Accepted Manuscript

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PII: S0304-405X(16)30048-4
DOI: 10.1016/j.jfineco.2016.03.009
Reference: FINEC 2649

To appear in: *Journal of Financial Economics*

Received date: 3 December 2014
Revised date: 19 June 2015
Accepted date: 17 July 2015

Please cite this article as: Robert Geske, Avanidhar Subrahmanyam, Yi Zhou, Capital structure effects on the prices of equity call options, *Journal of Financial Economics* (2016), doi: 10.1016/j.jfineco.2016.03.009

Capital structure effects on the prices of equity call options

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March 12, 2016

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ABSTRACT

We examine whether values of equity options traded on individual firms are sensitive to the firm's capital structure. We estimate the compound option (CO) model, which views equity as an option on the firm. Compared with the Black-Scholes model, the CO model with a term structure of volatility (TSV) reduces pricing errors by 20% on average. The compound option effect is particularly strong for highly levered firms and long-term options, in which the pricing improvement is up to 70% of the Black-Scholes error. Without a TSV, the CO model reduces pricing errors of in-the-money options by 12.74% on average and for out-of-the-money by 9.22%. We show that the CO model implies a market value of firm leverage and allows imputation of the firm's implied volatility, both of which have potential applications in corporate finance.

JEL classification: G12

Keywords: Derivatives, Options, Leverage, Stochastic volatility

1. Introduction

A firm's debt influences the values of securities held by the firm's residual claimants, i.e., equityholders. In turn, leverage must influence options on equity. Does incorporating the economic dependence of option values on firm leverage improve the pricing performance of option valuation models? In this paper, we investigate this question in a Black-Scholes-Merton no-arbitrage setting. We perform this analysis by testing the compound option (CO) model, which is a partial equilibrium, closed-form extension of the seminal Black and Scholes (1973) (BS) model. The CO model follows directly from Merton (1973), in which stock is considered to be an option on a levered corporation and, thus, an option on stock is an option on an option, or a compound option. So a CO is a nested sequence of options on options. BS were the first to show that all options are levered investments in the underlying optioned security or asset, in which the exercise price acts as the leverage. Most corporations have some form of direct or indirect debt obligation, which serves as the strike price of the firm's option to default on its debt. This debt induces stochastic total risk and systematic risk of the corporation's equity, and it has led to the practice of adjusting a levered firm's volatility and beta relative to an unlevered firm.[1] Surprisingly, however, very few tests have been conducted of option pricing models such as CO (Geske, 1979) that directly incorporate leverage based on economic principles.

Black (1976) shows empirically a negative correlation between stock price returns and stock volatility. This relation was first linked to leverage and formally analyzed by Christie (1982) and has been termed the leverage effect. Toft and Prucyk (1997) show using a sample of 138 firms over a 13-week period in 1994 that debt levels are related to option volatility. However, they do not investigate the extent of option pricing improvement attributable to leverage.

Some important option pricing papers have assumed an explicit exogenous stochastic

[1] A firm not directly issuing bonds has many indirect promised payouts (accounts payable, bank loans, leases, taxes, etc.).

process for stock volatility and a negative correlation between a stock's return and this stochastic volatility. Among these papers are Heston (1993), Bakshi et al. (1997), Pan (2002), and Bates (2000).[2] This paper takes option theory deeper into the theory of the firm. Assuming a conventional diffusion process for firm value with a deterministic firm volatility generates an implicit endogenous stochastic process for the stock return volatility. The advantages of an implicit endogenous stochastic volatility process are that, first, the process parameters do not require estimation and, second, the stochastic changes in the stock's volatility have a known economic cause, which is the firm's capital structure.[3]

Given that almost all firms are portfolios of assets and liabilities financed by some form of debt and equity, total firm volatility should be smaller and more stable than the component stock volatility. Following this reasoning, in both the Merton model of stock as a default option and the CO model for a call option on the stock that is a default option, if we assume the firm volatility is deterministic, then firm debt implies the stock volatility is greater than the firm volatility, depends on the stock price, and follows a particular stochastic function. As the stock price falls (rises), the firm's debt-equity ratio rises (falls), and this increased (decreased) risk is reflected by a rise (fall) in the variance of the returns on the stock. If the empirically observed negative relation is partially caused by debt, then strike price leverage could be both statistically and economically relevant to pricing equity options. This is what our paper demonstrates. Our implicit notion is that if debt is a relevant but omitted variable, then any exogenous option model parameters for the stock's stochastic process could be estimated with error. Thus, we attempt to isolate and analyze whether debt has an effect on equity option prices independent of other assumed complexities of the equity

[2]See also Scott (1987), Stein and Stein (1991), and Wiggins (1987). Heston and Nandi (2000) develop a closed-form generalized autoregressive conditional heteroscedasticity (GARCH) option valuation model that is easier to implement, exhibits the required negative relation between equity returns and volatility, and contains the Heston (1993) stochastic volatility model as a continuous time limit. They also compare with BS and demonstrate that their out-of-sample valuation errors are lower than the ad hoc modified version of BS. Liu et al. (2005) attempt to further disentangle the equity return distribution by incorporating diffusive and jump premia, stochastic interest rates, imprecise modeling, and uncertainty aversion.

[3]Thus, in the compound option setting, equity, as an option on the firm, exhibits stochastic volatility, just like a call option on equity.

return distribution such as stochastic equity volatility, stochastic systematic risk, stochastic interest rates, and stochastic jumps.

So by incorporating the face value of debt, which serves as the equity strike price, the CO model uses Modigliani and Miller (1958) (MM) to take equity option pricing theory deeper into the theory of the firm.[4] Application of Ito's lemma confirms that a levered firm's inferred stock return volatility cannot be constant as assumed by BS but, instead, is a function of the level of the stock price. As a firm's stock price changes, if the firm's management does not react, the firm's leverage ratio changes inversely, as does the equity risk measured by volatility. In this setting, the face value of firm debt, the implied market value of debt, and the duration of debt affect the stochastically changing shape of each firm's stock return distribution. The shape of the conditional equity return distribution at any time determines the model values for equity options with different strike prices and different times to expiration. We demonstrate that debt as the strike price for the equity default option used in the CO model setting can improve option valuation model accuracy.

This stochastic volatility effect for equity, when viewed as an option with a strike price equal to the aggregate face value of corporate debt (as in the CO model), is isomorphic to the similarly observed stochastic volatility of equity call option returns, either for many calls with different strike prices for a fixed stock price or for any fixed strike price as the stock price moves around the strike over time, both generating in-the-money (ITM) or out-of-the-money (OTM) options, either in cross section or over time. So, for an option on an option, the strike price of the first option generates stochastic volatility for that option's return, regardless of the underlying exhibiting deterministic volatility. A second option on the first option reacts directly to the changing stock volatility of the first option.

We believe that we are the first to empirically examine capital structure effects on the pricing of individual stock options using the closed-form CO model. We accomplish this by linking Compustat data on individual firms' debt to the OptionMetrics database, which

[4]Because at each time the stock price is known, given MM, the solution is for the implied market value of firm debt D, as the implied value of the firm, $V =$ known $S +$ unknown D.

contains detailed data on prices of options on individual stocks. Our paper is related to many other papers in the option pricing literature, all of which compare their extensions with the seminal BS model. For example, the implied binomial tree lattice approach was developed by Rubinstein (1985) and others to better fit the cross-sectional structure of option prices wherein the volatility can depend on the asset price and time. This lattice approach to an implied binomial tree produces a deterministic volatility function, and these implied tree lattice approaches have been shown to be as accurate at pricing options as ad hoc versions of BS in which the implied volatility is modified for strike price and time [see Dupire (1994) and Derman and Kani (1994)].

In applications beyond option pricing, we show that the CO model allows the estimation of the market value of firm leverage even when the firm's debt is not publicly traded and also implies volatility at the firm level. The former aspect allows one to assess how market value–based capital structure varies over time, and the latter allows one to assess how corporate actions such as real investment and mergers alter risk at firm level.

Our work serves to formalize how stock volatility that is induced by variations in leverage, rigorously addressed by Christie (1982), influences option values [see also Siriwardane and Engle (2015), who propose a structural generalized autoregressive conditional heteroscedasticity (GARCH) model that incorporates the effect of leverage on equity volatility]. In other work on the equity return–volatility relation, Bollerslev et al. (2006) find support for the negative relation between equity return and volatility using high frequency data. Figlewski and Wang (2000) report an asymmetric effect, in which the volatility-return relation is higher in an equity down-market than in an up-market. Duan and Wei (2009) find that the volatility-return dependence is also related to the systematic risk of the stock. Carr and Wu (2011) suggest that negative returns lead to disruptive market behaviors, increasing volatility. This literature is concerned with causality of the observed negative relation between stock returns (both expected and realized) and total risk of the stock, and it is not concerned with pricing options on the stock. This is appropriate because no-arbitrage option model values are independent of the expected return on the underlying asset, so changes in the stock expected returns do not affect option model values. Thus, research on the leverage effect is distinct from our focus, which is on whether adding debt to option models improves the option pricing accuracy within a no-arbitrage setting.[5]

The rest of the paper proceeds as follows. Section 2 describes the CO model and the BS model and their parsimonious implementation. Section 3 describes the data and explains how the necessary data inputs for CO and BS are calculated and used. Section 4 follows all previous empirical option model research and compares the model errors with respect to market prices for both the CO and the seminal BS models and reports on the economic and statistical significance of the CO model improvements. Section 5 describes some useful aspects of implied leverage. Section 6 concludes the paper.

2. Implementation of the CO and BS models

When valuing options to buy the stock of a firm with leverage, the compound option model always involves at least two correlated options, one for the managerial option to repay the debt and the other for the option to exercise for the stock. BS is concerned with a solitary option.[6] The compound option model, when applied to listed individual equity options, transforms the state variables underlying the option from the observable stock price and unobservable but implied stock volatility to the unobservable total market value of the firm, V, and the unobservable but implied firm volatility. The implied firm value is the sum of market values for both the observable equity price S and the unobservable but implied debt value D ($V = S + D$). In the compound option model, the stock price enters

[5]Hasanhodzic and Lo (2011) construct from Compustat a sample of 667 firms that they define as all equity (AE) and find that the volatility of the AE firms exhibits the same negative relation between price and volatility that is characteristic of the leverage effect. They puzzle over how this can be due to leverage and what is its cause. We believe it could be that in some strike price–leverage ranges the volatility of price changes, not returns, is close to constant, and so lower priced stocks exhibit higher return volatility.

[6]The CO model has been used for more complex debt structures involving both short-term and long-term debt and debt refinancing (Delianedis and Geske, 2010), and also for coupon debt (Geske, 1977; Eom et al., 2004).

only as the observable part of the unobservable firm value. Also, in the CO model, the volatility of the stock is random and inversely related to the value of the firm's equity and the firm's leverage. This interpretation of the compound option model introduces a method that enables the measurement of implied firm volatility and implied market debt value from both option and equity prices. The model is consistent with MM, allows for default on the debt and bankruptcy, and can include debt refinancing.[7] To develop the CO setting, and our estimation procedure, we use the following notation

C = current market value of an individual stock call option

S = current market value of the individual firm stock

D = current implied market value of the individual firm debt

V = current implied market value of the firm's securities (debt D + equity S)

V^* = critical total market value of the firm, where $V \geq V^*$ implies $S \geq K$

M = face value of market debt (debt outstanding for the firm)

K = strike price of option

r = risk-free rates of interest to dates T_i and T_d

$\sigma_{v_{T_i}}$ = instantaneous firm return volatility at expiration T_i and volatility bucket i

$\sigma_{v_{T_d}}$ = instantaneous firm return volatility at expiration T_d and volatility bucket d

$\sigma_{s_{T_i}}$ = instantaneous stock return volatility at expiration T_i and volatility bucket i

t = current time

T_i = specific expiration date of the option in bucket i

T_d = maturity of the debt

$N_1(.)$ = univariate cumulative normal distribution function

$N_2(\ldots)$ = bivariate cumulative normal distribution function

ρ = correlation between the two option exercise opportunities at T_1 and T_2

The CO model is derived from a partial equilibrium, self-financing, arbitrage-free portfolio formed with the option, the firm, and a risk-free bond. This differs from BS, in which the

[7]See Eom, Helwege, and Huang (2004) and Toft and Prucyk (1997), which both use models that take option valuation to the firm level and are able to imply the market value of each firm's debt and can include debt refinancing.

