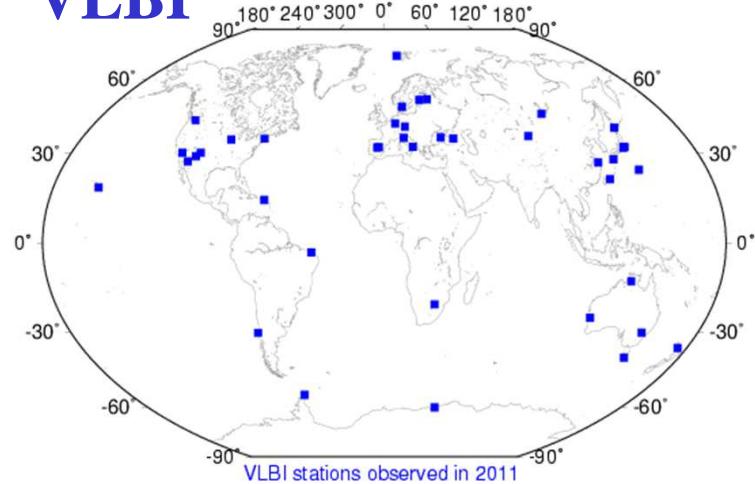
DORIS

Reference frame definition by individual techniques

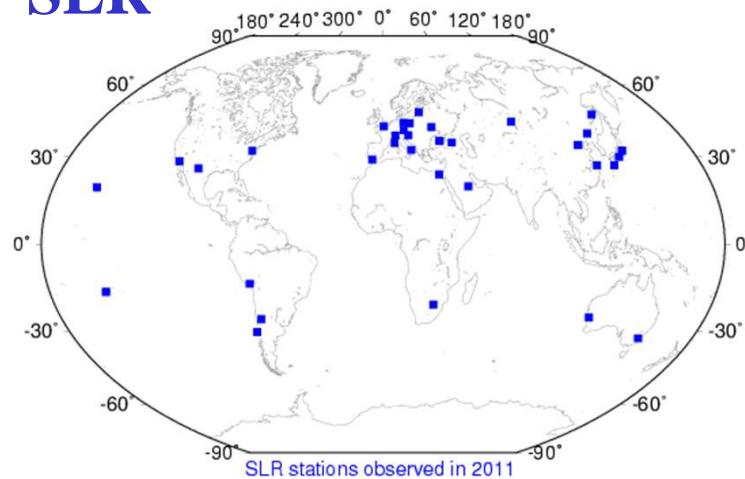
	Satellite Techniques	VLBI
Origin	Center of Mass	-
Scale	GM, c & Relativistic corrections	c Relativistic corrections
Orientation	Conventional	Conventional

Current networks: stations observed in 2011

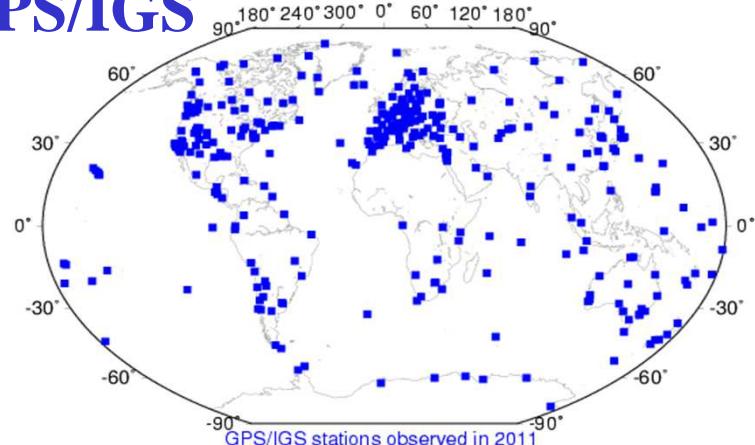
VLBI



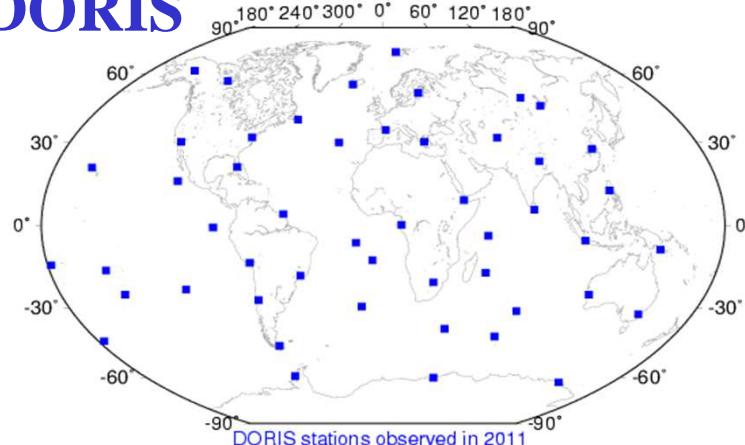
SLR



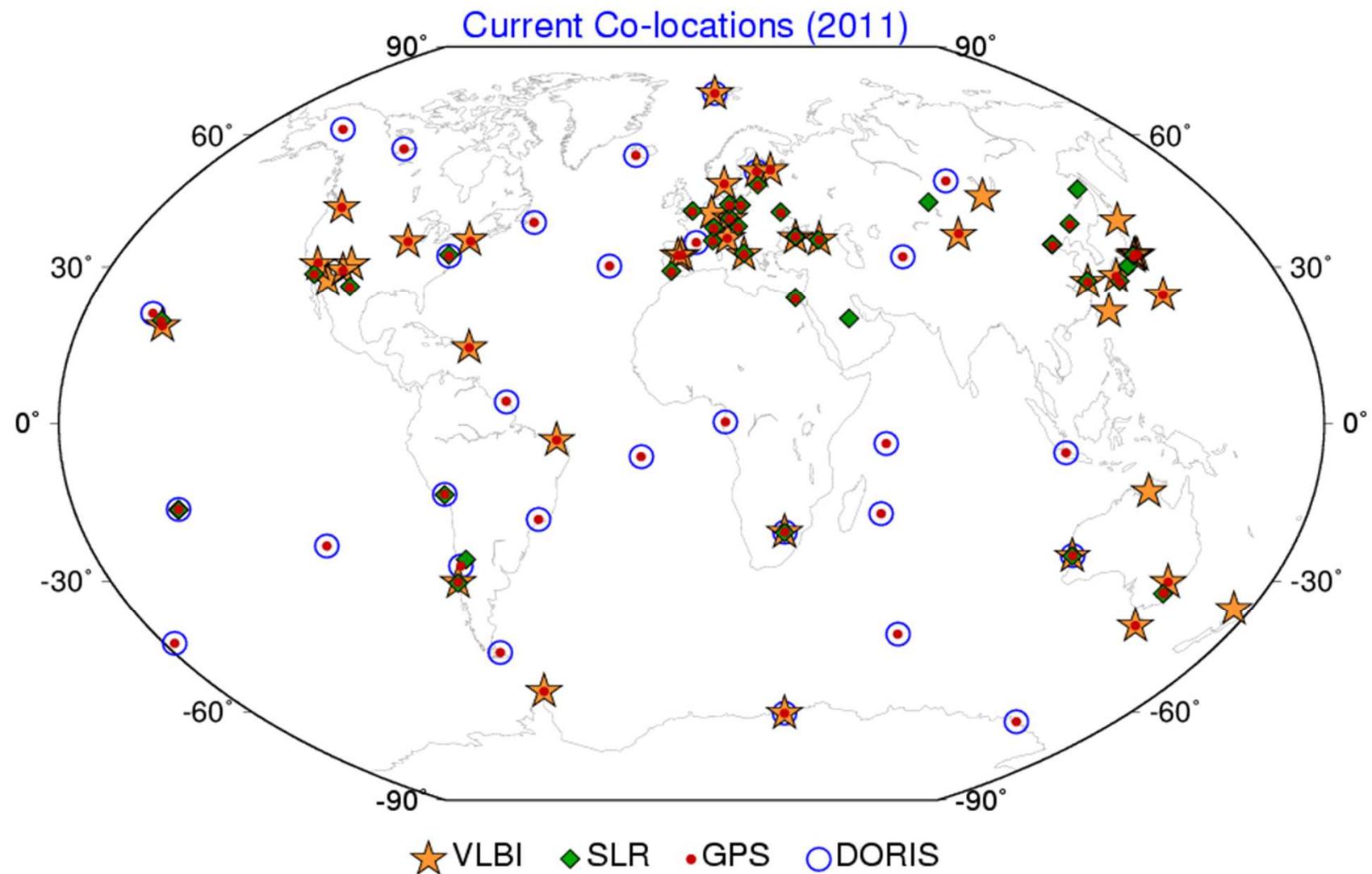
GPS/IGS



DORIS



Current Co-locations (2011)



International Association of Geodesy International Services

- International Earth Rotation and Reference Systems Service (IERS) (1988)
- Intern. GNSS Service (IGS) (1994)
- Intern. Laser Ranging Service (ILRS) (1998)
- Intern. VLBI Service (IVS) (1999)
- Intern. DORIS Service (IDS) (2003)

<http://www.iag-aig.org/>



International Terrestrial Reference System (ITRS)

Realized and maintained by the IERS



International Earth Rotation and Reference Systems Service (IERS)

Established in 1987 (started Jan. 1, 1988) by IAU and IUGG
to realize/maintain/provide:

- The International Celestial Reference System (ICRS)
- The International Terrestrial Reference System (ITRS)
- Earth Orientation Parameters (EOP)
- Geophysical data to interpret time/space variations in the ICRF, ITRF & EOP
- Standards, constants and models (i.e., conventions)

<http://www.iers.org/>

International Terrestrial Reference System (ITRS): Definition (IERS Conventions)

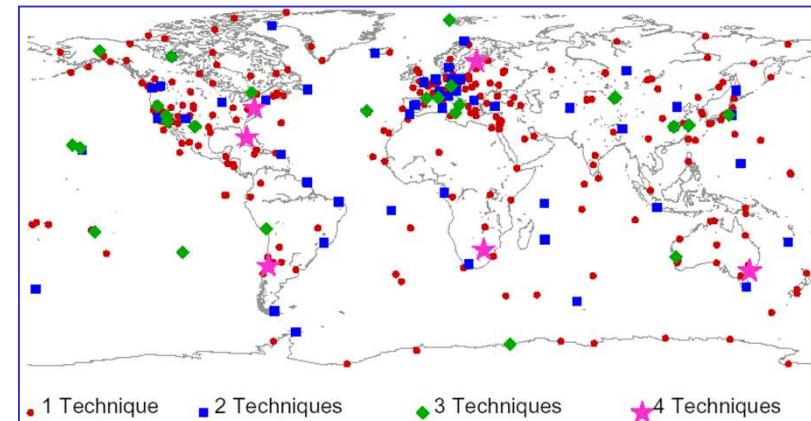
- **Origin:** Center of mass of the whole Earth, including oceans and atmosphere
- **Unit of length:** meter SI, consistent with TCG (Geocentric Coordinate Time)
- **Orientation:** consistent with BIH (Bureau International de l'Heure) orientation at 1984.0.
- **Orientation time evolution:** ensured by using a No-Net-Rotation-Condition w.r.t. horizontal tectonic motions over the whole Earth

$$h = \int_C \mathbf{X} \times \mathbf{V} dm = 0$$

International Terrestrial Reference System (ITRS)

- Realized and maintained by ITRS Product Center of the IERS
- Its realization is called International Terrestrial Reference Frame (ITRF)
- Set of station positions and velocities, estimated by combination of VLBI, SLR, GPS and DORIS individual TRF solutions
- Based on Co-location sites

Adopted by IUGG in 1991 for all Earth Science Applications



More than 800 stations located on more than 500 sites

Available: ITRF88,...,2000,
2005
Latest : ITRF2008

<http://itrf.ign.fr>

Co-location site

- Site where two or more instruments are operating
- Surveyed in three dimensions, using classical or GPS geodesy
- Differential coordinates (DX, DY, DZ) are available

$$DX_{(GPS, VLBI)} = X_{VLBI} - X_{GPS}$$

SLR/LLR



VLBI

GNSS



DORIS



Strengths : Contribution of Geodetic Techniques to the ITRF

Mix of techniques is fundamental to realize a frame that is stable in origin, scale, and with sufficient coverage

Technique Signal Source Obs. Type	VLBI Microwave Quasars Time difference	SLR Optical Satellite Two-way absolute range	GPS Microwave Satellites Range change	DORIS
Celestial Frame & UT1	Yes	No	No	No
Polar Motion	Yes	Yes	Yes	Yes
Scale	Yes	Yes	No (but maybe in the future!)	Yes
Geocenter ITRF Origin	No	Yes	Future	Future
Geographic Density	No	No	Yes	Yes
Real-time & ITRF access	Yes	Yes	Yes	Yes
Decadal Stability	Yes	Yes	Yes	Yes

How is the ITRF constructed ?

- **Input :**
 - Time series of mean station positions (at weekly or daily sampling) and daily EOPs from the 4 techniques
 - Local ties in co-location sites

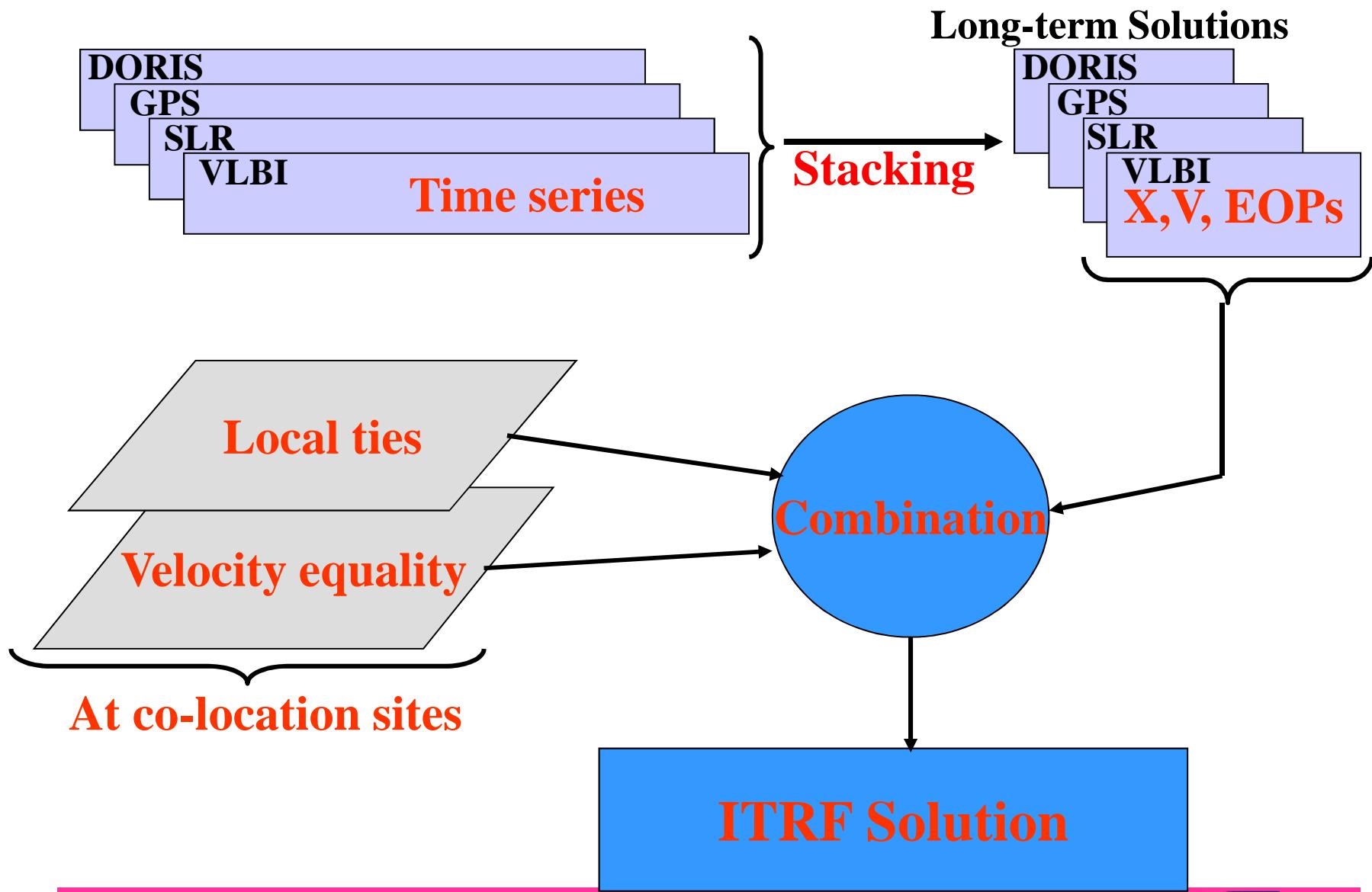
- **Output :**
 - Station positions at a reference epoch and linear velocities
 - Earth Orientation Parameters

