Baryon magnetic moments in large-Nc chiral perturbation theory: Effects of the decuplet-octet mass difference and flavor symmetry breaking

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Baryon magnetic moments in large-N_c chiral perturbation theory: Effects of the decuplet-octet mass difference on the leading order corrections

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Abstract

The baryon magnetic and transition magnetic moments are computed in heavy baryon chiral perturbation theory in the large-N_c limit, where N_c is the number of colors. One-loop nonanalytic corrections of orders $m_q^{1/2}$ and $m_q \ln m_q$ are analyzed and contributions of both intermediate octet and decuplet baryon states are included. The effects of the decuplet-octet mass difference are explicitly evaluated in corrections of order $m_q^{1/2}$ only. The resultant expressions are compared with the available experimental data and with other determinations in the context of conventional heavy baryon chiral perturbation theory for three light quarks flavors and at the physical value $N_c = 3$. The agreement reached is very good.

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I. INTRODUCTION

The SU(3) group theoretical approach to deal with baryon magnetic moments was first developed by Coleman and Glashow [1]; their analysis lead to the celebrated relations –named after them– among octet baryons in terms of two parameters, namely,

$$
\begin{aligned}
&\mu_{\Sigma^+}^{(0)} = \mu_p^{(0)}, \qquad && \mu_{\Sigma^-}^{(0)} + \mu_n^{(0)} = -\mu_p^{(0)}, \\
&2\mu_\Lambda^{(0)} = \mu_n^{(0)}, \qquad && \mu_{\Xi^-}^{(0)} = \mu_{\Sigma^-}^{(0)}, \\
&\mu_{\Xi^0}^{(0)} = \mu_n^{(0)}, \qquad && 2\mu_{\Lambda\Sigma^0}^{(0)} = -\sqrt{3}\mu_n^{(0)},
\end{aligned}
\tag{1}
$$

along with the isospin relation

$$
\mu_{\Sigma^+}^{(0)} - 2\mu_{\Sigma^0}^{(0)} + \mu_{\Sigma^-}^{(0)} = 0, \tag{2}
$$

where the superscript (0) will denote to the SU(3) symmetric values hereafter. Soon after the discovery of relations (1), experimental analyses found discrepancies by a few standard deviations from the SU(3) values. Since then, a number of methods have been used in order to improve the numerical predictions of Coleman and Glashow by including SU(3) breaking effects. Among these methods, heavy baryon chiral perturbation theory [2, 3] and the $1/N_c$ expansion of QCD [4, 5], where N_c is the number of colors, have been two schemes to understand the low-energy consequences of hadrons.

As a matter of fact, the combined use of chiral perturbation theory and the $1/N_c$ expansion is another calculational scheme which constrains the low-energy interactions of baryons with with the meson nonet[1] in a more effective way than each method alone [6]. Let us recall that in the chiral limit $m_q \to 0$ and mesons become massless Goldstone boson states; as a result, there is an expansion in powers of m_q/Λ_χ, where $\Lambda_\chi \sim 1$ GeV is the scale of chiral symmetry breaking. On the other hand, in the large-N_c limit, decuplet and octet baryons become degenerate, namely $\Delta \equiv M_T - M_B \propto 1/N_c \to 0$, where M_T and M_B denote the SU(3) invariant masses of the decuplet and octet baryon multiplets, respectively. It turns out that decuplet and octet baryon states constitute a single irreducible representation of the contracted spin-flavor symmetry of baryons in large-N_c QCD [4, 5]. Corrections about the large-N_c limit then appear in powers of $1/N_c$. All in all, the combined expansion in m_q/Λ_χ and $1/N_c$ requires to consider the double limit $m_q \to 0$ and $N_c \to \infty$.

[1] In the large-N_c limit the η' meson is related to the usual octet mesons π, K and η.

Caldi and Pagels [7], in the framework of chiral perturbation theory, found that corrections to baryon magnetic moments appear in the non-analytic forms $m_q^{1/2}$ and $m_q \ln m_q$, which can be obtained from meson-loop graphs. In heavy baryon chiral perturbation theory [2, 3], loop graphs have a calculable dependence on the ratio m_Π/Δ, where m_Π denotes the mass of meson $\Pi = \pi, K, \eta$. In order for the theory to be valid, the conditions $m_\Pi \ll \Lambda_\chi$ and $\Delta \ll \Lambda_\chi$ must be met, although the ratio m_Π/Δ is not constrained and can take any value [8].

In a previous paper [9] we computed one-loop corrections to baryon magnetic moments within a combined expansion in m_q and $1/N_c$. We considered contributions of orders $\mathcal{O}(m_q^{1/2})$ and $\mathcal{O}(m_q \ln m_q)$ to relative order $1/N_c^3$ in the $1/N_c$ expansion. The best way of approaching this problem was in the degeneracy limit $\Delta \to 0$. The resultant theoretical expressions agreed, order by order, with others obtained within baryon chiral perturbation theory [10–13] for octet and decuplet baryons and also for octet-octet and decuplet-octet transitions. Additionally, a comparison with the current experimental data [14] through a least-squares fit allowed us to get information about the free parameters of the theory. Although the predicted values obtained for all 27 possible magnetic moments were according to expectations, the fit somehow seemed to be not entirely satisfactory in the sense that the SU(3) invariants of chiral perturbation theory are not well reproduced in our approach.

It would be desirable to relax the restriction $\Delta \to 0$ and consider the more realistic case $\Delta \neq 0$. Indeed, in the present paper we do so as a second approximation in the contributions arising from loop graphs of order $\mathcal{O}(m_q^{1/2})$ *only*; we thus postpone the study of contributions of order $\mathcal{O}(m_q \ln m_q)$ for a future work [15]. Our motivation here is not really to be definitive about the determination of baryon magnetic moments in the combined scheme but rather to explore the effects $\Delta \neq 0$ has on the fit to experimental data. Any noticeably improvements should be observed in the best-fit values of the parameters in the fit and also in the value of χ^2.

This paper is organized ad follows. In Sec. II, apart from introducing our notation and conventions, we also provide an overview on the determination of baryon magnetic moments in large-N_c chiral perturbation theory. We start our discussion by defining the tree-level values and then, in Sec. III, we continue with getting one loop corrections. Specifically, we concentrate on corrections of order $\mathcal{O}(m_q^{1/2})$ by constructing the baryon operator which describes such contribution; the dependence on Δ is manifest at this level. We proceed further in order to achieve the reduction of this operator in the two flavor representations involved. The theoretical expressions obtained are thus compared with other determinations in the frame of chiral perturbation theory. In Sec. IV we carry out a least-squares fit in order to determine the best-fit parameters of the theory which allow us to predict numerical values of the unobserved magnetic moments; we then compare them with other numerical predictions. Finally, in Sec. V we discuss our findings. This work is complemented by Appendix A where we provide the reduction of the baryon operators to the order in $1/N_c$ discussed here.

II. BARYON MAGNETIC MOMENT IN LARGE-N_c CHIRAL PERTURBATION THEORY

The present analysis builds on earlier works, particularly on Refs. [4–6, 16] which established the mathematical groundwork on large-N_c QCD and the $1/N_c$ expansion for baryons, and also on Ref. [9], where baryon magnetic moments in large-N_c chiral perturbation theory in the degeneracy limit were discussed. Thus, we only give an outline of some relevant issues here.

The starting point of the analysis of Ref. [9] relies on the fact that, in the large-N_c limit, the baryon magnetic moments possess the same kinematical properties as the baryon axial-vector couplings so they are described in terms of the same operators. The $1/N_c$ expansion of the baryon axial vector operator A^{kc}, at the physical value $N_c = 3$, can be written as [5]

$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c} \mathcal{D}_2^{kc} + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{kc}, \tag{3}$$

where a_1, b_1, b_2 and c_3 are unknown coefficients not determined by the theory and

$$\mathcal{D}_2^{kc} = J^k T^c, \tag{4}$$

$$\mathcal{D}_3^{kc} = \{J^k, \{J^r, G^{rc}\}\}, \tag{5}$$

$$\mathcal{O}_3^{kc} = \{J^2, G^{kc}\} - \frac{1}{2}\{J^k, \{J^r, G^{rc}\}\}, \tag{6}$$

where J^k, T^c, and G^{kc} are the baryon spin, flavor and spin-flavor generators of the contracted spin-flavor symmetry SU(6). Successive higher order operators are obtained from the previous ones as $\mathcal{D}_n^{kc} = \{J^2, \mathcal{D}_{n-2}^{kc}\}$ and $\mathcal{O}_n^{kc} = \{J^2, \mathcal{O}_{n-2}^{kc}\}$ for $n \geq 4$. Besides, $\mathcal{D}_n^{kc}$ are diagonal operators with non-vanishing matrix elements only between states with the same spin whereas $\mathcal{O}_n^{kc}$ are purely off-diagonal operators with non-vanishing matrix elements only between states with different spin.