partial equilibrium, self-financing, risk-free no-arbitrage portfolio is formed with the option, the stock, and a risk-free bond. The boundary condition for exercise of a call option ($S \geq K$) is transformed from depending on the stock price and strike price to depending on the value of the firm, V, and on V^*, a critical firm value for exercise of the stock option, where $V^* = S^* + D^*$. S^* is known and equal to the relevant option strike price K. Thus, solving for V^* is equivalent to solving for D^*. The equity call option expires at day T_i and the debt default option expires at day T_d, where $T_d > T_i$. Thus, events that are expected to happen after the call option expires can affect the current value of the option. Given the above, if the firm value is described by a relative diffusion process, Eq (1) results for pricing individual stock call options[8]

$$C = VN_2(h_1 + \sigma_{v_{T_i}}, h_2 + \sigma_{v_{T_d}}; \rho) - Me^{-r_{T_d}(T_d - t)}N_2(h_1, h_2; \rho) - Ke^{-r_{T_i}(T_i - t)}N_1(h_1), \qquad (1)$$

where

$$h_1 = \frac{\ln(\frac{V}{V^*}) + (r_{T_i} - \frac{1}{2}\sigma^2_{v_{T_i}})(T_i - t)}{\sigma_{v_{T_i}}\sqrt{T_i - t}}, \qquad (2)$$

$$h_2 = \frac{\ln(\frac{V}{M}) + (r_{T_d} - \frac{1}{2}\sigma^2_{v_{T_d}})(T_d - t)}{\sigma_{v_{T_d}}\sqrt{T_d - t}}, \qquad (3)$$

and

$$\rho = \sqrt{\frac{T_i - t}{T_d - t}}. \qquad (4)$$

2.1. Implementation overview

To implement partial equilibrium, no-arbitrage models, market prices of the securities in the no-arbitrage portfolio must be used to imply the relevant model unknowns. The partial equilibrium option models of BS, Merton, and CO are all constructed by rebalancing a portfolio containing the following three key securities: (1) for BS, the stock S, the option C,

[8] See Geske (1979) for details.

and the risk-free bond [denoted $B(r_f)$], (2) for Merton, the firm's market value V, the stock as an option, S, and the risk-free bond $B(r_f)$, and (3) for CO, the firm's market value V, the option C, and the risk-free bond $B(r_f)$. As long as these three securities in each model are held in specific market value proportions, no-arbitrage exists between them. When BS, Merton, or CO is implemented in this no-arbitrage setting, researchers use the market prices of these three securities to imply all the required unknowns for these models to produce no-arbitrage option values. Thus, for BS, the three relevant securities are used to imply one BS unknown: σ_S, the stock return volatility. For Merton, the three relevant securities are used to imply two unknowns: the market value of the firm, V, and the firm return volatility, σ_V. For CO, the three relevant securities are used to imply three unknowns: the market value of the firm, V, the firm return volatility over the life of the call option $\sigma_{v_{T_i}}$, and the future market value of the firm at the option expiration date, V^*. Once these unknowns are implied by solving one for BS, two for Merton, or three equations for CO, they are then used with the other model inputs to produce the model no-arbitrage option values for the remaining options. If we assume a different firm volatility for the default option, $\sigma_{v_{T_d}}$, than for the volatility over the call option's life, $\sigma_{v_{T_i}}$, then a fourth equation is required.

In this paper, the CO and BS models are implemented with and without a term structure of volatility (TSV). We do this because the academic researchers and market practitioners have demonstrated that the BS model performs best with a TSV.[9] Here, we describe how we estimate the models with a TSV, and in Subsection 4.1, we discuss the estimation for the no-TSV case.

V^*, the critical firm value for option exercise, depends on each option's strike price, K_j, and each option's expiration date, T_i, for all strikes j and option expirations T_i, which

[9] A TSV is known to exist in the equity option market. Implementing the CO model and BS with a volatility term structure allows their models to have a similar number of parameters relative to each other and to the more complex models. However, the more complex models of Heston (1993), Bakshi, Cao, and Chen (1997) and Pan (2002) cannot easily utilize a volatility term structure. When a TSV is used, the BS model errors compare much more favorably with the Bakshi, Cao, and Chen model, especially in mean squared error because the Bakshi, Cao, and Chen models have five to nine parameters and the authors allow BS only one parameter, the implied volatility.

can be represented as $V^*(K_j, T_i)$. All options expiring within specific periods of days to expiration are grouped in buckets T_i, and options with specific days to expiration in the T_i range are valued with a volatility implied from the relevant term structure bucket T_i for the option and T_d for the debt. Thus, all compound call options depend on four unknowns: $C[V, V^*(K_j, T_i), \sigma_{v_{T_i}}, \sigma_{v_{T_d}}]$.

However, because $V = S + D$ and S is known, and because $V^* = S^* + D^*$ and each specific S^* is known and equal to a specific strike K_j, we can restate the four unknowns for compound call values as $C[D, D^*, \sigma_{v_{T_i}}, \sigma_{v_{T_d}}]$. At each specific option expiration T_i, if $V < V^*$, then $C = 0$, and if $V > V^*$, then $C = S - K$. The firm's implied volatility term structure is characterized as the relevant volatility to each option expiration date T_i, $\sigma_{v_{T_i}}$. The volatility to the single debt maturity date for each firm, T_d, is $\sigma_{v_{T_d}}$. The face value of a firm's debt outstanding is M, and T_d is the maturity of this debt.[10] The BS model has a similar stock volatility term structure, which for each expiration bucket T_i is represented as $\sigma_{s_{T_i}}$.

As specified, the events of exercising or not exercising the call option and the firm defaulting or not defaulting on debt are two correlated exercise opportunities at specific T_i for each call option expiration and at T_d for the debt default option. This correlation, measured by $\rho = \sqrt{\frac{T_i - t}{T_d - t}}$, where individual stock option expiration T_i is less than or equal to debt maturity, T_d, also affects stock option values.[11] Equity options with longer expirations or stocks with shorter debt maturities have T_i closer to T_d, which increases the correlation between the stock option and the default option, and this increased correlation reduces the pricing error of the CO model. These effects are in fact present in our analysis of the CO model pricing improvement for options with longer expirations. But this is not the only effect of

[10] There is nothing unusual in our implementation of BS, Merton (1974), and CO. We follow the standard practice from Merton (1974), implementing corporate debt as a zero coupon bond maturing at the duration of all promised payments outstanding, and Rubinstein (1985), implementing BS and CO. In this specification of the CO model with no coupons, debt is a zero coupon bond, so the implied market value of debt, D, can never exceed the debt face value, M. We check that this is always satisfied empirically.

[11] We find individual firm balance sheet duration always far exceeds listed and traded option expirations. No such case was found when $T_i > T_d$ in our sample of 16 years and more than two million option prices.

having expected events beyond the date the equity call option expires, which can affect the current value of the call. If the probability of default on the firm at day T_d increases, this can affect the current price of the equity call option.

To solve for four unknowns, $V = S + D$, V^*, $\sigma_{v_{T_i}}$, and $\sigma_{v_{T_d}}$, we utilize four equations: three versions of Eq. (1) for three call options and the Merton (1974) adapted Black-Scholes equation for stock as an option on the firm assets V, which is

$$S = VN(d_1) - Me^{-r_{T_d}}N(d_2) = (S + D)N(d_1) - Me^{-r_{T_d}}N(d_2), \tag{5}$$

where

$$d_1 = \frac{\ln(\frac{V}{M}) + (r_{T_d} + \frac{1}{2}\sigma^2_{v_{T_d}})T_d}{\sigma_{v_{T_d}}\sqrt{T_d}}, \tag{6}$$

and

$$d_2 = d_1 - \sigma_{v_{T_d}}\sqrt{T_d}. \tag{7}$$

Eq. (5) does not depend on V^* or $\sigma_{v_{T_i}}$, but it does depend on V and $\sigma_{v_{T_d}}$. However, Eq. (5) can be used with two other equations for CO option prices to simultaneously solve for the CO model's three unknowns, V, σ_V, and V^*.

To recognize the Black-Scholes model as a special case of the compound option model, consider that if the firm has no debt ($M = 0$, $V = S$, $\sigma_v = \sigma_s$), then Eq. (1) reduces to the following Black-Scholes equation for stock call options.

$$C = SN(d_1) - Ke^{-r_{T_i}}N(d_2), \tag{8}$$

where

$$d_1 = \frac{\ln(\frac{S}{K}) + (r_{T_i} + \frac{1}{2}\sigma^2_{s_{T_i}})T_i}{\sigma_{s_{T_i}}\sqrt{T_i}}, \tag{9}$$

and

$$d_2 = d_1 - \sigma_{s_{T_i}}\sqrt{T_i}. \tag{10}$$

By comparing the CO model with the BS model, we can directly examine the effects of adding debt to option values.

2.2. Implementation details

We now more explicitly describe how we solve for the BS and CO option values. In implementing these models with a TSV, we use different time to expiration buckets (T_i), which are defined in Subsection 3.5.

For BS, at each time, we have one equation and one unknown, which is $\sigma_{s_{T_i}}$. We use Eq. (8) and solve for the unknown implied volatility, $\sigma_{s_{T_i}}$, from the most at-the-money (ATM) option out of options expiring in the time bucket T_i. We produce a TSV of stock return volatility for each stock on each day. We use this implied $\sigma_{s_{T_i}}$ to value options expiring in the time bucket T_i, with different strike prices, K_j.

For CO values, at each time, we solve four equations for four unknowns, $V = S + D$, V^*, $\sigma_{v_{T_i}}$, and $\sigma_{v_{T_d}}$. We use Eq. (1) for the three closest to ATM options (one as close to ATM as possible, one the least OTM, and one the least ITM). We also use Eq. (5) for the stock as an option. What follows is a detailed description of our solution procedure.

In our implementation of CO, the firm debt is modeled as a zero coupon bond with time to expiration equal to the imputed duration of the firm's debt. At any time t, the current implied market value of firm's risky debt, D, is less than the risk-free present value of firm debt or implied $D < Me^{-rT_d}$.[12] So we set our initial guess for the current implied market value of the risky debt as $D = Me^{-rT_d}$. This initial guess for D gives a current firm value as $V = S + Me^{-rT_d}$. Furthermore, the volatility of a firm with leverage must be less than the volatility of the firm's equity, so we use the BS implied stock volatility as an upper bound for the firm volatility. With these two initial upper bounds for $V = S + D$ and σ_v, we can solve Eq. (5) for the initial guess for $\sigma_{v_{T_d}}$, the longest term firm volatility in the firm's volatility

[12] This implementation of Merton, where debt is a zero coupon bond, is similar to numerous other published papers such as Rubinstein (1985) and Eom, Helwege, and Huang (2004).

term structure. Given the above, Eq. (5) can be solved in a manner similar to BS for implied volatility, but in this case for the firm volatility, $\sigma_{v_{T_d}}$.

The initial guesses for V and $\sigma_{v_{T_d}}$ give two of the four unknown starting values we need for the solution to the four equations. We use our four equations with four market prices, C_1, C_2, C_3, the prices of the nearest ATM options with three known strike prices, K_1, K_2, K_3, and the stock price S and we iterate to solve for our four unknowns, D, D^*, $\sigma_{v_{T_i}}$, and $\sigma_{v_{T_d}}$ using the Matlab optimizer. We know that our solution each day must keep known S fixed, $D < Me^{-rT_d}$, and firm volatility $\sigma_{v_{T_i}} < \sigma_{s_{T_i}}$. We iterate D, D^*, $\sigma_{v_{T_i}}$, and $\sigma_{v_{T_d}}$ subject to these constraints. Once we solve for the four unknowns, we use them to price all the other options in the time bucket T_i. Then we move to the next time bucket, T_j, and use a similar procedure but keeping the firm value V and firm volatility for T_d, $\sigma_{v_{T_d}}$, the same. At any time t in each time bucket, we have the same firm value V, debt value D, and long-term firm volatility $\sigma_{v_{T_d}}$, but we have different values of D^*, $\sigma_{v_{T_i}}$, for CO and different $\sigma_{s_{T_i}}$ for BS.[13]

3. Data collection and variable construction for CO and BS models

This section describes the data we collected and how we used the data. Subsection 3.1 deals with nuances of our option data. Subsection 3.2 describes the stock dividend data and how we used this data to deal with potential early exercise problems. Subsection 3.3 describes the balance sheet data we collected and how we used this data to characterize each firm's debt as a zero coupon bond. Subsection 3.4 describes the bond data we obtained and how we used this data to construct the relevant interest rates for each option expiration date. Subsection 3.5 describes how we characterized the option expiration data and moneyness data in order to examine pricing errors in each separate categories of time to expiration and

[13] We have also used this solution procedure with the previous day's option and stock prices to predict the next day's option prices, and the results do not change materially.

moneyness. Subsection 3.6 describes and compares the averages of our data sets using the methods described for each of our relevant categories.

