Miguelangel García Castillo - 202115260

Camilo Andrés Castillo - 201813097

Punto 5 (teórico)

¿Cuál es el orden de $\mathcal{O}(h^k)$ de la aproximación?

$$f^4(x) = \frac{f'''(x+h) - f'''(x-h)}{2h} - \mathcal{O}(h^2)$$

$$= \frac{\left(\frac{f''(x+h) - f''(x)}{2h} - \mathcal{O}(h^2) - \frac{f''(x) - f''(x-h)}{2h} - \mathcal{O}(h^2)\right)}{2h} - \mathcal{O}(h^2)$$

$$= \frac{f''(x+2h) - 2f''(x) + f''(x-2h)}{4h^2} - \mathcal{O}(h^2)$$

$$= \frac{\frac{f'(x+3h) - f'(x+h)}{2h} + \frac{-2f'(x+h) + 2f'(x-h)}{2h} + \frac{(f'(x-h) - f'(x-3h))}{2h}}{4h^2} - \mathcal{O}(h^2)$$

$$= \frac{\frac{f'(x+3h) - f'(x+h) - 2f'(x+h) + 2f'(x-h) + f'(x-h) - f'(x-3h)}{2h}}{4h^2} - \mathcal{O}(h^2)$$

$$= \frac{f'(x+3h) - 3f'(x+h) + 3f'(x-h) - f'(x-3h)}{8h^3} - \mathcal{O}(h^2)$$

$$= \frac{\frac{f(x+4h) - f(x+2h)}{2h} + \frac{-3f(x+2h) + 3f(x)}{2h} + \frac{3f(x) - 3f(x-2h)}{2h} + \frac{-f(x-2h) + f(x-4h)}{2h}}{8h^3} - \mathcal{O}(h^2)$$

$$= \frac{\frac{f(x+4h) - f(x+2h) - 3f(x+2h) + 3f(x) + 3f(x) - 3f(x-2h) - f(x-2h) + f(x-4h)}{2h}}{8h^3} - \mathcal{O}(h^2)$$

$$= \frac{f(x+4h) - 4f(x+2h) + 6f(x) - 4f(x-2h) + f(x-4h)}{8h^4} - \mathcal{O}(h^2)$$

$$D_p = Derivada\ progresiva$$

$$D_p = f(x+h)$$
$$= f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^4(x) + \frac{h^5}{120} f^5(x) + \frac{h^6}{720} f^6(x)$$

$$D_r = Derivada\ regresiva$$

$$D_r = f(x-h)$$
$$= f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^4(x) - \frac{h^5}{120} f^5(x) + \frac{h^6}{720} f^6(x)$$

Cuando la derivada es par, la derivada progresiva y regresiva se suman. En caso de que sea impar, se restan ambas derivadas.

$$D_p + D_r = f(x+h) + f(x-h) = f(x) + h^2 f''(x) + \frac{h^4}{12} f^4(x) + \frac{h^6}{360} f^6(x)$$

$$f^4(x) = \frac{12\left(f(x+h) + f(x-h) - f(x) - h^2 f''(x) - \frac{h^6}{360} f^6(x)\right)}{h^4}$$
$$= \frac{12\big(f(x+h) + f(x-h) - f(x) - h^2 f''(x)\big)}{h^4} - \frac{h^2}{30} f^6(x)$$

Error:

$$\mathcal{O}(h^k) = \mathcal{O}(h^2) = \frac{h^4}{30h^2} f^6(x)$$