Now, in a complete analogy to expression (3), the $1/N_c$ expansion of the operator which yields baryon magnetic moments can be written as [9]

$$M^{kc} = m_1 G^{kc} + m_2 \frac{1}{N_c} \mathcal{D}_2^{kc} + m_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + m_4 \frac{1}{N_c^2} \mathcal{O}_3^{kc}, \tag{7}$$

where the series has also been truncated at $N_c = 3$. The magnetic moments are proportional to the quark charge matrix $\mathcal{Q} = \mathrm{diag}(2/3, -1/3, -1/3)$, so they can be separated into isovector and isoscalar components, M^{k3} and M^{k8}, respectively. Thus, the baryon magnetic moment operator can be defined as

$$M^k = M^{kQ} \equiv M^{k3} + \frac{1}{\sqrt{3}} M^{k8}. \tag{8}$$

Hereafter, the spin index k of M^k will be set to 3 whereas the flavor index Q will stand for $Q = 3 + (1/\sqrt{3})8$ so any operator of the form X^Q should be understood as $X^3 + (1/\sqrt{3})X^8$.

The baryon magnetic and transition magnetic moments at tree level can be straightforwardly computed from Eq. (8). The required matrix elements of the operators involved in such an expression are listed in Ref. [9] and will not be repeated here. Let us now proceed to discuss the leading one-loop corrections.

III. ONE-LOOP CORRECTIONS TO BARYON MAGNETIC MOMENTS

The one-loop diagrams that contribute to baryon magnetic moments are displayed in Figs. 1 and 2. Corrections arising from these loop graphs are order $\mathcal{O}(m_q^{1/2})$ and $\mathcal{O}(m_q \ln m_q)$, respectively.

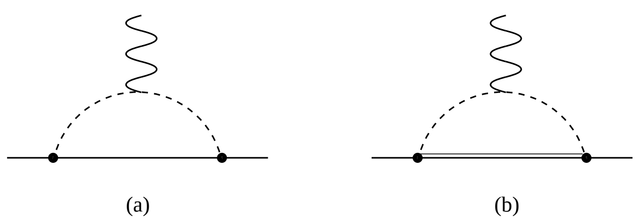

FIG. 1: Feynman diagrams which yield nonanalytic $m_q^{1/2}$ corrections to the magnetic moments of octet baryons. Dashed lines denote mesons and single and double solid lines denote octet and decuplet baryons, respectively. For decuplet baryons and decuplet-octet transitions the diagrams are similar

The analysis of baryon magnetic moments in the framework of large-N_c chiral perturbation theory presented in Ref. [9] was carried out in the degeneracy limit $\Delta \to 0$. We now intend to find out the effects $\Delta \neq 0$ has on the leading order corrections $\mathcal{O}(m_q^{1/2})$ and also on the overall fit to the experimental data. However, the calculation involving a non-vanishing Δ introduces a number of issues not discussed in Ref. [9]. First, one can discern that an immediate modification can be found in the baryon propagator in the loop integral of Fig. 1, which now has an explicit dependence on Δ. In order to deal with this issue, we can follow the approach implemented in the

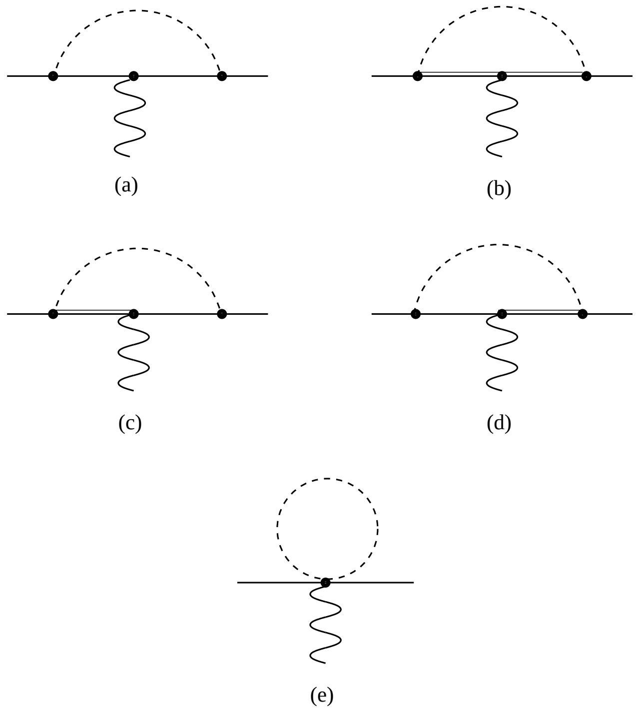

FIG. 2: Feynman diagrams which yield nonanalytic $m_q \ln m_q$ corrections to the magnetic moments of octet baryons. Dashed lines denote mesons and single and double solid lines denote octet and decuplet baryons, respectively. The wavefunction renormalization graphs are omitted in the figure but are nevertheless considered in the analysis. For decuplet baryons and decuplet-octet transitions the diagrams are similar.

analysis of flavor $\mathbf{27}$ nonanalytic corrections to the baryon masses presented in Ref. [6]. In this work, is was stated that in the chiral limit the baryon propagator is diagonal in spin so it can be expressed as

$$\frac{i\mathcal{P}_{\mathsf{j}}}{k^0 - \Delta_{\mathsf{j}}}, \tag{9}$$

where $\mathcal{P}_{\mathsf{j}}$ is a spin projector operator for spin $J = \mathsf{j}$, which satisfies by definition

$$\mathcal{P}_{\mathsf{j}}^2 = \mathcal{P}_{\mathsf{j}}, \tag{10a}$$

$$\mathcal{P}_{\mathsf{j}}\mathcal{P}_{\mathsf{j}'} = 0, \qquad \mathsf{j} \neq \mathsf{j}', \tag{10b}$$

and Δ_{j} stands for the difference of the hyperfine mass splitting for spin $J = \mathsf{j}$ and the external baryon, namely,

$$\Delta_{\mathsf{j}} = \mathcal{M}_{\text{hyperfine}}\big|_{J^2=\mathsf{j}(\mathsf{j}+1)} - \mathcal{M}_{\text{hyperfine}}\big|_{J^2=\mathsf{j}_{\text{ext}}(\mathsf{j}_{\text{ext}}+1)}. \tag{11}$$

The hyperfine mass splitting originates from the $1/N_c$ expansion of the baryon mass operator $\mathcal{M}$

which transforms as a $(0,1)$ under SU(2) × SU(3). It reads [6]

$$\mathcal{M} = m_{(0)}^{0,1} N_c \mathbb{1} + \sum_{n=2,4}^{N_c-1} m_{(n)}^{0,1} \frac{1}{N_c^{n-1}} J^n, \tag{12}$$

where the coefficients $m_{(n)}^{0,1}$ are dimensionful parameters of order $\mathcal{O}(\Lambda)$. The first term in Eq. (12) is the overall spin-independent mass of the baryon multiplet whereas the spin-dependent terms define $\mathcal{M}_{\text{hyperfine}}$. Thus, for p-wave pion emission Δ_{j} reduces to [6]

$$\Delta_{\mathrm{j}} = \begin{cases} \dfrac{1}{N_c} 2\,\mathrm{j}\, m_{(2)}^{0,1}, & \mathrm{j}_{\text{ext}} = \mathrm{j} - 1, \\ 0, & \mathrm{j}_{\text{ext}} = \mathrm{j}, \\ -\dfrac{1}{N_c} 2\,\mathrm{j}\, m_{(2)}^{0,1}, & \mathrm{j}_{\text{ext}} = \mathrm{j} + 1, \end{cases} \tag{13}$$

at leading order $1/N_c$ in the $1/N_c$ expansion.

