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Credit market frictions and capital structure dynamics ☆

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Abstract

We study the implications of credit market frictions for the dynamics of corporate capital structure and the risk of default of corporations. To do so, we develop a dynamic capital structure model in which firms face uncertainty regarding their ability to raise funds in credit markets and have to search for investors when seeking to adjust their capital structure. We provide a general analysis of shareholders' dynamic financing and default decisions, show when Markov perfect equilibria in financing and default barrier strategies exist, and when uniqueness can be achieved. We then use the model to generate a number of novel testable implications relating credit market frictions to target leverage, the pace and size of capital structure changes, creditor turnover, and the likelihood of default.

JEL classification: G12; G31; G32; G34

Keywords: Credit supply uncertainty; Dynamic capital structure; Default risk; Search and bargaining

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1. Introduction

Since the famous irrelevance theorem of Modigliani and Miller [48], financial economists have devoted much effort to understand the effects of frictions, such as corporate taxes and bankruptcy costs, on corporate capital structure.[1] Although we have learned much from this work, virtually all existing models implicitly assume that a firm's capital structure is entirely determined by its demand for debt or equity. That is, the supply of capital is perfectly elastic in these models so that corporate behavior and capital availability depend solely on firm characteristics.

This demand-driven approach has recently been called into question by a number of large-sample empirical studies.[2] These studies show that firms often face uncertainty regarding their future access to credit markets and that credit supply conditions are very important in determining capital structure decisions. Using a different research design, several surveys of corporate managers from around the globe have confirmed the findings of these large-sample studies. These surveys indicate that financing decisions are generally governed by the preferences of the suppliers of capital rather than by the demands of the users of capital and that capital supply has first order effects on firms' financing decisions (see Graham and Harvey [27], or Bancel and Mittoo [3]).

Our purpose in this paper is therefore twofold. First, we want to examine the importance of credit supply frictions in capital structure choice. Second, we seek to characterize their effects on the dynamics of leverage ratios and the pricing of risky debt. To this end, we build a dynamic model of financing decisions in which the Modigliani and Miller assumption of infinitely elastic supply of credit is relaxed and firms may have to search for investors when seeking to adjust their capital structure. As in Fisher, Heinkel, and Zechner [19] and Leland [38], we consider a firm with assets in place that generate a continuous stream of cash flows. The firm is levered because debt allows it to shield part of its profits from taxation. Leverage, however, is limited because debt financing increases the likelihood of costly financial distress and is subject to credit supply frictions. In our model, these frictions include not only issuance costs, as in prior contributions, but also search frictions.

In the model, management acts in the best interests of shareholders and makes three types of decisions to maximize equity value. First, it selects the firm's initial debt level. Second, it selects the firm's restructuring policy, i.e. the pace and size of capital structure changes. Third, it selects its default policy. Because the default and restructuring policies are selected after debt has been issued, management may have incentives to deviate from the policies conjectured by creditors at the time of debt issuance. In the paper, we therefore focus on Markov perfect equilibria in which the policies selected ex post by management coincide with those conjectured ex ante by creditors. We derive conditions under which such equilibria exist in barrier strategies, show when uniqueness can be achieved, and provide a full characterization of the associated financing and default decisions.

Our analysis emphasizes the role of credit supply frictions in affecting the time series of leverage ratios. In the model, firms are always on their optimal capital structure path but, due to

[1] A partial list of such models includes Leland [39,38], Fisher, Heinkel, and Zechner [19], Mello and Parsons [47], Collin-Dufresne and Goldstein [10], Duffie and Lando [14], Morellec [49], Strebulaev [54], Tserlukevich [56], Gomes and Schmid [24], Gorbenko and Strebulaev [25], Morellec and Schürhoff [51], or Hackbarth and Mauer [28].

[2] See Faulkender and Petersen [18], Becker [4], Leary [37], Massa and Zhang [44], Lemmon and Roberts [42], Ivashina and Scharfstein [35], Choi, Getmansky, Henderson, and Tookes [9], Becker and Ivashina [5], or Morellec, Valta, and Zhdanov [52].

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refinancing costs and search frictions, they restructure infrequently and do not keep their leverage at the target at all times. As a result, leverage is best described not just by a number, the target, but by its entire distribution. The model also reflects the interaction between credit supply uncertainty and firm characteristics, allowing us to produce a number of new predictions relating capital supply in credit markets to target leverage, the frequency of capital structure changes, creditor turnover, and the likelihood of default.

Specifically, in our framework, debt provides a tax benefit so that firms that perform well may seek to re-leverage. Because the surplus from changing the firm's capital structure is uncertain and restructuring is costly, the optimal policy is to re-leverage only when the firm's taxable income exceeds an endogenously determined threshold. We show that with search frictions, the firm balances the opportunity cost of restructuring early with the risk of not finding creditors in the future. As search frictions increase, the opportunity cost of waiting increases, leading to a decrease in the value-maximizing restructuring threshold. We also show that even though credit supply uncertainty makes it more difficult for firms to restructure, it may actually increase the frequency of capital structure changes. Another result of the paper is that as credit supply weakens, firms issue more debt when restructuring their capital structure. That is, because weaker credit supply makes it more difficult to re-leverage if profitability improves, the firm takes on more debt whenever it refinances.

After solving for optimal policy choices of shareholders in the presence of search frictions, we allow the firm to bargain over the terms of new debt issues with current creditors. We assume that mobilizing dispersed creditors is costly and analyze shareholders' decision to restructure with current creditors or search for new creditors. We show that in this more general model shareholders still follow a target capital structure policy and that the target does not depend on whether the firm restructures with new or existing creditors. We also show that the restructuring triggers and target leverage ratios implied by this bargaining process differ significantly from those of standard dynamic trade-off models. Finally, we relate creditor turnover to a number of firm and industry characteristics, such as cash flow volatility, credit supply, or refinancing costs.

The present paper relates to several contributions in the literature. Fisher, Heinkel, and Zechner [19], Leland [38], and Goldstein, Ju, and Leland [23] are the first to develop dynamic capital structure models with refinancing costs. They show that refinancing costs imply that firms rebalance their capital structure infrequently and are most often away from their target leverage. Strebulaev [54] simulates artificial data from a dynamic trade-off model to show that the financing behavior implied by this class of models is consistent with the data on financing decisions. Hackbarth, Miao, and Morellec [29] and Bhamra, Kuehn, and Strebulaev [6] extend these models to incorporate the effects of macroeconomic conditions on financing and default decisions. Morellec, Nikolov, and Schürhoff [50] examine the effects of agency conflicts on dynamic capital structure choice. Glover [22] uses a dynamic capital structure model to derive estimates of bankruptcy costs.

A growing literature examines the financing decisions of firms in models with roll-over debt structure. In these models, firms *costlessly* replace an exogenous fraction of their debt with newly issued debt at each point in time (see e.g. Leland [40], Leland and Toft [41], Hilberink and Rogers [30], Eom, Helwege, and Huang [16], He and Xiong [32], Cheng and Milbradt [8], or Schroth, Suarez, and Taylor [53]). In a recent contribution, Décamps and Villeneuve [11] show that there exists a unique equilibrium in default barrier strategies in these models when the liquidation value of assets is zero.

Our paper also relates to the recent literature that examines the effects of market liquidity on the pricing of risky bonds (see e.g. Ericsson and Renault [17], He and Xiong [32] or He and

Milbradt [31]). These models generally focus on the analysis of secondary market frictions on default risk and the pricing of risky bonds, given some exogenous financing and restructuring strategies. Our model considers instead the effects of funding liquidity, i.e. the ease with which funds can be raised from creditors, on the choice of corporate financing, restructuring, and default strategies.

A few theoretical papers have investigated the role of capital supply frictions for equity financed firms. In these papers, frictions in equity markets have been modeled either using search frictions, as in Hugonnier, Malamud, and Morellec [34], or using time-varying costs of equity financing, as in Bolton, Chen, and Wang [7]. None of these papers study frictions in debt markets.[3] Time varying costs of external finance imply that firms engage in precautionary fund raising and raise excessive funds when the cost is low. Similarly, search frictions imply that firms raise excessive funds anticipating future uncertain access to debt markets. One important difference between these two approaches when modeling capital structure choices is that search frictions lead to a larger cross-sectional dispersion of leverage ratios and debt issues. That is, while with costly financing firms always raise the same amount of funds whenever leverage deviates too much from the target, search frictions imply that deviations from the target may become arbitrarily large before firms raises debt.

Lastly, our paper also relates to Manso [46], who shows that when the cost of debt depends on the rating of a firm, the rating affects shareholders' default decision, which in turn affects the rating. As shown by Manso, these games of strategic complementarities generally have multiple equilibria.

Our paper advances the literature on dynamic capital structure choice in three important dimensions. First, the paper provides the first analysis of the effects of credit supply uncertainty on financing decisions. That is, although the Modigliani and Miller irrelevance does not hold on the demand side of the market in prior models, it is assumed to hold on the supply side. Second, our paper contributes to this literature by providing a general analysis of shareholders' dynamic optimization problem. In particular, while prior contributions always assumed the existence of an equilibrium, we provide the first formal analysis of Markov perfect equilibria in barrier strategies, show when equilibria in financing, restructuring, and default strategies exist, and when uniqueness can be achieved. Third, we develop a new analytical method based on discrete techniques that proves useful in showing existence and uniqueness of solutions in continuous time stopping/impulse control problems.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes firms' financing and default policies in the presence of credit supply uncertainty. Sections 4 extends the model to allow firms to bargain over the terms of new debt issues with current debtholders. Section 5 develops the empirical implications of the model. Section 6 concludes. All the proofs are gathered in the Online Appendix.

2. The model

This section presents our model of dynamic capital structure choice with uncertain credit supply. The model closely follows Fisher, Heinkel, and Zechner [19] and Leland [38]. Throughout the paper, assets are traded in complete and arbitrage-free markets. The default-free term structure is flat with an after-tax risk-free rate r, at which investors may lend and borrow freely.

[3] See however Morellec, Valta, and Zhdanov [52] who document the importance of capital supply uncertainty in debt market in explaining firms' investment and financing decisions in the US.

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Management acts in the best interest of shareholders and maximizes shareholder wealth when making policy choices. Time is continuous and uncertainty is modeled by a complete probability space $(\Omega, \mathscr{F}, \mathscr{Q}; \mathbb{F})$, where $\mathscr{Q}$ is the risk neutral probability measure and the filtration $\mathbb{F} = \{\mathscr{F}_t : t \geq 0\}$ satisfies the usual conditions.

We consider an infinitely lived firm with assets in place that generate a cash flow X_t at time t as long as the firm is in operation. This operating cash flow is independent of financing choices and governed by the process:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \quad X_0 > 0,$$

where $\mu < r$ and $\sigma > 0$ are constant parameters and $(B_t)_{t\geq 0}$ is a standard Brownian motion under $\mathscr{Q}$. Operating cash flows are taxed at a constant rate $\tau < 1$. As a result, firms have an incentive to issue debt to shield profits from taxation (due to the tax deductibility of interest payments). To stay in a simple time-homogeneous setting, we consider debt contracts that are characterized by a perpetual flow of coupon payments C and a principal P that shareholders have to repay in default (as in Leland [39], Duffie and Lando [14], or Manso [45]). Debt is issued at par and callable at market value. The proceeds from the debt issue are distributed to shareholders at the time of issuance.