3.1. Option data

We collect our option data from the Security file, the Security Price file, and the Option Price file from Ivy DB OptionMetrics. Our data span January 1996 through December 2011, 16 years (192 months) in total.

From the Security file, we obtain the *Security ID*, which is unique over the security's lifetime and is not recycled; *CUSIP*; *Index Flag*, which indicates whether the security is an index or not; and *Exchange Designator*, which indicates the current primary exchange for the security. We choose all securities that are individual stock equities and exclude all indices. We further select only the securities that are actively traded on the major exchanges. We also collect the information of the constituents of S&P 500 from the Wharton Research Data Services (WRDS)'s Compustat North America database.

From the Security Price file, we obtain the *Security ID* and *Close Price* for each date. We select security price–date records when there is trading in the underlying stock. From the Option Price file, we obtain the *Security ID*; *Strike Price*; *Expiration Date* of the option; *Call/Put Flag*; *Best Bid*, which is the best, or highest, closing bid price across all exchanges on which the option trades; *Best Offer*, which is the best, or lowest, closing ask price across all exchanges on which the option trades; *Last Trade Date*, which is the date on which the option last traded; *Volume*, which is the total volume for the option; and *Open Interest* for each date.[14] We filter the option price records based on the following rule: the date on which the option last traded is not missing, open interest is positive, bid price is positive and is strictly smaller than offer price, and the total volume of option contracts is positive.

We merge the selected data sets from the Security Price file and the Option Price file, and

[14]OptionMetrics did not lag the open interest by one day prior to November 28, 2000 but did lag the open interest by one day after November 28, 2000. We match open interest with the day it occurs.

we further merge the newly generated data set with the selected data set from the Security file. To minimize nonsynchronous trading problems, we retain option dates when both the security and the option were traded.

3.2. Dividends

The dividend information is obtained from the Center for Research in Security Prices (CRSP). We collect the following data: *CUSIP*; *Closing Price* to cross-check with the security price from OptionMetrics; *Declaration Date*, which is the date on which the board of directors declares a distribution; *Record Date* on which the stockholder must be registered as holder of record on the stock transfer records of the company to receive a particular distribution directly from the company; and *Payment Date*, which is the date upon which dividend checks are mailed or other distributions are made.

Because it is never optimal to exercise an American call option on a non-dividend-paying stock before the expiration date, it is convenient to work with options when no dividend is paid prior to expiration. To do this, all stocks in the sample are separated into two groups: The first group of stocks never pays any dividend between January 1996 and December 2011; the second group of stocks pays dividends in that period at least once. For the first group (473 firms in total), we use all the options written on these stocks in the entire sample period. For the second group (1,283 firms in total), we use all options that have no dividend payment during the remaining time to expiration. Thus, the call options we retain are American options but can be treated as quasi-European call options because these stocks do not pay dividends during the remaining time to expiration of the options. Finally, we check to see if arbitrage bounds are violated ($C \leq S - Ke^{-r_T T}$) and eliminate these option price records.

3.3. Balance sheet information

We use Compustat data to estimate the face value as well as the duration of debt. We match the Compustat Annual database from January 1996 to December 2011, by eight-digit CUSIP (Committee on Uniform Security Identification Procedures), with Ivy DB OptionMetrics. The balance sheet information we collect from Compustat is the book debt outstanding as categorized in maturity buckets. We impute different maturities to debt as follows. The debt to be matured in one year is defined as the total current liabilities (LCT), debt due in one year not included in current liabilities ($DD1$), accrued expense and deferred income ($AEDI$), deferred charges (DC), deferred federal tax ($TXDFED$), deferred foreign tax ($TXDFO$), deferred state tax ($TXDS$), and notes payable (NP). The total long-term debt is $DLTT$. Debt maturing in the second year is $DD2$; in the third year, $DD3$; in the fourth year, $DD4$; and in the fifth year, $DD5$. The debt imputed to be due in the seventh year is the capitalized lease obligation ($DCLO$), the debt of the consolidated subsidiary (DCS), the finance subsidiary (DFS), mortgage debt and other secured debt (DM), notes debt (DN) and other liabilities (LO), debentures (DD), contingent liabilities (CLG), long-term debt tied to the prime rate ($DLTP$), and all reported debt with maturity longer than five years ($DLTT - DD2 - DD3 - DD4 - DD5$).[15] In addition, we delete firms whose convertible debt ($DCVT$) is more than 3% of total assets (AT) or finance subsidiary (DFS) is 5% of total assets. This structure of debt outstanding permits the daily computation of the duration of debt and the amount due at the duration date.[16]

To make sure that the key debt information is not missing from the Compustat data, we check $DLTT$ and $DD1$ to $DD5$. If all of the six data items are missing, then we do not include this company's record. If only some of the data items are missing while others have positive values, then we set the missing items to zero and keep this company's record. We

[15] While leases are not an off–balance sheet item, if an off–balance sheet item can cause default, such as swaps not declared as hedges, then it should be included in our measure of debt and could impact the measure of both M and T_d, as well as the default risk. That it is not can cause some small errors in our measurement of debt.

[16] We use the standard Macaulay (1938) measure of duration with the interest rate computations described in Subsection 3.4.

exclude all utility, financial, and nonprofit firms.

3.4. Interest rate and discount rate

No-arbitrage option models require risk-free interest rates. The riskless interest rates appropriate to each option expiration and bond maturity were estimated by interpolating the effective market yields from two U.S. Treasury securities which span the relevant option expiration and debt maturity using six month, one, two, three, five, and ten year constant maturity bills and bonds from the Federal Reserve data for government securities. The interest rate for a particular maturity is computed by linearly interpolating between the two continuous rates whose maturities straddle the appropriate option expiration dates.

3.5. Characteristics of volatility terms and moneyness

We divide the option data into several categories according to either time to option expiration or moneyness. Five ranges of time to expiration are classified

1. Very near term (21 to 40 days)
2. Near term (41 to 60 days)
3. Middle term (61 to 110 days)
4. Far term (111 to 170 days)
5. Very far term (171 to 365 days)

Options with fewer than 21 days to expiration and more than 365 days to expiration are omitted.[17] The five ranges of option maturity classification define our firm and stock volatility buckets, and they are set such that the numbers of options in each category are relatively even.

[17] Rubinstein (1985) and others have used this classification for moneyness and time to expiration.

The ratio of the strike price to the current stock price is defined as the moneyness measure. The option contracts are classified into seven moneyness ranges

1. Very deep in-the-money (0.40 to 0.75)
2. Deep in-the-money (0.75 to 0.85)
3. In-the-money (0.85 to 0.95)
4. At-the-money (0.95 to 1.05)
5. Out-of-the-money (1.05 to 1.15)
6. Deep out-of-the-money (1.15 to 1.25)
7. Very deep out-of-the-money (1.25 to 2.50)

Again, we follow standard practice, e.g., Rubinstein (1985), and omit options with a ratio less than 0.40 or larger than 2.5 because their light trading frequency and possible nonsynchronicity of trading. We also delete options written on the stocks whose prices are equal or below $5.

3.6. Summary statistics

Table 1 describes the sample properties of the eligible individual stock call option prices. We report summary statistics for the bid-ask midpoint, the bid-ask spread (i.e., the ask price minus the bid-ask midpoint), the trading volume, and the total number of options, for all categories partitioned by moneyness and term of expiration. There are a total of 2,084,312 call option observations. ITM consists of 31.7% of the sample, ATM moneyness takes up 21.2% of the total sample, and OTM is 47.1% of the sample. The fact that there are more OTM than ITM stock call options could represent the practice of covered call writing using OTM options. The ATM moneyness sample is the smallest by definition as there is only one class of ATM, and three classes of ITM and OTM options.

Among other things, we observe as expected that very deep in-the-money options are the most expensive with the average price across all times to expiration around \$19.60 and that very deep out-of-the-money are the least expensive with the average price across all days to expiration around \$1.53. The average price of ATM options is \$3.08.

The very near term ATM options have the highest average per day trading volume of 731.55 in contracts (on one hundred shares). Across all times to expiration, the ATM options have the highest average per day trading volume of 724.55. ITM options' average per day trading volumes range from 149.00 to 255.13, and OTM options' average per day trading volumes vary from 449.02 to 654.28. The options that are deeper in the money and have longer time to expiration are more expensive and, thus, they have less than the average per day trading volume, which is consistent with both previous research and economic intuition.

4. Comparison with the Black-Scholes model

In this section, we consider a graph that illustrates the method of comparison between BS and CO models. We then present tables showing both the pricing errors of BS and CO and the pricing error improvement of CO over BS with respect to moneyness and time to expiration by calendar year and by leverage.

4.1. Model pricing error comparison

Fig. 1 is a representative graph of individual stock call option market prices (the dark line), BS model values (the dashed line), the CO model values (the lighter line), and moneyness, K/S. The BS model undervalues most ITM call options (low K/S) and overvalues most OTM call options (high K/S).[18] The figure also shows that the CO model has the

[18] Fig. 1 represents the most ubiquitous result. There are 15 different model distance comparisons: both over market, both under, one over while the other is under, one equal to the market while the other is either over or under, both equal to each other but either over or under, both equal to each other and equal to the market, and multiple cases for each situation in which the models are not equal to each other.

potential to improve or even eliminate these BS valuation errors because of the strike price leverage effect. The strike price creates a negative relation between the individual stock price relative to the strike price leverage (D/S). Thus, the CO model produces option values that are greater (less) than the BS values for ITM (OTM) individual stock call options.

We calculate the percentage pricing error of BS as $\frac{ABS(BS-Market)}{Market}$, the percentage pricing error of CO as $\frac{ABS(CO-Market)}{Market}$, and the improvement of the compound option model over Black-Scholes as $\frac{ABS(BS-Market)-ABS(CO-Market)}{ABS(BS-Market)}$. We present this analysis for all matched pairs of options for a variety of categories with different times to expiration and moneyness.

We also analyze the comparison with and without a term structure of volatility, to delineate the role of a TSV in influencing the efficacy of pricing. The BS model with a TSV is implemented by estimating the implied stock volatility from an at-the-money option for each option expiration period and then using this volatility to price all the options of the same option expiration period on the same day. The CO model with a TSV is implemented by first estimating the implied firm value V, the boundary condition firm value V^*, the firm volatility $\sigma_{V_{T_d}}$ at expiration T_d, the firm volatility $\sigma_{V_{T_i}}$ at expiration T_i from three option prices, and the stock price (for details, see Section2) for the closest option expiration term. For all the other option expiration terms, we keep V and $\sigma_{V_{T_d}}$ the same, but we recalibrate V^* and $\sigma_{V_{T_i}}$ from the two most at-the-money option prices for that option expiration term, and then we use all four parameters to price all the other options of the same option expiration term on the same day. The BS model without a TSV is implemented by estimating the implied stock volatility σ_S, which minimizes the sum of squared errors among all the option prices on the preceding day and then using this volatility to price all the options on the next day. The CO model without a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, and the firm volatility σ_V, which minimize the sum of squared errors among all the option prices on the preceding day, and then using the three parameters to price all the options on the next day.

4.2. Pricing error by calendar year, leverage, expiration, and moneyness

The pricing errors for both models are presented by percent error and by absolute percent error improvement (as defined in Subsection 4.1) of CO over BS in Tables 2 and 3. Tables 4–6 present dollar pricing error. We summarize the most important tabulated results, which are an analysis of CO and BS pricing errors presented in Table 2 for ITM options and Table 3 for OTM options, analyzed with respect to categorized data ranges for both leverage and time to expiration. Tables 2–6 represent the analysis with a TSV, and Tables 7–9 represent the same analysis without a TSV. The average of the total (over all option matched pairs) absolute pricing error improvement of CO over BS with a TSV is 20.0% for ITM call options presented in Table 2, Panel F, and is 19.7% for OTM options shown in Table 3, Panel F.

However, more important for this paper is how the errors relate to firms with different leverages and different times to expiration of the call option on individual stocks. Leverage is defined as the current implied market value of debt, D, divided by the current market value of the equity, S. Leverage and time to the debt default option relative to the equity call option expiration are the key variables of the CO model omitted by the BS model. The existence and importance of leverage arise from each firm's default option strike price relative to the current stock price. To see that the resultant leverage from this default strike price has an effect on the price of an equity call option, recall that the CO model's differences from the BS model are more economically significant the greater the leverage, which works through the debt strike price (i.e., debt face value), the current firm market value and stock market value relative to the debt strike price, and the model's implied stock volatility.