A realization of $\mathcal{P}_{\mathrm{j}}$ is given by [6]

$$\mathcal{P}_{\mathrm{j}} = \frac{\Pi_{\mathrm{j}\neq \mathrm{j}'}(J^2 - J_{\mathrm{j}'}^2)}{\Pi_{\mathrm{j}\neq \mathrm{j}'}(J_{\mathrm{j}}^2 - J_{\mathrm{j}'}^2)}, \tag{14}$$

i.e., the projection operators for spin J_{j} is given by the product over all $J_{\mathrm{j}'} = 1/2, 3/2, \ldots, N_c/2$ not equal to J_{j}. The general form of the spin projector (14) for arbitrary N_c can be found in Ref. [6]; however, here we just need the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ projectors for $N_c = 3$, which read

$$\mathcal{P}_{\frac{1}{2}} = -\frac{1}{3}\left(J^2 - \frac{15}{4}\right), \tag{15a}$$

$$\mathcal{P}_{\frac{3}{2}} = \frac{1}{3}\left(J^2 - \frac{3}{4}\right), \tag{15b}$$

where

$$\Delta_{\frac{1}{2}} = \begin{cases} 0, & \mathrm{j}_{\text{ext}} = \frac{1}{2}, \\ -\Delta, & \mathrm{j}_{\text{ext}} = \frac{3}{2}, \end{cases} \tag{16a}$$

$$\Delta_{\frac{3}{2}} = \begin{cases} \Delta, & \mathrm{j}_{\text{ext}} = \frac{1}{2}, \\ 0, & \mathrm{j}_{\text{ext}} = \frac{3}{2}, \end{cases} \tag{16b}$$

and

$$\Delta = \frac{3}{N_c} m_{(2)}^{0,1}. \tag{17}$$

It is straightforward to check that expressions (15) meet conditions (10).

The diagram Fig. 1 is thus given by the product of a baryon operator times a flavor tensor containing information about the loop integrals. Using the baryon propagator (9), the loop graphs of Fig. 1 can thus be expressed as

$$M^k_{\text{loop 1}} = \sum_{\text{j}} \epsilon^{ijk} A^{ia} \mathcal{P}_{\text{j}} A^{jb} \Gamma^{ab}(\Delta_{\text{j}}), \tag{18}$$

where A^{ia} and A^{jb} of Eq. (3) have been used at the meson-baryon vertices. Here $\Gamma^{ab}(\Delta_{\text{j}})$ is an antisymmetric tensor which explicitly depends on the difference of the hyperfine mass splitting Δ_{j}. It can be decomposed as

$$\Gamma^{ab}(\Delta_{\text{j}}) = A_0(\Delta_{\text{j}})\Gamma^{ab}_0 + A_1(\Delta_{\text{j}})\Gamma^{ab}_1 + A_2(\Delta_{\text{j}})\Gamma^{ab}_2, \tag{19}$$

where the tensors Γ^{ab}_i are written as [16]

$$\Gamma^{ab}_0 = f^{abQ}, \tag{20a}$$

$$\Gamma^{ab}_1 = f^{ab\overline{Q}}, \tag{20b}$$

$$\Gamma^{ab}_2 = f^{aeQ}d^{be8} - f^{beQ}d^{ae8} - f^{abe}d^{eQ8}. \tag{20c}$$

Let us recall that Γ^{ab}_0 and Γ^{ab}_1 are both SU(3) octets, except that the former transforms as the electric charge whereas the latter also transforms as the electric charge but rotated by π in isospin space. In turn, Γ^{ab}_2 breaks SU(3) as $\mathbf{10} + \overline{\mathbf{10}}$ [16].

On the other hand, the coefficients $A_i(\Delta_{\text{j}})$ are linear combinations of the functions $I(m_\pi, \Delta_{\text{j}})$ and $I(m_K, \Delta_{\text{j}})$ which result of doing the loop integrals; they read

$$A_0(\Delta_{\text{j}}) = \frac{1}{3}[I(m_\pi, \Delta_{\text{j}}) + 2I(m_K, \Delta_{\text{j}})], \tag{21a}$$

$$A_1(\Delta_{\text{j}}) = \frac{1}{3}[I(m_\pi, \Delta_{\text{j}}) - I(m_K, \Delta_{\text{j}})], \tag{21b}$$

$$A_2(\Delta_{\text{j}}) = \frac{1}{\sqrt{3}}[I(m_\pi, \Delta_{\text{j}}) - I(m_K, \Delta_{\text{j}})], \tag{21c}$$

where the loop integral is [10]

$$\left(\frac{8\pi^2 f^2}{M_N}\right) I(m, \Delta_{\text{j}}) = -\Delta_{\text{j}} \ln \frac{m^2}{\mu^2} + \begin{cases} 2\sqrt{m^2 - \Delta_{\text{j}}^2}\left[\dfrac{\pi}{2} - \tan^{-1}\dfrac{\Delta_{\text{j}}}{\sqrt{m^2 - \Delta_{\text{j}}^2}}\right], & |\Delta_{\text{j}}| \le m \\ \sqrt{\Delta_{\text{j}}^2 - m^2}\left[-2i\pi + \ln \dfrac{\Delta_{\text{j}} - \sqrt{\Delta_{\text{j}}^2 - m^2}}{\Delta_{\text{j}} + \sqrt{\Delta_{\text{j}}^2 - m^2}}\right], & |\Delta_{\text{j}}| > m, \end{cases} \tag{22}$$

where $f \sim 93$ MeV is the pion decay constant, M_N and m denote the nucleon and meson masses, respectively, and μ is the renormalization scale.

Thus, the one-loop correction arising from Fig. 1 can be decomposed into the pieces emerging from the flavor $\mathbf{8}$ and flavor $\mathbf{10}+\overline{\mathbf{10}}$ representations as follows,

$$M^k_{\text{loop 1}} = \sum_{\text{j}} \left[A_0(\Delta_{\text{j}}) M^{kQ}_{\mathbf{8},\text{loop 1}}(\mathcal{P}_{\text{j}}) + A_1(\Delta_{\text{j}}) M^{k\overline{Q}}_{\mathbf{8},\text{loop 1}}(\mathcal{P}_{\text{j}}) + A_2(\Delta_{\text{j}}) M^{kQ}_{\mathbf{10}+\overline{\mathbf{10}},\text{loop 1}}(\mathcal{P}_{\text{j}}) \right], \tag{23}$$

where the flavor contributions read

$$M^{kc}_{\mathbf{8},\text{loop 1}}(\mathcal{P}_{\text{j}}) = \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{\text{j}} A^{jb}, \tag{24}$$

and

$$M^{kc}_{\mathbf{10}+\overline{\mathbf{10}},\text{loop 1}}(\mathcal{P}_{\text{j}}) = \epsilon^{ijk}(f^{aec}d^{be8} - f^{bec}d^{ae8} - f^{abe}d^{ec8}) A^{ia} \mathcal{P}_{\text{j}} A^{jb}. \tag{25}$$

For computational purposes, a free flavor index c has been left in Eqs. (24) and (25). This free index can be set to $Q = 3 + (1/\sqrt{3})8$ once the operator reductions on the right-hand sides of such equations have been performed.

The correction $M^k_{\text{loop 1}}$, Eq. (23), to the SU(3) symmetric value of baryon magnetic moment, can be organized as

$$\begin{aligned} M^k_{\text{loop 1}} \; &= \; \mathcal{P}_{1/2} \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{1/2} A^{jb} \left[A_0(0) \Gamma^{ab}_0 + A_1(0) \Gamma^{ab}_1 + A_2(0) \Gamma^{ab}_2 \right] \mathcal{P}_{1/2} \\ & \quad \mathcal{P}_{1/2} \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{3/2} A^{jb} \left[A_0(\Delta) \Gamma^{ab}_0 + A_1(\Delta) \Gamma^{ab}_1 + A_2(\Delta) \Gamma^{ab}_2 \right] \mathcal{P}_{1/2}, \end{aligned} \tag{26}$$

for octet baryons,

$$\begin{aligned} M^k_{\text{loop 1}} \; &= \; \mathcal{P}_{3/2} \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{1/2} A^{jb} \left[A_0(-\Delta) \Gamma^{ab}_0 + A_1(-\Delta) \Gamma^{ab}_1 + A_2(-\Delta) \Gamma^{ab}_2 \right] \mathcal{P}_{3/2} \\ & \quad \mathcal{P}_{3/2} \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{3/2} A^{jb} \left[A_0(0) \Gamma^{ab}_0 + A_1(0) \Gamma^{ab}_1 + A_2(0) \Gamma^{ab}_2 \right] \mathcal{P}_{3/2}, \end{aligned} \tag{27}$$

for decuplet baryons

$$\begin{aligned} M^k_{\text{loop 1}} \; &= \; \mathcal{P}_{3/2} \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{1/2} A^{jb} \left[A_0(0) \Gamma^{ab}_0 + A_1(0) \Gamma^{ab}_1 + A_2(0) \Gamma^{ab}_2 \right] \mathcal{P}_{1/2} \\ & \quad \mathcal{P}_{3/2} \epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{3/2} A^{jb} \left[A_0(\Delta) \Gamma^{ab}_0 + A_1(\Delta) \Gamma^{ab}_1 + A_2(\Delta) \Gamma^{ab}_2 \right] \mathcal{P}_{1/2}, \end{aligned} \tag{28}$$

and for decuplet-octet transitions.