Firms whose conditions improve sufficiently may re-leverage by incurring a proportional cost q. Firms whose conditions deteriorate sufficiently may default on their debt obligations. In the model, default leads to the liquidation of the firm. At the chosen time of default, a fraction $\omega \in (0, 1)$ of assets is lost as a frictional cost and the value of the remaining assets is assigned to debtholders. Under these assumptions, the firm's debt structure remains fixed until the firm goes into default or calls its debt and restructures with newly issued debt.[4]

We are interested in building a model in which capital structure choices depend not only on firm characteristics but also on frictions in debt markets. Indeed, as documented by a series of recent empirical studies, credit supply conditions are very important in determining firms' financing decisions. For example, Faulkender and Petersen [18], Leary [37], and Sufi [55] provide evidence suggesting an important role for the supply of credit in determining leverage ratios. Lemmon and Roberts [42] find that negative shocks to the supply of credit lead to large declines in debt issuance. Massa and Zhang [44] find that the relative availability of bond and bank financing affect the firm's ability to borrow. Choi, Getmansky, Henderson, and Tookes [9] show that the issuance of convertible bonds is positively related to a number of capital supply measures. Morellec, Valta, and Zhdanov [52] show that capital supply uncertainty in debt markets has first order effects on firms' investment and financing decisions. Finally, a number of surveys indicate that financing decisions are generally governed by the preferences of the suppliers of capital rather than by the demands of the users of capital (see Graham and Harvey [27]).

These studies suggest that credit supply is a key determinant of financing decisions and that firms often face uncertainty regarding their access to credit markets. To capture this important feature of credit markets, we consider that it takes time to secure debt financing and that credit supply is uncertain. In particular, we assume that if a firm decides to issue debt, then it has

[4] While in principle management can both increase and decrease future debt levels, Gilson [21] finds that transaction costs discourage debt reductions outside of renegotiation. In addition, we show in Proposition 5 that there is no deleveraging along the optimal path.

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to search for creditors.[5] In the analysis below, we assume that creditors appear to firms with constant Poisson arrival rate $\lambda > 0$. That is, conditional on searching, the probability of getting financing from new creditors over each time interval $[t, t + dt]$ is λdt and the expected financing lag is $1/\lambda$.[6] We formalize this with the following assumption.

Assumption 1. There exists a Poisson process N_t with intensity $\lambda > 0$, independent of X_t, whose jumps represent the times at which the firm can restructure its debt. The public filtration $\mathbb{F}$ is generated by the processes X_t and N_t.

In Section 3, we start by formulating the dynamic capital structure problem of a firm that needs to search for outside debt investors when seeking to re-leverage and can only issue debt with these new investors. In Section 4, we generalize the model to allow the firm to search for new creditors or to bargain over the terms of new debt issues with current debtholders. To highlight the effects of credit market frictions on debt dynamics, we follow previous contributions by assuming that the firm's shareholders are able to finance temporary cash shortfalls so that the firm defaults when equity value is zero.

3. Capital structure and credit supply uncertainty

3.1. Equilibrium: definition and general properties

From a game-theoretical point of view, our model is equivalent to a continuous time, infinitely repeated game with one large, strategic player (the firm) and a large number of small, non-strategic players (creditors). The nature of the interaction can be explained as follows: Creditors have expectations over the firm's default policy that determine the rate at which they are willing to lend funds to the firm. The firm, acting strategically, takes creditors' expectations as given and optimally defaults when equity value falls to zero. The value of equity and the decision to default depend on the future fund raising opportunities for the firm, which are in turn determined by creditors' beliefs. This closes the equilibrium loop. We start by defining payoffs and policies of all players.

Definition 1. An admissible policy is a pair (a, δ), where $a = (a_t)_{t \geq 0}$ is a process adapted to the public filtration $\mathbb{F}$ satisfying $a_t \geq 1$ for all $t \geq 0$ and δ is a stopping time with respect to $\mathbb{F}$, such that the coupon payment C_t on the firm's debt evolves according to

$$dC_t = (a_t - 1)C_{t-} dN_t$$

and the firm defaults at the stopping time δ. We respectively denote by $\mathbb{A}$ and $\mathbb{S}$ the sets of such admissible restructuring and default policies.

[5] A growing body of literature argues that assets prices are more sensitive to supply shocks than standard asset pricing theory predicts. Search theory plays a key role in the formulation of models capturing this idea (see e.g. Duffie, Garleanu, and Pedersen [13], Duffie and Strulovici [15], or Vayanos and Weill [57]).

[6] One possible interpretation for our assumption is that the firm cannot find a single creditor with deep pockets and has to rely on a syndicate of banks or a group of debt investors as in He and Xiong [33]. The firm will then restructure once it has found sufficiently many creditors.

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Definition 2. An admissible policy $(a,\delta)\in\mathbb{A}\times\mathbb{S}$ is said to be Markovian if it is given by $a_t=a(X_t,C_{t-})$ and $\delta=\inf\{t\geq 0:(X_t,C_t)\in\Omega\}$ for some Borel-measurable function $a(x,c)$ and some Borel set $\Omega\subset\mathbb{R}^2$.

Denote by $D(X,C|(a,\delta))$ the value of the firm's *current debt* under the assumptions that the current coupon rate is C and that the firm follows the Markovian restructuring and default policies (a,δ). Prior to the conjectured default time, this value satisfies:

$$D\big(X_0,C\big|(a,\delta)\big)$$
$$=E_0\left[\int_0^{\iota\wedge\delta}e^{-rs}C\,ds+e^{-r\iota}1_{\{\iota\leq\delta\}}D\big(X_\iota,C\big|(a,\delta)\big)+e^{-r\delta}1_{\{\iota>\delta\}}(1-\omega)\phi_0X_\delta\right],$$

where ι denotes the first restructuring time, $\iota\wedge\delta\equiv\inf\{\iota,\delta\}$ is the first time that the firm either defaults or restructures, and ϕ_0 is implicitly defined by

$$E_t\left[\int_t^\infty e^{-r(s-t)}(1-\tau)X_s\,ds\right]=\phi_0X_t.$$

Since debt is callable at market value, its price is independent of the restructuring policy a selected by shareholders and only depends on the default policy δ of the firm. As a result, we can also write the value of debt prior to the conjectured default time as:

$$D\big(X_0,C\big|(a,\delta)\big)=D(X_0,C|\delta)=\frac{C}{r}\big(1-E_0\big[e^{-r\delta}\big]\big)+\phi_0(1-\omega)E_0\big[e^{-r\delta}X_\delta\big].\tag{1}$$

The first term on the right hand side of this equation captures the present value of the cash flows accruing to debtholders up to the default time. The second term represents their cash flow in default, i.e. the value of assets net of liquidation costs.

Because credit supply is uncertain, debtholders may be able to capture part of the restructuring surplus at refinancing dates. That is, we consider that once management and debt investors meet, they bargain over the terms of the new debt issue to determine the cost of debt or, equivalently, the proceeds from the debt issue. Specifically, denote firm value under the policy (a,δ) by $V(X,C|(a,\delta))$ and assume that shareholders may select a restructuring strategy a' that potentially differs from that anticipated by creditors at the time of debt issuance. We can then define the restructuring surplus $S(a',X,C|(a,\delta))$ by:

$$S\big(a',X,C\big|(a,\delta)\big)=V\big(X,a'C\big|(a,\delta)\big)-V\big(X,C\big|(a,\delta)\big)-q1_{\{a'>1\}}D\big(X,a'C\big|\delta\big),\tag{2}$$

where the last term on the right hand side represents the restructuring cost. Given a non-negative surplus, we consider below that the allocation of this surplus between shareholders and new creditors results from Nash bargaining. Denoting the bargaining power of creditors by $\eta\in(0,1)$, the amount π^* that creditors can extract at the time of a restructuring satisfies

$$\pi^*=\operatorname*{argmax}_{\pi\geq 0}\pi^\eta\big[S\big(a',X,C\big|(a,\delta)\big)-\pi\big]^{1-\eta}=\eta S\big(a',X,C\big|(a,\delta)\big).$$

This Nash bargaining solution determines uniquely the cost of new debt issues and allows us to write equity value under the restructuring and default policy (a,δ) as

$$E\big(X_0,C_0\big|(a,\delta)\big)=E_0\left[\int_0^\delta e^{-rs}\big[(1-\tau)(X_s-C_{s-})ds+H\big(a_s,X_s,C_{s-}|(a,\delta)\big)dN_s\big]\right],$$

where

$$H\big(a', X, C\big|(a,\delta)\big) = (1 - q1_{\{a'>1\}})D\big(X, a'C\big|\delta\big) - D(X, C|\delta) - \eta S\big(a', X, C\big|(a,\delta)\big),$$

represents the cash flow to shareholders following a change in the firm's coupon rate from C to $a'C$. This cash flow corresponds to the proceeds from the debt issue net of the flotation costs and of the surplus extraction by new debtholders.

As discussed above, our model is equivalent to an infinitely repeated game between the firm and the creditors. Such infinitely repeated games typically have multiple equilibria, particularly due to reputation effects (see Fudenberg and Levine [20], for the general theory and Diamond [12], for an application to debt markets). In the following, we therefore follow Maskin and Tirole [43] and confine ourselves to analyzing Markov perfect equilibria.

Definition 3. A Markov perfect equilibrium is a Markovian restructuring and default policy $(a^*, \delta^*) \in \mathbb{A} \times \mathbb{S}$ such that

$$\begin{aligned}&\big(a^*, \delta^*\big)\\ &\quad \in \underset{(a,\delta)\in\mathbb{A}(a^*,\delta^*)}{\operatorname{argmax}} E_0\left[\int_0^\delta e^{-rs}\big[(1-\tau)(X_s - C_{s-})ds + H\big(a_s, X_s, C_{s-}\big|(a^*,\delta^*)\big)dN_s\big]\right],\end{aligned} \tag{3}$$

where $\mathbb{A}(a^*, \delta^*)$ denotes the set of admissible strategies (a, δ) with $a \geq 1$ (not necessarily Markovian) satisfying the integrability condition

$$E_0\left[\int_0^\delta e^{-rs}\big(C_{s-} + \lambda\big|H\big(a_s, X_s, C_{s-}\big|(a^*,\delta^*)\big)\big|\big)ds\right] < \infty,$$

and where we assume that if a firm does not default at the conjectured default time δ^*, creditors keep believing for all $t > \delta^*$ that the firm will default in the next instant and, therefore, price the firm's debt as $D(X_t, C|\delta) = (1-\omega)\phi_0 X_t$.

While Markov perfect equilibria are free from reputation effects, we still cannot rule out multiplicity of such equilibria, since shareholders' optimal restructuring and default policies depend on the creditors beliefs about the default region Ω.[7] As a result, in all the existing literature on dynamic capital structure choice, it is assumed that shareholders follow barrier restructuring and default strategies whereby the firm defaults the first time that the cash flow shock goes down to a constant barrier $X_d(C)$ and seek to restructure its capital structure towards a constant target $T = C/X$ whenever the current value cash flow shock exceeds a barrier $X_u(C) > X_d(C)$. In particular, these models characterize Markov perfect equilibria in restructuring and default barrier strategies, defined as follows:

Definition 4. A Markov perfect equilibrium is in barrier strategies if

$$\begin{aligned}\delta &= \inf\big\{t \geq 0 : X_t \leq X_d(C_{t-})\big\},\\ a_t &= 1_{\{X_t < X_u(C_{t-})\}} + 1_{\{X_t \geq X_u(C_{t-})\}}(TX_t/C_{t-})\end{aligned}$$

[7] The reason is that, for general sets Ω, the function $D(X, C|(a,\delta))$ that creditors use for pricing debt obligations may exhibit non-monotonic behavior, leading to optimal default regions of non-barrier type.

for linear functions $X_u(C) \geq X_d(C)$ with $X_u(0) = X_d(0) = 0$ and some nonnegative constant target $T \in [1/X_u(1), 1/X_d(1)]$.

