COIM calculations

Miguel Sarraf

Contents

1 Definitions **2**

2 Simple operations **3**
2.1 Addition by scalar 3
2.2 Multiplication by scalar 4

3 Groupings **5**
3.1 Constant weighted sum 5

1 Definitions

For each kind of applied constrain, we need to determine which variables will be predicted (usually they will be some sort of calculation between the original variables). Moreover, we need to extract the formula to calculated the error for the original variables given the error for the predicted model.

For this sake, the following standard is defined:

- Capital letters: constants
- Small letters: variables
- Δx: the error related to variable x

It will be used the general error propagation formula:
$\Delta f(x_0, x_1, ...) = \sqrt{(\sum_{i=0}^{N} (\frac{\partial f}{\partial x_i})^2 \Delta x_i^2)}$

2 Simple operations

2.1 Addition by scalar

Constrain: $b = a + K$

Both variables have the same variance, so either of them will work similarly.

$a' = a$

Henceforth, the variables to be predicted are a'.

To retrieve the original values, we re-apply the constrain formulas:

$a = a'$

$b = a' + K$

Finally, we calculate the propagated error formula with the other formulas:

$\frac{\partial a}{\partial a'} = 1$

$\frac{\partial b}{\partial a'} = 1$

$\Delta a = \sqrt{1 \cdot \Delta a'^2} = \Delta a'$

$\Delta b = \sqrt{1 \cdot \Delta a'^2} = \Delta a'$

2.2 Multiplication by scalar

Constrain: $b = Ka$

For a better prediction, the value must not be too varying, so we choose, between the 2 variables, the one with less variance.

$a' = \begin{cases} b & |K| \leq 1 \\ a & |K| > 1 \end{cases}$

Henceforth, the variables to be predicted are a'.

To retrieve the original values, we re-apply the derived formulas:

$$\begin{cases} b = a' \quad a = \frac{a'}{K} & |K| \leq 1 \\ a = a' \quad b = a'K & |K| > 1 \end{cases}$$

Finally, we calculate the propagated error formula with the other formulas:

$\frac{\partial a}{\partial a'} = \begin{cases} \frac{1}{K} & |K| \leq 1 \\ 1 & |K| > 1 \end{cases}$

$\frac{\partial b}{\partial a'} = \begin{cases} 1 & |K| \leq 1 \\ K & |K| > 1 \end{cases}$

$\Delta a = \begin{cases} \sqrt{\frac{1}{K^2} \cdot \Delta a'^2} = \frac{\Delta a'}{K} & |K| \leq 1 \\ \sqrt{1 \cdot \Delta a'^2} = \Delta a' & |K| > 1 \end{cases}$

$\Delta b = \begin{cases} \sqrt{1 \cdot \Delta a'^2} = \Delta a' & |K| \leq 1 \\ \sqrt{K^2 \cdot \Delta a'^2} = K\Delta a' & |K| > 1 \end{cases}$

3 Groupings

3.1 Constant weighted sum

Constrain: $\sum_{i=0}^{n} W_i \cdot a_i = K$

For a better prediction, the value must not be too varying, so we divide them by their sum. Also, it is interesting to predict the relation between the variables, instead of the values themselves, so we divide all of the variables by the least varying one (called a_0).

$\sum_{i=0}^{N} W_i \cdot a_i = K$
$W_0 \cdot a_0 + \sum_{i=1}^{N} W_i \cdot a_i = K$
$\frac{W_0}{K} + \sum_{i=1}^{N} W_i \cdot \frac{a_i}{a_0 K} = \frac{1}{a_0}$
$a_i' = \frac{a_i}{a_0 K}$

Henceforth, the variables to be predicted are a_i'.

To retrieve the original values, we re-apply the derived formulas:

$\frac{W_0}{K} + \sum_{i=1}^{N} W_i \cdot a_i' = \frac{1}{a_0} \rightarrow a_0 = \frac{1}{\frac{W_0}{K} + \sum_{i=1}^{N} W_i \cdot a_i'} = \frac{K}{W_0 + K \sum_{i=1}^{N} W_i \cdot a_i'}$
$a_i' = \frac{a_i}{a_0 K} \rightarrow a_i = a_i' a_0 K$

Finally, we calculate the propagated error formula with the other formulas:

$\frac{\partial a_0}{\partial a_i'} = \frac{-K^2 \cdot W_i}{(W_0 + K \sum_{j=1}^{N} W_j \cdot a_j')^2} = -W_i \cdot (\frac{K}{W_0 + K \sum_{j=1}^{N} W_j \cdot a_j'})^2 = -W_i \cdot a_0^2$
$\frac{\partial a_i}{\partial a_0} = K((-\frac{1}{a_0^2 W_i})a_0 + (a_i')1) = K(-0\frac{1}{a_0 W_i} + \frac{a_i}{a_0 K}) = \frac{1}{a_0 W_i}(W_i a_i - K)$
$\frac{\partial a_i}{\partial a_i'} = K(1(a_0) + a_i'(-W_i \cdot a_0^2)) = K(a_0 - \frac{W_i \cdot a_i}{a_0 K} a_0^2) = a_0(K - W_i a_i)$
$\Delta a_0 = \sqrt{\sum_{j=1}^{N} W_j^2 \cdot a_0^4 \cdot \Delta a_j'^2} = a_0^2 \sqrt{\sum_{j=1}^{N} W_j^2 \cdot \Delta a_j'^2}$
$\Delta a_i = \sqrt{a_0^2 (K - W_i a_i)^2 \cdot \Delta a_i'^2 + \frac{1}{a_0^2 W_i^2} (K - W_i a_i)^2 \cdot \Delta a_0^2} =$
$|K - W_i a_i| \sqrt{a_0^2 \cdot \Delta a_i'^2 + \frac{1}{a_0^2 W_i^2} \cdot \Delta a_0^2} = |\frac{K - W_i a_i}{a_0 W_i}| \sqrt{a_0^4 W_i^2 \cdot \Delta a_i'^2 + \Delta a_0^2}$