To proceed further, let us note that a generic operator product like $\epsilon^{ijk} f^{abc} A^{ia} \mathcal{P}_{\text{j}} A^{jb}$ can be decomposed as $\alpha \epsilon^{ijk} f^{abc} A^{ia} A^{jb} + \beta \epsilon^{ijk} f^{abc} A^{ia} J^2 A^{jb}$, where α and β are some coefficients. The first summand in the expression mentioned previously corresponds to the degeneracy case $\Delta \to 0$

discussed in Ref. [9] whereas the second one is the new contribution to be dealt with in the present analysis. Now, in the product operators such as $\epsilon^{ijk}f^{abc}A^{ia}J^2A^{jb}$, $\epsilon^{ijk}f^{abe}d^{ec8}A^{ia}J^2A^{jb}$ and so on found in Eqs. (24) and (25), there will appear up to eight-body operators if we truncate the $1/N_c$ expansion of A^{kc} at the physical value $N_c = 3$. The leading order in $1/N_c$ is contained in the product $\epsilon^{ijk}f^{abc}G^{ia}J^2G^{jb}$ and similar terms with two G's, which will be proportional to the square of a_1, which is the leading parameter introduced in Eq. (3). In order to perform the current analysis on an equal footing as Ref. [9], we work out terms up to relative order $\mathcal{O}(1/N_c^3)$, which implies evaluating products up to seven-body operators in Eqs. (24) and (25). The contributions ignored will be proportional to b_3^2, c_3^2, and b_3c_3, which we consider small compared to the ones retained. Because the operator basis is complete [5], the reduction, although long and tedious, is always possible. In Appendix A we present the relevant reductions of baryon operators up to the order in $1/N_c$ required here.

Gathering together partial results, the *spin dependent* contributions to be combined with their spin independent counterparts given in Eqs. (35) and (36) of Ref. [9] are

(1) Flavor $\mathbf{8}$ representation

$$
\begin{aligned}
&\epsilon^{ijk}f^{abc}A^{ia}J^2A^{jb} = \\
&-\frac{1}{2}(N_c+N_f)a_1^2\,G^{kc} + \left[\frac{1}{2}(1+N_f)a_1^2 + \frac{3N_f}{N_c^2}a_1c_3\right]\mathcal{D}_2^{kc} \\
&+\left[-\frac{1}{8}(N_c+N_f)a_1^2 - \frac{N_f}{4N_c}a_1b_2 - \frac{N_c+N_f}{2N_c^2}a_1b_3 - \frac{3(N_c+N_f)}{2N_c^2}a_1c_3\right]\mathcal{D}_3^{kc} \\
&+\left[-\frac{1}{4}(N_c+N_f)a_1^2 - \frac{1+N_f}{N_c}a_1b_2 - \frac{3(N_c+N_f)}{N_c^2}a_1b_3 - \frac{N_c+N_f}{2N_c^2}a_1c_3\right]\mathcal{O}_3^{kc} \\
&+\left[\frac{1}{4}a_1^2 - \frac{N_f}{4N_c^2}b_2^2 - \frac{N_f-2}{2N_c^2}a_1b_3 + \frac{7N_f+12}{4N_c^2}a_1c_3\right]\mathcal{D}_4^{kc} \\
&+\left[-\frac{N_c+N_f}{4N_c^2}a_1c_3 - \frac{N_f}{2N_c^3}b_2b_3\right]\mathcal{D}_5^{kc} + \left[-\frac{1}{2N_c}a_1b_2 - \frac{N_c+N_f}{2N_c^2}a_1b_3\right. \\
&\left.-\frac{N_c+N_f}{4N_c^2}a_1c_3 - \frac{1+N_f}{N_c^3}b_2c_3\right]\mathcal{O}_5^{kc} + \frac{1}{2N_c^2}a_1c_3\mathcal{D}_6^{kc} - \frac{1}{2N_c^3}b_2c_3\mathcal{O}_7^{kc} \\
&+\mathcal{O}(\mathcal{D}_3J^2\mathcal{D}_3),
\end{aligned}
\tag{29}
$$

(2) Flavor $\mathbf{10}+\overline{\mathbf{10}}$ representation

$$
\begin{aligned}
&\epsilon^{ijk}(f^{aec}d^{be8} - f^{bec}d^{ae8} - f^{abe}d^{ec8})A^{ia}J^2A^{jb} = \\
&\quad \frac{1}{2}a_1^2\left(\{T^c,G^{k8}\} - \{G^{kc},T^8\}\right) - \frac{1}{N_c}a_1b_2\left(\{G^{kc},\{J^r,G^{r8}\}\} - \{G^{k8},\{J^r,G^{rc}\}\}\right) \\
&\quad + \frac{1}{2N_c^2}(-4a_1b_3 + 5a_1c_3)\left(\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\} - \{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\right) \\
&\quad + \left[-\frac{1}{4}a_1^2 - \frac{3}{N_c^2}a_1b_3 - \frac{1}{2N_c^2}a_1c_3\right]\left(\{J^2,\{G^{kc},T^8\}\} - \{J^2,\{G^{k8},T^c\}\}\right) \\
&\quad + \left[-\frac{5}{16N_c}a_1b_2 + \frac{3(N_c-6)}{16N_c^2}a_1b_3 - \frac{3(N_c-6)}{32N_c^2}a_1c_3 - \frac{1}{16N_c^3}b_2b_3 + \frac{(37N_c-138)}{16N_c^3}b_2c_3\right] \\
&\quad \times \Bigg[\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} - \{J^2,[G^{k8},\{J^r,G^{rc}\}]\} + \{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&\quad - \{[J^2,G^{k8}],\{J^r,G^{rc}\}\} - \{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\Bigg] \\
&\quad + \left[-\frac{1}{2N_c}a_1b_2 - \frac{1}{N_c^3}b_2c_3\right]\left(\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\} - \{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\}\right) \\
&\quad - \frac{1}{2N_c^3}b_2c_3\Bigg[\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\} - \{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\} \\
&\quad + \{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\} - \{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\} \\
&\quad - \{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\}\Bigg] \\
&\quad + \left[-\frac{1}{2N_c^2}a_1b_3 - \frac{1}{4N_c^2}a_1c_3\right]\left(\{J^2,\{J^2,\{G^{kc},T^8\}\}\} - \{J^2,\{J^2,\{G^{k8},T^c\}\}\}\right) \\
&\quad + \left[-\frac{1}{2N_c^2}a_1b_3 + \frac{1}{4N_c^2}a_1c_3\right]\left(\{J^2,\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\} - \{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\}\right) \\
&\quad - \frac{1}{2N_c^3}b_2c_3\left(\{J^2,\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}\} - \{J^2,\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\}\}\right) \\
&\quad + \mathcal{O}(\mathcal{D}_3J^2\mathcal{D}_3), \qquad (30)
\end{aligned}
$$

where the free flavor index c will be set to $Q = 3 + (1/\sqrt{3})8$ or $\overline{Q} = 3 - (1/\sqrt{3})8$ as required in Eq. (23). The symbol $\mathcal{O}(\mathcal{D}_3J^2\mathcal{D}_3)$ in Eqs. (29) and (30) means that, in the structures such as $\epsilon^{ijk}f^{abc}A^{ia}J^2A^{jb}$, $\epsilon^{ijk}f^{aec}d^{be8}A^{ia}J^2A^{jb}$ and so on we have included all terms up to seven-body operators, such as $\mathcal{D}_2J^2\mathcal{D}_3$, but have neglected contributions which are eight-body operators –like $\mathcal{D}_3J^2\mathcal{D}_3$– or higher. In addition, the operator $[J^2,[T^8,G^{kc}]]$ and its anticommutator with J^2 have been omitted in expresion (30) because they do not contribute to any observed magnetic moments.