Everywhere in the sequel, we refer to a Markov perfect equilibrium in barrier strategies simply as an equilibrium in barrier strategies. To see why it may not be optimal for shareholders to follow barrier strategies, consider the behavior of the firm in a region where creditors do not anticipate default. The firm's decision to deviate from the conjectured equilibrium behavior and to default early depends on the properties of the instantaneous expected cash flows that are given by

$$G(X, C|\delta) \equiv (1-\tau)(X-C) + \lambda \max_{a \geq 1} \big\{(1 - q 1_{a>1}) D(X, aC|\delta) - D(X, C|\delta)\big\}, \tag{4}$$

where maximization over a indicates that the firm has the option to change its capital structure when it meets new creditors and λ is the arrival rate of new creditors. Intuitively, shareholders will find it optimal to default only when the instantaneous expected cash flows $G(X, C|\delta)$ are sufficiently small. In particular, if these cash flows are monotone decreasing in C, we may expect that there exists a critical threshold of C/X beyond which shareholders decide to default. However, if $G(X, C|\delta)$ is non-monotonic, there might be several intervals on which $G(X, C|\delta)$ is sufficiently small to make it optimal for shareholders to default. The following proposition shows that this intuition is indeed correct.

Proposition 1. *If creditors conjecture a default policy δ such that the function $D(X, C|\delta)$ is homogeneous of degree one in (X, C) and the function $G(X, C|\delta)$, defined by decreasing in C, then the optimal default policy for problem (3) is of barrier type. If, in addition, the conjectured default policy is of barrier type, then both the optimal default and restructuring policies are of barrier type. In particular, all equilibria in which $D(X, C|\delta)$ is homogeneous of degree one and $G(X, C|\delta)$ is decreasing in C are in barrier strategies.*

While monotonicity of (4) has a clear intuitive meaning, it is not at all obvious that this condition indeed holds in equilibrium. Luckily enough, we show below that this monotonicity is indeed true for equilibria in barrier strategies. To construct such equilibria, we first pick a default threshold $X_d(C) \equiv X_d(1)C$, where $X_d(1)$ is a positive constant, and solve the following equilibrium problem: Given that debt holders believe that the firm defaults when the cash flow shock reaches the barrier $X_d(C)$ and the firm has to follow this default policy, what is the equilibrium restructuring strategy for shareholders? Having constructed the solution to this artificial problem, we start varying the default threshold to find a value $X_d(1)$ such that it is indeed optimal for shareholders to default at $X_d(C)$. By Proposition 1, this algorithm allows to uncover all Markov perfect equilibria in barrier strategies.

3.2. Equilibrium in barrier strategies

Fix a default threshold $X_d(C) \equiv X_d(1)C$ and denote by $\delta(X_d(1))$ the corresponding default time. Using Eq. (1) and standard derivations, we have that the corresponding debt value function $D(X, C|\delta(X_d(1)))$ is given by:

$$D\big(X, C\big|\delta\big(X_d(1)\big)\big) = \begin{cases} \frac{C}{r} - [\frac{C}{r} - (1-\omega)\phi_0 X_d(C)](\frac{X}{X_d(C)})^\beta, & \text{for } X > X_d(C), \\ (1-\omega)\phi_0 X, & \text{for } X \leq X_d(C), \end{cases}$$

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where $\alpha > 0$ and $\beta < 0$ denote the roots of the characteristic equation

$$Q(x;r) = \mu x + \frac{\sigma^2}{2}x(x-1) - r = 0.$$

Thus, when $X > X_d(C)$, the value of corporate debt is equal to the value of risk-free debt (first term on the right hand side) minus the change in value that occurs at the time of default (second term). When $X \leq X_d(C)$, the value if corporate debt is equal to the liquidation value of assets (even if the firm has not defaulted yet due to Assumption 3).

Because the firm's default and restructuring strategies are interrelated, we need to know when restructuring may be optimal for shareholders in order to characterize the optimal default strategy. In the model, the value maximizing restructuring strategy results from a trade-off between the additional tax benefits of debt that the firm can get by increasing its leverage ratio and the refinancing costs (including the potential surplus extraction by creditors). Proposition 2 below shows that when refinancing costs exceed potential tax savings, i.e. when $q \geq \tau$, it is never optimal for the firm to restructure its debt.

Proposition 2. *If $q \geq \tau$, then there exists a unique equilibrium in barrier strategies. In this equilibrium, restructuring never happens, default occurs at the threshold*

$$X_d^S(C) = \frac{\beta}{\beta - 1}\frac{r-\mu}{r}C \equiv X_d^S(1)C,$$

and firm value satisfies for $X \geq X_d^S(C)$

$$V^S(X,C) = \phi_0 X + \frac{\tau C}{r}\left[1 - \left(\frac{X}{X_d^S(C)}\right)^\beta\right] - \omega\frac{(1-\tau)\beta C}{r(1-\beta)}\left(\frac{X}{X_d^S(C)}\right)^\beta.$$

If issuance costs satisfy $q < \tau$, then there will always be restructuring in equilibrium.

Assume now that $\tau > q$ so that restructuring the firm's capital structure may be optimal. Given the default policy $\delta(X_d(1))$, a Markov perfect equilibrium with fixed default threshold $X_d(C)$ is a Markovian restructuring policy $a^* \in \mathbb{A}$ such that

$$a^* \in \underset{(a,\delta(X_d(1)))\in\mathbb{A}(a^*,\delta(X_d(1)))}{\operatorname{argmax}} E_0\left[\int_0^{\delta(X_d(1))} e^{-rs}(1-\tau)(X_s - C_{s-})ds\right]$$
$$+ E_0\left[\int_0^{\delta(X_d(1))} e^{-rs}H\big(a_s, X_s, C_{s-}\big|\big(a^*, \delta\big(X_d(1)\big)\big)\big)dN_s\right]. \tag{5}$$

We shall now use the above representation of shareholders' optimization problem to derive equity value. Consider first the equity value function $E(X, C|(a, \delta(X_d(1))))$ associated with the (possibly suboptimal) barrier strategy $(a, \delta(X_d(1)))$. In the region where the firm does not default (i.e. for $X > X_d(C)$), shareholders receive a cash flow $(1-\tau)(X_t - C)dt$ over each time interval $[t, t+dt]$ and have the option to restructure their debt when meeting new creditors. As a result, equity value satisfies:

$$\begin{aligned}
&rE\big(X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big) \\
&\quad = \mu X \frac{\partial E(X, C|(a, \delta(X_d(1))))}{\partial X} + \frac{\sigma^2}{2} X^2 \frac{\partial^2 E(X, C|(a, \delta(X_d(1))))}{\partial X^2} \\
&\qquad + (1-\tau)(X - C) + 1_{\{X > X_u(C)\}} \lambda(1-\eta) S\big(T, X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big)
\end{aligned}$$

where $S(T, X, C|(a, \delta(X_d(1))))$ is the restructuring surplus defined in Eq. (2). The left hand side of this equation represents the required return for investing in the firm's equity per unit of time. The right hand side is the sum of the cash flow generated by the firm's assets and the expected change in equity value that captures the effects of credit supply uncertainty with the term $1_{\{X > X_u(C)\}} \lambda(1-\eta) S(T, X, C|(a, \delta(X_d(1))))$.

This equation is solved subject to the following boundary conditions

$$\begin{aligned}
&\lim_{X \downarrow X_d(C)} E\big(X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big) = 0, \\
&\lim_{X \uparrow X_u(C)} E\big(X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big) = \lim_{X \downarrow X_u(C)} E\big(X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big), \\
&\lim_{X \uparrow X_u(C)} \frac{\partial}{\partial X} E\big(X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big) = \lim_{X \downarrow X_u(C)} \frac{\partial}{\partial X} E\big(X, C|\big(a, \delta\big(X_d(1)\big)\big)\big), \\
&\lim_{X \to \infty} \big(E\big(X, C\big|\big(a, \delta\big(X_d(1)\big)\big)\big)/X\big) = \kappa,
\end{aligned}$$

where $\kappa > 0$ is a constant. The first boundary condition shows that the value of assets net of liquidation costs goes to debtholders in default. The second and third conditions are continuity and smoothness conditions satisfied by equity value at the restructuring threshold. The last condition is a standard no-bubbles condition.

To complete the characterization of equity value, we need to derive the optimality conditions corresponding to the restructuring strategy that solves problem (5). In the Online Appendix, we establish the following result.

Proposition 3. *Assume that issuance costs are such that $q < \tau$. Then for any constant $X_d(1) > 0$ there exists a unique barrier restructuring strategy $\mathbf{P} = \mathbf{P}(X_d(1)) = (a^*, \delta(X_d(1)))$ that solves problem (5) and is characterized by*

$$V\big(X_u(C), C\big|\mathbf{P}\big) = \big\{E(X, TX|\mathbf{P}) + (1-q)D(X, TX|\mathbf{P})\big\}\big|_{X = X_u(C)},$$

and

$$T = \operatorname*{argmax}_{T > 0} \big\{E(X, TX|\mathbf{P}) + (1-q)D(X, TX|\mathbf{P})\big\}.$$

The first condition in Proposition 3 determines the value-maximizing threshold when seeking to restructure with new creditors. The second condition determines the target leverage ratio T. This target leverage ratio does not depend on X due to the homogeneity of the functions $E(X, C|\mathbf{P})$ and $D(X, C|\mathbf{P})$.

To the best of our knowledge, Proposition 3 is the first in the literature to rigorously establish the optimality of barrier policies.[8] Since the optimal policy involves two barriers (target level

[8] To the best of our knowledge, all dynamic capital structure models simply assume a barrier policy and then solve for the value-matching and smooth-pasting conditions without any verification of their optimality.

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and the restructuring threshold), proving Proposition 3 requires analyzing subtle properties of the value function and optimal policies. This is done in two steps. In the first step, we show that a solution to the Hamilton Jacobi Bellman (HJB) equation exists. To this end, we exploit the fact that restructuring happens at jumps times of the Poisson process N_t. This makes the structure of the problem similar to that of a discrete time stochastic control problem and allows us to apply contraction mapping techniques to establish the existence and uniqueness of a smooth solution to the HJB equation and the verification result. In the second step, we exploit the fine structure of the equation to show that the optimal policy is of barrier type. The details are reported in the Online Appendix.