CATREF combination model

$$\left\{ \begin{array}{l} X_s^i = X_c^i + (t_s^i - t_0) \dot{X}_c^i \\ \quad + T_k + D_k X_c^i + R_k X_c^i \\ \quad + (t_s^i - t_k) [\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i] \\ \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{array} \right.$$

$$\left\{ \begin{array}{lcl} x_s^p & = & x_c^p + R2_k \\ y_s^p & = & y_c^p + R1_k \\ UT_s & = & UT_c - \frac{1}{f} R3_k \\ \dot{x}_s^p & = & \dot{x}_c^p \\ \dot{y}_s^p & = & \dot{y}_c^p \\ LOD_s & = & LOD_c \end{array} \right.$$

ITRF Construction



SINEX Format

```
%=SNX 2.01 IGN 10:157:00000 IGN 04:003:00000 09:005:00000 C 01308 2 X V
```

```
*
```

+SITE/ID

```
*CODE PT __DOMES__ T _STATION DESCRIPTION_ APPROX_LON_ APPROX_LAT_ _APP_H_
ANKR A 20805M002 Ankara, Turkey 32 45 30.4 39 53 14.5 976.0
```

```
...
```

+SOLUTION/EPOCHS

```
*Code PT SOLN T Data_start_ Data_end_ Mean_epoch_
ANKR A 5 C 04:003:00000 08:133:00000 06:067:43200
```

```
...
```

+SOLUTION/ESTIMATE

```
*INDEX TYPE_ CODE PT SOLN _REF_EPOCH_ UNIT S __ESTIMATED VALUE____ _STD_DEV____
```

```
.....
```

19	STAX	ANKR	A	5	06:183:00000	m	2	0.412194852609284E+07	0.17234E-03
20	STAY	ANKR	A	5	06:183:00000	m	2	0.265218790321918E+07	0.12249E-03
21	STAZ	ANKR	A	5	06:183:00000	m	2	0.406902377621100E+07	0.16467E-03
22	VELX	ANKR	A	5	06:183:00000	m/y	2	-.668839830148651E-02	0.14215E-03
23	VELY	ANKR	A	5	06:183:00000	m/y	2	-.270320979559104E-02	0.10069E-03
24	VELZ	ANKR	A	5	06:183:00000	m/y	2	0.971313341105308E-02	0.13542E-03

```
.....
```

+SOLUTION/MATRIX_ESTIMATE L COVA

```
*PARA1 PARA2 __PARA2+0_____ __PARA2+1_____ __PARA2+2_____
```

```
1 1 0.150471439320574E-06
```

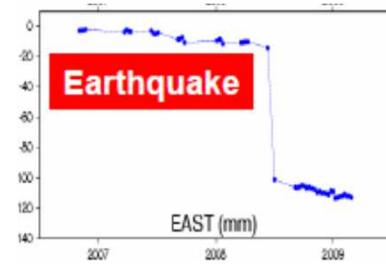
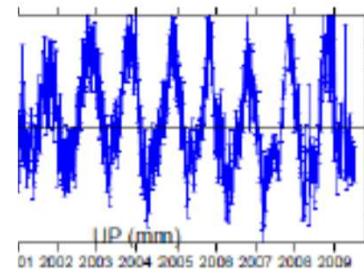
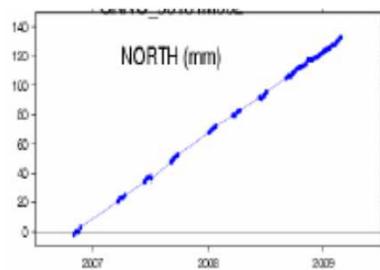
```
2 1 -.140657602040892E-06 0.176947767515801E-06
```

```
3 1 -.115071650206259E-06 0.127287839143953E-06 0.122184056413112E-06
```



Power of station position time series

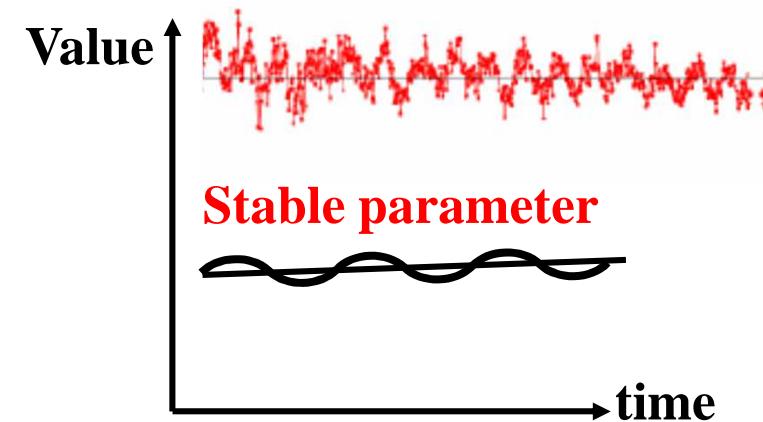
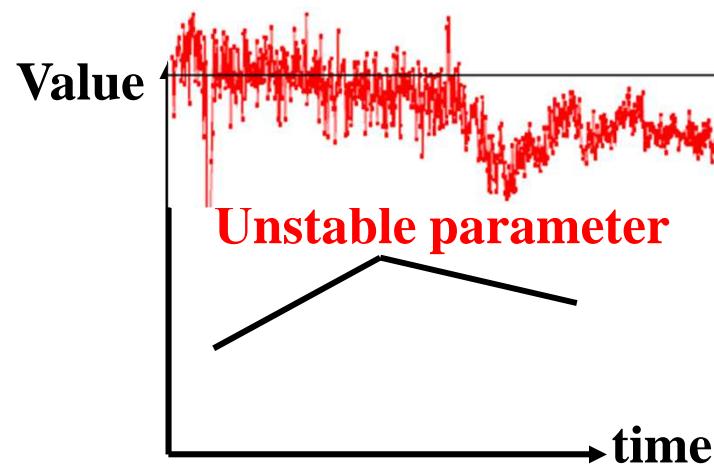
- Monitor station behavior
 - Linear, non-linear (seasonal), and discontinuities



- Monitor time evolution of the frame physical parameter (origin and scale)
- Estimate a robust long-term secular frame

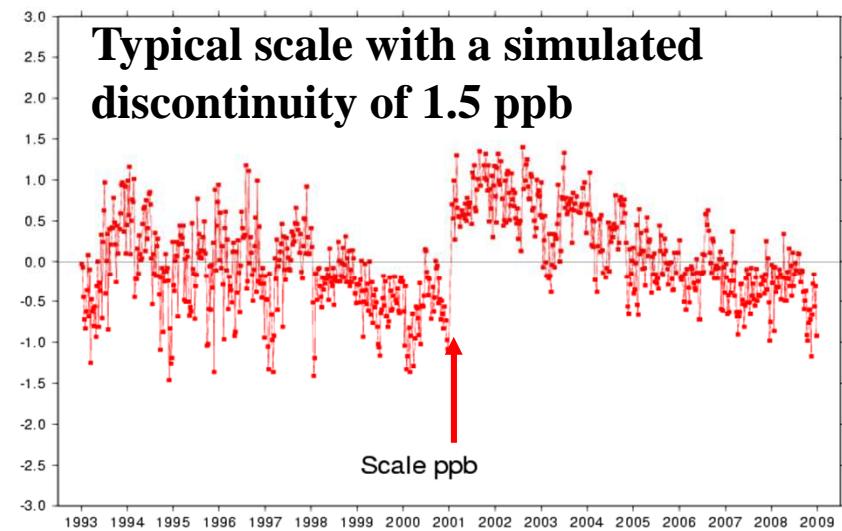
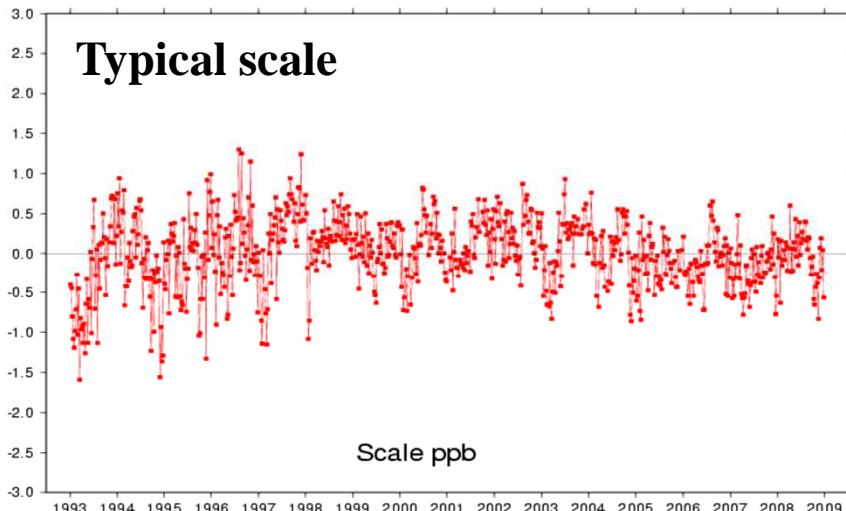
ITRF and Science Requirement

- Long-term **stable** ITRF: 0.1 mm/yr
==> **Stable**: linear behaviour of the TRF parameters, i.e. with no discontinuity :
 - Origin Components: 0.1 mm/yr
 - Scale 0.01 ppb/yr (0.06 mm/yr)

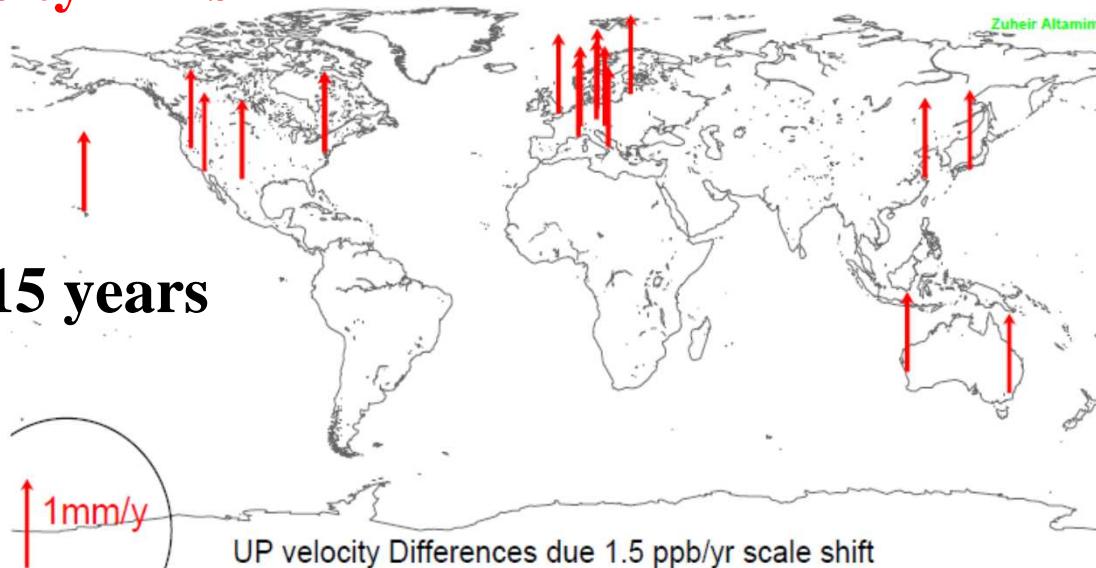


But stability also means TRF site position predictability

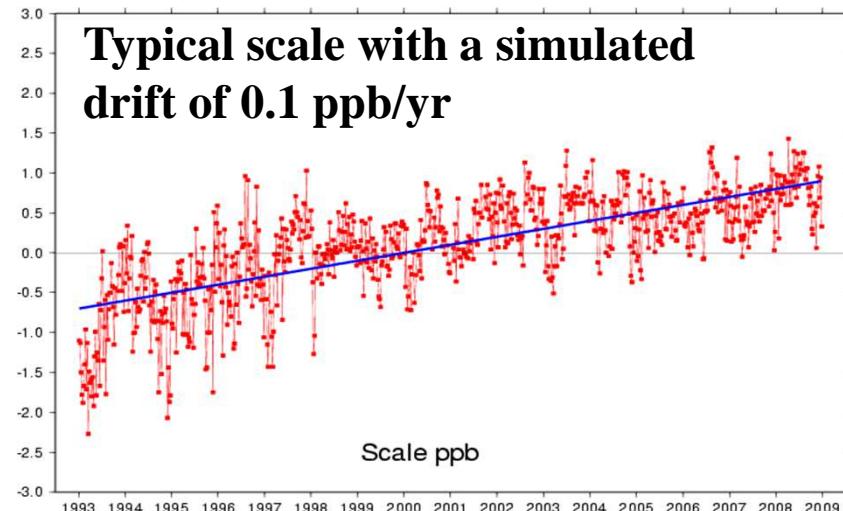
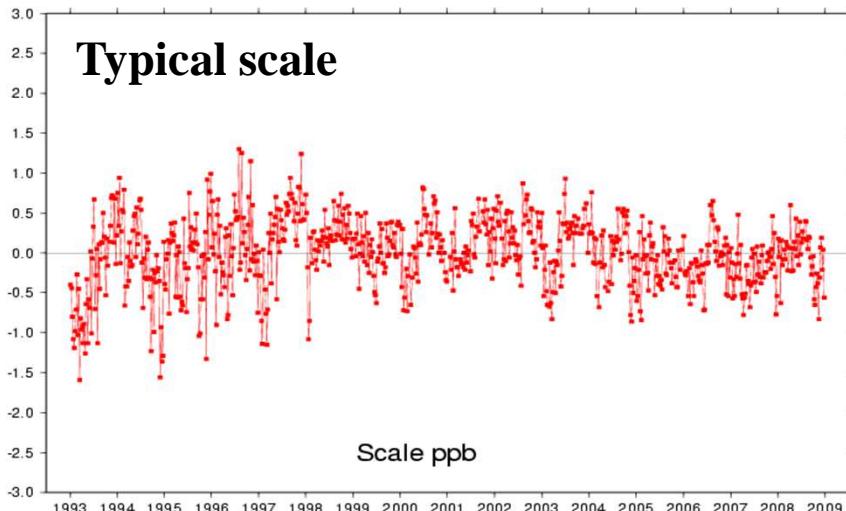
Impact of 1.5 ppb scale discontinuity