Notice also that Eqs. (29) and (30) have been rearranged to exhibit explicitly leading and subleading terms in $1/N_c$. It is simple to realize that these expressions yield matrix elements at most of order $\mathcal{O}(N_c^2)$ if we take into account that baryons with spins of order unity have matrix elements of the flavor generators T^c that are $\mathcal{O}(1)$, $\mathcal{O}(\sqrt{N_c})$, and $\mathcal{O}(N_c)$ for $c = 1,2,3$, $c = 4,5,6,7$, and $c = 8$, respectively, and matrix elements of the spin-flavor generators G^{kc} that are $\mathcal{O}(1)$, $\mathcal{O}(\sqrt{N_c})$, and $\mathcal{O}(N_c)$ for $c = 1,2,3$, $c = 4,5,6,7$, and $c = 8$, respectively [5]. In addition, f, Δ and M_N are $\mathcal{O}(\sqrt{N_c})$, $\mathcal{O}(1/N_c)$ and $\mathcal{O}(N_c)$, respectively, so the one-loop contribution $M_{\text{loop 1}}^k$, Eq. (23), is order $\mathcal{O}(N_c)$. In the limit of small m_s, the symmetry breaking part of $M_{\text{loop 1}}^k$ is $\mathcal{O}(m_s^{1/2})$ so the overall contribution of Eq. (23) to baryon magnetic moments is $\mathcal{O}(m_s^{1/2}N_c)$ and there is little wonder that this contribution is dominant over the one of Fig. 2.

At this stage, analytical expressions for all 27 possible baryon magnetic and transition magnetic moments can readily be obtained by evaluating the matrix elements of the baryon operators indicated in Eqs. (26)-(28) between baryon SU(6) symmetric states. Most matrix elements are listed in Ref. [9], except for a few ones which result from anticommutators of some of the already existing operators with J^2, whose matrix elements can be trivially evaluated. As as example, for μ_{Σ^-} one finds

$$\begin{aligned}
\mu_{\Sigma^-}^{\text{(loop 1)}} = & \left[\frac{7}{18}a_1^2 + \frac{2}{9}a_1b_2 + \frac{1}{18}b_2^2 + \frac{7}{27}a_1b_3 + \frac{2}{27}b_2b_3\right] I(m_\pi, 0) \\
& + \left[\frac{1}{36}a_1^2 - \frac{1}{18}a_1b_2 + \frac{1}{36}b_2^2 + \frac{1}{54}a_1b_3 - \frac{1}{54}b_2b_3\right] I(m_K, 0] \\
& + \left[-\frac{1}{18}a_1^2 - \frac{1}{18}a_1c_3\right] I(m_\pi, \Delta) + \left[-\frac{1}{9}a_1^2 - \frac{1}{9}a_1c_3\right] I(m_K, \Delta), \qquad (31)
\end{aligned}$$

which in the limit $\Delta \to 0$ reduces to the value already found [9]. Theoretical expressions like (31) are quite useful when comparing our results with the ones obtained in the framework of chiral perturbation theory [10–13]. It has been already shown that there is a one-to-one correspondence between the parameters of the $1/N_c$ baryon chiral Lagrangian at $N_c = 3$ [6] and the octet and decuplet chiral Lagrangian [2, 3]. The baryon-pion couplings are related to the coefficients of the $1/N_c$ expansion of A^{ia}, Eq. (3), at $N_c = 3$ by

$$D = \frac{1}{2}a_1 + \frac{1}{6}b_3, \qquad (32a)$$

$$F = \frac{1}{3}a_1 + \frac{1}{6}b_2 + \frac{1}{9}b_3, \qquad (32b)$$

$$\mathcal{C} = -a_1 - \frac{1}{2}c_3, \qquad (32c)$$

$$\mathcal{H} = -\frac{3}{2}a_1 - \frac{3}{2}b_2 - \frac{5}{2}b_3. \qquad (32d)$$

For octet baryons, the magnetic moments computed in Ref. [10] can be rewritten as

$$\mu_i = \alpha_i + \sum_{X=\pi,K} \beta_i^{(X)} I(m_X,0) + \sum_{X=\pi,K} \beta_i^{\prime(X)} I(m_X,\Delta) + \sum_{X=\pi,K,\eta} \frac{1}{32\pi^2 f^2}(\overline{\gamma}_i^{(X)} - 2\overline{\lambda}_i^{(X)}\alpha_i) m_X^2 \ln\frac{m_X^2}{\mu^2}, \tag{33}$$

where α_i corresponds to the tree-level value of baryon i, $\beta_i^{(X)}$ and $\beta_i^{\prime(X)}$ are the contributions arising from loop graphs of Fig. 1 and the remaining coefficients come from loop graphs of Fig. 2. For μ_{Σ^-}, the corresponding chiral coefficients listed in Ref. [10] read

$$\beta_{\Sigma^-}^{(\pi)} = \frac{2}{3}D^2 + 2F^2, \qquad \beta_{\Sigma^-}^{(K)} = (D-F)^2, \qquad \beta_{\Sigma^-}^{\prime(\pi)} = -\frac{1}{18}\mathcal{C}^2, \qquad \beta_{\Sigma^-}^{\prime(K)} = -\frac{1}{9}\mathcal{C}^2. \tag{34}$$

Under identifications (32), the above chiral coefficients coincide with their corresponding analogues in Eq. (31). The same agreement if found in all expressions for octet baryons. As for decuplet baryons and decuplet-octet transitions, the comparison is not as simple as in the previous case so we prefer to perform a numerical comparison instead. This will be discussed in the next section.

On the other hand, corrections of order $\mathcal{O}(m_q^{1/2} N_c)$ with non-vanishing Δ have some important effects on the Coleman-Glashow relations referred to in the introductory section. First, the term that comes along with A_0, $M_{\mathbf{8},\text{loop 1}}^{kQ}$ in Eq. (23), yields baryon magnetic moments that satisfy relations (1) whereas violations to them are due to the terms that accompany to A_1 and A_2, which are $M_{\mathbf{8},\text{loop 1}}^{k\overline{Q}}$ and $M_{\mathbf{10}+\overline{\mathbf{10}},\text{loop 1}}^{kQ}$, respectively. For instance, for the first relation one has

$$\begin{aligned}\mu_{\Sigma^+}^{\text{(loop 1)}} - \mu_p^{\text{(loop 1)}} = {} & \left[-\frac{11}{36}[I(m_K,0) - I(m_\pi,0)] - \frac{5}{18}[I(m_K,\Delta) - I(m_\pi,\Delta)]\right] a_1^2 \\ & - \frac{1}{18}[I(m_K,0) - I(m_\pi,0)]a_1 b_2 + \frac{1}{36}[I(m_K,0) - I(m_\pi,0)]b_2^2 \\ & - \frac{11}{54}[I(m_K,0) - I(m_\pi,0)]a_1 b_3 - \frac{1}{54}[I(m_K,0) - I(m_\pi,0)]b_2 b_3 \\ & - \frac{5}{18}[I(m_K,\Delta) - I(m_\pi,\Delta)]a_1 c_3. \end{aligned} \tag{35}$$

Analogous results are obtained for the remaining relations and will not be listed here.

In addition, we can also verify that the sum rules derived by Caldi and Pagels [7] are also satisfied for $\Delta \neq 0$ in our approach, namely,

$$\mu_{\Sigma^+}^{\text{(loop 1)}} + 2\mu_\Lambda^{\text{(loop 1)}} + \mu_{\Sigma^-}^{\text{(loop 1)}} = 0, \tag{36}$$

$$\mu_{\Xi^0}^{\text{(loop 1)}} + \mu_{\Xi^-}^{\text{(loop 1)}} + \mu_n^{\text{(loop 1)}} - 2\mu_\Lambda^{\text{(loop 1)}} + 2\mu_p^{\text{(loop 1)}} = 0, \tag{37}$$

and

$$\mu_{\Lambda}^{\text{(loop 1)}} - \sqrt{3}\mu_{\Lambda\Sigma^0}^{\text{(loop 1)}} - \mu_{\Xi^0}^{\text{(loop 1)}} - \mu_{n}^{\text{(loop 1)}} = 0. \tag{38}$$

In turn, the relation

$$\mu_{\Sigma^+}^{\text{(loop 1)}} - 2\mu_{\Sigma^0}^{\text{(loop 1)}} + \mu_{\Sigma^-}^{\text{(loop 1)}} = 0, \tag{39}$$

also holds to this order.