Using Proposition 3, we can now solve for the equity value function associated with a fix default threshold $X_d(C)$ as

$$\begin{aligned} E(X, C|\mathbf{P}) = {} & 1_{\{X > X_u(C)\}} V_s(X, C|\mathbf{P}) + 1_{\{X_d(C) < X \le X_u(C)\}} V_{ns}(X, C|\mathbf{P}) \\ & - 1_{\{X > X_d(C)\}} D(X, C|\mathbf{P}) \end{aligned}$$

where

$$\begin{aligned} V_s(X, C|\mathbf{P}) &= \frac{1 - \tau + \lambda^* A_1(\mathbf{P})}{r - \mu + \lambda^*} X + \frac{\tau C}{r + \lambda^*} + A_2(\mathbf{P}) C \left(\frac{X}{X_u(C)}\right)^{\psi}, \\ V_{ns}(X, C|\mathbf{P}) &= \phi_0 X + \frac{\tau C}{r} + A_3(\mathbf{P}) C \left(\frac{X}{X_d(C)}\right)^{\beta} + A_4(\mathbf{P}) C \left(\frac{X}{X_u(C)}\right)^{\alpha} \end{aligned}$$

where $\lambda^* = \lambda(1 - \eta)$, the constant ψ is the negative root of the equation $Q(x; r + \lambda^*) = 0$ and the four constants $A_i(\mathbf{P})$, the target T, and the restructuring threshold $X_u(1)$ solve

$$\begin{aligned} & A_3(\mathbf{P}) = V_{ns}(1, T|\mathbf{P}) - q D(1, T|\mathbf{P}), \\ & V_{ns}\big(X_d(C), C\big|\mathbf{P}\big) = (1 - \omega)\phi_0 X_d(C), \\ & V_{ns}\big(X_u(C), C\big|\mathbf{P}\big) = V_s\big(X_u(C), C\big|\mathbf{P}\big), \\ & V_s\big(X_u(C), C\big|\mathbf{P}\big) = V_{ns}\big(X_u(C), T X_u(C)\big|\mathbf{P}\big) - q D\big(X_u(C), T X_u(C)\big|\mathbf{P}\big), \end{aligned}$$

and

$$\begin{aligned} \frac{\partial}{\partial X} V_{ns}(X, C|\mathbf{P}) \bigg|_{X = X_u(C)} &= \frac{\partial}{\partial X} V_s(X, C|\mathbf{P}) \bigg|_{X = X_u(C)} \\ \frac{\partial}{\partial C} V_{ns}(X, C|\mathbf{P}) \bigg|_{X = 1, C = T} &= -q \frac{\partial}{\partial C} D(X, C|\mathbf{P}) \bigg|_{X = 1, C = T}. \end{aligned}$$

These equations show that equity value can be written as the difference between firm value and the value of outstanding debt. The value of the firm depends on whether the cash flow shock is in the restructuring region ($V_s(X, C|\mathbf{P})$ for $X > X_u(C)$) or in the inaction region ($V_{ns}(X, C|\mathbf{P})$ for $X < X_u(C)$). Firm value in the inaction region ($X < X_u(C)$) is given by the sum of the present value of cash flows from assets in place (first term) and tax savings (second term), plus the change in firm value occurring when the firm defaults (third term), plus the change in firm value occurring when the cash flow shock reaches the restructuring threshold (last term). Firm value in the restructuring region ($X > X_u(C)$) can be interpreted similarly. Finally, we also establish the following result in the Online Appendix:

Proposition 4. *For all $X_d(1) > X_d^S(1)$, the equity value satisfies*

$$\frac{\partial}{\partial X} E\big(X, C \big| \mathbf{P}\big(X_d(1)\big)\big)\Big|_{X=X_d(C)} > 0, \tag{6}$$

so that defaulting slightly later leads to a local increase in equity value.

Proposition 4 shows that under the value-maximizing restructuring strategy, the continuation value of equity is larger when the firm has the option to change its capital structure in the future. This in turn implies that shareholders will always default later than in the static model in which no restructuring is allowed.

In the model, we assume that there is no deleveraging along the optimal path. While this assumption is commonly justified by transactions costs, it is possible to show that reducing debt is never optimal for shareholders if debt holders are dispersed and have rational expectations.[9] Indeed, in this case agency problems associated with debt cannot be resolved through collective negotiations. As a result, the firm is forced to buy back debt in an open market and each creditor can decide to sell his claims back to the firm or to hold on to them.

Suppose that creditors and investors rationally anticipate that the firm will reduce the coupon rate from C to $C - \Delta$ in the next instant. In any barrier strategy equilibrium, the value of each unit of debt, $\frac{D(X,C)}{C}$, is monotone decreasing in C because default probability is monotone increasing in C. As a result, to prevent arbitrage opportunities the bond price will rise immediately to $D(X, C - \Delta)$, i.e. to a level that reflects the lower default risk following the recapitalization.[10] This in turn implies that buying back Δ units of debt costs $\frac{\Delta}{C-\Delta} D(X, C - \Delta)$ to the firm and that equity value changes from $E(X, C)$ to $E(X, C - \Delta) - \frac{\Delta}{C-\Delta} D(X, C - \Delta)$. The following proposition shows that such debt reductions are always suboptimal for shareholders.

Proposition 5. *In any equilibrium in barrier strategies, $E(X, C - \Delta) - \frac{\Delta}{C-\Delta} D(X, C - \Delta)$ is monotone decreasing in Δ.*

3.3. Existence of equilibria in barrier strategies

Having constructed equity value for a given default boundary, we now have to find a constant $X_d(1)$ that maximizes equity value. We expect such an $X_d(1)$ to satisfy

$$\frac{\partial}{\partial X} E\big(X, C \big| \mathbf{P}\big(X_d(1)\big)\big)\Big|_{X=X_d(C)} = 0. \tag{7}$$

This smooth-pasting condition is clearly necessary. Indeed, if the derivative was positive (negative), shareholder could increase equity value in a small neighborhood of $X_d(C)$ by slightly decreasing (increasing) $X_d(C)$. Proposition 6 below shows that this condition is also sufficient and thus provides a complete characterization of equilibria in barrier strategies.

Proposition 6. *The policy $\mathbf{P}(X_d(1)) = (a^*, \delta(X_d(1)))$ is a Markov perfect equilibrium if and only if condition (7) is satisfied.*

9 Admati, DeMarzo, Hellwig, and Pfleiderer [1] prove an analogous result in a one-period model.

10 If there was no such adjustment, investors would be able to buy debt at the low price $D(X, C)$ before the recapitalization and sell these bonds immediately afterwards at the price $D(X, C - \Delta)$. This activity would automatically raise the bond price until such an arbitrage opportunity is no longer available.

As in the case with barrier restructuring strategies, almost all existing literature until now simply assumes that the smooth pasting condition (7) determines the optimal default boundary. However, to verify this, one needs to show that the equity value is indeed nonnegative in the no-default region. The proof of this result is based on monotonicity of the function (4) for the case of barrier default policies.

Proposition 6 shows that the existence/uniqueness of an equilibrium in barrier strategies is equivalent to the existence/uniqueness of a solution to the smooth-pasting condition (7). As we show below, this equation does not always have a solution. In order to characterize Markov perfect equilibria in barrier strategies, we therefore start by analyzing the special case in which it is costless to refinance and derive conditions under which an equilibrium in barrier strategies exists when $q = 0$. We then turn to the analysis of positive issuance costs and provide a general characterization of the firm's policy choices.

Assume first that issuing debt is costless, in that $q = 0$. In this case, it is optimal to start searching for creditors whenever the cash flow shock reaches a new maximum and the restructuring threshold satisfies $X_u(1) = 1/T$. Define the constants $\kappa_1(\lambda)$ and $\kappa_2(\lambda)$ by

$$\kappa_1(\lambda) \equiv \frac{(1-\psi)\alpha\lambda(1-\eta)\mu - r(r-\mu+\lambda(1-\eta))(\alpha-\psi)}{(r-\mu)(r+\lambda(1-\eta))(1-\psi)(\alpha-\beta)},$$

$$\kappa_2(\lambda) \equiv \frac{1}{\alpha-1}\big[1+(1-\beta)\kappa_1(\lambda)\big].$$

A direct calculation shows that $\alpha\mu < r$ and therefore $\kappa_1(\lambda) < 0 < \kappa_2(\lambda)$. Hence, there exists a unique solution J_0 to the equation $f(y) = 0$ where

$$f(y) \equiv 1 + \kappa_1(\lambda)y^{-\beta} + \kappa_2(\lambda)y^{-\alpha}. \tag{8}$$

Denote by $J > J_0$ the unique solution to the equation $g(y) = 0$ where

$$g(y) \equiv \beta\omega - \frac{\beta\omega}{\tau} + (1-\beta)\big[1+\kappa_1(\lambda)y^{-\beta}\big] + \big[(\beta-\alpha)\omega + (1-\beta)\big]\kappa_2(\lambda)y^{-\alpha}. \tag{9}$$

We then have the following result.

Proposition 7. *When $q = 0$, an equilibrium in barrier strategies exists if and only if the corporate tax rate τ is below the critical tax rate τ^* defined by*

$$\tau^* \equiv \frac{\beta}{\beta - \kappa_2(\lambda)(\alpha-\beta)J_0^{-\alpha}} \in (0, 1).$$

In this case, there exists a unique equilibrium in barrier strategies and the corresponding default and restructuring thresholds satisfy

$$\frac{1}{X^*_{u,0}(1)} = T^*_0 = \frac{1}{JX^*_{d,0}(1)} = \frac{r\omega\phi_0}{\rho J|f(J)|}.$$

Having characterized optimal policy choices when $q = 0$, we now turn to the analysis of equilibria in barrier strategies with positive issuance costs. It follows from the proof of Proposition 7 that, when $\tau < \tau^*$, equity value satisfies

$$\frac{\partial}{\partial X}E\big(X, C\big|\mathbf{P}\big(X_d(1)\big)\big)\Big|_{q=0, X=X_d(C)} < 0,$$

for any constant $X_d(1) < X^*_{d,0}(1)$. Thus, for any $X_d(1) < X^*_{d,0}(1)$, equity value is negative in a right neighborhood of $X_d(C)$ when $q = 0$. As shown in Lemma A.5 in the Online Appendix, equity value is monotone decreasing in the issuance cost q. Thus, it follows that

$$E\big(X, C \big| \mathbf{P}\big(X_d(1)\big)\big) \le 0$$

for any $q > 0$ in a right neighborhood of $X_d(C)$. This in turn implies that for any $X_d(1) < X^*_{d,0}(1)$ and $q > 0$, we have

$$\frac{\partial}{\partial X} E\big(X, C \big| \mathbf{P}\big(X_d(1)\big)\big)\bigg|_{X = X_d(C)} < 0.$$

This fact, together with condition (6) and the intermediate value theorem, implies that the smooth pasting condition (7) has a solution in $(X^*_{d,0}(C), X^S_d(C))$ for any $q > 0$ when $\tau < \tau^*$. Furthermore, standard implicit function type arguments imply that this solution is unique when q is sufficiently small. This leads to the main result of this section.

Theorem 1. *Assume that issuance costs are positive in that $q > 0$. If the corporate tax rate satisfies $\tau < \tau^*$, then there exists a Markov perfect equilibrium in barrier strategies, which is unique when q is sufficiently small.*

Theorem 1 provides sufficient conditions for a Markov perfect equilibrium in barrier strategies to exist. In such equilibria, shareholders default the first time that the cash flow shock decreases to an endogenous threshold $X^*_d(C) \equiv X^*_d(1)C$. In addition, they issue new debt the first time that the cash flow shock is above $X^*_u(C)$ and shareholders meet new debt investors. Even though Theorem 1 does not establish uniqueness of the equilibrium for large q, extensive numerical simulations suggest that uniqueness does hold. If there were multiple equilibria, the equilibrium with the minimal default threshold $X_d(1)$ would be the most desirable outcome from the social welfare perspective. Indeed, when the cost q is sufficiently small, firm value with a fixed default policy, $V(X, C|\mathbf{P}(X_d(1)))$, is monotone decreasing in $X_d(1)$ and hence the minimal threshold equilibrium maximizes firm value.