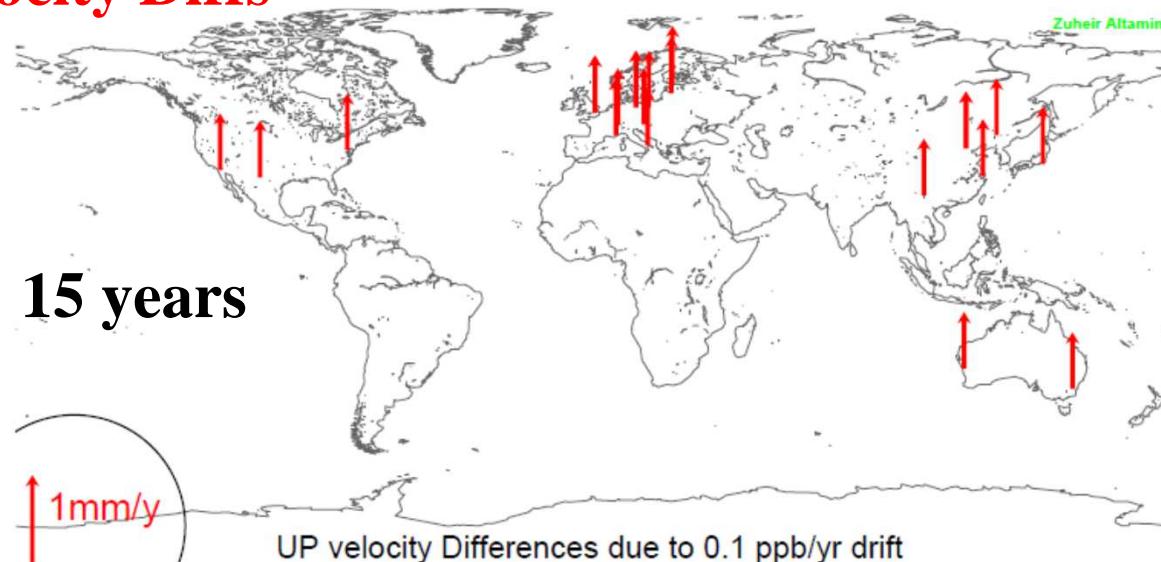
Vertical velocity Diffs



Impact of 0.1 ppb/yr scale drift



Vertical velocity Diffs

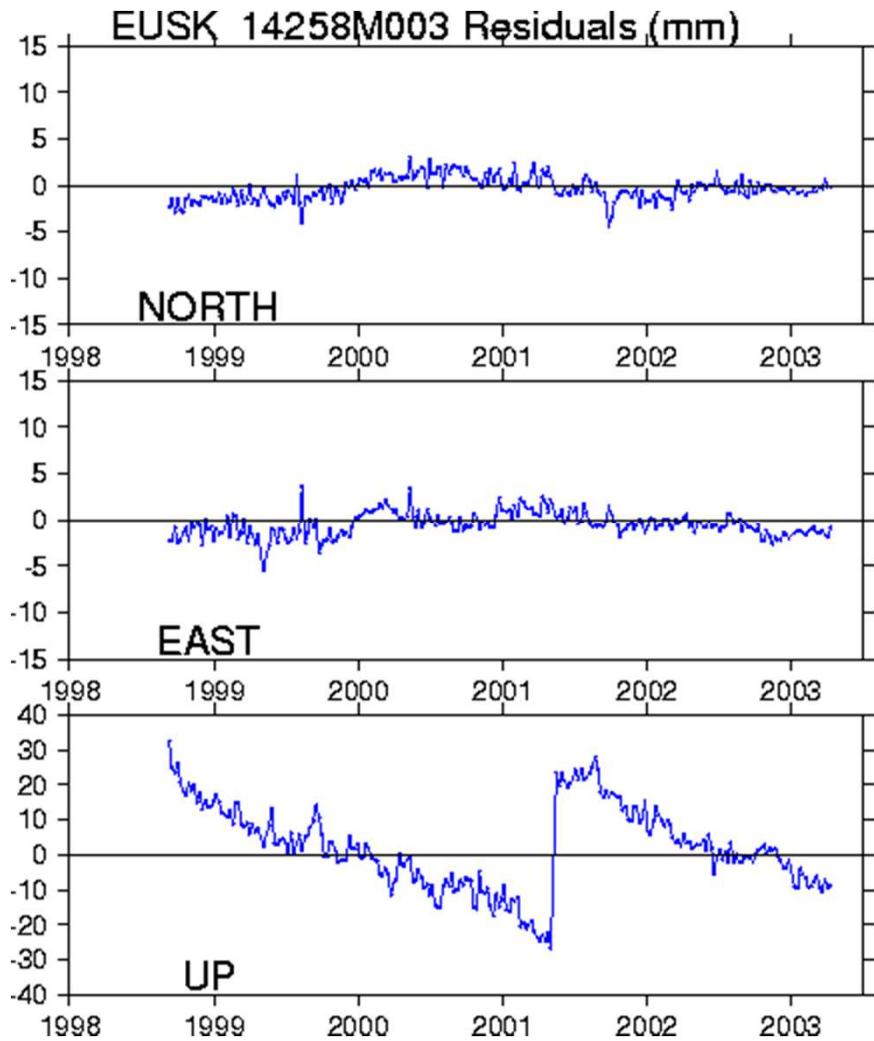


Data-span: 15 years

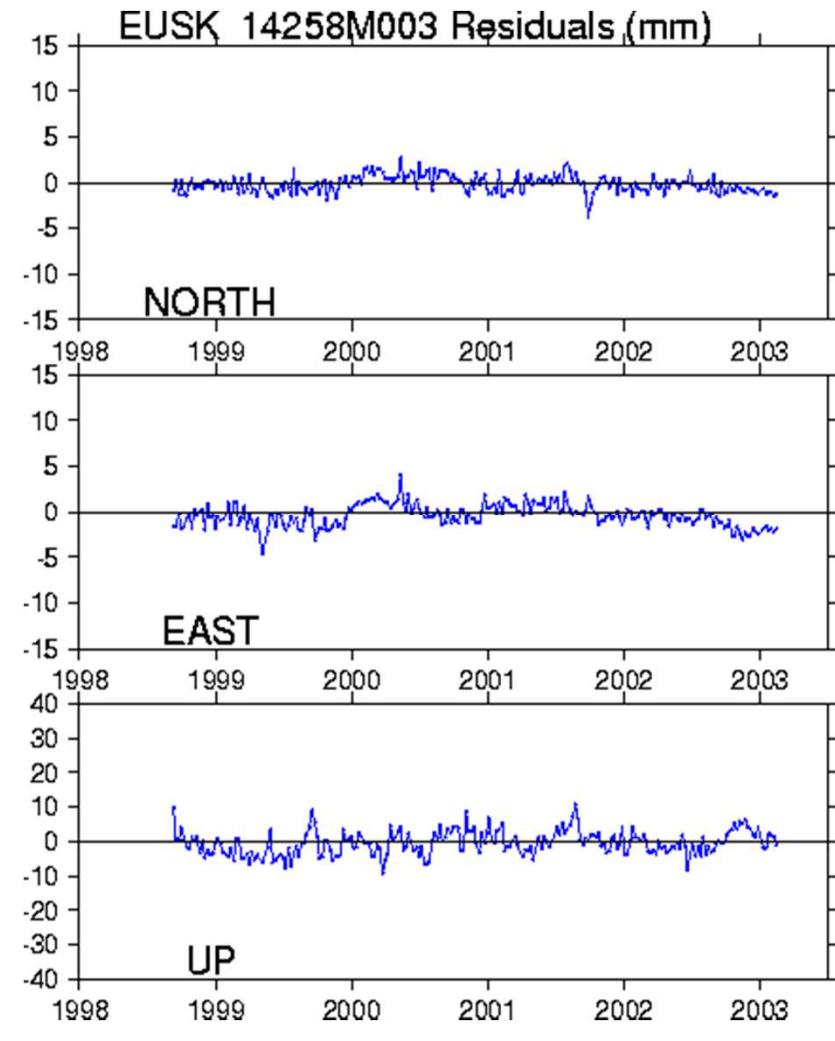
Some examples of discontinuities and seasonal variations

Discontinuity due to equipment change

Before

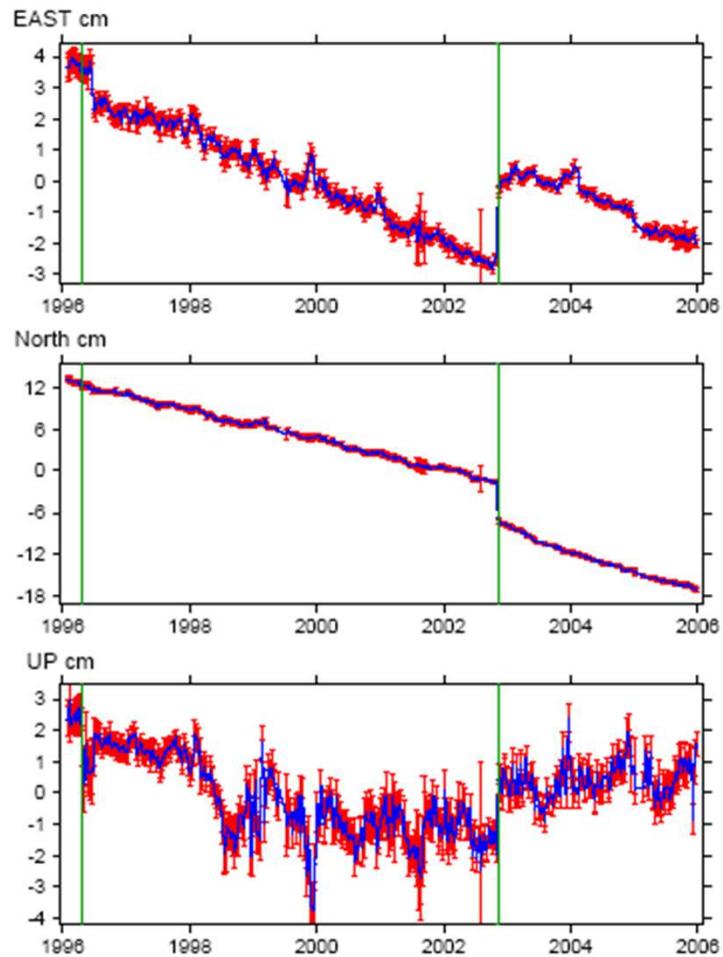


After

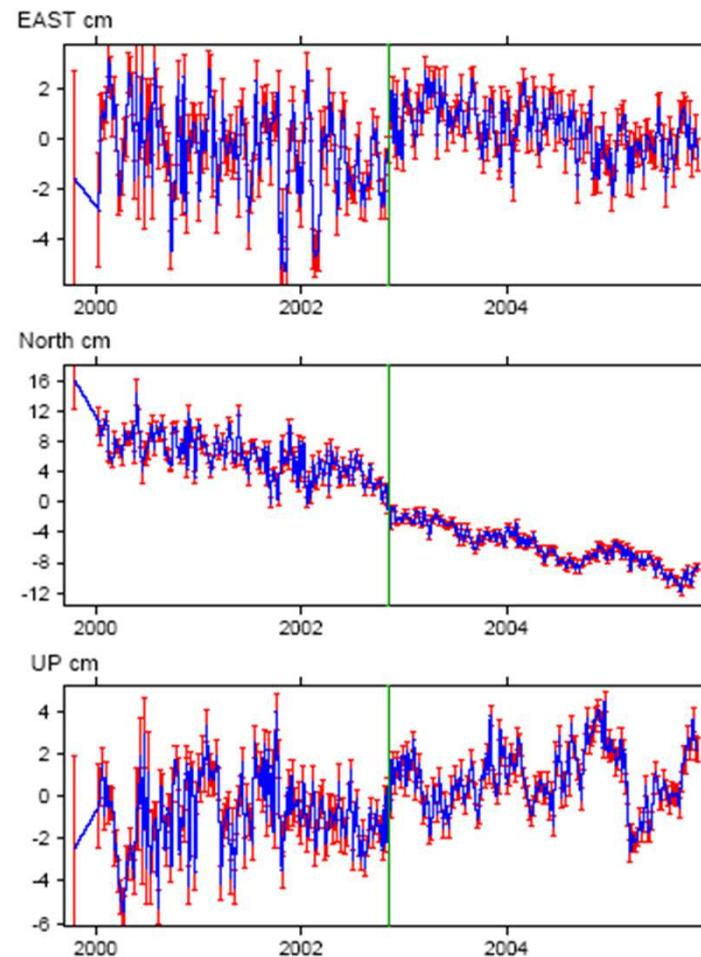


Denali Earthquake (Alaska)

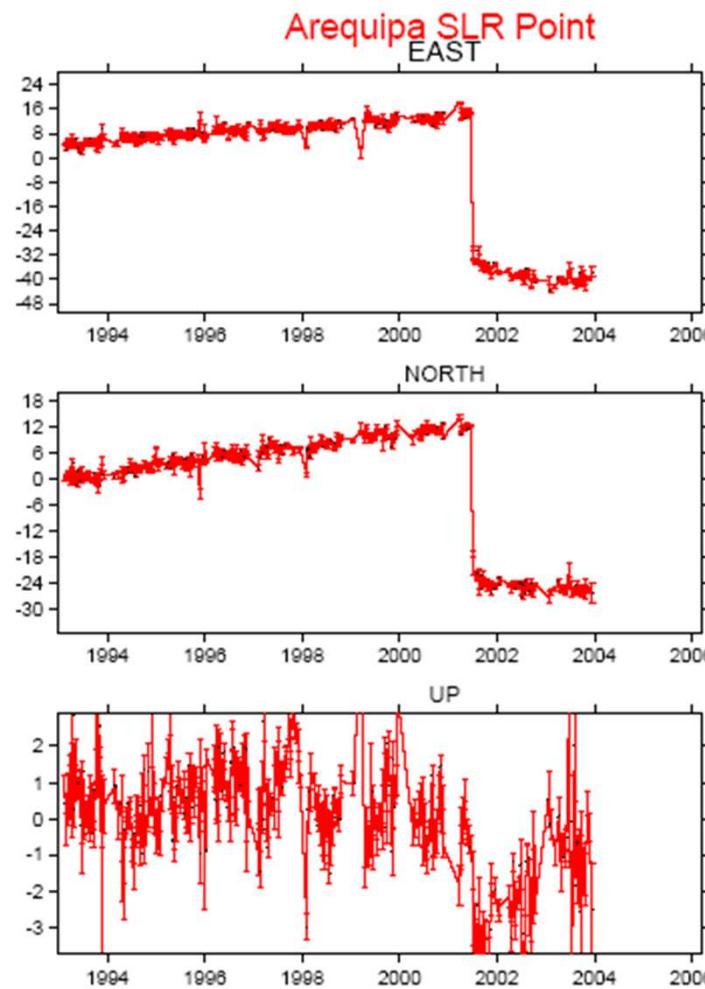
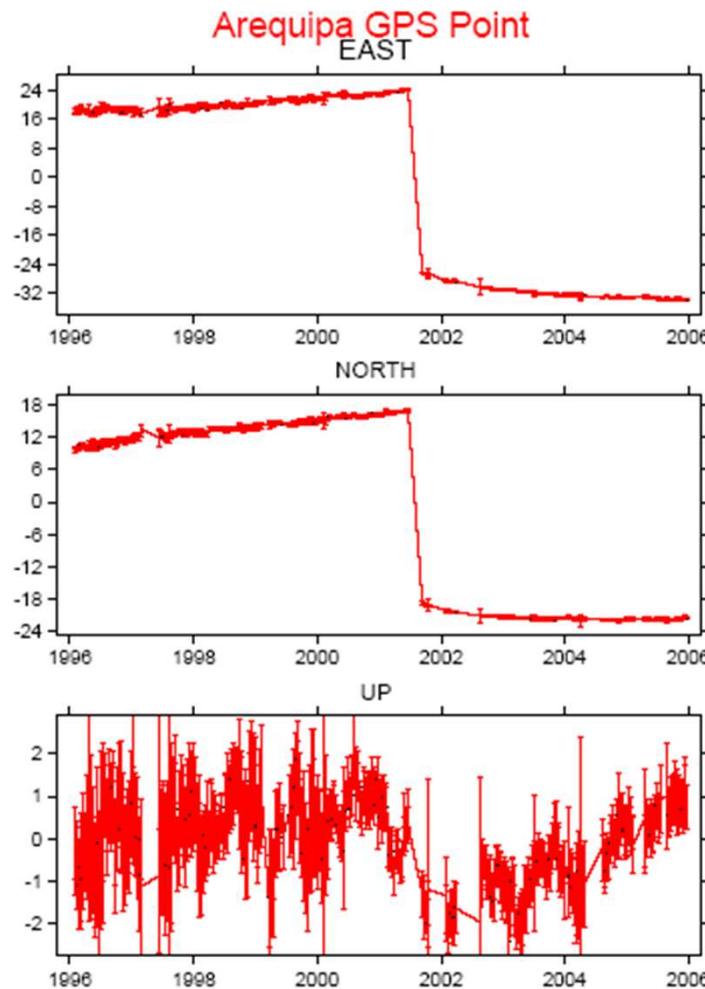
GPS