For option time to expiration, T_i, relative to the debt default option expiration, T_d, the longer the time to expiration of the call option, the more time for the leverage to have an option price effect. Also, the inclusion in equity option pricing of many future years of potential firm events beyond the expiration of the equity call at T_i out to the expiration of the stock default option at T_d (e.g., seven years), such as changes in the firm leverage, default probability, and both firm and equity risk, can be expected to change the current price of an equity call option. Further, the bivariate correlation between the exercise of the call option and the exercise of the stock default option is determined by the ratio of the square root of time to call option expiration divided by the time to the default option expiration ($\rho = \sqrt{T_i/T_d}$). On any day for any given firm, the time T_d to the stock default option expiration is fixed and the times to various traded call option expirations, T_i, vary. So on any given day, this correlation increases as the time to the individual call option expiration increases.

4.2.1. Comparisons of BS and CO

In Panel F of Tables 2 and 3, we present the pricing error analysis with respect to firm leverage and option days to expiration. For both ITM options in Table 2 and OTM options in Table 3, the absolute percent pricing error improvement of CO over BS becomes economically and statistically more significant and increases monotonically with respect to increases in both the amount of leverage and the days to call option expiration.[19] This result confirms that the capital structure of an individual firm is relevant to pricing equity call options on the firm's stock.

In Table 2, Panel F, for ITM options, the total average percent pricing error improvement for different leverage categories ranges monotonically from 13.0% up to 56.0% for the smallest leverage category of 10%–20% up to the largest leverage category of 151%–200%. Also, in Panel F, for different days to option expiration, the total average pricing error improvement increases monotonically from 15.1% up to 34.5% for options expiring in the nearest expiration bucket of 21–40 days up to options expiring in the farthest expiration bucket of 171–365 days. Furthermore, the strength of this monotonicity is represented by the fact that it holds not just on average across all leverages and times to call option expiration, but also on each component of the average. Thus, holding each leverage category constant, the improvement is monotonic across each days to expiration category. As the CO model predicts, the largest

[19]The statistical significance is discussed in Subsection 4.3.

improvement of CO over BS should occur in the highest leverage category (151%–200%) and the longest days to expiration category (171–365 days), which it does at 67%, and the smallest improvement of 8.6% should and does occur in the smallest leverage category (10%–20%) and the shortest days to expiration category (21–40 days).[20]

Table 3, Panel F, for OTM options, shows results similar to ITM options in Table 2, Panel F. The total average percent pricing error improvement for different leverage categories ranges monotonically from 14.8% up to 26.7% for the smallest leverage category of 10%–20% up to the largest leverage category of 151%–200%. Also, Panel F shows that for different days to option expiration the total average pricing error improvement increases monotonically from 12.6% for options expiring in the nearest expiration bucket of 21–40 days, up to 31.7% for options expiring in the farthest expiration bucket of 171–365 days. Again the strength of this monotonicity is represented by the fact that holding each option days to option expiration category constant, the improvement is monotonic across leverage categories, and holding each leverage category constant, the improvement is monotonic across each days to expiration category. The largest OTM improvement of CO over BS is 47.9%–50.7%, which occurs in the longest days to expiration category (171–365 days) and the two highest leverage categories (101%–150% and 151%–200%), and the smallest improvement of 8.7% is in the smallest leverage category (10%–20%) and the shortest days to expiration category (21–40 days). This monotonicity and range of absolute pricing error improvement is up to 50%-70% for OTM and ITM options.

Tables 2 and 3 illustrate the usual empirical result that the percent pricing error is much smaller for the ITM options than for the OTM options. While it could be obvious that ITM options have smaller pricing errors because they behave more like stock than OTM options, the CO improvement over the BS model for ITM options can still be very large (i.e., 67%) because the stock itself is an option. Table 2 Panel B, for ITM options, shows that the BS percent pricing errors across all categories range from 2.24% to 4.70%, and the CO percent

[20] This diminished improvement in lower leverage categories simply reflects that the CO model converges to BS whenever the strike price of the default option approaches zero, either because the dollars of debt owed goes to zero or the time of debt repayment goes to infinity.

pricing errors in Panel D range from 1.11% to 2.39%. The total percent pricing errors for ITM options increase monotonically with respect to time to expiration for the BS model, shown in Table 2 Panel B, and this same comparison for the CO model, shown in Panel D, shows these errors tend to decrease monotonically. Table 3, Panel B, for OTM options, shows that the BS percent pricing errors range from 20.95% to 33.84%, and the CO percent pricing errors in Panel D range from 13.49% to 30.03%. Also, Table 2 Panel B, shows for ITM that the BS total percent pricing errors are monotonically increasing in both leverage and time to option expiration, and Panel D shows that the CO total percent pricing errors tend to decrease in both time to option expiration and leverage. These results are similar for Table 3, Panels B and D, for OTM options.

Table 2 and Table 3, Panel A, also suggest that the improvements of the CO over the BS model is greater for both ITM and OTM call options in the years following the Long-Term Capital Management crisis (1999–2000) and the years of the high-tech bubble collapse (2000–2001). Furthermore, the improvement around the recent Great Recession appears greater for ITM call options the years during and following the crisis (2007–2009), and for the OTM options the improvement appears larger for the years preceding the crisis (2004–2007). Because improvement of CO over BS is leverage-related, this suggests that expected impending leverage changes are more important to OTM options and current existing leverage is more important to ITM options. This is also related to the fact that the ITM options behave much more like stock than the OTM options.

Table 4 presents a univariate sort of the average dollar pricing error of BS and CO models with respect to days to expiration and moneyness, respectively, for both ITM and OTM call options. Panel A is the pricing error by days to expiration, and Panel B is the pricing error by moneyness. Both Panels A and B confirm the result represented in Fig. 1 that both models underprice ITM options and overprice the OTM options. Panel A demonstrates for ITM and OTM options that while both model dollar errors increase with respect to days to expiration, the BS dollar errors are larger and increase more in every case. Panel B demonstrates that while both models' dollar errors increase with in and out of the moneyness, the BS errors are again larger in each case.

Table 5 and 6 delve deeper into the Table 4 dollar error results by adding leverage to both the days to expiration results presented in Table 4, Panel A, and to the moneyness results presented in Panel B. Table 5 presents a double sort result of the dollar pricing error of BS and CO first by leverage and then by days to expiration, for both ITM and OTM call options. While both models' dollar errors increase with respect to days to expiration, the BS errors increase more. What is new here is that while both models' total dollar errors increase with respect to days to expiration for both ITM and OTM options, for the three higher leverage categories from 60% to 200% the CO model dollar errors decrease monotonically with respect to days to expiration for both ITM and OTM options. Also, while the CO model total errors decrease monotonically with respect to all leverage categories for both ITM and OTM options, the BS errors do not show a consistent pattern with respect to leverage, first increasing as leverage increases and then decreasing as leverage continues to increase. Furthermore, the BS dollar errors are never less than the CO errors for any leverage days to expiration category and are much larger for the higher leverage longer days to expiration category.

Table 6 presents a double sort result of the dollar pricing error of BS and CO models, first by leverage and then by moneyness, for both ITM and OTM call options. This table again shows that while both BS and CO models undervalue ITM options and overvalue OTM options, the CO model option is superior for all moneyness and leverage categories. While both models' dollar errors increase with respect to moneyness, the BS errors increase much more. What is new in Table 6 is that for the highest leverage category of 151% to 200% the CO model dollar errors decrease with respect to moneyness for both ITM and OTM options. Also, while the CO model total dollar errors decrease monotonically with respect to all leverage categories for both ITM and OTM options, the BS errors do not show a consistent pattern with respect to leverage, first increasing as leverage increases and then decreasing as leverage continues to increase. The finding that the BS model does not show a consistent pattern with increasing leverage when total errors are sorted over either days to expiration in Table 5 or moneyness in Table 6 is consistent with the fact that BS model omits leverage.

4.2.2. Comparisons without a term structure of volatility

We now compare BS and CO without a TSV. Table 7 presents a univariate sort of the average dollar pricing error of BS and CO with respect to days to expiration and moneyness, respectively, for both ITM and OTM call options (the analog of Table 4).[21] Panel A is the pricing error by days to expiration, and Panel B is the pricing error by moneyness. Both Panels A and B again confirm the result represented in Fig. 1 that both models underprice ITM options and overprice the OTM options. Panel A demonstrates for ITM and OTM options that while both model dollar errors increase with respect to days to expiration, the BS dollar errors are both larger and increase more in every case. Panel B demonstrates that while both models' errors increase with in and out of the moneyness, the BS errors are again larger in each case. Both Panels A and B show the ITM errors for CO decrease for the longest expiring options (171–365 days).

Tables 8 and 9 delve deeper into the Table 7 dollar error results (these are the analogs of Tables 5 and 6). Table 8 again presents a double sort result of the dollar pricing error of BS and CO, first by leverage and then by days to expiration, for both ITM and OTM call options. While both models' dollar errors increase with respect to days to expiration, the BS errors increase more. Here, without a TSV, the CO model does not show a decrease in pricing errors as time to option expiration increases, which it did with a TSV for both ITM and OTM options, for higher leverage categories from 60% to 200%. Also, the CO model's total errors decrease monotonically with respect to all leverage categories for only the OTM options, while before with a TSV the CO errors were decreasing monotonically for ITM

[21] For brevity, we provide only selected comparisons to the TSV case in Subsection 4.2.1. The other cases are available upon request.

options as well. The BS errors do not show a consistent pattern with respect to leverage. Furthermore, the BS dollar errors are never less than the CO errors for any leverage days to expiration category and are much larger for the higher leverage longer days to expiration category.

Table 9 presents a double sort result of the dollar pricing error of BS and CO models, first by leverage and then by moneyness, for both ITM and OTM call options. This table again shows that while both BS and CO models undervalue ITM options and overvalue OTM options, the CO model option is superior for all moneyness and leverage categories. While both models' dollar errors increase with respect to moneyness, the BS errors increase much more. The finding that the BS model (with or without a TSV) does not show a consistent pattern with increasing leverage when total errors are sorted over either days to expiration in Table 5 or moneyness in Table 6 is consistent with the fact that BS model omits leverage.

Implementing both the BS and CO models with and without a TSV allows us to quantify the pricing improvements of adding a TSV to these no-arbitrage option pricing models. Tables 6 and 9 are informative about the pricing improvement from using a TSV. Comparing BS with and without a TSV, the ITM average BS pricing errors increase by 11.3%, from -0.194 to -0.216 without a TSV, or the reverse conclusion is that if a TSV is added, the BS average ITM errors are reduced by 10.2% (from -0.216 to -0.194). A similar comparison for the CO model shows that the ITM average pricing errors across moneyness and leverage are -0.152 with a TSV, and if the TSV is not used, the errors increase to -0.180. This is either an 18.4% increase in average ITM pricing errors by not implementing the CO model with a TSV or a 15.6% decrease in average ITM pricing errors by implementing the CO model with a TSV.

For OTM options, the BS average overpricing errors across all levels of moneyness and leverage increase 24.7% without a TSV, from 0.146 to 0.182. The CO average OTM overpricing errors increase by 34.2% without a TSV, from 0.111 to 0.149 over the same categories of moneyness and leverage. While the CO pricing errors exhibit a larger percentage increase than the BS pricing errors without a TSV, the CO average OTM pricing errors are 0.149, again less than the BS average OTM errors of 0.182.[22]

We have established that individual firm's debt and its capital structure, D/S, are relevant to improving the pricing accuracy of equity call options. While all past empirical analysis focused on pricing errors with respect to moneyness and time to equity option expiration, the results presented here show that both sources of error are better understood when firm leverage is included.

Finally, while the CO model prices options better than the BS model with and without a TSV, in all instances over this diverse economic time period, the BS model performs fairly well.

4.3. Alternative testing and statistical significance

We also perform other nonparametric tests: a signed rank test, a sign test, and a Kruskal-Wallis test (for two independent samples, i.e., the Mann-Whitney U Test). All tests show similar results that the CO model improvements over BS with and without a TSV are significant at a p-value smaller than the 0.001% for all categories of leverage, days to expiration, moneyness, and calendar years.[23]

[22] In the Internet Appendix, we estimate CO and BS values for two additional cases. The first involves the case in which we use only one implied parameter, respectively equity and firm volatility, for each of BS and CO and no TSV. We use credit default swap (CDS) data to obtain discount rates for valuing debt. This requires estimation of default spreads for most firms by imposition of credit ratings on firms that have CDS quotes, and then interpolation for firms of similar ratings without CDS quotes and for firm debt durations that differ from the CDS maturities. In the second case, we imply equity volatility, σ_S, for BS and V, V^*, and σ_V for CO, without a TSV, just as in the no-TSV procedure in Subsection 4.2.2, but we assume that $\sigma_{V_{T_i}} \neq \sigma_{V_{T_d}}$, i.e., that firm volatility, which in almost all cases matures years after the current equity call options, is different from the firm volatility that is used to price shorter-horizon listed call options. In the first case, CO model performance is affected by errors induced by credit rating imputations and interpolations required to obtain discount rates, and the CO model suffers relative to BS. This exercise underlines the need to use the traded securities utilized to construct the no-arbitrage portfolios, as in BS implementations. In the second case, the errors reduce relative to the no-TSV case in the paper, and CO continues to outperform BS.