In the Online Appendix, we also show that Markov perfect equilibria in barrier strategies may fail to exist when tax benefits are very large. That is, we show that when $\tau > \tau^*$, there exists a cutoff level q^* for issuance costs satisfying

$$q^* \equiv \inf\left\{ q > 0 : \inf_{X_d(1) > 0} \frac{\partial}{\partial X} E\big(X, C \big| \mathbf{P}\big(X_d(1)\big)\big)\bigg|_{X = X_d(C)} < 0 \right\},$$

such that an equilibrium in barrier strategies exists if and only if $q > q^*$.

To aid in the intuition of this result, suppose that q is sufficiently small that issuing new debt is almost costless. Suppose also that debtholders believe that the firm will default very late, that is assume that $X_d(1)$ is very small. Let $\tilde{X}_d(1)$ be the default policy selected by shareholders (which is of barrier type by Proposition 1). Suppose now that we start decreasing $X_d(1)$ and make it small so that debtholders are willing to invest in bonds as if they were almost riskless. Then, because issuing debt is cheap, the firm will find it optimal to issue very large amounts of debt. This will increase the expected surplus from restructuring and significantly increase equity value when the tax benefits of debt are sufficiently high. This is turn will make it optimal to default even later, so that $\tilde{X}_d(1) < X_d(1)$. Thus, an equilibrium (that is, beliefs $X_d(1)$ for which $\tilde{X}_d(1) = X_d(1)$) will fail to exist. In Section 5, we show that for the typical US firm, τ^* is generally above 99%

(that is, well beyond the statutory corporate tax rate). In the following, we thus focus on the case $\tau < \tau^*$ that is described in Theorem 1.

When there are no issuance costs, in that $q = 0$, the equilibrium characterization can be reduced to a single algebraic equation for the default threshold. This characterization, together with an application of the implicit function theorem, allow us to establish explicit comparative statics results for the case when the cost q of debt issuance is sufficiently small. In particular, we show in the Online Appendix that the following is true.

Proposition 8. *$X_d(1)$ is monotone decreasing in λ and increasing in q and η. Furthermore, if q is sufficiently small, then:*

- *$X_d(1)$ is monotone decreasing in τ;*
- *T^* and $\frac{1}{X_u(1)}$ are increasing in τ and η and decreasing in λ;*
- *$\frac{1}{T^* X_d(1)}$ and $\frac{X_u(1)}{X_d(1)}$ are decreasing in τ and η and increasing in λ.*

The intuition behind these results is as follows. Increasing λ and decreasing q or η clearly increases the value of the restructuring options to shareholders and equity value and, hence, decreases the selected default threshold. Similarly, it is straightforward to show that the equity value normalized by taxes, $E(X, C)/(1 - \tau)$ is monotone increasing in τ because, for this normalized value, τ is exactly the tax benefit of debt. As a result, increasing τ reduces the selected default threshold. Consider next the effects of the parameters on target leverage T^* and the restructuring threshold $X_u(1)$. Higher tax benefits and higher bargaining cost η induce the firm to take on more debt ex ante, i.e. to increase T^*. Similarly, a low value for λ implies that the firm can access debt markets only infrequently, and hence issues more debt driven by precautionary motives. Another prediction of the model is that an increase in the tax rate leads the firm to restructure more frequently whereas an increase in credit supply leads the firm to restructure at higher profitability levels.

The quantities $\log X_u(1) - \log X_d(1)$ and $\log(1/T^*) - \log X_d(1)$ are the *distances to default* when the firm is at the search boundary and at the target leverage respectively. Our results imply that, while higher tax benefits τ reduce the default threshold, they also *reduce the distance to default* when the firm is at the target leverage. This implies that at the moment of a restructuring, debt is actually more risky when tax benefits are higher (since the firm balances the marginal tax benefits of debt with the marginal default costs). The effect of λ is opposite: Higher credit supply λ reduces default threshold and increases the distance to default at the target.

To conclude this section, note that when the firm needs to search for creditors, restructuring generally occurs at an inefficient time compared to an environment in which capital supply is infinite. A number of questions naturally arise in such a context. First, can the firm contact current debtholders to restructure its capital structure instead of looking for new investors? Second, what are the terms of the new debt contract if the firm restructures with current creditors? The next section answers these and other related questions.

4. Restructuring with existing creditors

In this section, we generalize our model to allow the firm to bargain over the terms of new debt issues with current debtholders. Because bonds are generally held dispersedly and many of the loans issued by large firms are syndicated, mobilizing current creditors to issue new debt may be costly, with the cost increasing in the amount of outstanding debt. In the following, we

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model this market imperfection by assuming that contacting existing debt holders is costly and let ϵC_{t-} denote the cost of mobilizing these creditors, where $\epsilon > 0$ and C_{t-} is the total outstanding coupon when the firm contacts existing creditors.[11]

If shareholders can contact existing creditors every time they want to restructure, then the set of restructuring times is no longer restricted to the set $\mathscr{N}$ of jump times of the Poisson process N. In particular, the set of strategies available to shareholders now consists in pairs (b, γ), where γ is the default time and $b \geq 1$ is a process such that

$$b_t = 1 + \sum_{k=1}^{\infty} 1_{\{\tau_k = t\}} \xi_k$$

for some increasing sequence of restructuring times $\tau_k \in \mathbb{S}$ and some sequence of nonnegative random variables $\xi_k \in \mathscr{F}_{\tau_k}$ that represent the relative increase in the firm's coupon payment. The corresponding dynamics of the firm's coupon payment C_t are then given by

$$dC_t = (b_t - 1)C_{t-} = \sum_{k=1}^{\infty} 1_{\{\tau_k = t\}} \xi_k C_{t-},$$

where ξ_k corresponds to restructuring with new creditors when $\tau_k \in \mathscr{N}$.

Assume that debt holders anticipate that shareholders will use a strategy (b, γ) as above. Let $\theta \in [0, 1]$ denote the bargaining power of existing creditors. Nash bargaining implies that the part of the restructuring surplus that accrues to creditors at restructuring dates is

$$\pi^* = (\eta 1_{\{\tau_k \in \mathscr{N}\}} + \theta 1_{\{\tau_k \notin \mathscr{N}\}}) \big[S\big(b, X, C \big| (b, \gamma)\big) - \epsilon C 1_{\{\tau_k \notin \mathscr{N}\}} \big],$$

where S is defined as in Eq. (2) with $V_{\mathbf{b}}(X, C|(b, \gamma))$ – the value of the firm when contacting existing creditors at a cost is possible – replacing V. For a given strategy (b, γ) the value of the firm's equity, denoted by $E_{\mathbf{b}}(X, C|(b, \gamma))$, is then defined by

$$\begin{aligned} &E_{\mathbf{b}}\big(X, C \big| (b, \gamma)\big) \\ &\quad = E_0 \Bigg[\int_0^{\gamma} e^{-rt} (1 - \tau)(X_t - C_{t-}) dt + \sum_{k=1}^{\infty} 1_{\{\tau_k \leq \gamma\}} e^{-r\tau_k} H_{\mathbf{b}}\big(b_{\tau_k}, X_{\tau_k}, C_{\tau_k -} \big| (b, \gamma)\big) \Bigg], \end{aligned}$$

where

$$\begin{aligned} &H_{\mathbf{b}}\big(b', X, C \big| (b, \gamma)\big) \\ &\quad = (1 - q 1_{\{b' > 1\}}) D\big(X, b'C \big| (b, \gamma)\big) - D\big(X, C \big| (b, \gamma)\big) \\ &\qquad - \eta S\big(b', X, C \big| (b, \gamma)\big) \mathbf{1}_{\tau_k \in \mathscr{N}} - \big[\theta S\big(b', X, C \big| (b, \gamma)\big) + (1 - \theta)\epsilon C\big] \mathbf{1}_{\tau_k \notin \mathscr{N}}. \end{aligned}$$

When $\theta = 0$ and $\epsilon = 0$, there is no surplus extraction by current creditors and no cost of collective action. In this case, our model reduces to the models of Fisher, Heinkel, and Zechner [19] and Leland [38], in which the supply of credit is infinitely elastic at the correct price and the firm's capital structure is entirely determined by demand factors.

A Markov perfect equilibrium for the model in which the firm can restructure with current creditors can be defined as follows:

[11] The assumption of linear cost is made purely for convenience. The proof presented in the Online Appendix shows that our results hold for a large class of cost functions.

Definition 5. A Markovian strategy (b^*, γ^*) is a Markov perfect equilibrium if

$$\left(b^*, \gamma^*\right) \in \underset{(b,\gamma)\in\mathbb{B}(b^*,\gamma^*)}{\operatorname{argmax}} E_0\left[\int_0^\gamma e^{-rt}(1-\tau)(X_t - C_{t-})dt\right] + E_0\left[\sum_{k=1}^{\infty} 1_{\{\tau_k \le \gamma\}} e^{-r\tau_k} H_{\mathbf{b}}\left(b_{\tau_k}, X_{\tau_k}, C_{\tau_k -}\middle|\left(b^*, \gamma^*\right)\right)\right],$$

where $\mathbb{B}(b^*, \gamma^*)$ denotes the set of strategies (b, γ) such that

$$E_0\left[\int_0^\gamma e^{-rt} C_{t-} dt + \sum_{k=1}^{\infty} 1_{\{\tau_k \le \gamma\}} e^{-r\tau_k} \left| H_{\mathbf{b}}\left(b_{\tau_k}, X_{\tau_k}, C_{\tau_k -}\middle|\left(b^*, \gamma^*\right)\right)\right|\right] < \infty.$$

As in Section 3, we will only consider equilibria in barrier strategies. In the current model, the firm can either search for new investors or renegotiate outstanding debt with existing creditors when seeking to adjust its capital structure. Each strategy is associated with a different cost, i.e. a monetary cost when negotiating with current creditors and a waiting cost when searching for new creditors. Therefore, we expect the equity value-maximizing restructuring strategy to be characterized by two thresholds (barriers), $X_{\mathbf{b}u}(C)$ and $\bar{X}_{\mathbf{b}u}(C) \ge X_{\mathbf{b}u}(C)$, such that the optimal policy is (i) to search for new creditors (and restructure with them if they are found) when $X \ge X_{\mathbf{b}u}(C)$; (ii) to immediately contact current creditors if $X \ge \bar{X}_{\mathbf{b}u}(C)$. In the latter case, restructuring occurs exactly at $\bar{X}_{\mathbf{b}u}(C)$. In the former case, restructuring may not occur at $X_{\mathbf{b}u}(C)$ since the firm needs to find debt investors before changing its capital structure. More formally, barrier strategies are defined as follows when the firm can restructure with existing creditors[12]:

Definition 6. A Markov perfect equilibrium is in barrier strategies if

$$\gamma = \inf\left\{t \ge 0 : X_t \le X_{\mathbf{b}d}(C_{t-})\right\},$$
$$b_t = 1_{\{X_t < X_{\mathbf{b}u}(C_{t-})\}} + 1_{\{X_t \ge X_{\mathbf{b}u}(C_{t-}), t\in\mathscr{N}\}\cup\{X_t \ge \bar{X}_{\mathbf{b}u}(C_{t-})\}}(T_{\mathbf{b}} X_t / C_{t-})$$

for linear functions $\bar{X}_{\mathbf{b}u}(C) \ge X_{\mathbf{b}u}(C) \ge X_{\mathbf{b}d}(C)$ with $\bar{X}_{\mathbf{b}u}(0) = X_{\mathbf{b}u}(0) = X_{\mathbf{b}d} = 0$ and some nonnegative constant target $T_{\mathbf{b}} \in [1/\bar{X}_{\mathbf{b}u}(1), 1/X_{\mathbf{b}d}(1)]$.