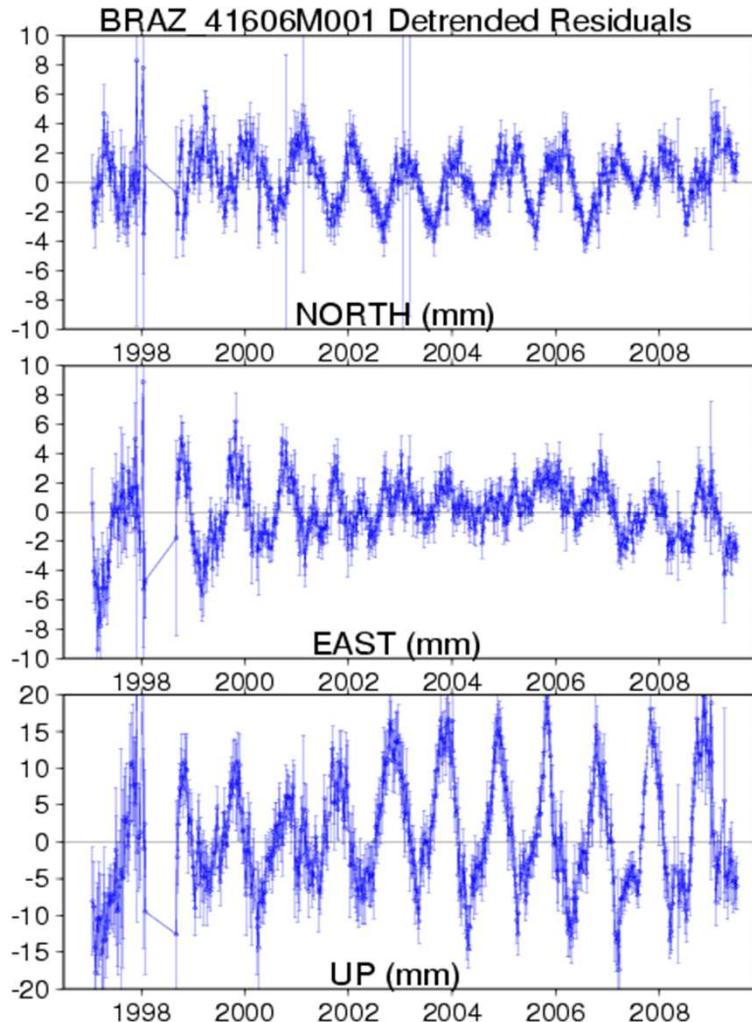
DORIS



Arequipa Earthquake



Example of seasonal variations BRAZ GPS antenna



ITRF2008

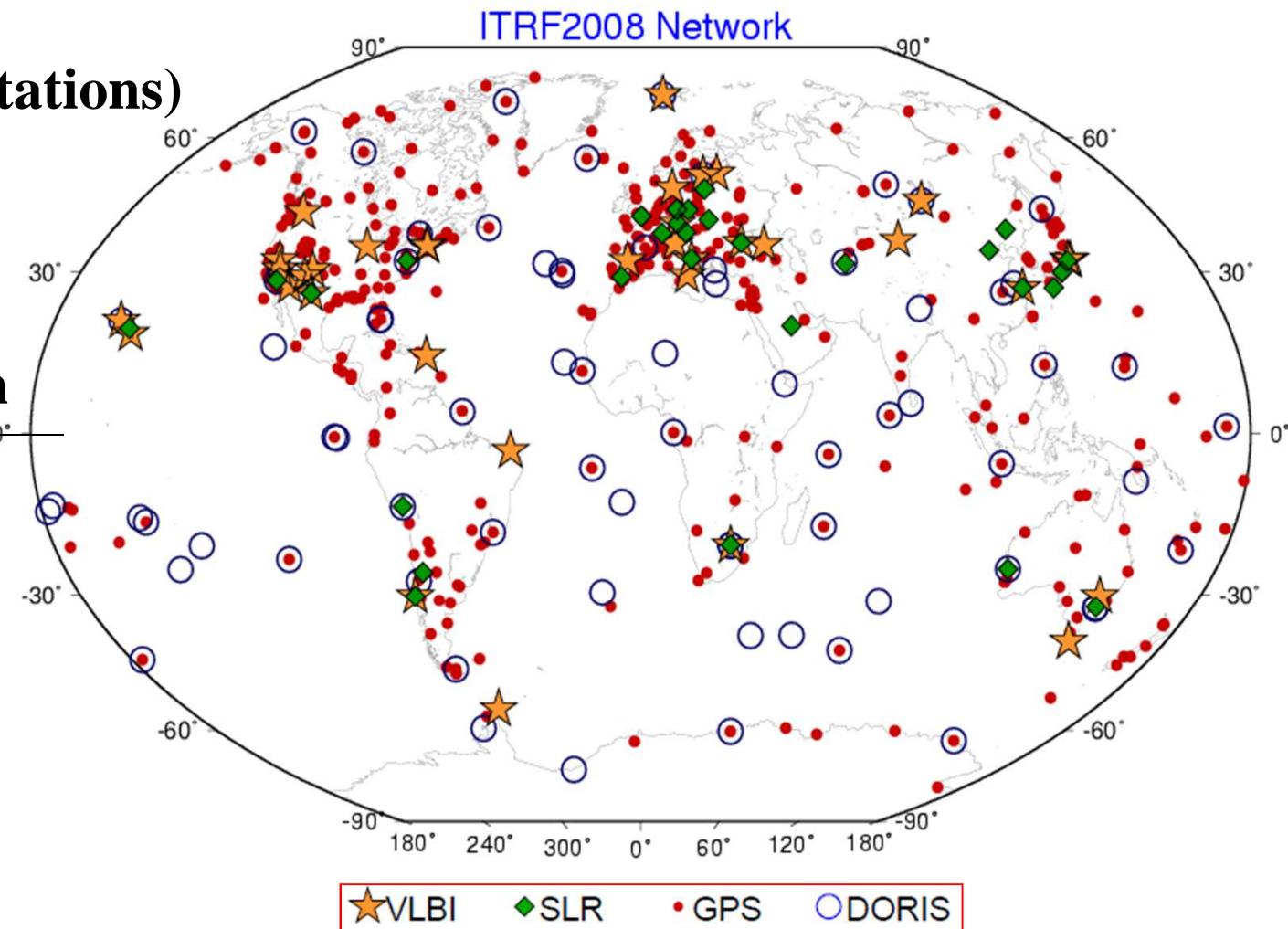
- Time Series of Station Positions :
 - Daily (VLBI)
 - Weekly (GPS, SLR & DORIS)
- and Earth Orientation Parameters:
 - Polar Motion (x_p, y_p)
 - Universal Time (UT1) (Only from VLBI)
 - Length of Day (LOD) (Only from VLBI)

ITRF2008 Network

580 sites (920 stations)