[23] These statistical tables are available upon request.

5. Useful aspects of implied firm parameters

To this point the paper has demonstrated results from the implementation of the CO model, which is consistent with the MM no-arbitrage theory of capital structure to price equity call options. The main focus of the paper is to see if the CO model can produce equity call option values that are closer to the market prices when compared with the seminal no-arbitrage BS model, that omits capital structure parameters. We have established this result in Section 4, both with and without at TSV. Here we provide a discussion about potential uses of the implied capital structure parameters from the CO model. These CO implied parameters are D, the implied market value of the firm debt; $D^*_{T_i}$, the implied threshold for equity option exercise; $\sigma_{v_{T_i}}$, the firm implied volatility to option expiration date T_i; and $\sigma_{v_{T_d}}$, the implied firm volatility to the default option expiration date T_d. In the equations that include firm value V, we substitute known S plus unknown D. We do this to ensure that the optimizer always iterates with the known stock price. Once we have the implied debt value, D, we obtain the implied debt-equity ratio (leverage), the implied firm value, and the implied total risk of the firm, σ_v. These are forward-looking implied parameters derived from liquid and efficient option, stock, and treasury bond market prices and, thus, should have the research advantages associated with forward-looking implied estimates [viz. Jackwerth (2000)]. Next consider experiments that would be done with these implied capital structure variables.

We first demonstrate how the implied leverage of a corporation is related to the level of the equity in the firm. As per the leverage effect, the two should be inversely related. The implied leverage is obtained by implying the market value of the firm debt, D, using the CO model, and then simply dividing the implied D by the observed S to obtain implied leverage of the firm. We present graphs to compare this measure of implied market leverage with the actual market price of the firm's equity monthly over our entire sample period.

Figs. 2 and 3 show the implied leverage graphed on the left vertical axis (dashed line) and the market price of the firm's equity graphed on the right vertical axis (solid line), and on the horizontal axis we present time from January 1996 to December 2011. Fig. 2 is a graph for IBM, and Fig. 3 is a graph for Apple. Both figures demonstrate an inverse relation between our implied measure of leverage and the actual market stock price over the entire sample. This would be the expected result as long as the firm does not refinance when the stock price changes.

Next we believe that if this relation is true for individual stocks, it should be true for an index of stocks that is a weighted sum of individual stocks. So, in Fig. 4, we report the implied leverage of four hundred firms from the Standard & Poor's (S&P) 500 and the market price level of the same four hundred S&P firms over our sample time period. We use only four hundred firms here in our sample because we exclude financial firms, nonprofit firms, and utilities. This figure also demonstrates that the relation between our implied measure of leverage and the market price level of this aggregate portfolio of four hundred firms are inversely related just as these variables are for individual firms.

These figures each support the notion that as the price of each firm's equity changes, if the firm does not refinance its debt, the market (not book) leverage and the firm's equity will be inversely related, and basic financial economics says the leverage and the equity's riskiness will be positively related. So, to summarize, this positive relation between market leverage and equity risk, discovered by Black (1976) and analyzed by Christie (1982), is independent of the reason or cause for the equity price change. The equity price change could have occurred because the firm's expected cash flows changed while the firm risk remained constant (a first moment shift in the numerator of discounted cash flows) or the firm risk changed while the firm's expected cash flows remained the same (a second moment shift in the denominator of discounted cash flows), or a combination of the two.[24]

The ability to back out risk at the firm level (i.e., σ_v) using efficient, no-arbitrage prices of

[24]The relation between a stock's expected return and its risk (the leverage effect) is not this paper's concern. For no-arbitrage option models, expected return is irrelevant. If the firm risk changed randomly, this could obscure but will not eliminate the strike price effect. Also, the CO model does not accommodate the case of stochastic firm volatility.

options and stocks would allow researchers to better examine interesting and novel questions. For example, consider the following: Do loan market rates reflect the total risk of a firm's assets? In legal corporate cases, should forward-looking, implied measures of firm value and firm asset risk be considered relevant? Are there regime shifts in individual firm asset risks and in total market risk? Do the firms that win a leveraged buyout competition have higher or lower asset risk than the loser? Can research better separate components of equity total risk from the firm's asset risk and its leverage? For firms in the same industry, how different are their measured asset risks and why do their asset risks differ?

6. Conclusions

This paper empirically tests the compound option approach to equity option valuation using a sample of more than two million option prices and 16 years of data. To the best of our knowledge, no previous work has used actual debt from financial statements for valuing equity call options to test the CO approach. In this no-arbitrage setting, equity is a default option on the firm's assets, and the call trading in option markets is an option on the firm's equity. This approach is consistent with the Modigliani and Miller no-arbitrage capital structure theory, takes option pricing deeper into the theory of the firm, and gives an economic role for firm debt and its resultant leverage to effect option prices.

While some can a priori believe the CO model would outperform the Black-Scholes model because of the inclusion of firm debt, this belief requires qualification. The CO model includes additional parameters: the firm's debt obligations and their payment due dates. These additional parameters must be converted to firm debt face values, M, and durations, T_d, to represent the zero coupon bond value and the maturity of the default option.[25] Despite the extra unknowns, we show that the CO model outperforms the BS model and offers interesting intuition for the improved pricing accuracy. The improvements are significant both economi-

[25] If debt payouts (coupons, sinking funds, bank debt) are not aggregated but are allowed at different times, a more complex CO model could be used (e.g., Eom, Helwege, and Huang, 2004 and Geske, 1977).

cally and statistically when pricing individual stock options and can be accomplished simply and parsimoniously because the compound option model implies an implicit endogenous instead of explicit exogenous stock stochastic volatility process, using contemporaneous, liquid market prices for the equity and options on the equity. The improvement stems from the fact that the CO model incorporates the strike price effects of leverage on asset prices in a way that is consistent with Modigliani and Miller. The model embeds a stochastic process for the stock volatility that characterizes how debt can cause the individual stock risk to change stochastically and inversely with the leverage.

We demonstrate that the CO improvements over BS are material for all strikes and all times to expiration, and we also show, as expected, that the improvements are greater, the greater the market leverage in each firm. In addition, the improvements are greater, the longer the time to expiration of the equity call option. Furthermore, we demonstrate that the option pricing improvements of CO model relative to BS hold both with and without a TSV, and we characterize how much more pricing improvement is gained by both models when a TSV is added to each model. This improvement arises through the relation between the time to expiration of the default option and the time to expiration of the equity call option. Thus, the CO model explicitly allows for events expected to occur after the call option expires to affect the current call option value.

The compound option model allows calculation of implied firm value, implied firm leverage from leverage, and implied firm volatility from option prices, which has promising applications in areas such as mergers and acquisitions, when assessing firm risk following a merger could be important. Finally, because our pricing improvements obtain for individual equity options, they should also hold for equity index options in which the market debt is simply a daily weighted average of individual firms' debt. Applying this methodology to index options is left for future research.

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Table 1:

Sample properties of individual stock options

The reported numbers are the average bid-ask midpoint price, the average per day trading volume, and the total number of options, for all categories partitioned by moneyness and days to expiration. The sample period extends from January 1996 through December 2011. S denotes the spot individual stock price, and K is the exercise price. ITM, ATM and OTM denote in-the-money, at-the-money, and out-of-the money options, respectively.

Moneyness	Days to expiration					
K/S	21–40	41–60	61–110	111–170	171–365	Subtotal
ITM	\$16.52	\$20.39	\$21.32	\$20.37	\$21.34	\$19.60
$[0.40-0.75)$	162.52	32.52	30.94	19.63	30.38	149.00
	26,980	15,578	18,649	14,488	15,884	91,579
ITM	\$8.96	\$11.66	\$12.36	\$13.22	\$14.38	\$10.78
$[0.75-0.80)$	168.01	33.59	36.02	16.83	35.80	161.91
	79,431	29,500	22,792	15,056	12,648	159,427
ITM	\$5.53	\$7.14	\$8.43	\$10.07	\$11.60	\$6.62
$[0.85-0.95)$	259.18	60.31	53.18	28.27	40.29	255.13
	257,808	71,638	38,715	23,833	18,442	410,436
ATM	\$2.52	\$3.75	\$5.62	\$8.85	\$11.52	\$3.08
$[0.95-1.05]$	731.55	94.41	78.80	42.85	43.53	724.55
	357,054	47,739	19,576	10,061	7,445	441,875
OTM	\$0.92	\$1.64	\$2.80	\$4.45	\$5.87	\$1.70
$(1.05-1.15]$	666.30	71.78	69.69	45.65	80.56	654.28
	279,427	100,615	47,496	29,607	22,045	479,190
OTM	\$0.64	\$1.13	\$1.94	\$3.13	\$4.31	\$1.62
$(1.15-1.25]$	479.06	68.03	52.54	54.86	76.32	458.32
	92,822	52,435	35,149	27,207	24,005	231,618
OTM	\$0.61	\$0.86	\$1.29	\$2.01	\$2.66	\$1.53
$(1.25-2.50]$	536.00	71.43	43.66	46.70	66.92	449.02
	53,303	46,757	55,541	53,967	60,619	270,187
Subtotal	\$3.34	\$4.44	\$5.84	\$6.59	\$7.54	\$4.41
	565.91	69.51	57.52	38.43	59.34	554.21
	1,146,825	364,262	237,918	174,219	161,088	2,084,312

Table 2:

Call in-the-money (ITM) absolute model percent pricing error improvement with a term structure of volatility (TSV)

Panels A, B, C, D, E, and F report the average percent pricing errors of Black and Scholes (BS) and compound option (CO) model valuations and the average absolute percent pricing error improvement, for all categories partitioned by term of expiration, for each calendar year and every leverage group. ITM denotes in-the-money options. The BS pricing error is defined as $\frac{ABS(BS-Market)}{Market}$. The CO pricing error is defined as $\frac{ABS(CO-Market)}{Market}$. The average absolute pricing error improvement is defined as $\frac{ABS(BS-Market)-ABS(CO-Market)}{ABS(BS-Market)}$. BS with a TSV is implemented by estimating the implied stock volatility from an at-the-money (ATM) option for each option expiration period and then using this volatility to price all the options of the same option expiration period on the same day. CO with a TSV is implemented by first estimating the implied firm value V, the boundary condition firm value V^*, the firm volatility $\sigma_{V_{T_d}}$ at expiration T_d, and the firm volatility $\sigma_{V_{T_i}}$ at expiration T_i from three option prices and the stock price (for details, see Section2) for the closest option expiration term. For all the other option expiration terms, we keep V and $\sigma_{V_{T_d}}$ the same, but we recalibrate V^* and $\sigma_{V_{T_i}}$ from the two most ATM option prices for that option expiration term, and then we use all four parameters to price all the other options of the same option expiration term on the same day. Leverage D/E is defined as the current implied market value of debt, D, divided by the current market value of equity E.