Similarly, for decuplet baryons we also find that the sum rules introduced in Ref. [17] are also satisfied,

$$\mu_{\Delta^{++}}^{\text{(loop 1)}} - \mu_{\Delta^+}^{\text{(loop 1)}} - \mu_{\Delta^0}^{\text{(loop 1)}} + \mu_{\Delta^-}^{\text{(loop 1)}} = 0, \tag{40}$$

$$\mu_{\Sigma^{*+}}^{\text{(loop 1)}} - 2\mu_{\Sigma^{*0}}^{\text{(loop 1)}} + \mu_{\Sigma^{*-}}^{\text{(loop 1)}} = 0, \tag{41}$$

and

$$\mu_{\Delta^{++}}^{\text{(loop 1)}} - 3\mu_{\Delta^+}^{\text{(loop 1)}} + 3\mu_{\Delta^0}^{\text{(loop 1)}} - \mu_{\Delta^-}^{\text{(loop 1)}} = 0, \tag{42}$$

whereas for transition magnetic moments, one also has

$$\mu_{\Delta^+ p}^{\text{(loop 1)}} - \mu_{\Delta^0 n}^{\text{(loop 1)}} = 0, \tag{43}$$

and

$$\mu_{\Sigma^{*+}\Sigma^+}^{\text{(loop 1)}} - 2\mu_{\Sigma^{*0}\Sigma^0}^{\text{(loop 1)}} + \mu_{\Sigma^{*-}\Sigma^-}^{\text{(loop 1)}} = 0. \tag{44}$$

In summary, the introduction of a non-vanishing Δ does not modify the sum rules between magnetic moments derived in previous works. We are now in a position of performing a fit to the experimental data [14] in order to provide numerical estimates of our findings. This is now discussed in the next section.

IV. FIT TO EXPERIMENTAL DATA

We are ready to perform a numerical comparison of the theoretical expressions obtained here with the available experimental data [14] through a least-squares fit. In order to perform this comparison on an equal footing as in Ref. [9], we also use the very same 11 baryon magnetic moments. This information is displayed in Table I (second column from left to right).

The analytic expressions used are written in terms of two sets of parameters: the first one is constituted by a_1, b_2, b_3 and c_3 arising from the $1/N_c$ expansion of A^{kc}, Eq. (3), and the latter is formed by $m_1, \ldots, m_4$ arising from the $1/N_c$ expansion of M^{kc}, Eq. (7). The first set plays an important role in the leading order corrections discussed here.

As for the least-squares fit, the actual theoretical expressions we use are given as

$$\mu_B = \alpha_0(\mu_B^{(0)} + \mu_B^{(\text{loop 2})}) + \mu_B^{(\text{loop 1})}, \tag{45}$$

where a scale factor $\alpha_0 = 2\mu_p^{\text{exp}}$ is incorporated in order to ensure that the best-fit parameters m_i are order one at most. For the present analysis, $\mu_B^{(\text{loop 1})}$ is the contribution obtained here which includes a non-vanishing Δ whereas $\mu_B^{(\text{loop 2})}$ is the contribution already discussed in Ref. [9] in the degeneracy limit.

In order to get a meaningful χ^2, we need to add a theoretical error to each magnetic moment. In the analysis of Ref. [9] the estimated error was $0.05\,\mu_N$. If we add this now, the fit would indeed produce a small χ^2, but the analysis would be strongly biased by the determination of $\mu_{\Delta^{++}}$ used. We prefer to add a more conservative estimate of $0.03\,\mu_N$, assuming that the error we introduce with the approximation implemented is $\sim 1/N_c^3$.

Without furher ado, best-fit parameters found are

$$\begin{aligned}
a_1 &= 0.98 \pm 0.09, & m_1 &= 1.31 \pm 0.03, \\
b_2 &= -0.94 \pm 0.14, & m_2 &= 0.34 \pm 0.12, \\
b_3 &= -1.05 \pm 0.14, & m_3 &= -0.22 \pm 0.09, \\
c_3 &= -0.80 \pm 0.12, & m_4 &= 0.05 \pm 0.17,
\end{aligned} \tag{46}$$

with $\chi^2 = 13.75$ for three degrees of freedom and the quoted errors come from the fit only. The higher contributions to χ^2 come this time from μ_Λ $(\Delta\chi^2 = 2.43)$, μ_{Ξ^0} $(\Delta\chi^2 = 2.81)$ and $\mu_{\Delta^{++}}$ $(\Delta\chi^2 = 2.74)$. The relative high χ^2 almost doubles the one got in Ref. [9], which is a consequence of our working assumptions. For instance, theoretical errors of $0.04\,\mu_N$ and $0.05\,\mu_N$ added would yield $\chi^2 = 2.30$ and $\chi^2 = 1.61$, respectively. In consequence, there would be a shifting in the central values of the best-fit parameters but the strongest effect would be observed in a significant increase in the quoted errors with respect to the case previously discussed, particularly in m_4 (the fits produce $m_4 = -0.79 \pm 1.27$ and $m_4 = -0.72 \pm 1.52$, respectively).

As for the fit itself, the best-fit parameters obtained agree with expectations. We notice that the parameters of the first set are order $\mathcal{O}(1)$, as expected. As for the second set, with the introduction of the scale α_0, the values are in good agreement with the $1/N_c$ predictions: The leading order parameter m_1 is order $\mathcal{O}(1)$, whereas m_2, and m_3 and m_4 are roughly suppressed by $1/N_c$ and $1/N_c^2$, respectively, relative to the leading order parameter.

In addition, we find numerically that $F = 0.05$, $D = 0.31$, $\mathcal{C} = -0.58$ and $\mathcal{H} = 2.56$ for the first set referred to above and $\mu_F = 0.47$, $\mu_D = 0.62$, $\mu_C = 0.64$ and $\mu_H = -2.68$ for the second set. Indeed, these values still do not coincide with the determinations in heavy baryon chiral perturbation theory [2, 3, 10], which causes some worry.

In Table I, the third column (from left to right) displays the predicted magnetic moments within the combined expansion in m_q and $1/N_c$. These predicted values consist of contributions arising from tree level and loop graphs from Fig. 1, Fig. 2(a-d), and Fig. 2(e). The predictions are in good agreement with the measured ones. We are able to also provide some predictions of the unmeasured magnetic moments. They are in overall good agreement with other determinations in the framework of the $1/N_c$ expansion [17] and chiral perturbation theory [12]. We only mention that, for instance, $\mu_{\Sigma^{*0}} = 0$ at tree level and up to corrections of order $\mathcal{O}(m_q^{1/2})$, but a non-vanishing contribution is picked up due to terms of order $\mathcal{O}(m_q \ln m_q)$. We also note in passing that, for the transition $\Sigma^{*0}\Lambda$ although its predicted magnetic moment is in magnitude comparable to the one reported in Ref. [17], its carries the opposite sign.

V. CONCLUDING REMARKS

In this paper we evaluated the magnetic moments of baryons within large-N_c chiral perturbation theory, including one-loop corrections of orders $\mathcal{O}(m_q^{1/2})$ and $\mathcal{O}(m_q \ln m_q)$ by following the lines of Ref. [9]. The present analysis differs from the previous one in the sense that here we consider the effects that a non-vanishing baryon decuplet-octet mass difference Δ introduces in the correction of order $\mathcal{O}(m_q^{1/2})$. In the large-N_c limit, $\Delta \propto 1/N_c$ so the degeneracy case constitutes a very good first approximation. However, a more realistic situation would be to consider $\Delta \neq 0$. Because both types of corrections involve a rather different mathematical complication, as a second approximation we study the case indicated above and postpone the analysis of the other one [15].

In a complete parallelism to Ref. [9], we constructed the baryon operator that describes the order $\mathcal{O}(m_q^{1/2})$ correction to baryon magnetic moments. This correction arises from the Feynman diagrams depicted in Fig. 1. The explicit dependence on Δ is contained in the definition of the baryon propagator (9). After a long, tedious, but otherwise standard calculation, we obtain the *spin-dependend* terms, (29) and (30), which have to be combined with the spin-independent ones already computed in Ref. [9]. Expressions like (31) are thus obtained for $\mu_B^{\text{(loop 1)}}$ for all 27 possible

TABLE I: Numerical values of baryon magnetic moments found in this work and comparison with the available experimental data. Comparisons with other determinations is also included. The entries are given in nuclear magnetons.