To characterize Markov perfect equilibria in barrier strategies, we proceed as in Section 3. We first fix a default policy $X_{\mathbf{b}d}(C) \equiv X_{\mathbf{b}d}(1)C$ for some constant $X_{\mathbf{b}d}(1) > 0$ and then find the associated equilibrium restructuring policy b. The tractability of the analysis in the previous sections was largely due to the fact that restructurings only happened at Poisson times. This made the problem effectively discrete and allowed us to use the convenient machinery of contraction mappings. Introducing bargaining with existing creditors breaks this Poisson-type "discreteness". In this case, the optimal stopping/impulse control problem of shareholders becomes much more complex because it is essentially equivalent to that of a firm with infinitely many "real options" to restructure with current creditors.

[12] Since contacting existing creditors at time t entails a fixed cost ϵC_{t-}, we assume in this definition that the target capital structure is the same independently of the identity of the financiers (existing or new). We verify this conjecture formally in the Online Appendix.

While problems with a finite number of real options can be solved using backward induction (see, e.g., Hugonnier, Malamud, and Morellec [34]), the problem with infinitely many options is much more complex because the exercise value of the option depends on the (stationary) optimal exercise policy, which itself depends on the exercise value. This leads to a non-standard fixed point problem, and the contraction techniques used in the previous section are not directly applicable here. In particular, even though one can formally write down the HJB equation for this problem, showing that this HJB equation does have a regular solution is difficult.

To overcome this problem, we develop a new analytical technique that deals with such option exercise problems. Specifically, we first assume that shareholders can only contact current creditors at the jump times of a Poisson process with some fixed intensity Λ and solve for the corresponding policy. In this case, contraction mapping techniques can be used to establish a verification result, i.e. to show that firm value is a smooth solution of the HJB equation. Using techniques from the theory of second order ODEs (the maximum principle) then allows us to show that the optimal policy in the approximate (fixed Λ) policy is indeed of barrier type. We then take the limit $\Lambda \to \infty$ and show that the value function of the finite Λ problem converges to a finite limit that indeed satisfies the corresponding "free exercise" HJB equation.

An important step of the proof is to show that the limiting function indeed satisfies the smooth-pasting condition at the boundary of the option exercise region. This requires computing subtle asymptotics for limiting behavior of the corresponding ODE solutions. Then, standard verification results can be used to show that the limiting function is indeed the "true" value function and that the optimal policy is also of barrier type (being the limit of barrier-type policies).

Going through these steps allows us to establish the following result.

Proposition 9. *For any $X_{\mathbf{b}d}(1) > 0$ there exists a Markov perfect equilibrium with fixed default policy that is given by some barrier strategy $\mathbf{P} = \mathbf{P}(X_{\mathbf{b}d}(1))$. The strategy $\mathbf{P}(X_{\mathbf{b}d}(1))$ is a Markov perfect equilibrium if and only if it satisfies*

$$\frac{\partial}{\partial X} E_{\mathbf{b}}\big(X, C \big| \mathbf{P}\big(X_{\mathbf{b}d}(1)\big)\big)\bigg|_{X = X_{\mathbf{b}d}(C)} = 0. \tag{10}$$

Using Proposition 7, we can now derive equity value under the value-maximizing default and restructuring strategy. In the region $(X_{\mathbf{b}d}(C), \bar{X}_{\mathbf{b}u}(C))$, shareholders receive a cash flow $(1-\tau)(X_t - C)dt$ over each time interval $[t, t+dt]$ and have the option to change the firm's capital structure when meeting new creditors. As a result, the equity value function associated with the barrier strategy $\mathbf{P} = \mathbf{P}(X_{\mathbf{b}d}(1))$ satisfies:

$$\begin{aligned} rE_{\mathbf{b}}(X, C|\mathbf{P}) &= \mu X \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}) + \frac{\sigma^2}{2} X^2 \frac{\partial^2}{\partial X^2} E_{\mathbf{b}}(X, C|\mathbf{P}) + (1-\tau)(X - C) \\ &\quad + 1_{\{X \geq X_{\mathbf{b}u}(C)\}} \lambda^* \big[V_{\mathbf{b}}(X, T_{\mathbf{b}} X|\mathbf{P}) - V_{\mathbf{b}}(X, C|\mathbf{P}) - qD(X, T_{\mathbf{b}} X|\mathbf{P})\big] \end{aligned}$$

where $V_{\mathbf{b}}(X, C|\mathbf{P}) = E_{\mathbf{b}}(X, C|\mathbf{P}) + D(X, C|\mathbf{P})$ denotes firm value and the last term on the right hand side reflects the effects of credit supply uncertainty on equity value. This equation is solved subject to the following conditions at the default and restructuring thresholds:

$$\begin{aligned} &E_{\mathbf{b}}\big(X_{\mathbf{b}d}(C), C\big|\mathbf{P}\big) = 0, \\ &V_{\mathbf{b}}\big(\bar{X}_{\mathbf{b}u}(C), C\big|\mathbf{P}\big) = \big\{V_{\mathbf{b}}(X, T_{\mathbf{b}}X|\mathbf{P}) - qD(X, T_{\mathbf{b}}X|\mathbf{P}) - \epsilon C\big\}\big|_{X = \bar{X}_{\mathbf{b}u}(C)}, \\ &\frac{\partial}{\partial X} V_{\mathbf{b}}(X, C|\mathbf{P})\bigg|_{X = \bar{X}_{\mathbf{b}u}(C)} = \frac{\partial}{\partial X}\big\{V_{\mathbf{b}}(X, T_{\mathbf{b}}X|\mathbf{P}) - qD(X, T_{\mathbf{b}}X|\mathbf{P}) - \epsilon C\big\}\bigg|_{X = \bar{X}_{\mathbf{b}u}(C)}. \end{aligned}$$

The first and second boundary conditions are the value-matching conditions that apply to equity value at the default and restructuring thresholds. The third boundary condition is the smooth-pasting condition that applies when restructuring with existing creditors at $\bar{X}_{\mathbf{b}u}$. The value-matching condition at $\bar{X}_{\mathbf{b}u}$ implies that there is no surplus at the time of a restructuring. This in turn implies that the bargaining power of existing creditors θ has no bearing on the equilibrium.

In addition to these boundary conditions, equity value satisfies:

$$V_{\mathbf{b}}\big(X_{\mathbf{b}u}(C), C\big|\mathbf{P}\big) = \big(V_{\mathbf{b}}(X, T_{\mathbf{b}}X|\mathbf{P}) - qD(X, T_{\mathbf{b}}X|\mathbf{P})\big)\big|_{X=X_{\mathbf{b}u}(C)}$$
$$\lim_{X\uparrow X_{\mathbf{b}u}(C)} E_{\mathbf{b}}(X, C|\mathbf{P}) = \lim_{X\downarrow X_{\mathbf{b}u}(C)} E_{\mathbf{b}}(X, C|\mathbf{P}),$$
$$\lim_{X\uparrow X_{\mathbf{b}u}(C)} \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}) = \lim_{X\downarrow X_{\mathbf{b}u}(C)} \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}),$$

at the search threshold. The first condition determines the value-maximizing threshold when seeking to restructure with new creditors. The last two conditions are continuity and smoothness conditions. In these conditions, the target leverage ratio $T_{\mathbf{b}}$ satisfies

$$T_{\mathbf{b}} = \operatorname*{argmax}_{T_{\mathbf{b}}>0}\big\{E_{\mathbf{b}}(X, T_{\mathbf{b}}X|\mathbf{P}) + (1-q)D(X, T_{\mathbf{b}}X|\mathbf{P})\big\},$$

and the default threshold satisfies the smooth pasting condition (10).

As in the model with search, we need to solve the smooth pasting condition (10) for $X_{\mathbf{b}d}(1)$ to prove existence/uniqueness of an equilibrium in barrier strategies. To this end, we use monotonicity arguments similar to those used in the proof of Theorem 1. In the model, equity value for a firm with a fixed default policy is monotone decreasing in q and ϵ. Therefore, an equilibrium exists as soon as it exists when $q = \epsilon = 0$.[13] Since the latter case corresponds to the search model with an infinite intensity, we define

$$\kappa_1(\infty) \equiv \lim_{\lambda\to\infty} \kappa_1(\lambda) = \frac{\alpha\mu - r}{(r-\mu)(\alpha-\beta)},$$
$$\kappa_2(\infty) \equiv \lim_{\lambda\to\infty} \kappa_2(\lambda) = \frac{1+(1-\beta)\kappa_1(\infty)}{\alpha-1}.$$

We can now define the unique solution $\bar{J}_0$ to the equation

$$1 + \kappa_1(\infty)\bar{J}_0^{-\beta} + \kappa_2(\infty)\bar{J}_0^{-\alpha} = 0,$$

and the critical tax rate $\tau^*(\infty) < \tau^*$ by

$$\tau^*(\infty) \equiv \frac{-\beta}{-\beta + (\alpha-\beta)\kappa_2(\infty)\bar{J}_0^{-\alpha}} \in (0, 1).$$

We then have the following result:

Theorem 2. *Assume that $q > 0$ and that shareholders can raise new debt from existing creditors by paying a cost of collective action ϵC_{t-}. If the corporate tax rate satisfies $\tau < \tau^*(\infty)$, then there exists a Markov perfect equilibrium in barrier strategies. Furthermore, this equilibrium is unique if ϵ and q are sufficiently small.*

[13] The proof of this result is similar to that for Theorem 1. See Lemma C.9 in the Online Appendix for details.

Theorem 2 provides sufficient conditions for a Markov perfect equilibrium in barrier strategies to exist when the firm can restructure with existing creditors. As in Section 3, the level of the corporate tax rate is key in determining whether an equilibrium exists. While in our base case environment, the corporate tax rate does satisfy the restriction $\tau < \tau^*(\infty)$ of Theorem 2, it is also interesting to characterize firm behavior when this restriction is not satisfied. In the Online Appendix, we show that if the corporate tax rate satisfies $\tau > \tau^*(\infty)$, then there exists a threshold $q^*_{\mathbf{b}}$ satisfying

$$q^*_{\mathbf{b}} \equiv \inf\left\{ q > 0 : \inf_{X_{\mathbf{b}d}(1)>0} \frac{\partial}{\partial X} E_{\mathbf{b}}\big(X, C \big| \mathbf{P}\big(X_{\mathbf{b}d}(1)\big)\big)\Big|_{X=X_{\mathbf{b}d}(C)} < 0 \right\},$$

such that a Markov perfect equilibrium in barrier strategies exists if and only if $q > q^*_{\mathbf{b}}$. This threshold is monotone increasing in the meeting intensity λ, decreasing in the bargaining power η, and satisfies $q^*_{\mathbf{b}} \geq q^*$. This implies again that there may not exist an equilibrium in barrier strategies for the model when tax benefits of debt are very large.

