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Punto 5 (teórico)

¿Cuál es el orden de $\mathcal{O}(h^k)$ de la aproximación?

$$f^4(x) = \frac{f'''(x+h) - f'''(x-h)}{2h} - \mathcal{O}(h^2)$$

$$= \frac{\left(\frac{f''(x+h) - f''(x)}{2h} - \mathcal{O}(h^2) - \frac{f''(x) - f''(x-h)}{2h} - \mathcal{O}(h^2)\right)}{2h} - \mathcal{O}(h^2)$$

$$= \frac{f''(x+2h) - 2f''(x) + f''(x-2h)}{4h^2} - \mathcal{O}(h^2)$$

$$= \frac{\frac{f'(x+3h) - f'(x+h)}{2h} + \frac{-2f'(x+h) + 2f'(x-h)}{2h} + \frac{\left(f'(x-h) - f'(x-3h)\right)}{2h}}{4h^2}$$

$$- \mathcal{O}(h^2)$$

$$= \frac{\frac{f'(x+3h) - f'(x+h) - 2f'(x+h) + 2f'(x-h) + f'(x-h) - f'(x-3h)}{2h}}{4h^2}$$

$$- \mathcal{O}(h^2)$$

$$= \frac{f'(x+3h) - 3f'(x+h) + 3f'(x-h) - f'(x-3h)}{8h^3} - \mathcal{O}(h^2)$$

$$= \frac{\frac{f(x+4h) - f(x+2h)}{2h} + \frac{-3f(x+2h) + 3f(x)}{2h} + \frac{3f(x) - 3f(x-2h)}{2h} + \frac{-f(x-2h) + f(x-4h)}{2h}}{8h^3}$$

$$- \mathcal{O}(h^2)$$

$$= \frac{\frac{f(x+4h) - f(x+2h) - 3f(x+2h) + 3f(x) + 3f(x) - 3f(x-2h) - f(x-2h) + f(x-4h)}{2h}}{8h^3}$$

$$- \mathcal{O}(h^2)$$

$$= \frac{f(x+4h) - 4f(x+2h) + 6f(x) - 4f(x-2h) + f(x-4h)}{8h^4} - \mathcal{O}(h^2)$$

$$D_p = Derivada\ progresiva$$

$$D_p = f(x + h)$$

$$= f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \frac{h^4}{24}f^4(x) + \frac{h^5}{120}f^5(x) + \frac{h^6}{720}f^6(x)$$

$$D_r = Derivada\ regresiva$$

$$D_r = f(x - h)$$

$$= f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \frac{h^4}{24}f^4(x) - \frac{h^5}{120}f^5(x) + \frac{h^6}{720}f^6(x)$$

$$D_p + D_r = f(x + h) + f(x - h) = f(x) + h^2 f''(x) + \frac{h^4}{12}f^4(x) + \frac{h^6}{360}f^6(x)$$

$$f^4(x) = \frac{12\left(f(x+h) + f(x-h) - f(x) - h^2 f''(x) - \frac{h^6}{360}f^6(x)\right)}{h^4}$$

$$= \frac{12\left(f(x+h) + f(x-h) - f(x) - h^2 f''(x)\right)}{h^4} - \frac{h^2}{30}f^6(x)$$

Error:

$$\mathcal{O}(h^k) = \mathcal{O}(h^2) = \frac{h^4}{30h^2}f^6(x)$$