Baryon	Experimental Data	Total	Tree level	Loop 1	Loop 2(a-d)	Loop 2(e)	Ref. [17]	Ref. [12]
n	-1.913 ± 0.000	-1.955	-2.310	-0.046	0.125	0.276		
p	2.793 ± 0.000	2.771	3.786	-0.091	-0.090	-0.834		
Σ^-	-1.160 ± 0.025	-1.186	-1.476	0.111	0.135	0.043		
Σ^0		0.619	1.155	-0.036	-0.064	-0.437	$0.77(10)$	
Σ^+	2.458 ± 0.010	2.424	3.786	-0.183	-0.263	-0.916		
Ξ^-	-0.651 ± 0.003	-0.683	-1.476	0.059	0.094	0.639		
Ξ^0	-1.250 ± 0.014	-1.306	-2.310	0.150	0.064	0.791		
Λ	-0.613 ± 0.004	-0.566	-1.155	0.036	0.117	0.437		
$\Lambda\Sigma^0$	1.61 ± 0.08	1.576	2.001	-0.039	-0.021	-0.364		
Δ^{++}	6.14 ± 0.51^a	5.294	7.203	-0.020	-0.225	-1.664		$6.04(13)$
Δ^+		2.350	3.601	-0.041	-0.202	-1.008	$3.04(13)$	$2.84(2)$
Δ^0		-0.594	0.000	-0.062	-0.180	-0.352	$0.00(10)$	$-0.36(9)$
Δ^-		-3.538	-3.601	-0.083	-0.158	0.303	$-3.04(13)$	$-3.56(20)$
Σ^{*+}		2.525	3.601	0.021	-0.441	-0.656	$3.35(13)$	$3.07(12)$
Σ^{*0}		-0.267	0.000	0.000	-0.267	0.000	$0.32(11)$	0
Σ^{*-}		-3.059	-3.601	-0.021	-0.093	0.656	$-2.70(13)$	$-3.07(12)$
Ξ^{*0}		0.178	0.000	0.062	-0.237	0.352	$0.64(11)$	$0.36(9)$
Ξ^{*-}		-2.559	-3.601	0.041	-0.007	1.008	$-2.36(14)$	$-2.56(6)$
Ω^-	-2.02 ± 0.05	-2.038	-3.601	0.103	0.099	1.361		-2.02
$\Delta^+ p$	3.51 ± 0.09	3.462	3.530	1.665	-1.090	-0.643		
$\Delta^0 n$		3.462	3.530	1.665	-1.090	-0.643	$3.51(11)$	
$\Sigma^{*0}\Lambda$		-2.728	-3.057	-1.760	1.533	0.557	$2.93(11)$	
$\Sigma^{*0}\Sigma^0$		1.430	1.765	1.687	-1.354	-0.667	$1.39(11)$	
$\Sigma^{*+}\Sigma^+$		2.885	3.530	3.071	-2.728	-0.988	$2.97(11)$	
$\Sigma^{*-}\Sigma^-$		-0.024	0.000	0.303	0.019	-0.345	$-0.19(11)$	
$\Xi^{*0}\Xi^0$		2.743	3.530	3.071	-2.869	-0.988	$2.96(12)$	
$\Xi^{*-}\Xi^-$		-0.113	0.000	0.303	-0.070	-0.345	$-0.19(11)$	

[a]Value reported in Ref. [18].

magnetic moments, which, together with the tree-level value $\mu_B^{(0)}$ and $\mu_B^{(\text{loop 2})}$ from Fig. 2, make up the magnetic moment of baryon B.

The final analytical expressions were compared with the experimental data [14] through a least-squares fit and also crosschecked with other calculations within the $1/N_c$ expansion [17] and chiral perturbation theory [10, 12]. We should stress that for octet baryons the comparison between analytical expressions was possible whereas for the other cases it was performed through numerical estimates. The overall comparison has been a successful one.

As for the least-squares fit, we also found evidence here that the invariant couplings F, D, $\mathcal{C}$ and $\mathcal{H}$ [related to the parameters of the $1/N_c$ expansion of the axial current by Eq. (32)], neither at tree level nor one-loop corrected, produce physically admissible fits. We thus allowed all eight parameters (a_1, b_2, b_3, and c_3 and $m_1, \ldots m_4$) enter as free ones in the analysis. The best-fit parameters found, listed in (46), agree very well with expectations. These parameters are then used to provide the numerical values of unmeasured magnetic moments displayed in Table I and then compared with other calculations. Again, there is an overall good agreement. Unfortunately, we still do not reproduce the numerical values of the SU(3) invariants F, D, $\mathcal{C}$ and $\mathcal{H}$ and μ_F, μ_D, μ_C and μ_T which appear in chiral perturbation theory. Certainly, there are some improvements in the determinations of $\mathcal{H}$ and μ_T as compared with Ref. [9]. Further improvement should come from new and/or additional measurements on transition magnetic moments.

We should point out that the numerical values listed in Table I are a consequence of our working assumptions. In other words, the theoretical error added to the observed magnetic moments plays an important role in the fit itself. Either a larger or a smaller error has a strong impact on the output. As mentioned in the previous section, the error we added is a conservative estimate and it should reflect somehow the degree of precision we achieve in the present analysis. What is important to state is that a non-vanishing Δ leads to simplicity, stability and robustness of the fitting, which was hardly attainable in Ref. [9]. The preliminary results displayed in Table I should at any rate be improved by adding a non-vanishing Δ to the correction of order $\mathcal{O}(m_q \ln m_q)$. This problem, however, requires a non-negligible effort and will be presented in a future work [15].

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Appendix A: Reduction of baryon operators

1. Flavor $\mathbf{8}$ representation

The reduction of the baryon operator $\epsilon^{ijk} f^{abc} A^{ia} J^2 A^{jb}$, up to relative order $\mathcal{O}(1/N_c^3)$ in the $1/N_c$ expansion, for the individual contributions yields

$$\epsilon^{ijk} f^{abc} G^{ia} J^2 G^{jb} = -\frac{1}{2}(N_c+N_f) G^{kc} + \frac{1}{2}(N_f+1)\mathcal{D}_2^{kc} - \frac{1}{8}(N_c+N_f)\mathcal{D}_3^{kc} - \frac{1}{4}(N_c+N_f)\mathcal{O}_3^{kc} + \frac{1}{4}\mathcal{D}_4^{kc}, \tag{A1}$$

$$\epsilon^{ijk} f^{abc} (G^{ia} J^2 \mathcal{D}_2^{jb} + \mathcal{D}_2^{ia} J^2 G^{jb}) = -\frac{1}{4} N_f \mathcal{D}_3^{kc} - (N_f+1)\mathcal{O}_3^{kc} - \frac{1}{2}\mathcal{O}_5^{kc}, \tag{A2}$$

$$\epsilon^{ijk} f^{abc} \mathcal{D}_2^{ia} J^2 \mathcal{D}_2^{jb} = -\frac{1}{4} N_f \mathcal{D}_4^{kc}, \tag{A3}$$

$$\begin{aligned} \epsilon^{ijk} f^{abc} (G^{ia} J^2 \mathcal{D}_3^{jb} + \mathcal{D}_3^{ia} J^2 G^{jb}) \ = \ & -\frac{1}{2}(N_c+N_f)\mathcal{D}_3^{kc} - 3(N_c+N_f)\mathcal{O}_3^{kc} - \frac{1}{2}(N_f-2)\mathcal{D}_4^{kc} \\ & - \frac{1}{2}(N_c+N_f)\mathcal{O}_5^{kc}, \end{aligned} \tag{A4}$$

$$\begin{aligned} \epsilon^{ijk} f^{abc} (G^{ia} J^2 \mathcal{O}_3^{jb} + \mathcal{O}_3^{ia} J^2 G^{jb}) = {} & 3N_f \mathcal{D}_2^{kc} - \frac{3}{2}(N_c+N_f)\mathcal{D}_3^{kc} - \frac{1}{2}(N_c+N_f)\mathcal{O}_3^{kc} \\ & + \frac{1}{4}(7N_f+12)\mathcal{D}_4^{kc} - \frac{1}{4}(N_c+N_f)\mathcal{D}_5^{kc} - \frac{1}{4}(N_c+N_f)\mathcal{O}_5^{kc} + \frac{1}{2}\mathcal{D}_6^{kc}, \end{aligned} \tag{A5}$$

$$\epsilon^{ijk} f^{abc} (\mathcal{D}_2^{ia} J^2 \mathcal{D}_3^{jb} + \mathcal{D}_3^{ia} J^2 \mathcal{D}_2^{jb}) = -\frac{1}{2} N_f \mathcal{D}_5^{kc}, \tag{A6}$$

$$\epsilon^{ijk} f^{abc} (\mathcal{D}_2^{ia} J^2 \mathcal{O}_3^{jb} + \mathcal{O}_3^{ia} J^2 \mathcal{D}_2^{jb}) = -(N_f+1)\mathcal{O}_5^{kc} - \frac{1}{2}\mathcal{O}_7^{kc}. \tag{A7}$$