Panel A: Pricing error of BS model by calendar year

Year	21–40	41–60	61–110	111–170	171–365	Subtotal
1996	2.36	2.13	2.03	2.06	2.16	2.19
1997	2.26	2.00	1.80	1.87	1.78	2.02
1998	2.43	2.93	2.38	2.37	2.09	2.50
1999	1.97	1.91	2.14	1.77	1.80	1.96
2000	1.86	1.63	1.59	1.50	1.45	1.70
2001	2.37	2.42	2.39	2.56	2.17	2.38
2002	3.18	3.68	3.58	3.82	3.52	3.43
2003	2.63	3.18	3.13	3.34	3.50	3.00
2004	1.95	2.39	2.32	2.56	2.76	2.24
2005	1.72	1.96	2.07	2.16	2.42	1.94
2006	1.53	1.76	1.89	2.07	2.41	1.79
2007	1.88	2.24	2.36	2.74	2.65	2.23
2008	2.65	2.84	2.98	2.94	3.25	2.82
2009	2.38	2.72	2.77	2.90	3.32	2.64
2010	2.55	2.92	3.08	3.35	3.27	2.85
2011	2.93	3.27	3.32	3.18	3.00	3.11
Subtotal	2.40	2.71	2.71	2.74	2.81	2.59

Panel B: Pricing error of BS model by leverage

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	2.24	2.50	2.47	2.53	2.58	2.40
21%–30%	2.42	2.76	2.85	2.81	2.86	2.64
31%–60%	2.52	2.86	2.84	2.97	3.01	2.73
61%–100%	2.59	2.86	2.87	3.03	3.07	2.77
101%–150%	2.72	3.17	3.06	3.24	3.72	2.99
151%–200%	3.28	4.57	4.70	3.43	3.84	4.23
Subtotal	2.40	2.71	2.71	2.74	2.81	2.59

Panel C: Pricing error of CO model by calendar year

Year	21–40	41–60	61–110	111–170	171–365	Subtotal
1996	2.19	1.88	1.70	1.64	1.48	1.89
1997	2.10	1.78	1.56	1.48	1.38	1.77
1998	2.20	2.59	1.99	1.84	1.33	2.15
1999	1.72	1.58	1.74	1.21	1.00	1.59
2000	1.49	1.23	1.14	0.95	0.71	1.27
2001	1.94	1.88	1.82	1.76	1.23	1.83
2002	2.68	3.07	2.71	2.56	2.27	2.72
2003	2.31	2.74	2.54	2.48	2.45	2.47
2004	1.73	2.07	1.87	1.82	1.90	1.84
2005	1.54	1.71	1.71	1.67	1.79	1.65
2006	1.34	1.50	1.51	1.55	1.75	1.47
2007	1.66	1.93	1.91	2.18	2.05	1.87
2008	2.08	2.18	2.23	2.14	2.38	2.16
2009	1.89	2.13	2.11	2.02	2.25	2.02
2010	2.23	2.52	2.53	2.49	2.24	2.38
2011	2.55	2.84	2.77	2.43	2.07	2.62
Subtotal	2.05	2.27	2.18	2.04	1.96	2.12

Panel D: Pricing error of CO model by leverage

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	2.07	2.27	2.17	2.11	2.11	2.14
21%–30%	2.14	2.40	2.39	2.16	2.09	2.24
31%–60%	2.05	2.28	2.11	1.97	1.80	2.09
61%–100%	1.81	1.90	1.77	1.47	1.25	1.77
101%–150%	1.72	1.96	1.57	1.49	1.72	1.74
151%–200%	1.78	2.78	2.81	1.11	1.22	2.43
Subtotal	2.05	2.27	2.18	2.04	1.96	2.12

Panel E: Absolute percent pricing error improvement by calendar year

Year	21–40	41–60	61–110	111–170	171–365	Subtotal
1996	8.5	15.8	21.8	28.0	39.4	18.5
1997	9.0	15.1	18.8	25.9	26.4	16.2
1998	11.8	18.6	22.3	28.5	44.3	19.4
1999	14.5	21.6	31.9	38.9	54.0	24.9
2000	21.6	28.4	32.8	43.9	55.9	29.8
2001	20.2	25.9	28.3	37.4	51.7	27.2
2002	16.3	18.3	25.2	34.7	36.3	21.5
2003	12.7	16.0	20.3	28.8	33.0	18.7
2004	11.5	15.3	21.6	32.8	33.2	18.1
2005	11.1	14.3	21.4	27.8	30.4	17.1
2006	13.0	16.7	22.9	28.5	32.4	19.0
2007	12.2	16.1	21.1	24.4	24.8	17.4
2008	21.6	24.4	27.6	30.7	30.9	24.8
2009	20.8	22.7	25.0	32.3	34.9	23.7
2010	12.9	15.2	19.8	28.5	35.8	17.4
2011	13.5	14.3	19.1	27.0	34.7	17.2
Subtotal	15.1	18.1	22.6	29.7	34.5	20.0

Panel F: Absolute percent pricing error improvement by leverage

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	8.6	11.2	15.4	21.5	23.4	13.0
21%–30%	13.2	15.6	20.2	28.5	32.7	17.9
31%–60%	19.1	22.5	28.7	37.5	45.5	25.2
61%–100%	30.5	35.6	42.2	55.7	63.5	37.6
101%–150%	36.0	41.0	49.8	55.1	65.7	42.6
151%–200%	49.2	57.0	58.7	66.9	67.4	56.0
Subtotal	15.1	18.1	22.6	29.7	34.5	20.0

Table 3:

Call out-of-the-money (OTM) absolute model percent pricing error improvement with a term structure of volatility (TSV)

Panels A, B, C, D, E and F report the average percent pricing errors of Black and Scholes (BS) and compound option (CO) model valuations and the average absolute percent pricing error improvement, for all categories partitioned by term of expiration, for each calendar year and every leverage group. OTM denotes out-of-the-money options. The BS pricing error is defined as $\frac{ABS(BS-Market)}{Market}$. The CO pricing error is defined as $\frac{ABS(CO-Market)}{Market}$. The average absolute pricing error improvement is defined as $\frac{ABS(BS-Market)-ABS(CO-Market)}{ABS(BS-Market)}$. BS with a TSV is implemented by estimating the implied stock volatility from an at-the-money (ATM) option for each option expiration period and then using this volatility to price all the options of the same option expiration period on the same day. CO with a TSV is implemented by first estimating the implied firm value V, the boundary condition firm value V^*, the firm volatility $\sigma_{V_{T_d}}$ at expiration T_d, and the firm volatility $\sigma_{V_{T_i}}$ at expiration T_i from three option prices and the stock price (for details, see Section2) for the closest option expiration term. For all the other option expiration terms, we keep V and $\sigma_{V_{T_d}}$ the same, but we recalibrate V^* and $\sigma_{V_{T_i}}$ from the two most ATM option prices for that option expiration term, and then we use all four parameters to price all the other options of the same option expiration term on the same day. Leverage D/E is defined as the current implied market value of debt, D, divided by the current market value of equity E.

Panel A: Pricing error of BS model by calendar year

Year	21–40	41–60	61–110	111–170	171–365	Subtotal
1996	23.66	17.71	17.58	10.54	9.72	17.05
1997	21.97	14.31	13.25	9.55	7.45	14.72
1998	23.14	22.47	14.70	11.79	9.60	18.33
1999	19.45	13.80	39.88	8.19	7.84	19.66
2000	18.08	12.74	12.90	6.47	5.45	13.21
2001	20.25	18.24	19.79	20.60	16.49	19.10
2002	34.62	34.92	36.22	34.00	33.84	34.74
2003	30.46	29.05	29.73	21.10	22.17	27.08
2004	24.17	20.78	19.61	17.93	16.63	20.34
2005	23.80	19.75	19.45	16.92	15.83	19.87
2006	19.91	18.51	18.50	19.44	18.42	19.04
2007	24.42	21.12	23.99	26.58	26.78	24.21
2008	34.29	35.34	40.02	33.32	32.99	35.33
2009	29.92	29.46	32.94	32.21	39.82	31.72
2010	30.55	29.08	30.35	26.55	30.03	29.60
2011	37.93	37.40	38.32	25.51	20.25	34.85
Subtotal	29.74	28.67	30.12	24.16	24.83	28.23

Panel B: Pricing error of BS model by leverage

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11-20	27.45	26.63	26.79	20.95	21.69	25.54
21-30	29.80	29.31	30.50	23.60	23.96	28.32
31-60	31.09	29.98	31.31	28.76	29.08	30.28
61-100	33.84	30.76	33.01	26.32	26.34	31.17
101-150	33.77	29.44	29.37	22.66	31.80	30.51
151-200	30.37	35.10	25.57	26.41	29.05	30.10
Subtotal	29.74	28.67	30.12	24.16	24.83	28.23

Panel C: Pricing error of CO model by calendar year

Year	21–40	41–60	61–110	111–170	171–365	Subtotal
1996	24.43	17.81	17.83	8.42	7.03	16.62
1997	22.58	13.92	12.63	6.86	4.72	13.92
1998	23.13	21.38	12.98	8.98	5.59	16.92
1999	19.37	12.50	35.32	5.86	5.44	17.94
2000	17.77	11.36	10.17	3.94	3.03	11.72
2001	17.94	14.98	15.49	15.55	11.24	15.27
2002	30.33	30.60	30.65	27.36	26.18	29.19
2003	27.60	25.39	24.96	16.40	16.45	22.90
2004	22.40	17.60	15.89	12.79	10.82	16.69
2005	23.78	16.98	15.17	11.83	10.99	16.97
2006	18.27	14.99	14.08	14.12	12.21	15.28
2007	22.55	17.83	18.97	19.16	19.18	19.75
2008	30.35	31.56	35.11	28.27	27.41	30.94
2009	25.51	24.34	25.88	24.66	29.20	25.57
2010	27.68	24.38	23.94	18.49	20.04	24.21
2011	34.09	32.44	32.43	20.79	14.93	30.05
Subtotal	26.96	24.73	25.05	18.59	18.13	23.88

Panel D: Pricing error of CO model by leverage

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11-20	25.86	24.38	23.78	17.41	17.37	22.91
21-30	27.21	25.55	25.33	18.04	17.62	24.10
31-60	27.36	25.02	25.25	21.62	20.71	24.80
61-100	29.10	23.50	24.71	17.13	15.66	23.95
101-150	29.04	22.35	19.93	13.49	15.39	22.69
151-200	26.61	30.03	15.42	14.55	13.32	22.25
Subtotal	26.96	24.73	25.05	18.59	18.13	23.88

Panel E: Absolute percent pricing error improvement by calendar year

Year	21–40	41–60	61–110	111–170	171–365	Subtotal
1996	0.5	8.1	11.0	28.8	34.7	13.7
1997	1.0	10.8	14.6	29.8	35.9	14.9
1998	4.9	10.5	18.5	27.8	40.3	15.8
1999	7.1	18.1	24.1	33.5	34.9	18.8
2000	10.1	21.9	27.3	40.8	42.9	23.0
2001	17.7	22.7	24.7	29.5	40.6	26.0
2002	14.5	13.5	17.5	19.9	24.1	17.5
2003	13.2	16.4	20.7	24.9	26.0	19.3
2004	12.3	19.8	24.9	31.9	37.3	23.6
2005	9.3	19.8	29.9	34.6	35.7	23.1
2006	14.0	24.4	29.2	34.0	37.0	25.4
2007	12.3	20.5	25.7	29.9	31.6	22.2
2008	15.2	15.7	19.5	23.7	25.0	18.3
2009	16.2	17.6	22.5	25.4	26.2	19.9
2010	13.5	18.3	23.0	30.8	34.5	20.8
2011	12.1	15.3	18.5	24.6	31.6	17.4
Subtotal	12.6	17.2	21.5	28.0	31.7	19.7

Panel F: Absolute percent pricing error improvement by leverage

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11-20	8.7	12.1	16.1	22.8	24.6	14.8
21-30	12.3	17.0	21.0	27.4	30.9	19.4
31-60	15.9	20.3	25.4	31.3	36.0	23.2
61-100	17.8	26.4	31.9	43.2	49.6	29.0
101-150	17.8	25.6	30.1	37.3	50.7	27.3
151-200	15.2	19.3	34.8	37.0	47.9	26.7
Subtotal	12.6	17.2	21.5	28.0	31.7	19.7

Table 4:

Pricing error by days to expiration and moneyness with a term structure of volatility (TSV)

The reported numbers are the average pricing error defined as the market price minus the model value, for all categories partitioned by time to expiration and moneyness. ITM and OTM denote in-the-money and out-of-the money options, respectively. Black and Scholes (BS) with a TSV is implemented by estimating the implied stock volatility from an at-the-money (ATM) option for each option expiration period, and then using this volatility to price all the options of the same option expiration period on the same day. Compound option (CO) with a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, the firm volatility $\sigma_{V_{T_d}}$ at expiration T_d, and the firm volatility $\sigma_{V_{T_i}}$ at expiration T_i from three option prices and the stock price (for details, see Section2) for the closest option expiration term. For all the other option expiration terms, we keep V and $\sigma_{V_{T_d}}$ the same, but we recalibrate V^* and $\sigma_{V_{T_i}}$ from the two most ATM option prices for that option expiration term, and then we use all four parameters to price all the other options of the same option expiration term on the same day.