We complete this section with an analog of Proposition 8.

Proposition 10. $X_{\mathbf{b}d}(1)$ *is monotone decreasing in* λ, *and increasing in* q, η, *and* ϵ. *Furthermore, if* q *and* ϵ *are sufficiently small, then:*

- $X_{\mathbf{b}d}(1)$ *is monotone decreasing in* τ;
- $T^*_{\mathbf{b}}$, $\frac{1}{X_{\mathbf{b}u}(1)}$, *and* $\frac{1}{\bar{X}_{\mathbf{b}u(1)}}$ *are increasing in* τ;
- $\frac{1}{T^*_{\mathbf{b}} X_{\mathbf{b}d}(1)}$, $\frac{X_{\mathbf{b}u}(1)}{X_{\mathbf{b}d}(1)}$, *and* $\frac{\bar{X}_{\mathbf{b}u}(1)}{X_{\mathbf{b}d}(1)}$ *are decreasing in* τ.

In contrast to the results for the case without bargaining with existing creditors (Proposition 8), we cannot establish the monotonicity of the restructuring thresholds with respect to λ. The reason is that, in the limit when $\epsilon \to 0$, the value of the firm becomes independent of λ because the firm always restructures with existing creditors. Hence, we cannot exploit the implicit function theorem-based argument used to prove Proposition 8.

5. Corporate financing with credit supply frictions

5.1. Calibration of parameter values

This section examines the empirical predictions of the model for financing policies, creditor turnover, and the decision to default. To do so, we need to select parameter values for the initial value of the firm's cash flows X_0, the risk free interest rate r, the tax advantage of debt τ, liquidation costs ω, the physical and risk neutral growth rates of the firm's income m and μ, the volatility of the cash flow shock σ, refinancing costs q, the cost of collective action ϵ, and the bargaining power of outside creditors η. In what follows, we select parameter values that roughly reflect a typical US firm. These parameter values are reported in Table 1.

Consider first the parameters governing operating cash flows. We set the initial value of cash flows to $X_0 = 1$. This is without loss of generality since the homogeneity of our model implies that the quantities of interest do not depend on X_0. The main parameters describing the cash flow dynamics are (m, μ, σ). Morellec, Nikolov, and Schürhoff [50] construct estimates for these variables using data from Compustat, CRSP, and the Institutional Brokers' Estimate System (IBES). They find that for the average firm in their sample, the risk-neutral growth rate, the physical

Table 1
Benchmark parameter values.

Symbol	Interpretation	Value
A. Firm specific parameters:		
r	Interest rate	0.042
μ	Risk-neutral cash flow rate	0.0067
m	Real cash flow rate	0.0824
σ	Cash flow volatility	0.2886
τ	Corporate tax rate	0.15
ω	Liquidation cost	0.45
B. Credit market parameters:		
λ	Arrival rate of creditors	3.00
q	Proportional issuance cost	0.01
ϵ	Fixed issuance cost	0.025
η	Bargaining power of outside creditors	0.50
θ	Bargaining power of inside creditors	0.50
C. Implied parameters:		
τ^*	Maximal tax rate in the search model	0.9934
$\tau^*(\infty)$	Maximal tax rate in the model with outside creditors	0.8116

This table gives the benchmark parameter values that we use in our numerical illustrations of the model. The bargaining power of inside creditors is set equal to that of outside creditors. This is without loss of generality since the surplus from issuing debt to inside creditors is always equal to zero in equilibrium.

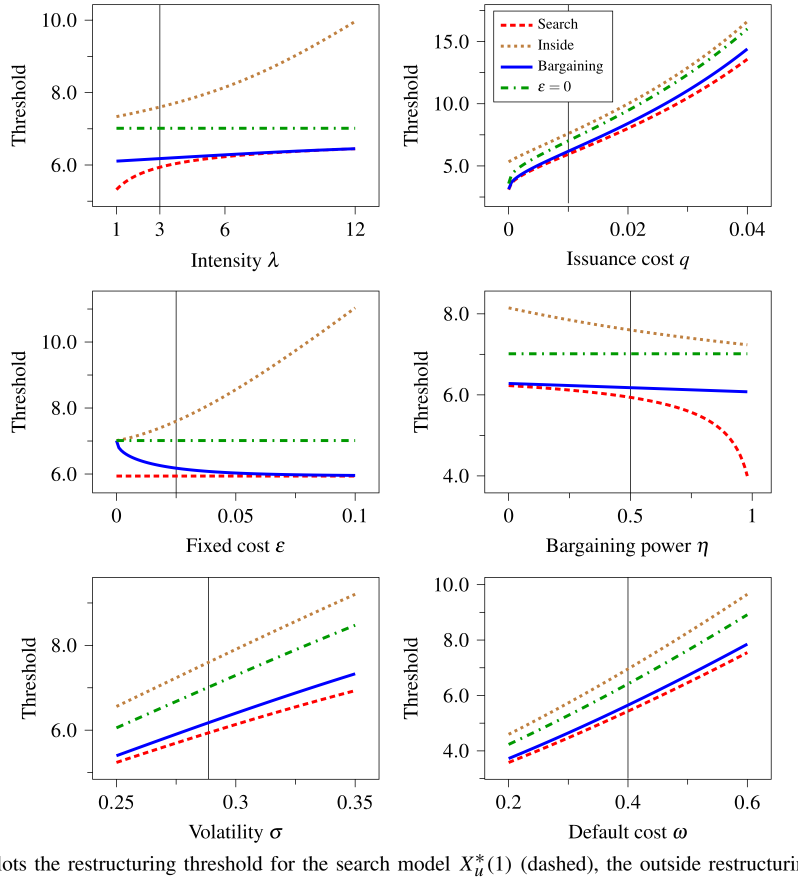
growth rate, and the volatility of the cash flow process are respectively given by $\mu = 0.67\%$, $m = 8.24\%$, and $\sigma = 28.86\%$.

The risk-free rate $r = 4.2\%$ is calibrated to the one-year treasury rate. The tax advantage of debt captures corporate and personal taxes and is set equal to $\tau = 15\%$. This corresponds to a tax environment in which the corporate tax rate is set at the highest possible marginal tax rate of 35% and the tax rates on dividends and interest income are set to 11.6% and 29.3%, consistent with the estimates in Graham [26]. Liquidation costs are defined as the firm's going concern value minus its liquidation value, divided by its going concern value. We base the calibration of liquidation costs on Glover [22] and set $\omega = 45\%$.

Several empirical studies provide estimates of issuance costs as a function of the amount of debt being issued. The model, however, is written in terms of debt issuance cost q as a fraction of total debt outstanding. In the following, we set the cost of debt issuance to $q = 1\%$. This produces a cost of debt issuance representing 2% of the issue size at the search threshold, consistent with the values found in the empirical literature (see Altinkilic and Hansen [2], and Kim, Palia, and Saunders [36]). We also assume that $\epsilon = 2.5\%$, which implies that the cost of collective action represents 25% of the total restructuring cost when the firm restructures with current creditors and check robustness by varying ϵ. Finally, we set the bargaining power of new creditors to $\eta = 50\%$ and also check robustness by varying η. These input parameter values imply that $\tau^* = 99.34\%$ and $\tau^*(\infty) = 81.16\%$.

5.2. *Optimal restructuring strategies*

We start by analyzing the effects of credit supply uncertainty on shareholders' restructuring strategy and creditor turnover. In the model, restructuring is endogenous and occurs the first time the cash flow process reaches the region $[X_{\mathbf{b}u}(C), \infty)$ and the firm finds new debt investors or



This figure plots the restructuring threshold for the search model $X_u^*(1)$ (dashed), the outside restructuring threshold $X_{\mathbf{b}u}(1)$ (solid), the inside restructuring threshold $X_{\mathbf{b}c}(1)$, and the restructuring threshold for the frictionless model with $\epsilon = 0$ (dash-dotted) as functions of the arrival rate of outside creditors λ, the proportional issuance cost q, the fixed issuance cost ϵ, the bargaining power of outside creditors η, the volatility of the firm's cash flows σ and the cost of default ω. In each panel the vertical line indicates the base case value of the parameter that is being varied.

Fig. 1. Restructuring thresholds.

upon reaching $\bar{X}_{\mathbf{b}u}(C)$, in which case the firm restructures with existing creditors. Credit supply uncertainty therefore affects dynamic capital structure choice through its effects on $X_{\mathbf{b}u}(C)$ and $\bar{X}_{\mathbf{b}u}(C)$. Credit supply uncertainty also implies that, in contrast with standard dynamic capital structure models, there exists some time series variation in the size of capital structure changes in our model.

To illustrate these effects, Fig. 1 plots the restructuring triggers as functions of the arrival rate of creditors λ, the share η of the surplus captured by new debtholders, the volatility of the cash flow shock σ, the tax rate τ, liquidation costs ω, and issuance costs q. In the figure, the dashed line represents the restructuring threshold with uncertain credit supply and without bargaining with current creditors, i.e. $X_u(C)$, the solid line represents the search threshold in the model with credit supply uncertainty and bargaining with current creditors, i.e. $X_{\mathbf{b}u}(C)$, and the dotted line represents the restructuring threshold with current creditors, i.e. $\bar{X}_{\mathbf{b}u}(C)$. Finally, the dot-dashed line represents the restructuring threshold when $\epsilon = 0$, i.e. in models like Fisher, Heinkel, and Zechner [19] or Leland [38].

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Fig. 1 shows that the search threshold in the model with bargaining, $X_{\mathbf{b}u}(C)$, is always above the search threshold in the model without bargaining, $X_u(C)$. This is due to the fact that when the firm can contact current creditors to restructure, the value of the option to look for new creditors is lower, so that shareholders exercise this option later. The figure also shows that the search threshold in the model with bargaining is lower than the restructuring threshold $\bar{X}_{\mathbf{b}u}(C)$ since shareholders need to pay a cost of collective action when restructuring with current creditors. Accordingly, the wedge between these two thresholds increases as ϵ increases. As shown by the figure, this wedge also increases as λ increases since the option value of restructuring with existing creditors decreases with the strength of credit supply.

Another result illustrated by the figure is that the search and restructuring thresholds increase with credit supply λ and decrease with the share of the restructuring surplus that new creditors can capture η. In particular, as the arrival rate of investors decreases, the opportunity cost of waiting to restructure increases (as the likelihood of finding investors decreases), so that the selected search threshold decreases. That is, when the firm has to find investors to restructure its debt, it balances the opportunity cost of early restructuring with the opportunity cost of waiting (the risk of not finding creditors to refinance). In addition, as η increases, the selected restructuring threshold decreases, reflecting shareholders' incentives to reduce the potential cash flow transfers towards new creditors at restructuring dates.

To illustrate the effects of credit market frictions on the frequency of capital structure changes, Fig. 2 plots the probability of a restructuring over a 3-year horizon (Panel A.) as well as the unconditional probability of a restructuring (Panel B.) as functions of the arrival rate of creditors λ, the cost of collective action ϵ, and issuance costs q. In the figure, the dotted line represents the probability of restructuring with current creditors, the dashed line represents the probability of restructuring with new creditors, and the solid line represents the probability of restructuring with new or existing creditors. Finally, the dot-dashed line represents the probability of restructuring with current creditors when $\epsilon = 0$. The Online Appendix shows how to compute these probabilities.