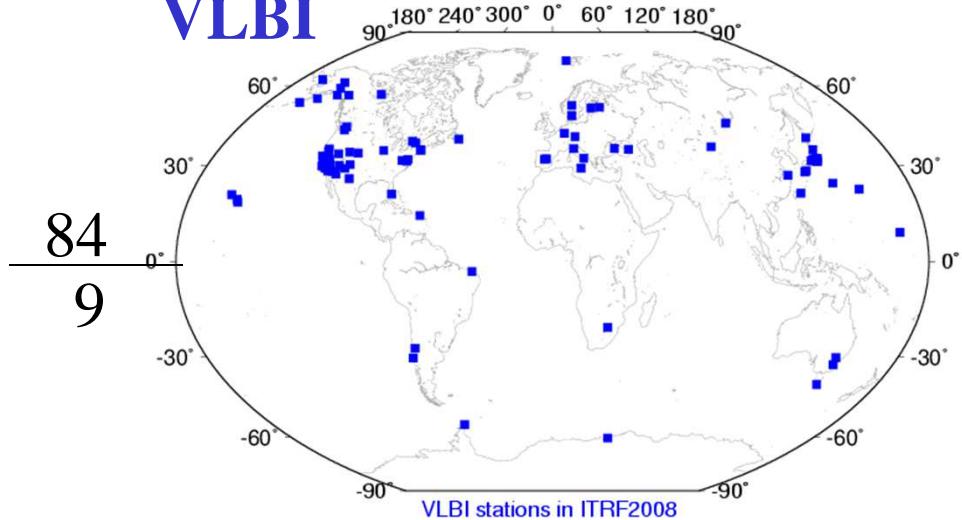
461 Sites North

118 Sites South

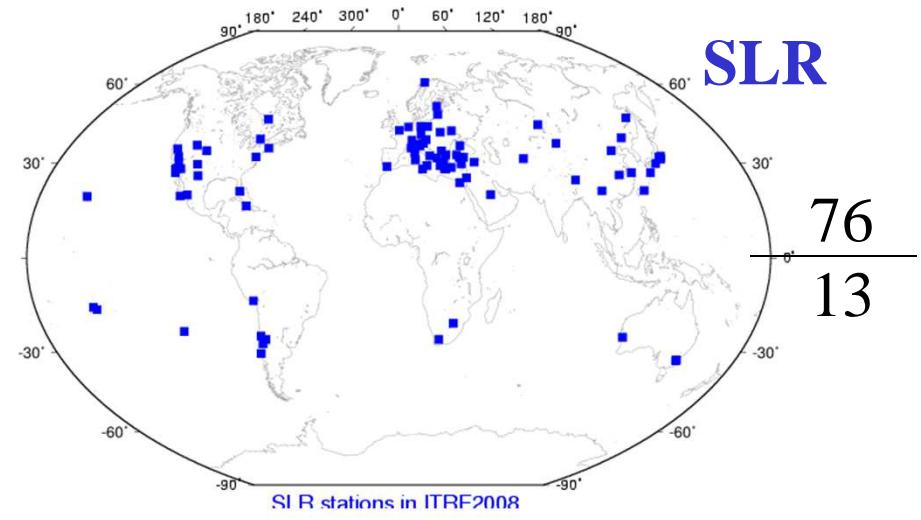


ITRF2008: Site distribution per technique

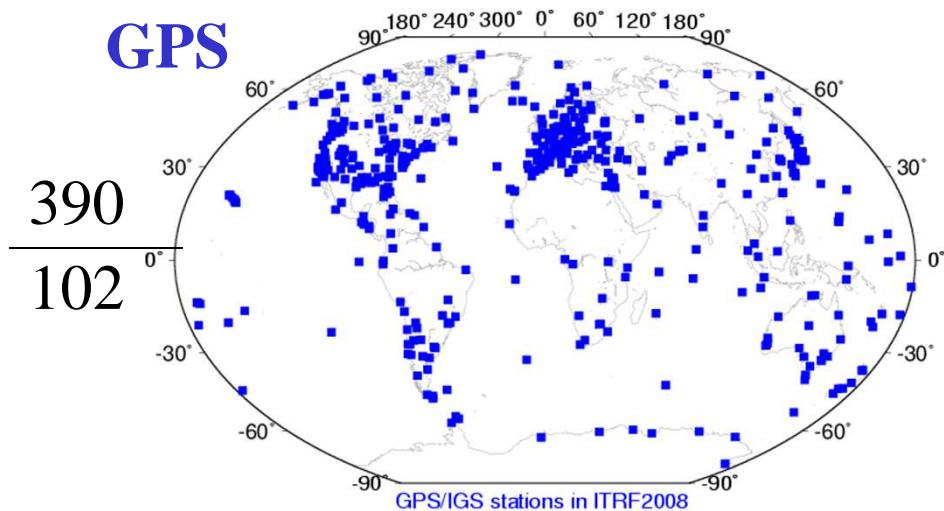
VLBI



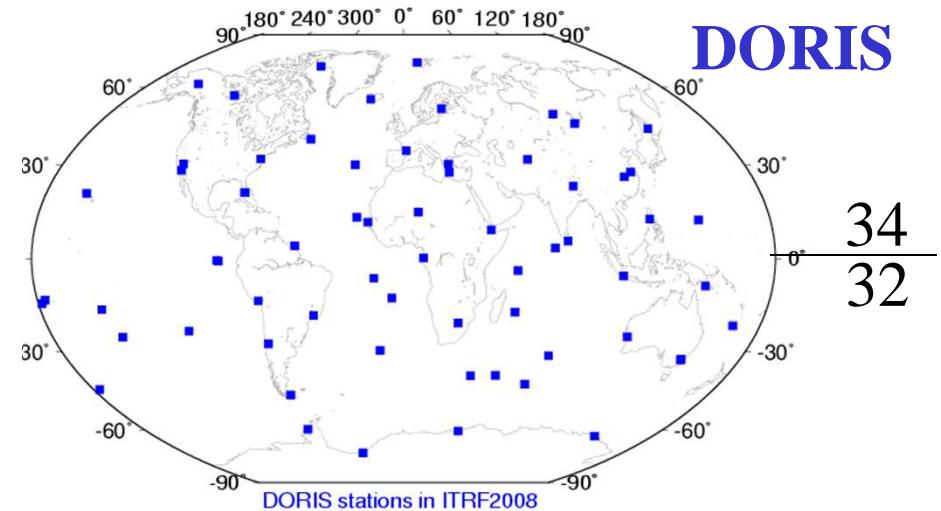
SLR



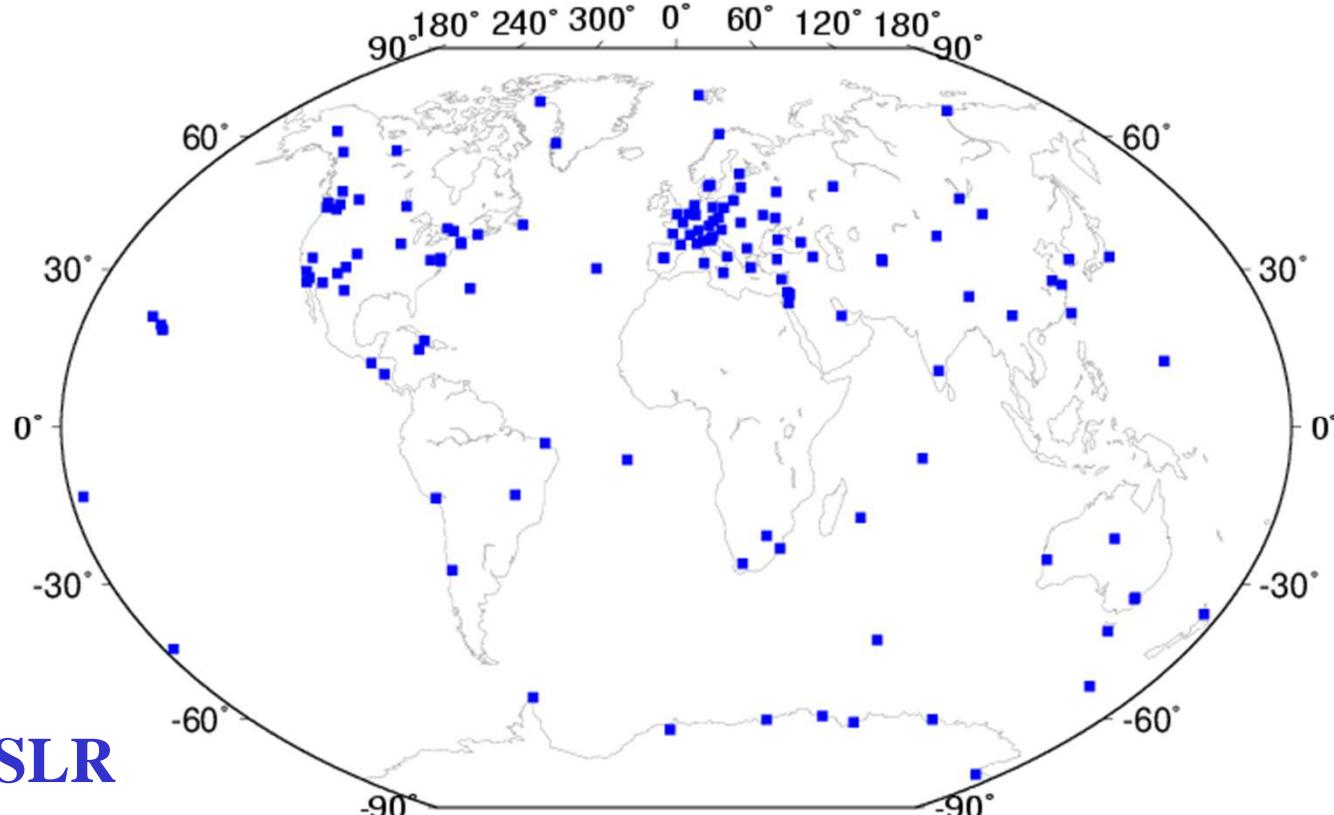
GPS



DORIS



ITRF2008 Datum Specification

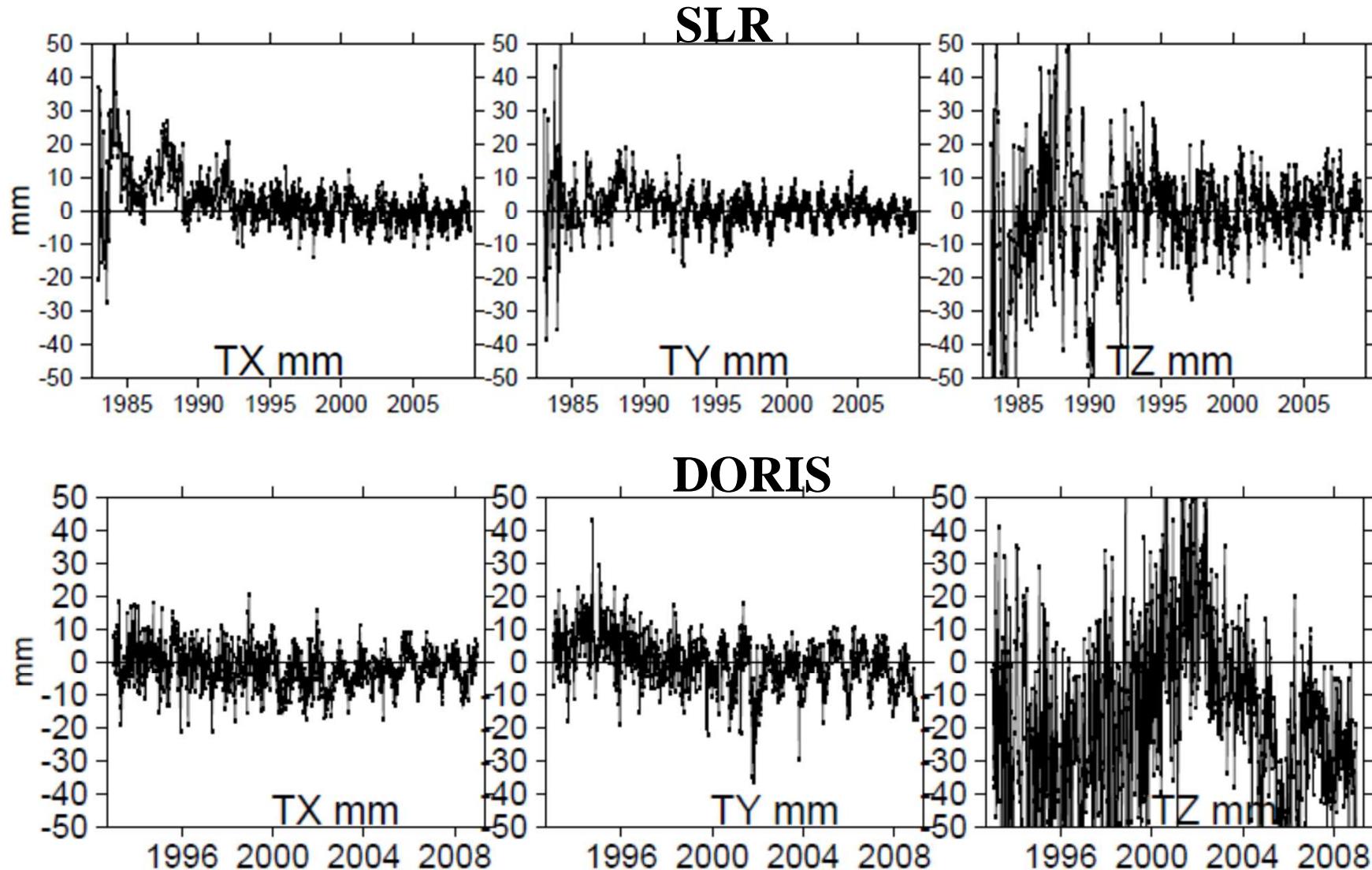


- Origin: SLR
- Scale : Mean of SLR & VLBI
- Orientation: Aligned to ITRF2005

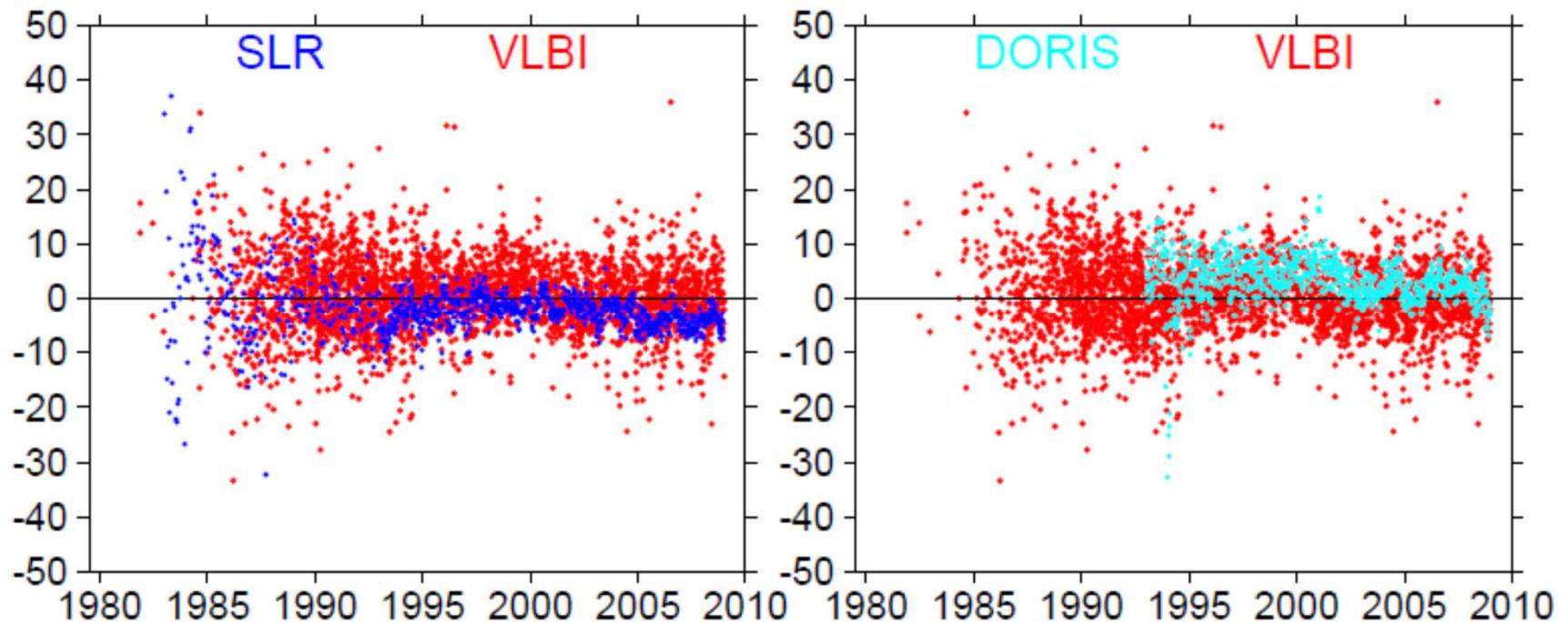
using 179 stations located at 131 sites:

104 at northern hemisphere and 27 at southern hemisphere

SLR & DORIS origin components wrt ITRF2008



Scales wrt ITRF2008



Transformation Param Fm ITRF2008 To ITRF2005

Tx	Ty	Tz	Scale
mm	mm	mm	ppb
-0.5	-0.9	-4.7	0.94
± 0.2	± 0.2	± 0.2	± 0.03

**At epoch
2005.0**

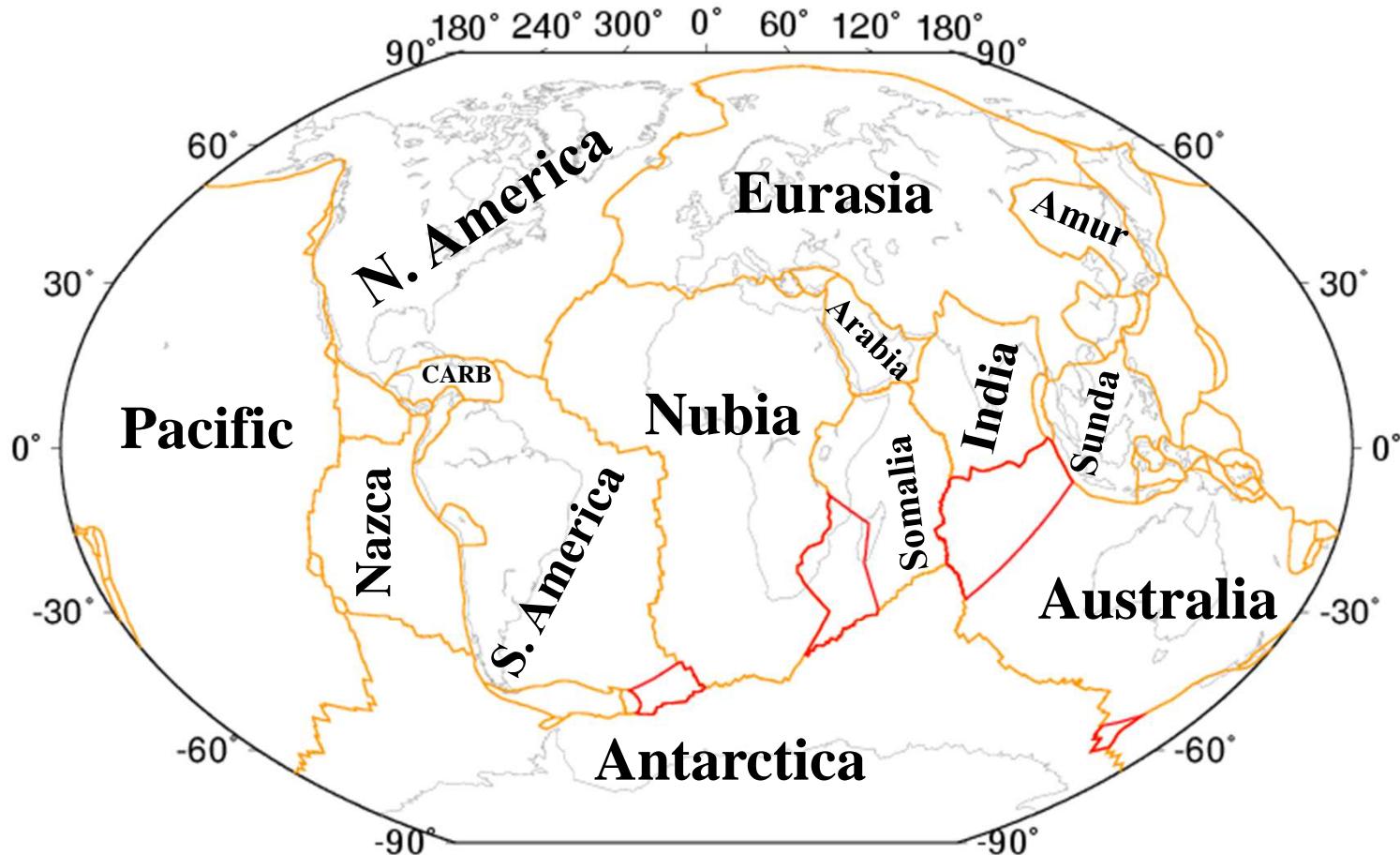
Tx rate	Ty rate	Tz rate	Scale rate
mm/yr	mm/yr	mm/yr	ppb/yr
0.3	0.0	0.0	0.00
± 0.2	± 0.2	± 0.2	± 0.03

How to estimate an absolute plate rotation pole ?

$$\dot{X} = \omega_p \times X$$

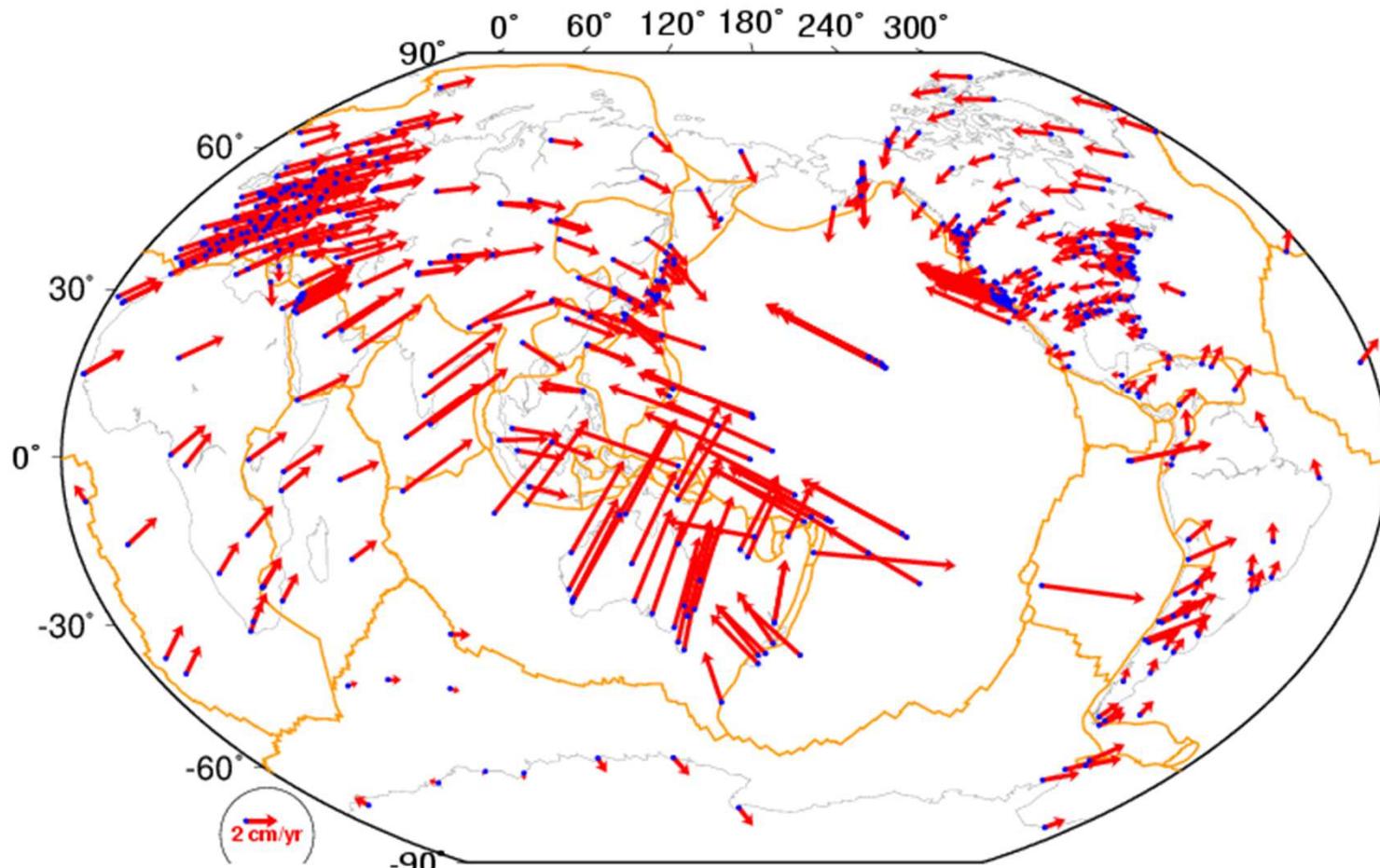
- TRF definition
- Number and distribution over sites over the plate
- Quality of the implied velocities
- Level of rigidity of the plate

Plate boundaries: Bird (2003) and MORVEL, DeMets et al. (2010)