Panel A: Pricing error by days to expiration

	21–40	41–60	61–110	111–170	171–365	Subtotal
ITM BS error	−0.144	−0.193	−0.226	−0.261	−0.304	−0.194
ITM CO error	−0.121	−0.159	−0.176	−0.185	−0.202	−0.152
OTM BS error	0.056	0.118	0.163	0.232	0.314	0.146
OTM CO error	0.041	0.097	0.129	0.173	0.223	0.111

Panel A: Pricing error by moneyness

	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
ITM BS error	−0.242	−0.251	−0.231	−0.199	−0.159	−0.194
ITM CO error	−0.169	−0.180	−0.172	−0.158	−0.131	−0.152
OTM BS error	0.113	0.134	0.169	0.197	0.190	0.146
OTM CO error	0.095	0.105	0.123	0.138	0.130	0.111

Table 5:

Pricing error by leverage and days to expiration with a term structure of volatility (TSV)

The reported numbers are the average pricing error defined as the market price minus the model value, for all categories partitioned by leverage and time to expiration jointly. ITM and OTM denote in-the-money and out-of-the money call options, respectively. Black and Scholes (BS) with a TSV is implemented by estimating the implied stock volatility from an at-the-money (ATM) option for each option expiration period, and then using this volatility to price all the options of the same option expiration period on the same day. Compound option (CO) with a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, the firm volatility $\sigma_{V_{T_d}}$ at expiration T_d, and the firm volatility $\sigma_{V_{T_i}}$ at expiration T_i from three option prices and the stock price (for details, see Section2) for the closest option expiration term. For all the other option expiration terms, we keep V and $\sigma_{V_{T_d}}$ the same, but we recalibrate V^* and $\sigma_{V_{T_i}}$ from the two most ATM option prices for that option expiration term, and then we use all four parameters to price all the other options of the same option expiration term on the same day. Leverage D/E is defined as the current implied market value of debt, D, divided by the current market value of equity E.

Panel A: ITM BS pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	−0.146	−0.197	−0.215	−0.223	−0.248	−0.187
21%–30%	−0.146	−0.197	−0.221	−0.241	−0.290	−0.191
31%–60%	−0.142	−0.195	−0.265	−0.362	−0.433	−0.217
61%–100%	−0.133	−0.177	−0.205	−0.242	−0.282	−0.173
101%–150%	−0.134	−0.155	−0.158	−0.195	−0.240	−0.154
151%–200%	−0.165	−0.074	−0.118	−0.149	−0.167	−0.137
Subtotal	−0.144	−0.193	−0.226	−0.261	−0.304	−0.194

Panel B: ITM CO pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	−0.132	−0.176	−0.181	−0.173	−0.187	−0.160
21%–30%	−0.126	−0.168	−0.177	−0.176	−0.201	−0.155
31%–60%	−0.112	−0.150	−0.197	−0.248	−0.273	−0.160
61%–100%	−0.090	−0.117	−0.122	−0.099	−0.087	−0.102
101%–150%	−0.080	−0.082	−0.060	−0.036	−0.055	−0.073
151%–200%	−0.100	−0.030	−0.031	−0.034	−0.034	−0.050
Subtotal	−0.121	−0.159	−0.176	−0.185	−0.202	−0.152

Panel C: OTM BS pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	0.060	0.119	0.167	0.210	0.264	0.141
21%–30%	0.053	0.116	0.133	0.204	0.278	0.131
31%–60%	0.060	0.124	0.193	0.304	0.418	0.177
61%–100%	0.045	0.107	0.138	0.187	0.311	0.124
101%–150%	0.041	0.099	0.120	0.170	0.338	0.112
151%–200%	0.035	0.088	0.176	0.181	0.202	0.112
Subtotal	0.056	0.118	0.163	0.232	0.314	0.146

Panel D: OTM CO pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	0.049	0.104	0.142	0.167	0.202	0.115
21%–30%	0.039	0.096	0.101	0.151	0.203	0.100
31%–60%	0.042	0.101	0.153	0.227	0.297	0.133
61%–100%	0.024	0.077	0.088	0.103	0.162	0.074
101%–150%	0.018	0.060	0.065	0.071	0.152	0.057
151%–200%	0.025	0.056	0.118	0.109	0.102	0.066
Subtotal	0.025	0.097	0.129	0.173	0.223	0.111

Table 6:

Pricing error by leverage and moneyness with a term structure of volatility (TSV)

The reported numbers are the average pricing error defined as the market price minus the model value, for all categories partitioned by leverage and moneyness jointly. ITM and OTM denote in-the-money and out-of-the money call options, respectively. Black and Scholes (BS) with a TSV is implemented by estimating the implied stock volatility from an at-the-money (ATM) option for each option expiration period, and then using this volatility to price all the options of the same option expiration period on the same day. Compound option (CO) with a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, the firm volatility $\sigma_{V_{T_d}}$ at expiration T_d, and the firm volatility $\sigma_{V_{T_i}}$ at expiration T_i from three option prices and the stock price (for details, see Section2) for the closest option expiration term. For all the other option expiration terms, we keep V and $\sigma_{V_{T_d}}$ the same, but we recalibrate V^* and $\sigma_{V_{T_i}}$ from the two most ATM option prices for that option expiration term, and then we use all four parameters to price all the other options of the same option expiration term on the same day. Leverage D/E is defined as the current implied market value of debt, D, divided by the current market value of equity E.

Panel A: ITM BS pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	−0.233	−0.242	−0.223	−0.192	−0.150	−0.187
21%–30%	−0.249	−0.244	−0.229	−0.198	−0.159	−0.191
31%–60%	−0.253	−0.289	−0.259	−0.224	−0.183	−0.217
61%–100%	−0.262	−0.224	−0.207	−0.178	−0.143	−0.173
101%–150%	−0.228	−0.208	−0.193	−0.152	−0.130	−0.154
151%–200%	−0.134	−0.172	−0.197	−0.127	−0.017	−0.105
Subtotal	−0.242	−0.251	−0.231	−0.199	−0.159	−0.194

Panel B: ITM CO pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	−0.181	−0.194	−0.185	−0.166	−0.133	−0.160
21%–30%	−0.180	−0.176	−0.174	−0.163	−0.135	−0.155
31%–60%	−0.153	−0.189	−0.175	−0.166	−0.144	−0.160
61%–100%	−0.120	−0.103	−0.110	−0.107	−0.093	−0.102
101%–150%	−0.109	−0.096	−0.083	−0.070	−0.067	−0.073
151%–200%	−0.033	−0.039	−0.032	−0.034	−0.039	−0.035
Subtotal	−0.169	−0.180	−0.172	−0.158	−0.131	−0.152

Panel C: OTM BS pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	0.112	0.128	0.155	0.177	0.191	0.141
21%–30%	0.109	0.132	0.169	0.172	0.097	0.131
31%–60%	0.125	0.157	0.211	0.263	0.267	0.177
61%–100%	0.101	0.109	0.130	0.160	0.184	0.124
101%–150%	0.084	0.103	0.117	0.155	0.187	0.112
151%–200%	0.084	0.104	0.118	0.185	0.120	0.112
Subtotal	0.113	0.134	0.169	0.197	0.190	0.146

Panel D: OTM CO pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	0.100	0.109	0.124	0.137	0.140	0.115
21%–30%	0.094	0.105	0.126	0.119	0.037	0.100
31%–60%	0.102	0.118	0.148	0.186	0.208	0.133
61%–100%	0.076	0.068	0.069	0.076	0.096	0.074
101%–150%	0.057	0.059	0.049	0.058	0.062	0.057
151%–200%	0.069	0.074	0.061	0.035	0.027	0.066
Subtotal	0.095	0.105	0.123	0.138	0.130	0.111

Table 7:

Pricing error by days to expiration and moneyness without a term structure of volatility (TSV)

The reported numbers are the average pricing error defined as the market price minus the model value, for all categories partitioned by time to expiration and moneyness. ITM and OTM denote in-the-money and out-of-the money options, respectively. Black and Scholes (BS) without a TSV is implemented by estimating the implied stock volatility σ_S, which minimizes the sum of squared errors among all the option prices on the preceding day, and then using this volatility to price all the options on the next day. Compound option (CO) without a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, and the firm volatility σ_V, which minimize the sum of squared errors among all the option prices on the preceding day, and then the three parameters are used to price all the options on the next day.

Panel A: Pricing error by days to expiration

	21–40	41–60	61–110	111–170	171–365	Subtotal
ITM BS error	−0.154	−0.240	−0.264	−0.287	−0.295	−0.216
ITM CO error	−0.134	−0.211	−0.220	−0.220	−0.206	−0.180
OTM BS error	0.097	0.129	0.223	0.332	0.444	0.182
OTM CO error	0.075	0.128	0.175	0.216	0.274	0.149

Panel A: Pricing error by moneyness

	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
ITM BS error	−0.245	−0.254	−0.240	−0.218	−0.196	−0.216
ITM CO error	−0.188	−0.196	−0.190	−0.182	−0.171	−0.180
OTM BS error	0.132	0.175	0.243	0.306	0.393	0.182
OTM CO error	0.102	0.104	0.150	0.174	0.205	0.149

Table 8:

Pricing error by leverage and days to expiration without a term structure of volatility (TSV)

The reported numbers are the average pricing error defined as the market price minus the model value, for all categories partitioned by leverage and time to expiration jointly. ITM and OTM denote in-the-money and out-of-the money options, respectively. Black and Scholes (BS) without a TSV is implemented by estimating the implied stock volatility σ_S, which minimizes the sum of squared errors among all the option prices on the preceding day, and then using this volatility to price all the options on the next day. Compound option (CO) without a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, and the firm volatility σ_V, which minimize the sum of squared errors among all the option prices on the preceding day, and then the three parameters are used to price all the options on the next day. Leverage D/E is defined as the current implied market value of debt, D, divided by the current market value of equity E.

Panel A: ITM BS pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	−0.156	−0.243	−0.239	−0.246	−0.201	−0.204
21%–30%	−0.156	−0.241	−0.262	−0.271	−0.342	−0.218
31%–60%	−0.152	−0.242	−0.304	−0.387	−0.425	−0.237
61%–100%	−0.149	−0.229	−0.292	−0.292	−0.330	−0.213
101%–150%	−0.145	−0.226	−0.234	−0.224	−0.311	−0.197
151%–200%	−0.185	−0.189	−0.277	−0.247	−0.265	−0.218
Subtotal	−0.154	−0.240	−0.264	−0.287	−0.295	−0.216

Panel B: ITM CO pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	−0.143	−0.224	−0.207	−0.198	−0.140	−0.179
21%–30%	−0.137	−0.215	−0.223	−0.210	−0.261	−0.186
31%–60%	−0.127	−0.206	−0.245	−0.291	−0.296	−0.191
61%–100%	−0.111	−0.176	−0.224	−0.182	−0.181	−0.155
101%–150%	−0.098	−0.160	−0.148	−0.088	−0.159	−0.126
151%–200%	−0.131	−0.098	−0.198	−0.138	−0.150	−0.138
Subtotal	−0.134	−0.211	−0.220	−0.220	−0.206	−0.180

Panel C: OTM BS pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	0.110	0.141	0.241	0.333	0.451	0.194
21%–30%	0.094	0.135	0.213	0.328	0.387	0.177
31%–60%	0.094	0.124	0.235	0.352	0.492	0.186
61%–100%	0.078	0.098	0.168	0.310	0.457	0.151
101%–150%	0.057	0.105	0.198	0.257	0.394	0.113
151%–200%	0.045	0.106	0.192	0.218	0.281	0.100
Subtotal	0.097	0.129	0.223	0.332	0.444	0.182

Panel D: OTM CO pricing errors by leverage and days to expiration

D/E	21–40	41–60	61–110	111–170	171–365	Subtotal
11%–20%	0.079	0.143	0.195	0.239	0.305	0.171
21%–30%	0.082	0.131	0.182	0.220	0.268	0.147
31%–60%	0.072	0.124	0.168	0.210	0.257	0.141
61%–100%	0.060	0.094	0.117	0.126	0.171	0.112
101%–150%	0.054	0.071	0.169	0.193	0.114	0.107
151%–200%	0.036	0.073	0.126	0.159	0.146	0.090
Subtotal	0.075	0.128	0.175	0.216	0.274	0.149

Table 9:

Pricing error by leverage and moneyness without a term structure of volatility (TSV)

The reported numbers are the average pricing error defined as the market price minus the model value, for all categories partitioned by leverage and moneyness jointly. ITM and OTM denote in-the-money and out-of-the money options, respectively. Black and Scholes (BS) without a TSV is implemented by estimating the implied stock volatility σ_S, which minimizes the sum of squared errors among all the option prices on the preceding day, and then using this volatility to price all the options on the next day. Compound option (CO) without a TSV is implemented by estimating the implied firm value V, the boundary condition firm value V^*, and the firm volatility σ_V, which minimize the sum of squared errors among all the option prices on the preceding day, and then the three parameters are used to price all the options on the next day. Leverage D/E is defined as the current implied market value of debt, D, divided by the current market value of equity E.