In the model, credit supply uncertainty has two opposing effects on the likelihood of capital structure changes. First, for any given restructuring threshold, an increase in λ makes it easier for the firm to find creditors and to restructure. Second, an increase in λ leads to an increase in the selected restructuring threshold. Fig. 2 shows that in our base case environment the first effect dominates so that the frequency of restructuring increases with the arrival rate of creditors λ. The figure also shows that as the cost of collective action increases, the overall probability of restructuring over a finite horizon first decreases and then increases. This is due to two opposing effects. On the one hand, an increase in ϵ leads to an increase of the cost of restructuring with existing creditors. This in turn leads to an increase in the restructuring threshold with existing creditors (as shown in Fig. 1) and to a decrease in the likelihood of a restructuring with existing creditors. On the other hand, an increase in ϵ leads to a decrease in the outside restructuring threshold $X_{\mathbf{b}u}(C)$ (as illustrated in Fig. 1) and, hence, to an increase in the likelihood of a restructuring with new creditors. Another result illustrated by Fig. 2 is that the frequency of capital structure changes decreases with issuance costs q, consistent with economic intuition.

Fig. 2, Panel B., also shows that the frequency of creditor changes – i.e. the creditor turnover – depends on the various frictions faced by firms when seeking to restructure. In particular, the figure shows that the creditor turnover increases with the cost of collective action ϵ and with credit supply λ. It also shows that the cost of financing q increases the unconditional probability of refinancing with existing creditors since the cost of collective action becomes a smaller fraction of the total cost of financing as issuance costs increase. The quantitative effects of ϵ and λ on

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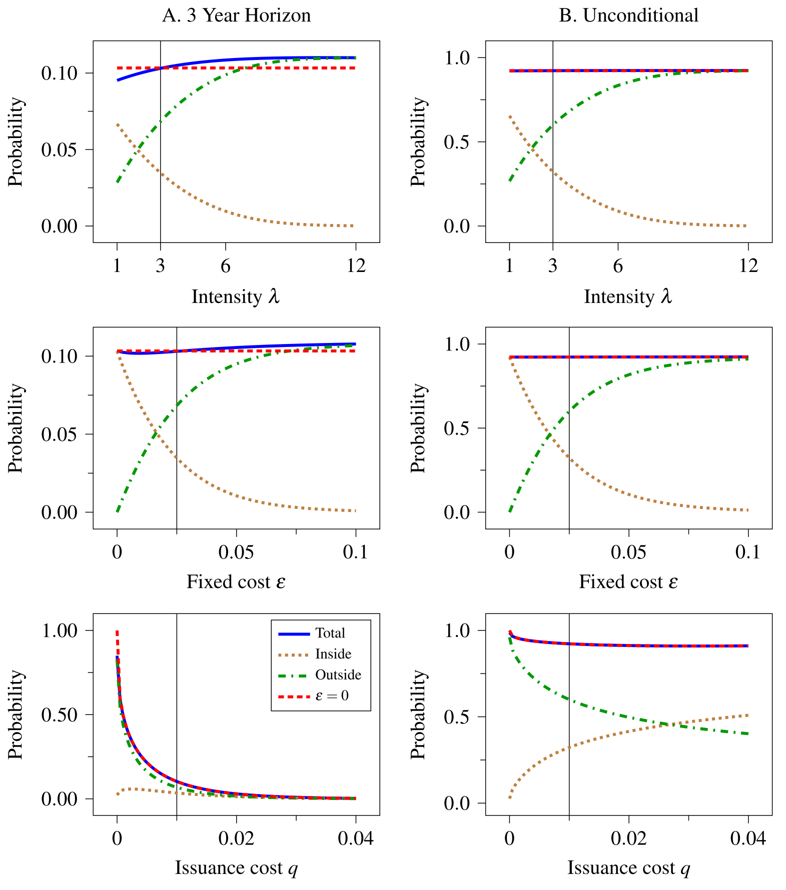


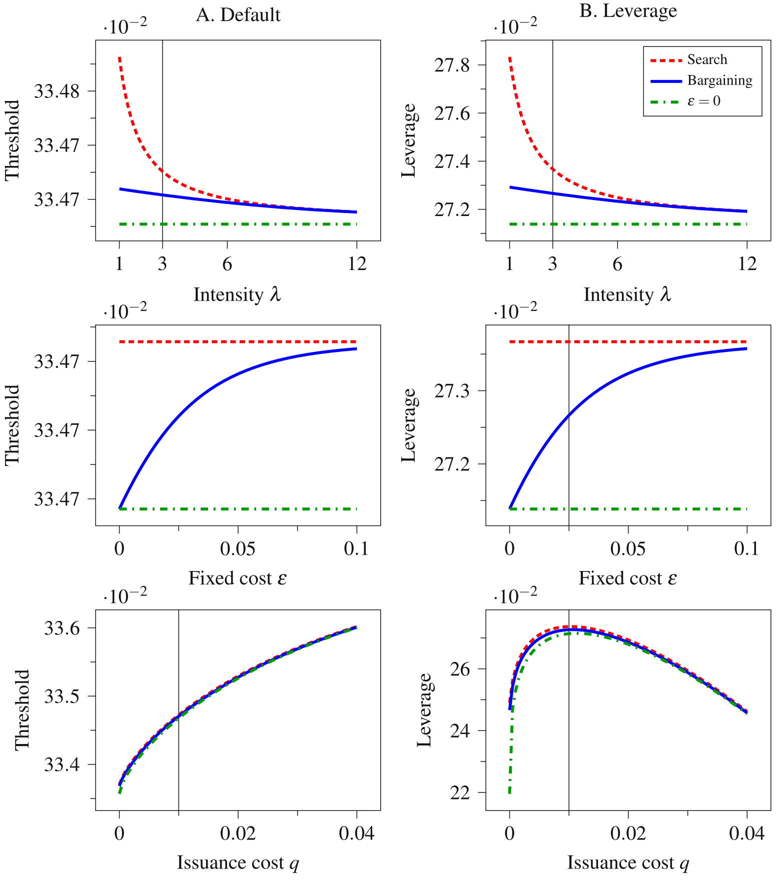
This figure plots the probability of restructuring at a horizon of three years (Panel A.) and the unconditional probability of restructuring (Panel B.) as functions of the arrival rate of outside creditors λ, the fixed issuance cost ϵ and the proportional issuance cost q. The dotted (dashed) line gives the probability of restructuring with inside (outside) creditors, the solid line gives the probability of restructuring with either inside or outside creditors and the dot-dashed line gives probability of restructuring in the frictionless model with $\varepsilon = 0$. In each panel the vertical line indicates the base case value of the parameter that is being varied.

Fig. 2. Probability of restructuring before default.

creditor turnover are large. Notably, the unconditional creditor turnover increases from 30.66% to 81.77% when ϵ is raised from 1% to 5%, while it increases from 26.63% to 78.07% when λ increases from 1 to 5.

5.3. Optimal capital structure and default decisions

We now turn to the analysis of leverage and default decisions. In the following, we focus on financing decisions made by shareholders at refinancing points (i.e. optimal leverage). When making such decisions, the objective of shareholders is to maximize the value of equity after the issuance of corporate debt (i.e. $E(X, XT|\mathbf{P})$) plus the proceeds from the debt issue. Fig. 3 Panel A. plots the value-maximizing leverage ratio as a function of the arrival rate of investors λ,



This figure plots the default threshold (Panel A.) and the target leverage ratio (Panel B.) for the search model (dashed), the model with inside creditors (solid) and the frictionless model with $\epsilon = 0$ (dot-dashed) as functions of the arrival rate of outside creditors λ, the fixed issuance cost ϵ and the proportional issuance cost q. In each panel the vertical line indicates the base case value of the parameter that is being varied.

Fig. 3. Default and leverage.

issuance costs q, and the cost of collective action ϵ. Input parameter values for Fig. 3 are set as in our base case and the leverage ratio is defined by:

$$L^* = \frac{D(1, T|\mathbf{P})}{D(1, T|\mathbf{P}) + E(1, T|\mathbf{P})}.$$

The figure shows that, as credit supply weakens, firms issue more debt when restructuring their capital structure. This result is consistent with the idea that, with weaker credit supply, it will be more difficult to restructure if economic conditions improve so that the firm needs to take on more debt ex ante. This result implies that the reduction in the implicit cost of search delays due to the higher debt level outweights the additional default costs borne by the firm. Another result illustrated by Fig. 3 is that the optimal leverage ratio increases with the cost of collective action ϵ. However, the quantitative effect is very small. Finally, as in prior studies (see

Strebulaev [54], or Morellec, Nikolov, and Schürhoff [50]), the leverage ratio decreases with bankruptcy costs and cash flow volatility (not shown).

Interestingly, the target leverage ratio first increases and then decreases with restructuring costs. That is, firms with low and high refinancing costs prefer lower leverage than those with intermediate costs. This non-monotonicity is due to two opposing effects. First, as restructuring costs increase, shareholders find it optimal to delay changes in capital structure, making it optimal to issue more debt at restructuring dates. Second, as restructuring costs increase, the cost of issuing new debt increases, making it optimal to issue less debt at restructuring dates. The figure shows that the first effect dominates for low values of q while the second effect dominates for large values of q.

The leverage levels reported in the figure are consistent with those in prior studies. In our base case environment, the optimal leverage at refinancing points in the model with search is 27.36%. This value lies between the optimal leverage ratio in a dynamic model with perfectly liquid bond markets (in which firms issue debt conservatively as the supply of credit is certain) and the optimal leverage ratio in a static model (in which firms issue debt aggressively as they will not be able to issue additional debt in the future).

Panel A. of Fig. 3 investigates the effects of credit market frictions on shareholders' default decision. In the model, the default policy that maximizes equity value balances the present value of the cash flows that shareholders receive in continuation with the cash flow that they receive in liquidation. By reducing the value of shareholder's restructuring options (and therefore the continuation value of equity), credit supply uncertainty leads shareholders to default earlier. The quantitative effect is however very small as the value of the restructuring options is small when the firm is close to default.[14]

6. Conclusion

Following Modigliani and Miller [48], extant theoretical research in corporate finance generally assumes that capital markets are frictionless so that corporate behavior and capital availability depend solely on firm characteristics. This demand-driven approach has recently been called into question by a large number of empirical studies. These studies document the central role of supply conditions in credit markets in explaining corporate policy choices and highlight the need for an improved understanding of the precise role of supply in firms' financing decisions.

This paper takes a first step in constructing a dynamic model of financing decisions with capital supply effects by considering a setup in which firms face uncertainty regarding their future access to credit markets. The model provides an explicit characterization of the optimal default and financing policies for a firm acting in the best interests of incumbent shareholders and shows that credit market frictions have first-order effects on corporate policy choices. The analysis in the paper yields a wide range of empirical implications relating supply conditions in the credit markets to firms' default risk, dynamic financing policies, the timing of security issues, creditor turnover, and the role of firm characteristics in shaping corporate policies. Overall, the analysis demonstrates that accounting for both demand and supply factors is critical to understanding firms' capital structure decisions.

[14] By contrast, He and Xiong [32] and He and Milbradt [31] develop models in which liquidity shocks in the secondary market for corporate bonds affect shareholders' default decision by increasing the cost of debt but have no bearing on restructuring strategies and optimal leverage, which are fixed exogenously.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2014.09.021.

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