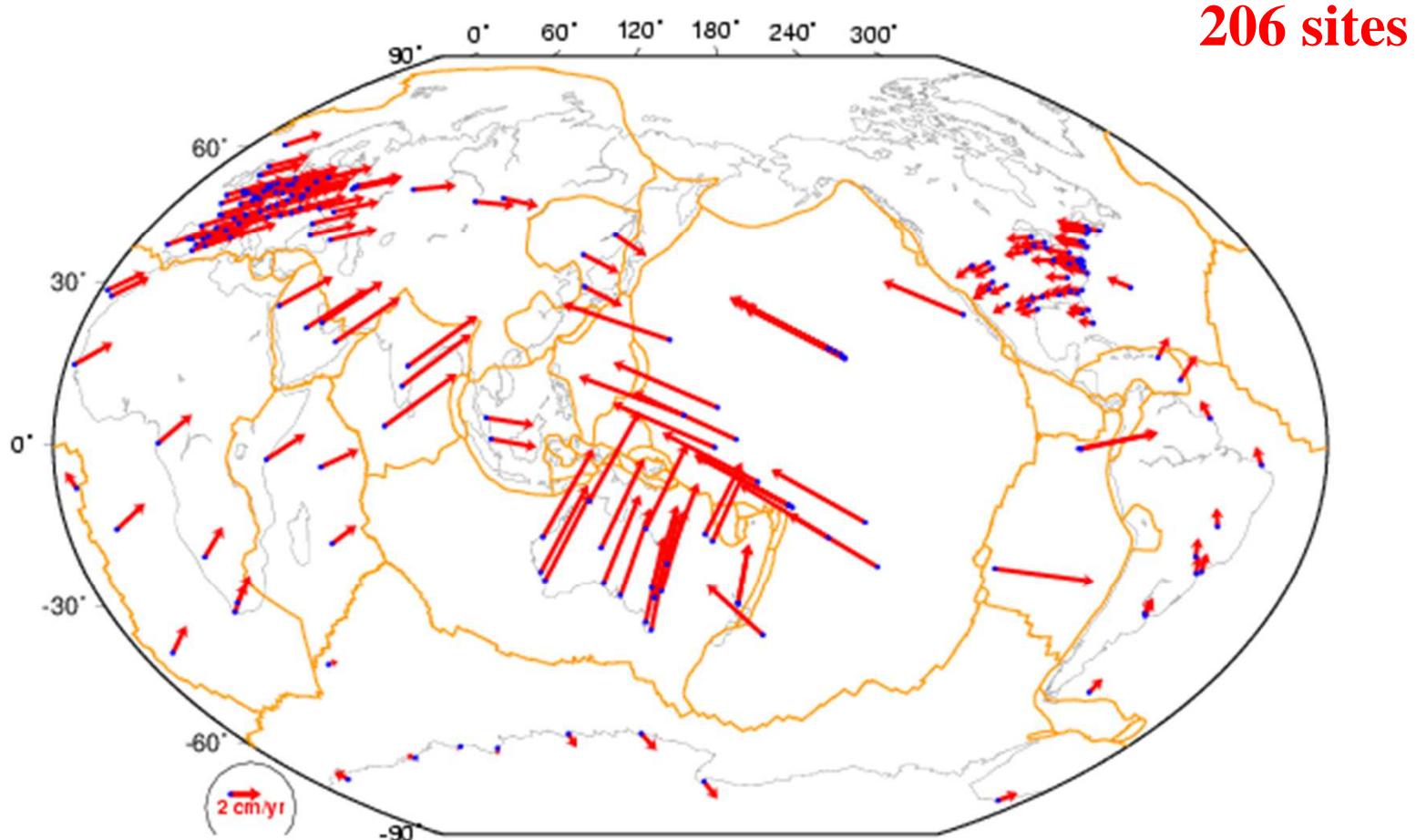


ALL ITRF2008 Site Velocities: time-span > 3 yrs

509 sites



ITRF2008-PMM: Selected Site Velocities



ITRF2008 Plate Motion Model

Inversion model:

$$\dot{X}_i = \omega_p \times X_i + \dot{T}$$

Results:

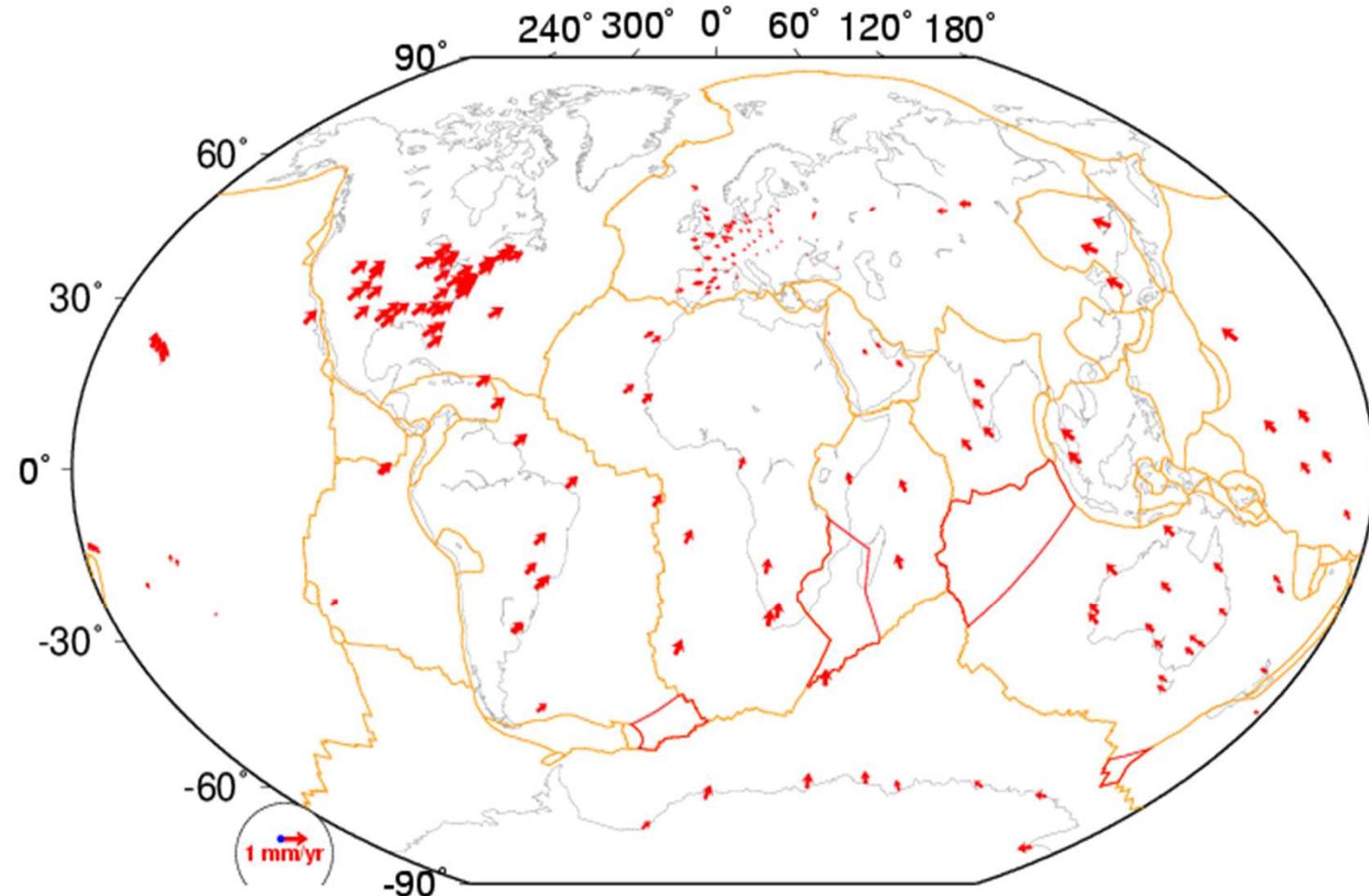
- Angular velocities for 14 plates
- Translation rate components

Table 2. Translation rate components

Number of sites			\dot{T}_x	\dot{T}_y	\dot{T}_z
	Total	EUR	NOAM		
206	69	44	0.41 ±0.27	0.22 ±0.32	0.41 ±0.30

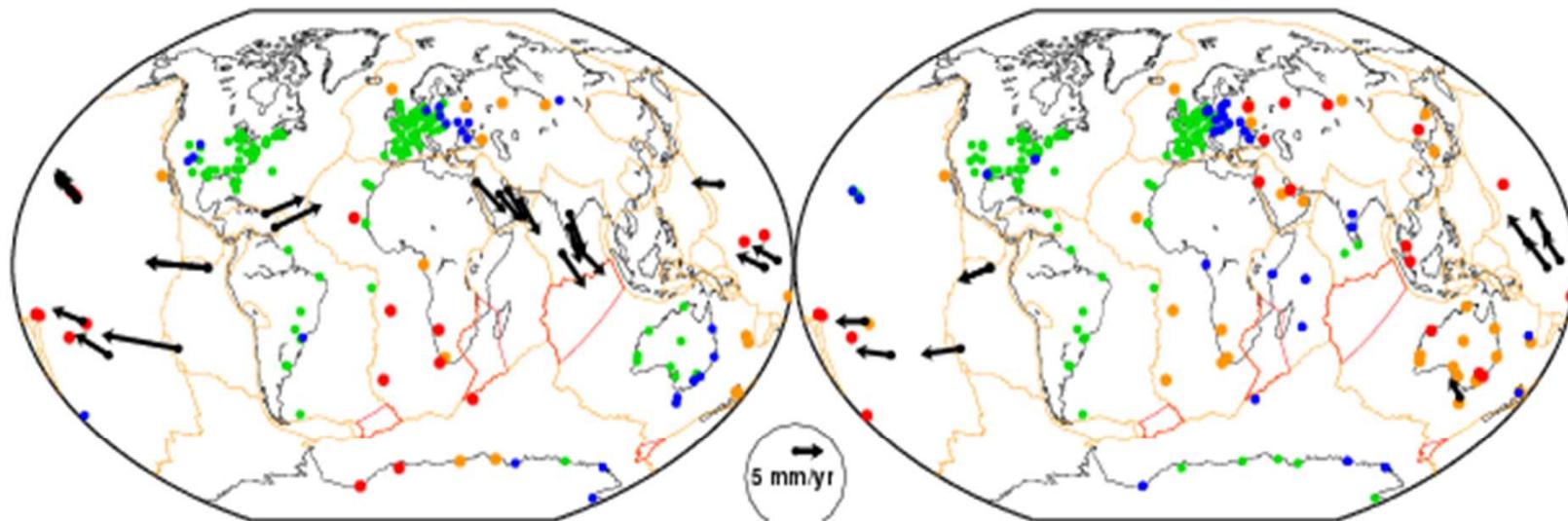
- More details in JGR paper by Altamimi et al. (2012)

Impact of the translation rate



Comparison between ITRF2008 & NNR-NUVEL-1 & NNR-MORVEL56

After rotation rate transformation



NNR-NUVEL-1A

RMS:

East : 2.5 mm/yr

North: 2.1 mm/yr

- Green: < 2 mm/yr
- Blue : 2-3 mm/yr
- Orange: 3-4 mm/yr
- Red : 4-5 mm/yr
- ←● Black : > 5 mm/yr

NNR-MORVEL56

RMS:

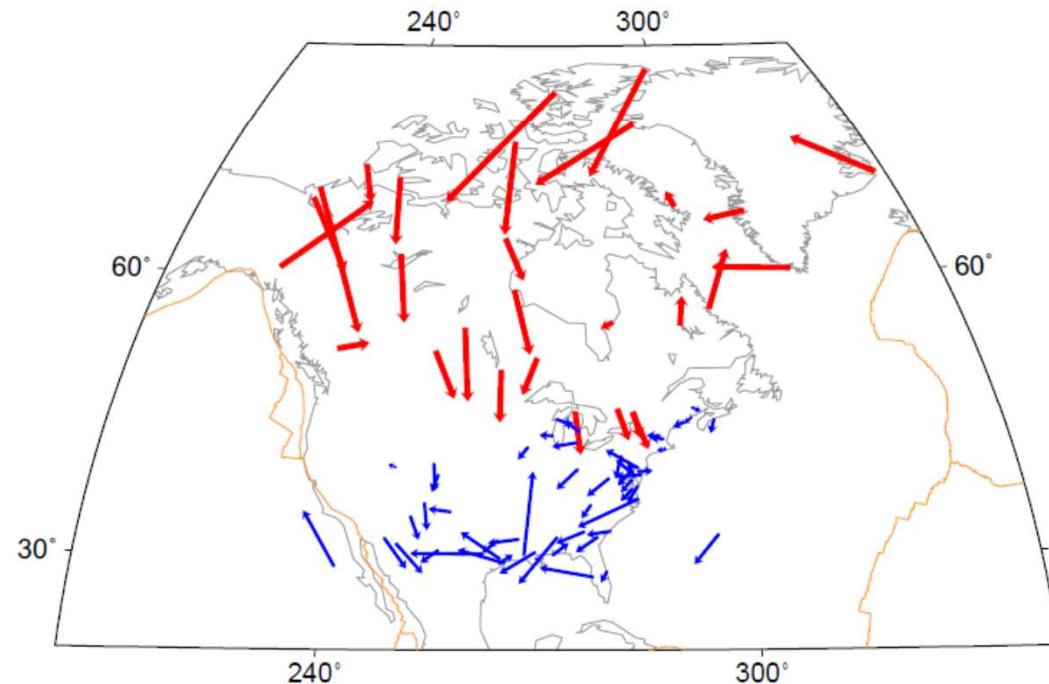
East : 1.8 mm/yr

North: 1.9 mm/yr

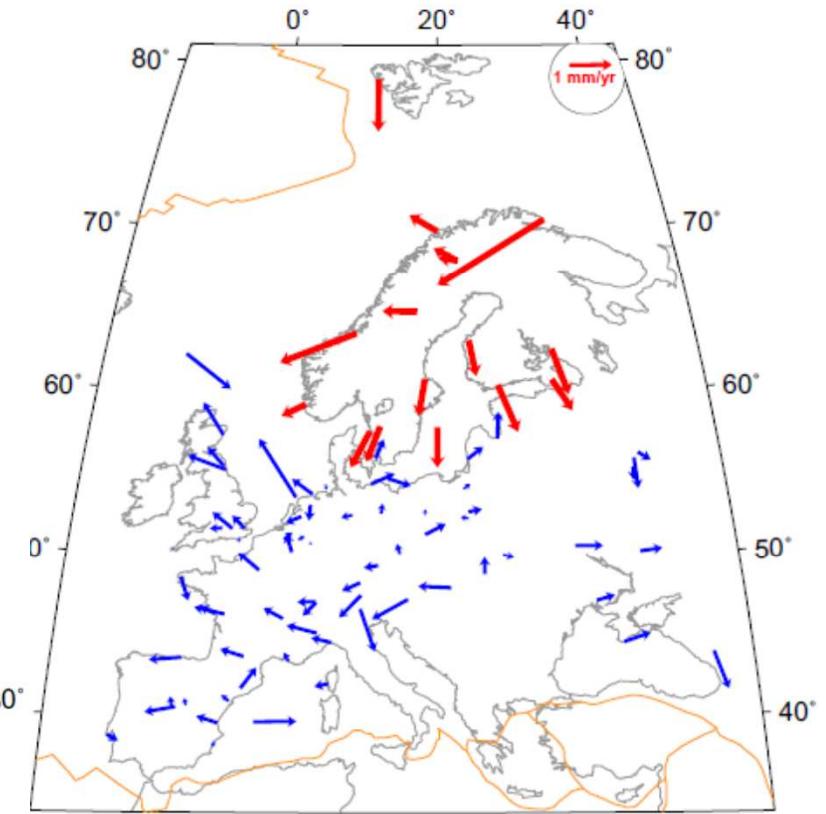
Plate motion and Glacial Isostatic Adjustment