2. Flavor $\mathbf{10}+\overline{\mathbf{10}}$ representation

Now, the reduction of the baryon operator $\epsilon^{ijk}(f^{aec}d^{be8} - f^{bec}d^{ae8} - f^{abe}d^{ec8}) A^{ia} J^2 A^{jb}$ leads to

$$\begin{aligned} & \epsilon^{ijk}(f^{aec}d^{be8} - f^{bec}d^{ae8} - f^{abe}d^{ec8}) G^{ia} J^2 G^{jb} = -\frac{1}{2}\{G^{kc},T^8\} + \frac{1}{2}\{G^{k8},T^c\} \\ & + \frac{1}{N_f}[J^2,[T^8,G^{kc}]] - \frac{1}{4}\{J^2,\{G^{kc},T^8\}\} + \frac{1}{4}\{J^2,\{G^{k8},T^c\}\} + \frac{1}{2N_f}\{J^2,[J^2,[T^8,G^{kc}]]\}, \end{aligned} \tag{A8}$$

$$\epsilon^{ijk}(f^{aec}d^{be8}-f^{bec}d^{ae8}-f^{abe}d^{ec8})(G^{ia}J^2\mathcal{D}_2^{jb}+\mathcal{D}_2^{ia}J^2G^{jb}) = \frac{N_c+N_f}{N_f}[J^2,[T^8,G^{kc}]]
- \{G^{kc},\{J^r,G^{r8}\}\} + \{G^{k8},\{J^r,G^{rc}\}\} - \frac{5}{16}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}
+ \frac{5}{16}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} + \frac{N_c+N_f}{2N_f}\{J^2,[J^2,[T^8,G^{kc}]]\} - \frac{5}{16}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}
+ \frac{5}{16}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} - \frac{1}{2}\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\} + \frac{1}{2}\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\}
+ \frac{5}{16}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}, \tag{A9}$$

$$\epsilon^{ijk}(f^{aec}d^{be8}-f^{bec}d^{ae8}-f^{abe}d^{ec8})\mathcal{D}_2^{ia}J^2\mathcal{D}_2^{jb} = 0, \tag{A10}$$

$$\epsilon^{ijk}(f^{aec}d^{be8}-f^{bec}d^{ae8}-f^{abe}d^{ec8})(G^{ia}J^2\mathcal{D}_3^{jb}+\mathcal{D}_3^{ia}J^2G^{jb}) = -3\{J^2,\{G^{kc},T^8\}\}
+ 3\{J^2,\{G^{k8},T^c\}\} - 2\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\} + 2\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} + \frac{6}{N_f}\{J^2,[J^2,[T^8,G^{kc}]]\}
+ \frac{3}{16}(N_c-6)\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} - \frac{3}{16}(N_c-6)\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}
+ \frac{3}{16}(N_c-6)\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} - \frac{3}{16}(N_c-6)\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}
- \frac{3}{16}(N_c-6)\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} - \frac{1}{2}\{J^2,\{J^2,\{G^{kc},T^8\}\}\}
+ \frac{1}{2}\{J^2,\{J^2,\{G^{k8},T^c\}\}\} - \frac{1}{2}\{J^2,\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\} + \frac{1}{2}\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\}
+ \frac{1}{N_f}\{J^2,\{J^2,[J^2,[T^8,G^{kc}]]\}\}, \tag{A11}$$

$$\epsilon^{ijk}(f^{aec}d^{be8}-f^{bec}d^{ae8}-f^{abe}d^{ec8})(G^{ia}J^2\mathcal{O}_3^{jb}+\mathcal{O}_3^{ia}J^2G^{jb}) = -\frac{1}{2}\{J^2,\{G^{kc},T^8\}\}
+ \frac{1}{2}\{J^2,\{G^{k8},T^c\}\} + \frac{5}{2}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\} - \frac{5}{2}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} + \frac{1}{N_f}\{J^2,[J^2,[T^8,G^{kc}]]\}
- \frac{3}{32}(N_c-6)\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} + \frac{3}{32}(N_c-6)\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}
- \frac{3}{32}(N_c-6)\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} + \frac{3}{32}(N_c-6)\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}
+ \frac{3}{32}(N_c-6)\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} - \frac{1}{4}\{J^2,\{J^2,\{G^{kc},T^8\}\}\}
+ \frac{1}{4}\{J^2,\{J^2,\{G^{k8},T^c\}\}\} + \frac{1}{4}\{J^2,\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\} - \frac{1}{4}\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\}
+ \frac{1}{2N_f}\{J^2,\{J^2,[J^2,[T^8,G^{kc}]]\}\}, \tag{A12}$$

$$\epsilon^{ijk}(f^{aec}d^{be8} - f^{bec}d^{ae8} - f^{abe}d^{ec8})(\mathcal{D}_2^{ia}J^2\mathcal{D}_3^{jb} + \mathcal{D}_3^{ia}J^2\mathcal{D}_2^{jb}) = -\frac{1}{16}\{J^2, [G^{kc}, \{J^r, G^{r8}\}]\}$$
$$+ \frac{1}{16}\{J^2, [G^{k8}, \{J^r, G^{rc}\}]\} - \frac{1}{16}\{[J^2, G^{kc}], \{J^r, G^{r8}\}\} + \frac{1}{16}\{[J^2, G^{k8}], \{J^r, G^{rc}\}\}$$
$$+ \frac{1}{16}\{J^k, [\{J^m, G^{mc}\}, \{J^r, G^{r8}\}]\}, \tag{A13}$$

$$\epsilon^{ijk}(f^{aec}d^{be8} - f^{bec}d^{ae8} - f^{abe}d^{ec8})(\mathcal{D}_2^{ia}J^2\mathcal{O}_3^{jb} + \mathcal{O}_3^{ia}J^2\mathcal{D}_2^{jb}) =$$
$$\frac{N_c+N_f}{N_f}\{J^2, [J^2, [T^8, G^{kc}]]\} + \frac{1}{2N_f}(N_c+N_f)\{J^2, \{J^2, [J^2, [T^8, G^{kc}]]\}\}$$
$$- \{J^2, \{G^{kc}, \{J^r, G^{r8}\}\}\} + \{J^2, \{G^{k8}, \{J^r, G^{rc}\}\}\}$$
$$+ \frac{1}{16}(37N_c - 138)\{J^2, [G^{kc}, \{J^r, G^{r8}\}]\} - \frac{1}{16}(37N_c - 138)\{J^2, [G^{k8}, \{J^r, G^{rc}\}]\}$$
$$+ \frac{1}{16}(37N_c - 138)\{[J^2, G^{kc}], \{J^r, G^{r8}\}\} - \frac{1}{16}(37N_c - 138)\{[J^2, G^{k8}], \{J^r, G^{rc}\}\}$$
$$- \frac{1}{16}(37N_c - 138)\{J^k, [\{J^m, G^{mc}\}, \{J^r, G^{r8}\}]\} - \frac{1}{2}\{J^2, \{J^2, [G^{kc}, \{J^r, G^{r8}\}]\}\}$$
$$+ \frac{1}{2}\{J^2, \{J^2, [G^{k8}, \{J^r, G^{rc}\}]\}\} - \frac{1}{2}\{J^2, \{[J^2, G^{kc}], \{J^r, G^{r8}\}\}\}$$
$$+ \frac{1}{2}\{J^2, \{[J^2, G^{k8}], \{J^r, G^{rc}\}\}\} + \frac{1}{2}\{J^2, \{J^k, [\{J^m, G^{mc}], \{J^r, G^{r8}\}\}]\}$$
$$- \frac{1}{2}\{J^2, \{J^2, \{G^{kc}, \{J^r, G^{r8}\}\}\}\} + \frac{1}{2}\{J^2, \{J^2, \{G^{k8}, \{J^r, G^{rc}\}\}\}\}. \tag{A14}$$

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