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Punto 5 (teórico)

¿Cuál es el orden de  $\mathcal{O}(h^k)$  de la aproximación?

$$\begin{aligned} f^4(x) &= \frac{f'''(x+h) - f'''(x-h)}{h} - \mathcal{O}(h^2) \\ &= \frac{\left( \frac{f''(x+h) - f''(x)}{h} - \mathcal{O}(h^2) - \frac{f''(x) - f''(x-h)}{h} - \mathcal{O}(h^2) \right)}{2h} - \mathcal{O}(h^2) \\ &= \frac{f''(x+2h) - 2f''(x) + f''(x-2h)}{h^2} - \mathcal{O}(h^2) \\ &= \frac{\frac{f'(x+3h) - f'(x+h)}{h} + \frac{-2f'(x+h) + 2f'(x-h)}{h} + \frac{(f'(x-h) - f'(x-3h))}{h}}{h^2} \\ &\quad - \mathcal{O}(h^2) \\ &= \frac{f'(x+3h) - f'(x+h) - 2f'(x+h) + 2f'(x-h) + f'(x-h) - f'(x-3h)}{2h} \\ &\quad - \mathcal{O}(h^2) \\ &= \frac{f'(x+3h) - 3f'(x+h) + 3f'(x-h) - f'(x-3h)}{h^3} - \mathcal{O}(h^2) \\ &= \frac{\frac{f(x+4h) - f(x+2h)}{h} + \frac{-3f(x+2h) + 3f(x)}{h} + \frac{3f(x) - 3f(x-2h)}{h} + \frac{-f(x-2h) + f(x-4h)}{h}}{h^3} \\ &\quad - \mathcal{O}(h^2) \\ &= \frac{f(x+4h) - f(x+2h) - 3f(x+2h) + 3f(x) + 3f(x) - 3f(x-2h) - f(x-2h) + f(x-4h)}{2h} \\ &\quad - \mathcal{O}(h^2) \\ &= \frac{f(x+4h) - 4f(x+2h) + 6f(x) - 4f(x-2h) + f(x-4h)}{h^4} - \mathcal{O}(h^2) \end{aligned}$$

$D_p = \text{Derivada progresiva}$

$$D_p = f(x + h)$$

$$= f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \frac{h^4}{24}f^{(4)}(x) + \frac{h^5}{120}f^{(5)}(x) + \frac{h^6}{720}f^{(6)}(x)$$

$D_r = \text{Derivada regresiva}$

$$D_r = f(x - h)$$

$$= f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \frac{h^4}{24}f^{(4)}(x) - \frac{h^5}{120}f^{(5)}(x) + \frac{h^6}{720}f^{(6)}(x)$$

$$D_p + D_r = f(x + h) + f(x - h) = 2f(x) + h^2f''(x) + \frac{h^4}{12}f^{(4)}(x) + \frac{h^6}{360}f^{(6)}(x)$$

$$\begin{aligned} f^{(4)}(x) &= \frac{12 \left( f(x + h) + f(x - h) - f(x) - h^2f''(x) - \frac{h^6}{360}f^{(6)}(x) \right)}{h^4} \\ &= \frac{12(f(x + h) + f(x - h) - f(x) - h^2f''(x))}{h^4} - \frac{h^2}{30}f^{(6)}(x) \end{aligned}$$

Error:

$$\mathcal{O}(h^k) = \mathcal{O}(h^2) = \frac{h^4}{30h^2}f^{(6)}(x)$$