Blue : points used

Red : points rejected



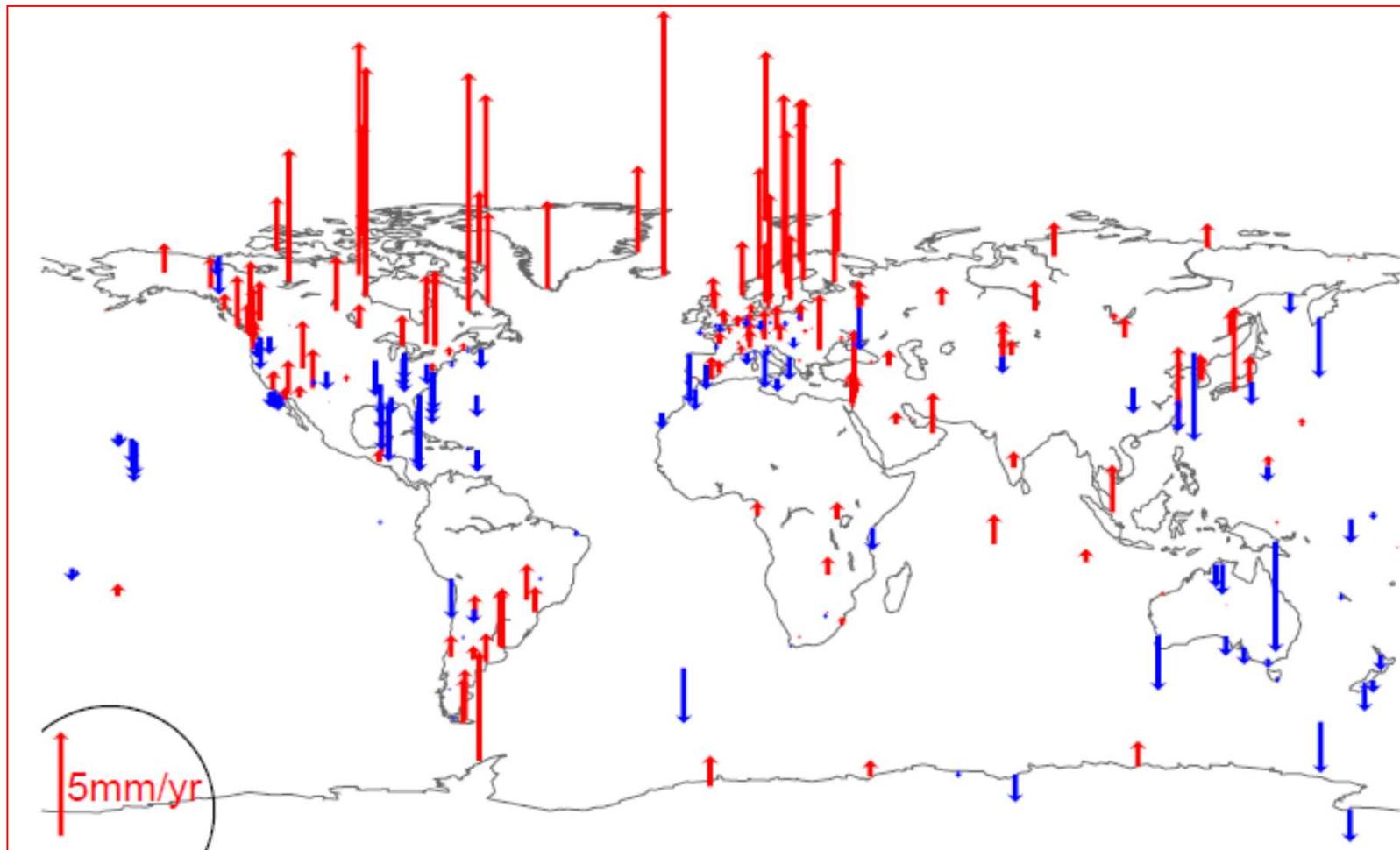
NOAM



EURA

Residual velocities after removing NOAM & EURA rotation poles

ITRF2008 Vertical velocity field



ITRF transformation parameters

Table 4.1: Transformation parameters from ITRF2008 to past ITRFs. “ppb” refers to parts per billion (or 10^{-9}). The units for rates are understood to be “per year.”

ITRF								
Solution	T1 (mm)	T2 (mm)	T3 (mm)	D (ppb)	R1 (mas)	R2 (mas)	R3 (mas)	Epoch
ITRF2005	-2.0	-0.9	-4.7	0.94	0.00	0.00	0.00	2000.0
rates	0.3	0.0	0.0	0.00	0.00	0.00	0.00	
ITRF2000	-1.9	-1.7	-10.5	1.34	0.00	0.00	0.00	2000.0
rates	0.1	0.1	-1.8	0.08	0.00	0.00	0.00	
ITRF97	4.8	2.6	-33.2	2.92	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF96	4.8	2.6	-33.2	2.92	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF94	4.8	2.6	-33.2	2.92	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF93	-24.0	2.4	-38.6	3.41	-1.71	-1.48	-0.30	2000.0
rates	-2.8	-0.1	-2.4	0.09	-0.11	-0.19	0.07	
ITRF92	12.8	4.6	-41.2	2.21	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF91	24.8	18.6	-47.2	3.61	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF90	22.8	14.6	-63.2	3.91	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF89	27.8	38.6	-101.2	7.31	0.00	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	
ITRF88	22.8	2.6	-125.2	10.41	0.10	0.00	0.06	2000.0
rates	0.1	-0.5	-3.2	0.09	0.00	0.00	0.02	

Access & alignment to ITRF

- Direct use of ITRF coordinates
- Use of IGS Products (Orbits, Clocks): all expressed in ITRF
- Alternatively:
 - Process GNSS data together with IGS/ITRF global stations in free mode
 - Align to ITRF by
 - Constraining station coordinates to ITRF values at the central epoch of the observations
 - Using minimum constraints approach

Transformation from an ITRF to another at epoch t_c

- **Input :** \mathbf{X} (ITRFxx, epoch t_c)
- **Output:** \mathbf{X} (ITRFyy, epoch t_c)
- **Procedure:**
 - Propagate ITRF transformation parameters from their epoch (2000.0, slide 72) to epoch t_c , for both ITRFxx and ITRFyy:

$$P(t_c) = P(2000.0) + \dot{P}(t_c - 2000.0)$$

- Compute the transformation parameters between ITRFxx and ITRFyy, by subtraction;
- Transform using the general transformation formula given at slide 8:

$$\mathbf{X}(\text{ITRFyy}) = \mathbf{X}(\text{ITRFxx}) + \mathbf{T} + \mathbf{D} \cdot \mathbf{X}(\text{ITRFxx}) + \mathbf{R} \cdot \mathbf{X}(\text{ITRFxx})$$

How to express a GPS network in the ITRF ?

- Select a reference set of ITRF/IGS stations and collect RINEX data from IGS data centers;
- Process your stations together with the selected ITRF/IGS ones:
 - Fix IGS orbits, clocks and EOPs
 - Eventually, add minimum constraints conditions in the processing
==> Solution will be expressed in the ITRFyy consistent with IGS orbits
 - Propagate official ITRF station positions at the central epoch (t_c) of the observations:
$$X(t_c) = X(t_0) + \dot{X}(t_c - t_0)$$
 - Compare your estimated ITRF station positions to official ITRF values and check for consistency!

From the ITRF to Regional Reference Frames

- Purpose: geo-referencing applications ($\sigma \sim \text{cm}$)
- There are mainly two cases/options to materialize a regional reference frame:
 1. Station positions at a given epoch, eventually updated frequently. Ex.: North & South Americas
 2. Station positions & minimized velocities or station positions & deformation model. Ex.: Europe (ETRS89) New Zealand, Greece (?)
 - Case 1 is easy to implement (see previous slide)
 - Case 2 is more sophisticated & needs application of:
 - Transformation formula (ETRS89)
 - Deformation model

GNSS and their associated reference systems

<u>GNSS</u>	<u>Ref. System/Frame</u>
• GPS (broadcast orbits)	WGS84
• GPS (precise IGS orbits)	ITRS/ITRF
• GLONASS	PZ-90
• GALILEO	ITRS/ITRF/GTRF
• COMPASS	CGCS 2000
• QZSS	JGS
• All are ‘aligned’ to the ITRF	
• WGS84 \approx ITRF at the decimeter level	
• GTRF \approx ITRF at the mm level	
• σ -Position using broadcast ephemerides = 150 cm	

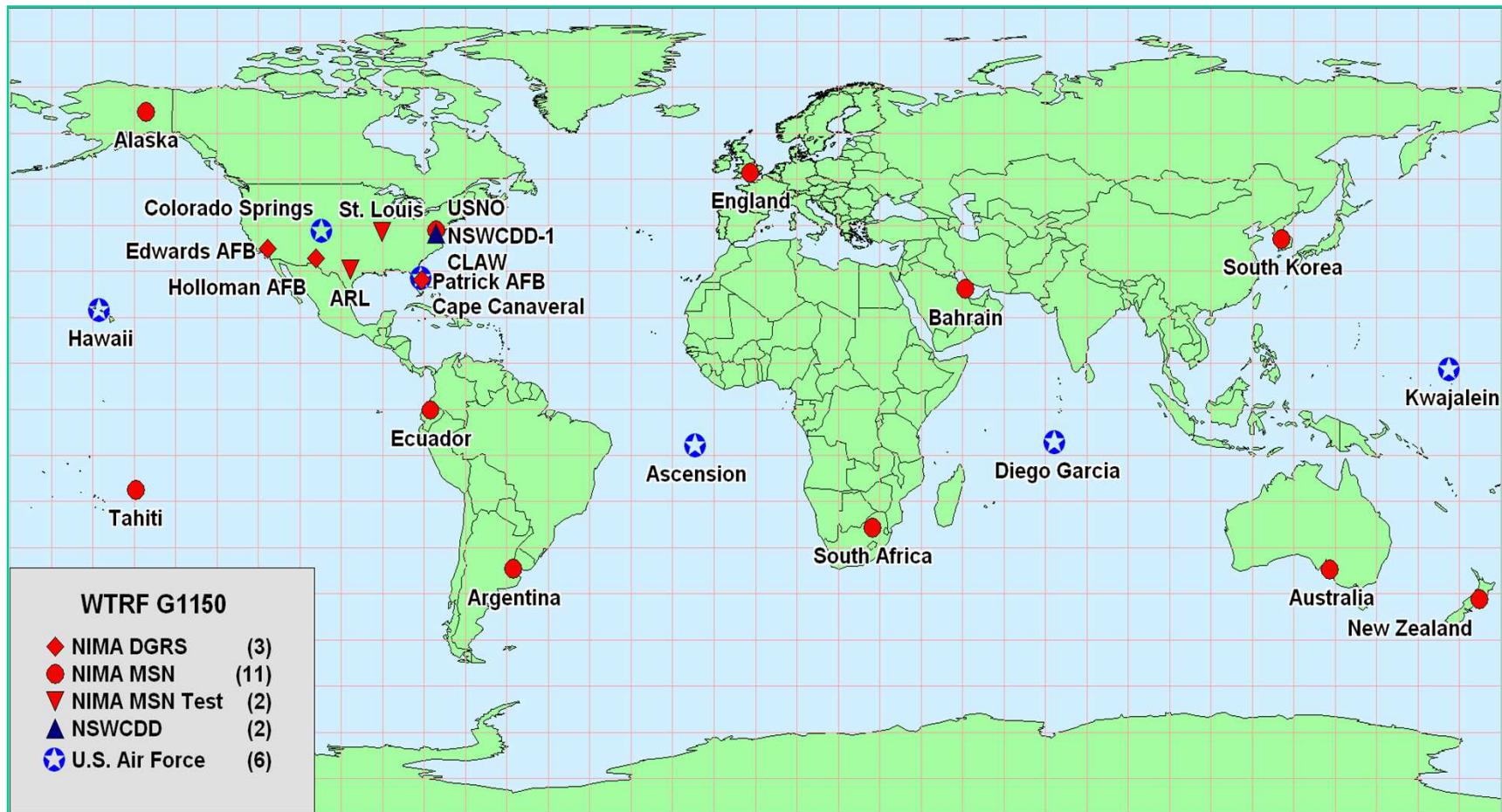
The World Geodetic System 84 (WGS 84)

- Collection of models including Earth Gravitational model, geoid, transformation formulae and set of coordinates of permanent DoD GPS monitor stations
- WGS 60...66...72...84
- Originally based on TRANSIT satellite DOPPLER data

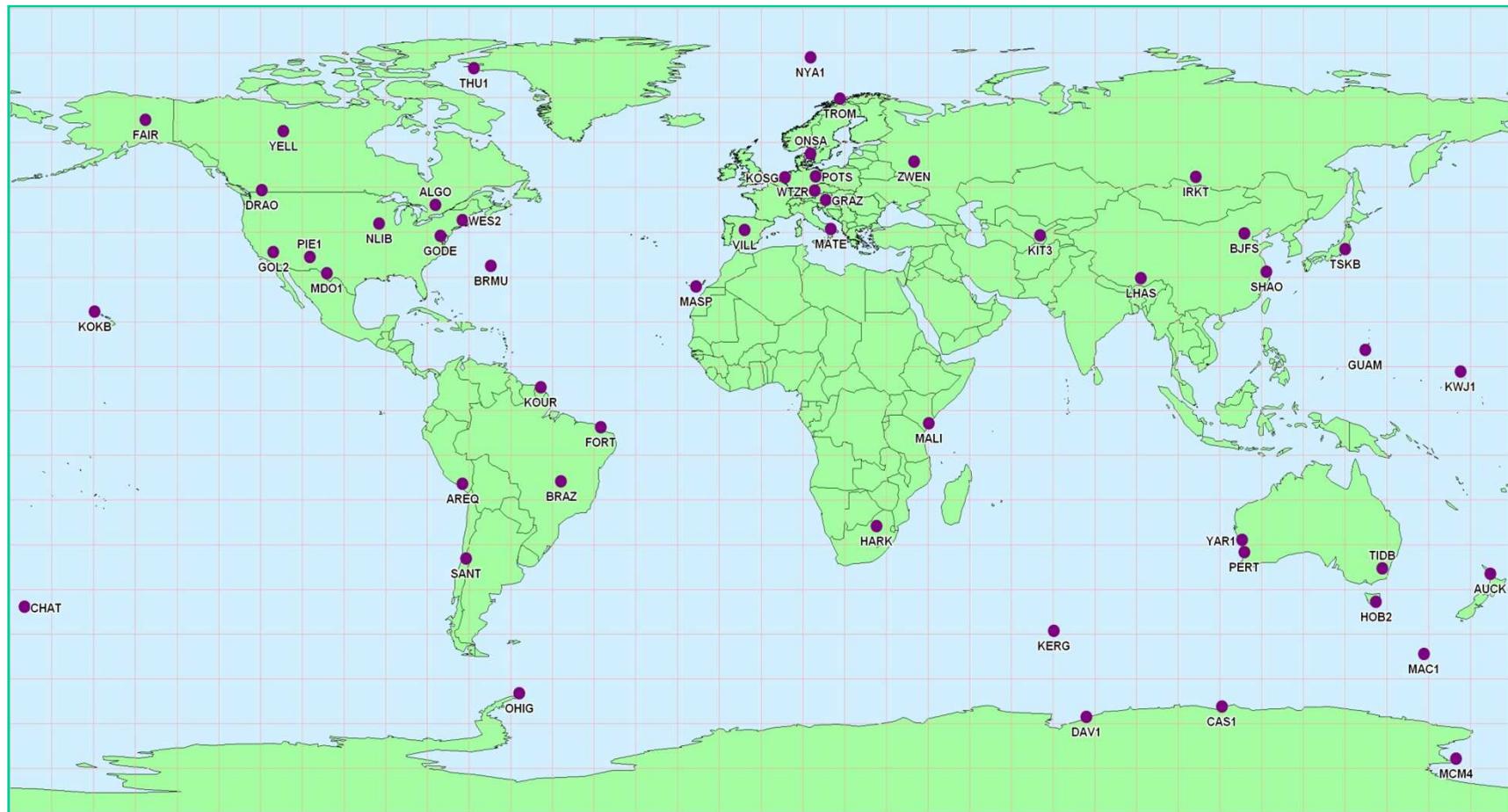
The World Geodetic System 84 (WGS 84)

- Recent WGS 84 realizations based on GPS data:
 - G730 in 1994
 - G873 in 1997
 - G1150 in 2002
 - G1674 in 2012 (aligned to ITRF2008)
- Coincides with any ITRF at 10 cm level
- No official Transf. Param. With ITRF

WGS 84-(G1150)