Panel A: ITM BS pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	−0.227	−0.233	−0.221	−0.206	−0.186	−0.204
21%–30%	−0.257	−0.248	−0.239	−0.221	−0.202	−0.218
31%–60%	−0.263	−0.305	−0.273	−0.238	−0.211	−0.237
61%–100%	−0.296	−0.253	−0.243	−0.215	−0.191	−0.213
101%–150%	−0.239	−0.218	−0.227	−0.200	−0.176	−0.197
151%–200%	−0.163	−0.183	−0.305	−0.237	−0.164	−0.218
Subtotal	−0.245	−0.254	−0.240	−0.218	−0.196	−0.216

Panel B: ITM CO pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	−0.185	−0.192	−0.178	−0.172	−0.169	−0.179
21%–30%	−0.200	−0.194	−0.192	−0.188	−0.180	−0.186
31%–60%	−0.191	−0.227	−0.206	−0.191	−0.180	−0.191
61%–100%	−0.188	−0.157	−0.164	−0.156	−0.148	−0.155
101%–150%	−0.154	−0.122	−0.135	−0.128	−0.119	−0.126
151%–200%	−0.071	−0.084	−0.221	−0.153	−0.095	−0.138
Subtotal	−0.188	−0.196	−0.190	−0.182	−0.171	−0.180

Panel C: OTM BS pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	0.143	0.182	0.247	0.304	0.390	0.194
21%–30%	0.127	0.175	0.247	0.303	0.377	0.177
31%–60%	0.130	0.178	0.256	0.333	0.421	0.186
61%–100%	0.118	0.151	0.204	0.258	0.386	0.151
101%–150%	0.100	0.113	0.158	0.253	0.365	0.113
151%–200%	0.080	0.084	0.178	0.180	0.208	0.170
Subtotal	0.132	0.175	0.243	0.306	0.393	0.182

Panel D: OTM CO pricing errors by leverage and moneyness

D/E	[0.40–0.60)	[0.60–0.70)	[0.70–0.80)	[0.80–0.90)	[0.90–0.95)	Subtotal
11%–20%	0.108	0.118	0.152	0.177	0.240	0.171
21%–30%	0.107	0.115	0.123	0.160	0.206	0.147
31%–60%	0.106	0.110	0.133	0.140	0.187	0.141
61%–100%	0.105	0.108	0.156	0.157	0.164	0.102
101%–150%	0.098	0.108	0.140	0.144	0.149	0.067
151%–200%	0.096	0.094	0.117	0.128	0.129	0.040
Subtotal	0.102	0.104	0.150	0.174	0.205	0.149

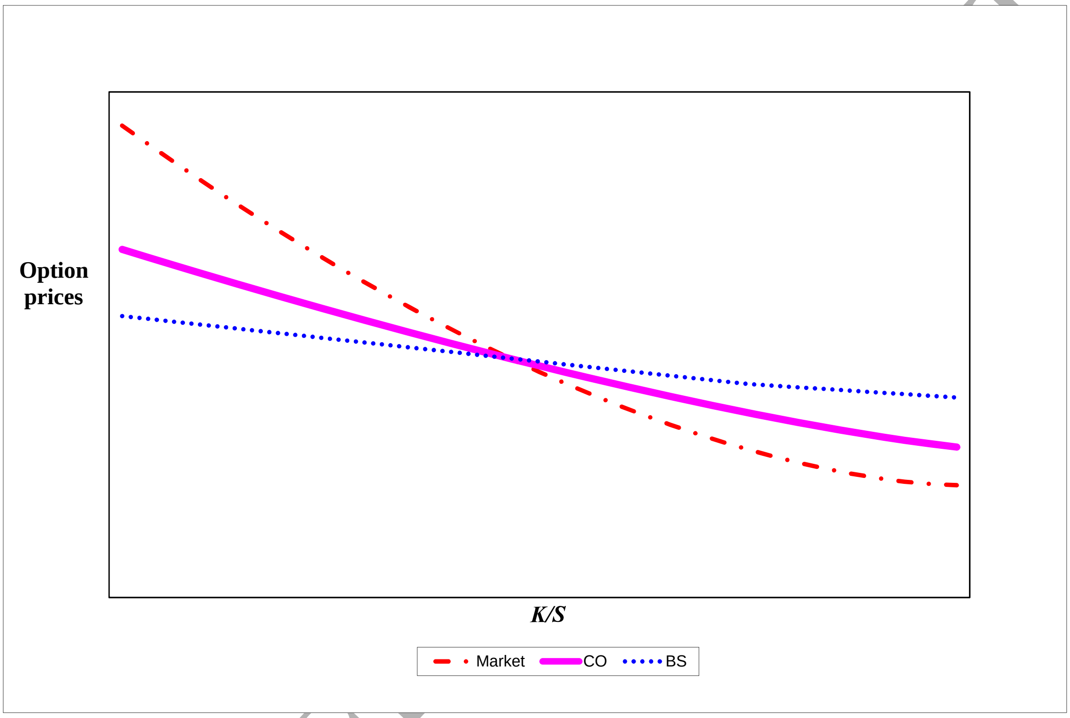

Fig. 1. **Pricing errors from the compound option (CO) and Black and Scholes (BS) models.** The graph shows the pricing errors (defined as market price minus the model value) for the CO and BS models, for a fixed stock price with options with different strike prices. The BS model undervalues in-the-money (ITM) call options and overvalues out-of-the-money (OTM) call options. The CO model undervalues less than the BS model for ITM call options and overvalues less than BS model for OTM call options. K is the exercise price. S is the spot individual stock price.

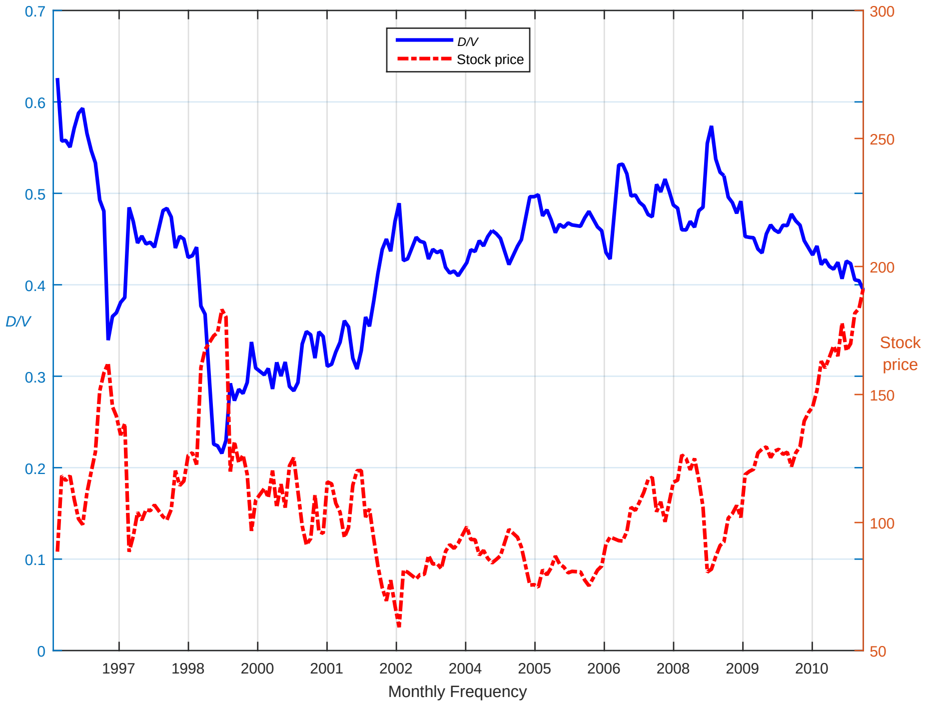

Fig. 2. **Implied leverage and stock price for IBM.** The graph shows the implied leverage graphed on the left vertical axis and the market price of the firm's equity graphed on the right vertical axis for IBM. This figure demonstrates an inverse relation between the implied measure of leverage and the actual market stock price. D/V is defined as the current implied market value of debt, D, divided by the current market value of firm V.

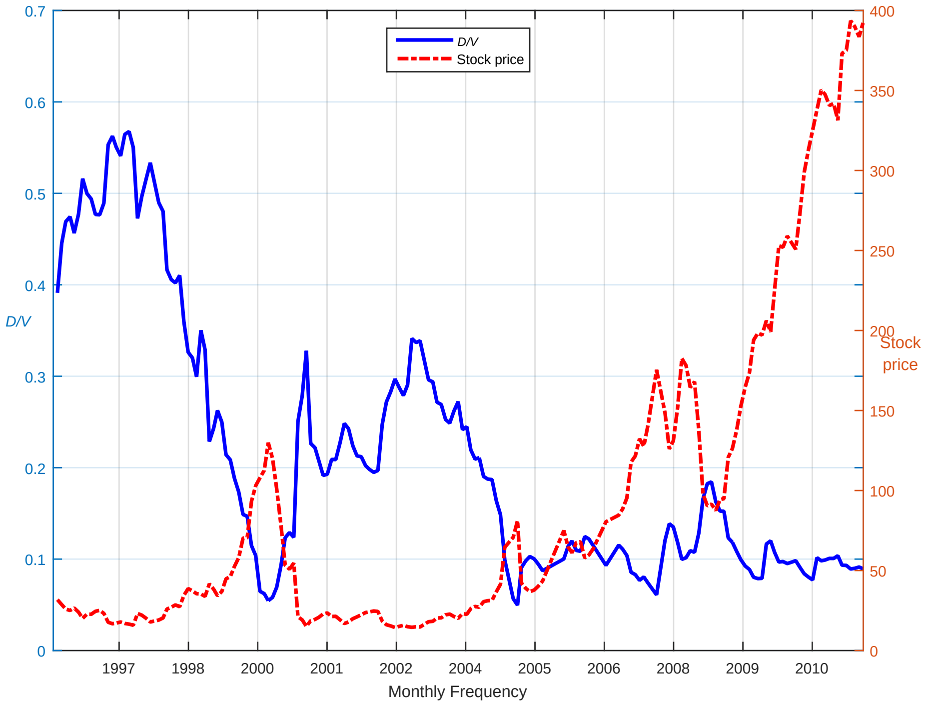

Fig. 3. **Implied leverage and stock price for Apple.** The graph shows the implied leverage graphed on the left vertical axis and the market price of the firm's equity graphed on the right vertical axis for Apple. This figure demonstrates an inverse relation between the implied measure of leverage and the actual market stock price. D/V is defined as the current implied market value of debt, D, divided by the current market value of firm V.

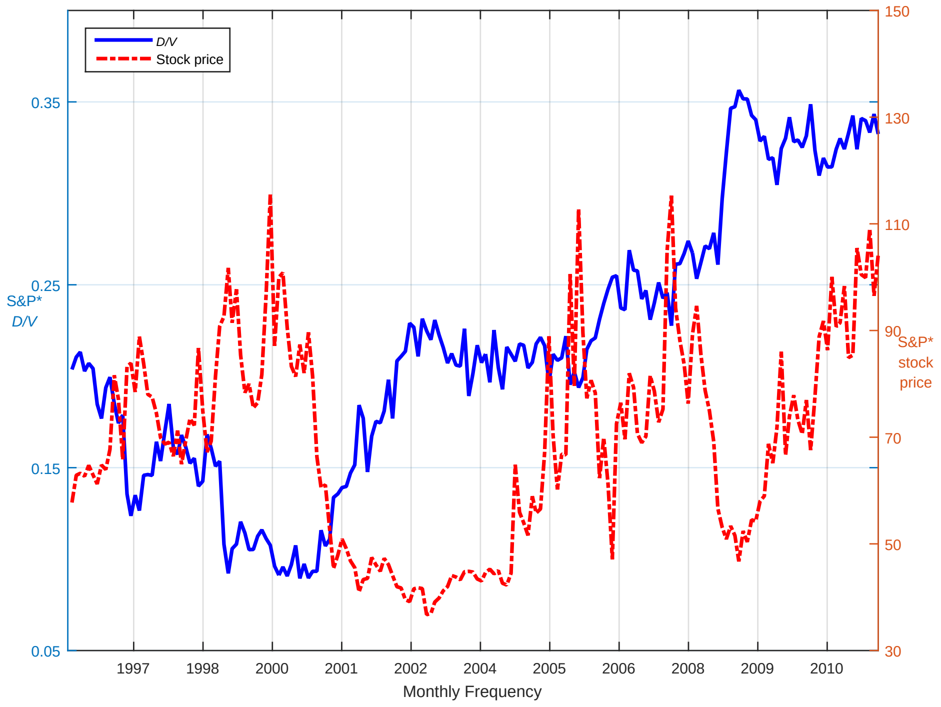

Fig. 4. **Implied leverage and stock price for S&P* firms.** The graph shows the implied leverage graphed on the left vertical axis and the market price of equity graphed on the right vertical axis for the S&P* firms (the Standard & Poor's 500 firms that are not financial, utility, or nonprofit firms). This figure demonstrates an inverse relation between the implied measure of leverage and the actual market stock price. D/V is defined as the current implied market value of debt of the S&P* firms, divided by the current market value of the the S&P* firms.