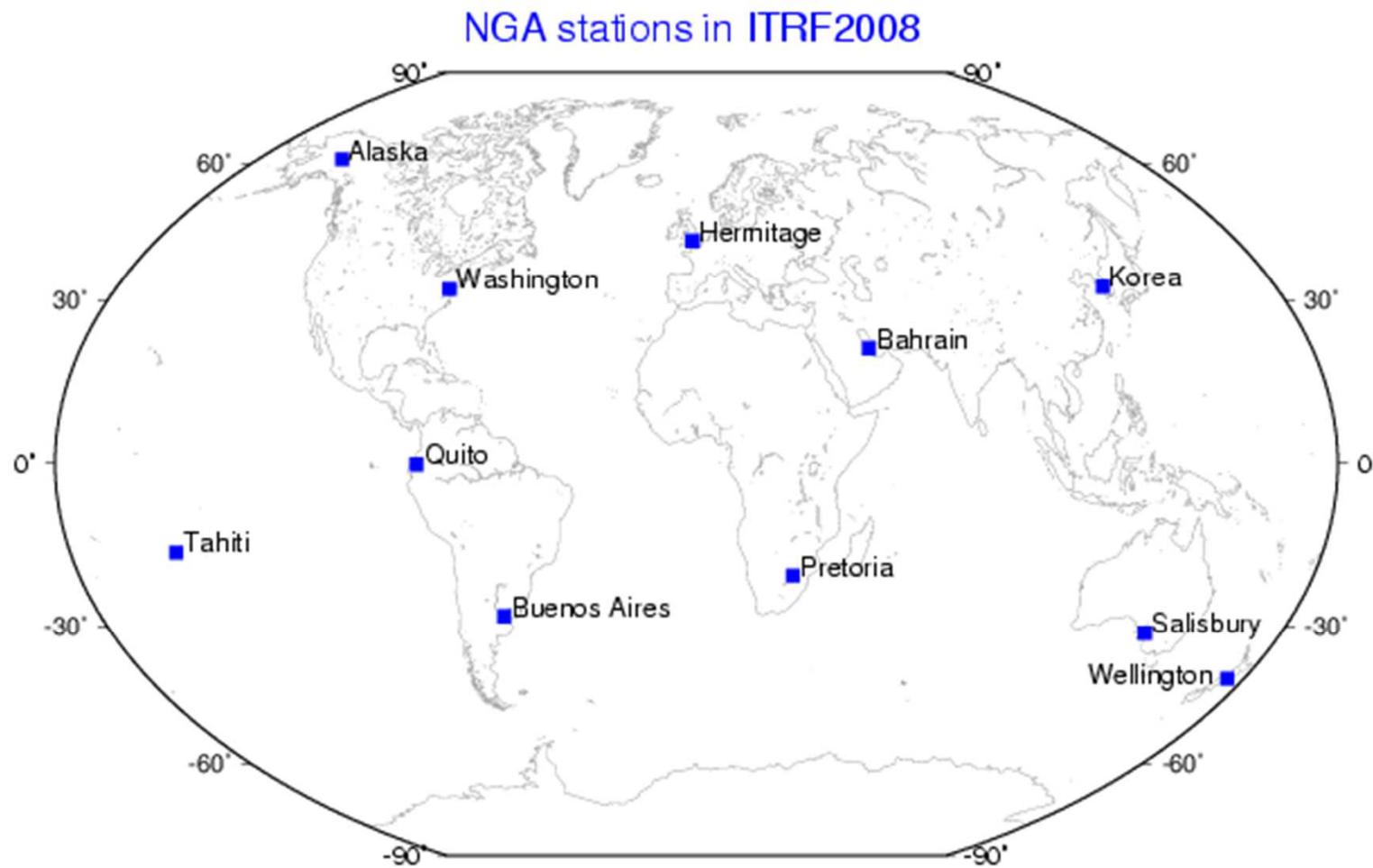
WGS 84-(G1150)



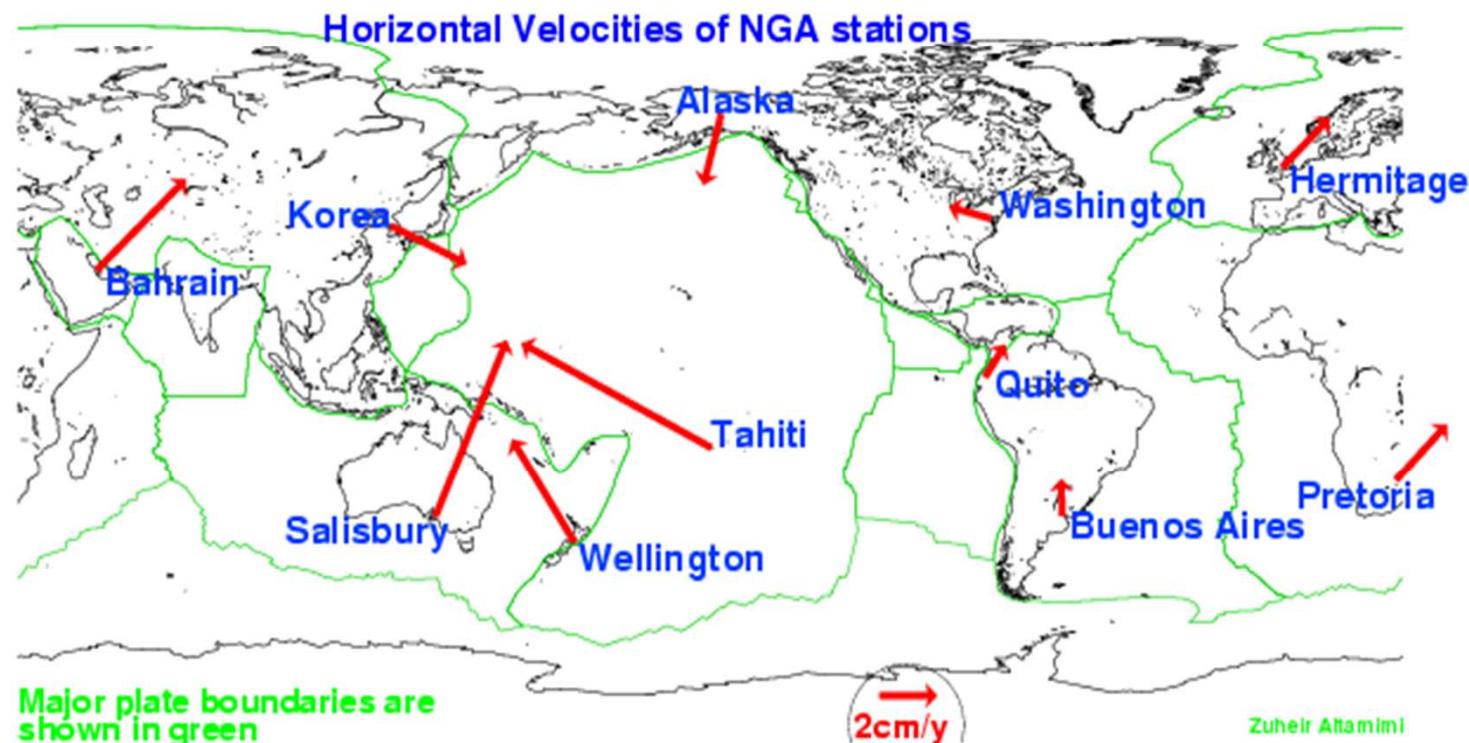
- Coordinates of ~20 stations fixed to ITRF2000
- No station velocities

WGS84 - NGA Stations in ITRF2008

NGA: National Geospatial-Intelligence Agency



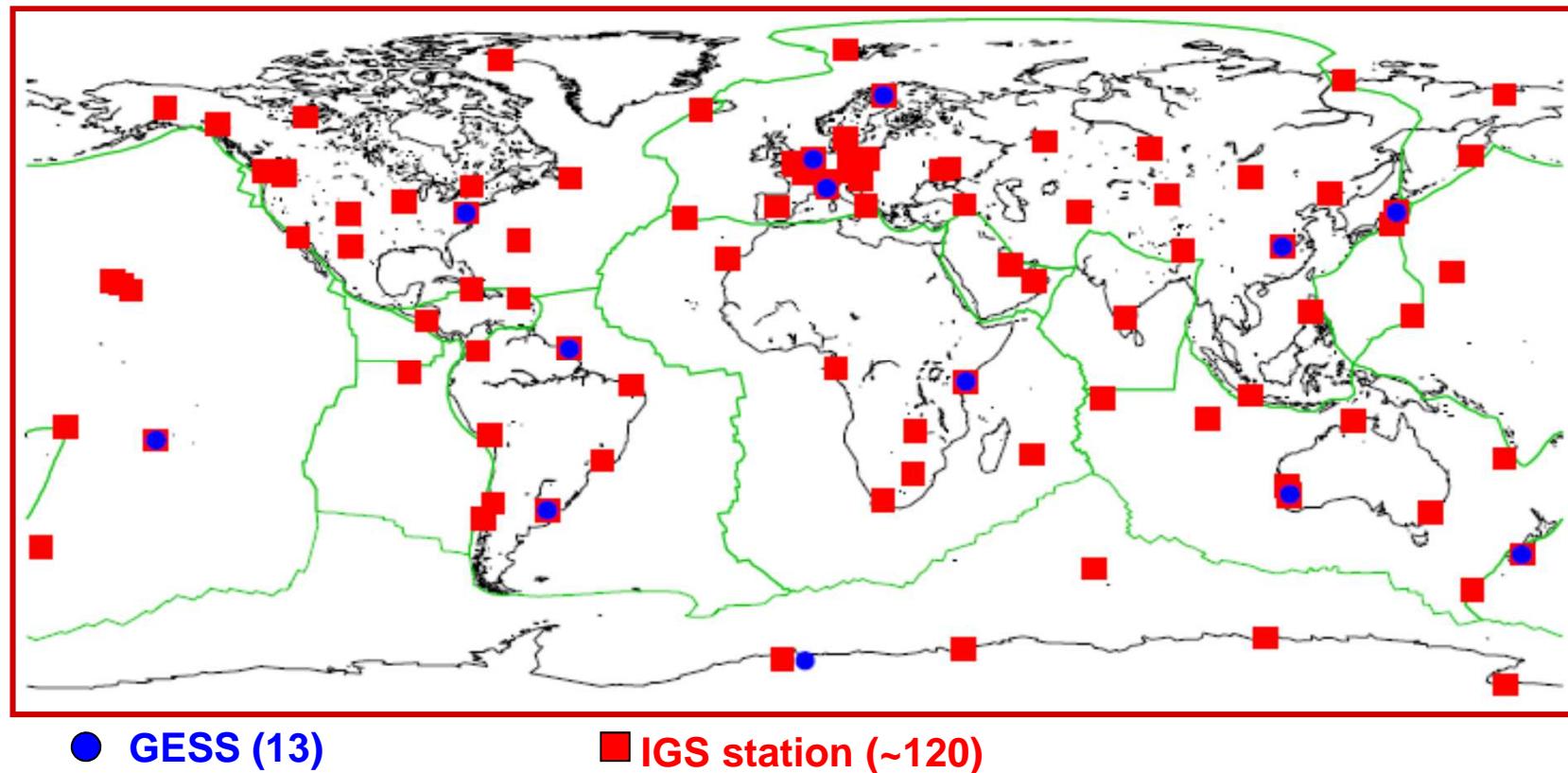
WGS84 - NGA Stations in ITRF2008



Galileo Terrestrial Reference Frame (GTRF)

- Galileo Geodesy Service Provider (GGSP)
- GGSP Consortium (GFZ, AIUB, ESOC, BKG, IGN)
 - Define, realize & maintain the GTRF
 - GTRF should be "compatible" with the ITRF at 3 cm level
 - Liaison with IERS, IGS, ILRS
- GTRF is a realization of the ITRS

The GTRF Experience



- Initial GSS positions & velocities are determined using GPS observations
- Subsequent GTRF versions using GPS & Galileo observations
- Ultimately Galileo Observations only

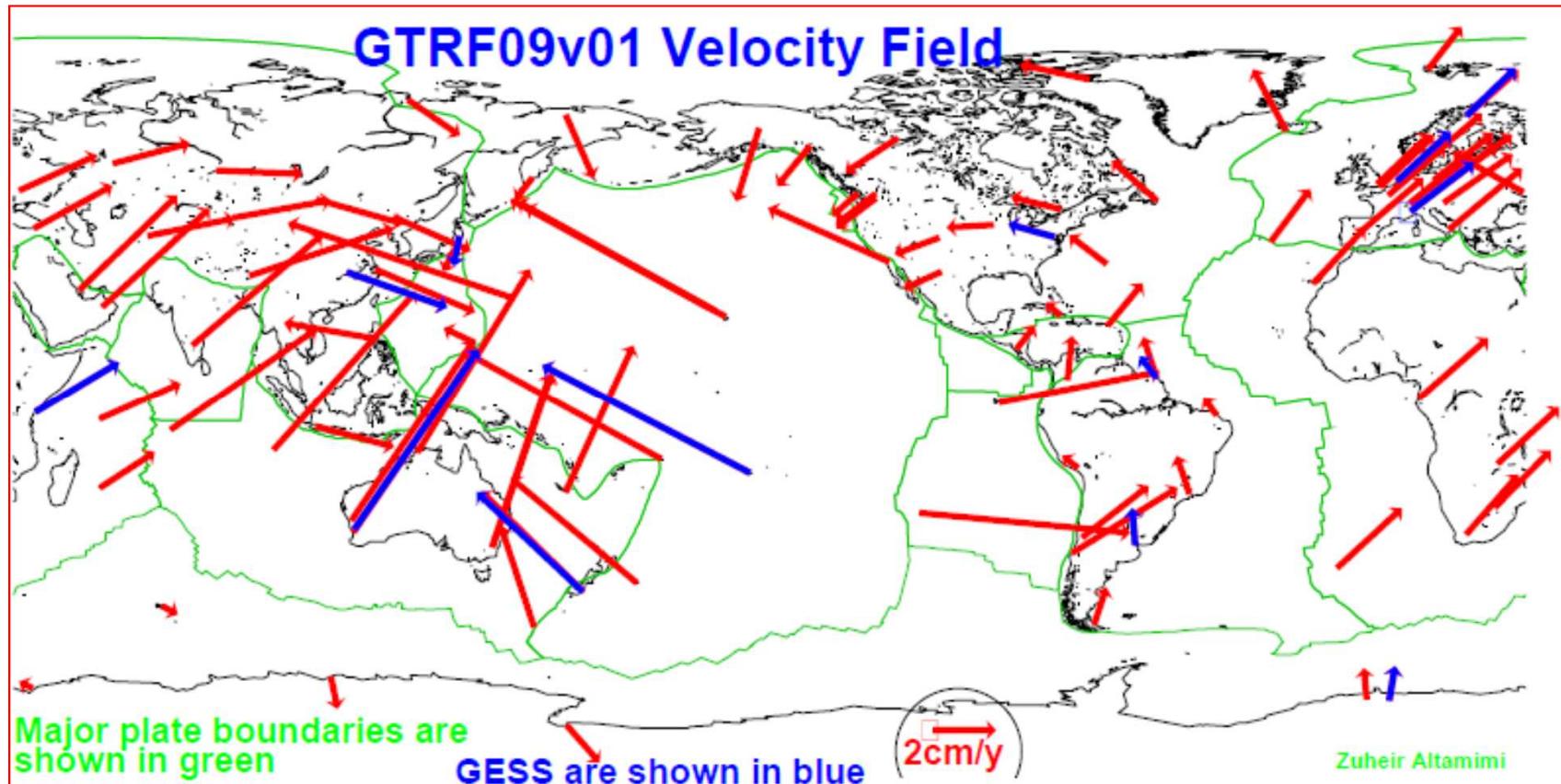
Combination Strategy

- Use Normal Equations from the 3 ACs
- Adequate for weighting
- Weekly and cumulative solutions are transformed into the ITRF using Minimum Constraints

$$X_R = X_c + A\theta \xrightarrow{\theta = \theta} (A^T A)^{-1} A^T (X_R - X_c) = 0$$

↑ ↑
ITRF Combined Solution
 (GTRF)

GTRF09v01 horizontal velocities



Comparison of GTRF09v01 to ITRF2005

- Transformation parameters

	T1 mm	T2 mm	T3 mm	D 10 ⁻⁹	R1 mas	R2 mas	R3 mas	Epoch y
<hr/>								
ITRF2005	0.3	-0.3	-0.2	-0.02	-0.003	-0.007	-0.006	7:360
±	0.2	0.2	0.2	0.03	0.007	0.008	0.008	
Rates	0.0	-0.1	-0.1	0.01	-0.001	-0.002	-0.001	
±	0.2	0.2	0.2	0.03	0.007	0.008	0.008	

==> Perfect GTRF alignment to the ITRF at the sub-mm level

- RMS difference between stations coordinates and velocities

	N	WRMS-Pos.			Epoch	WRMS-Vel.		
		E	N	U		E	N	U
		mm		y		mm/y		
<hr/>								
ITRF2005	89	1.0	1.2	2.6	7:360	0.3	0.3	0.6

Conclusion (1/2)

- The ITRF
 - is the most optimal global RF available today
 - gathers the strengths of space geodesy techniques
 - more precise and accurate than any individual RF
- Using the ITRF as a common GNSS RF will facilitate the interoperability
- Well established procedure available to ensure optimal alignment of GNSS RFs to ITRF
- To my knowledge: most (if not all) GNSS RFs are already “aligned” to ITRF
- GNSS RFs should take into account station velocities

Conclusion (2/2)

WGS84, PZ90, GTRF

**Are all connected to (compatible with)
a unique system: the ITRS**