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Misrepresentation and Capital Structure: Quantifying the Impact on Corporate Debt Value

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Abstract

Securities class actions typically involve some misrepresentation by a firm that overstates its true value. In securities class actions econometric models are used to assess damages to shareholders. However, studies on measuring damages for debt-holders is limited. Using structural models and leveraging the relationship between equity and firm value, we use observable equity information to determine firm and debt values and hence the effect of misrepresentation on corporate debt values. We find that the misrepresentation impact on debt value depends on the debt's credit risk. Generally, the debt for higher-leverage firms is more sensitive to the misrepresentation impact than for lower-leverage firms and junior debt is more affected by fraud than senior debt. Our proposed methodology allows for debt damages assessments consistent with standard methods for assessing equity damages.

Our findings have important consequences for damages assessment and the allocation of settlement awards in securities class actions. In some jurisdictions damages awarded are net of any hedge or risk-limitation transaction. Since corporate securities such as bonds and stocks are often held in portfolios for hedging purposes, measuring the effect of misrepresentation on all of the firm's issuances is essential to accurately computed damages awards. Additionally our approach provides a consistent methodology for computing damages for securities that do not trade on public markets. A case study of a recent securities class action illustrates our methodology.

Keywords: Damages; Misrepresentation; Securities Class Actions; Capital Structure; Valuation With Observable Information;
JEL Codes: G120, G130, G180, G300, G320, K220.

1. Introduction

A variety of illegal activities such as Ponzi schemes, insider trading, market manipulation and misrepresentation can lead to regulatory enforcement proceedings, criminal persecution and/or securities class action lawsuits. This paper concerns the assessment of damages in secondary market securities class actions as a result of misrepresentation by a firm, including its directors and officers. Examples of misrepresentation include the overstatement of earnings, the failure to properly disclose the risks of potential liabilities, and accounting irregularities. Typically,

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misrepresentation results in an overstatement of firm value. For firms whose shares trade in an efficient secondary market, the share price of the firm quickly drops when the misrepresentation is revealed. Using well-established econometric methods, the share price drop is used to assess potential damages to investors who transacted in the share during the class period — the time between the start of the misrepresentation and its revelation.[2]

Firms use many vehicles to finance their operations including common and preferred shares, warrants, various debt instruments and employee stock options. The instruments used determine a firm's capital structure and the firm value equals the total value of the component instruments in the capital structure and the expected bankruptcy costs. Fraud affects the value of the entire firm, not just the value of equity. Therefore, assessing damages only due to shareholders can lead to a significant understatement of the losses incurred by investors across all of the firm's issued securities. For example, the recent case of Sino-Forest in which equityholders were completely wiped out and control of the firm's residual value reverted to bondholders, clearly shows that fraud can induce losses to debtholders.[39] In most instances of misrepresentation equityholders are not completely wiped out, but the misrepresentation still affects the debt value.

In this paper we use structural models to measure the impact of misrepresentation on the value of a firm's debt. Using a relationship between equity and firm value we show how observable equity information can be used to determine firm value and hence the value of debt in the capital structure. Thus the effect of misrepresentation on firm value and debt value can be measured from the observable drop in share price. This leads to a proposed methodology for debt damages assessment consistent with the standard method for assessing damages to equity investors. Furthermore, trades involving corporate bonds happen much less frequently than for common shares and hence corporate debt markets are typically less efficient than equity markets. For example, during the class period there could either be i) no trades involving that firm's debt (i.e., no information about bond value change); or ii) very few trades in an inefficient market (i.e., potentially unreliable information about bond value change). In such situations, the modelling framework presented here allows one to compute bond value changes consistent with the observed share price change. Additionally in some jurisdictions (e.g., Province of Ontario, Canada) damages awarded are net of any hedge or risk-limitation transaction. Since corporate securities such as bonds, stocks and warrants are often held in portfolios for hedging purposes (e.g., short equity position to hedge a long bond position), measuring the effect of misrepresentation on all of the firm's issuances is essential to accurately computed damages awards. Hence, our findings and proposed methodology have important consequences for damages assessment and allocation of settlement awards in securities class actions.[3]

Our study shows that misrepresentation has a significant impact on the value of all debt components in the capital structure. We find that the debt for higher-leverage firms is more sensitive to the misrepresentation impact than for lower-leverage firms and junior debt is more affected by fraud than senior debt. Furthermore, the First Passage Time model (the Black and Cox [5] model or the one-factor Longstaff and Schwartz [35] model[4]) produces security values that are more sensitive to fraud size than the canonical Merton model [36]. This result is unsurprising, since it is well known that the relatively inflexible Merton model generates credit spreads that are

[2] In this paper we use the terms misrepresentation and fraud interchangeably, although we recognize there are important legal distinctions between them.

[3] Note that this paper concerns only the impact of fraud on the value of firm-issued securities, not on the value of third-party issued securities, such as exchange-traded single-name equity options.

[4] We refer to this model as the First Passage Time model in the rest of the paper.

too small to reflect actual credit spreads (i.e., it overprices bonds), hence leaving little opportunity for significant changes in bond prices due to fraud-induced drops in share price.

In addition to our main findings, we explicitly discuss bankruptcy costs in the First Passage Time model Furthermore, we are able to reduce a system of two nonlinear equations, used to connect the unobservable firm value and firm value volatility to observable equity value and equity volatility, into one equation. This technique improves the ability to solve the non-linear system, leading to a more efficient method for connect observable equity value to firm value and it provides explicit justification for use of the maximum likelihood method [14] for the capital structure models considered here.

The paper is organised as follows. The rest of Section 1 discusses a standard econometric method for assessing the impact of fraud on share price and also introduces the "constant percentage change" model. Section 2 discusses the First Passage Time model with bankruptcy costs. Section 3 gives the connection between the unobservable firm value and the observable share price, critical to measuring the impact of fraud on debt value. In Section 4 we investigate the effect of fraud on debt value and propose a methodology to compute the damages for a bond. A summary and concluding discussion is given in Section 5.

1.1. The Value Line and Constant Percentage Change Model

An efficient market[5] is one in which publicly-available information and signals are quickly evaluated and reflected in market prices. Stock markets such as NYSE and TSX are considered efficient markets. There are many different ways to measure market efficiency available in the literature [3], [21], [41], [23]. Both firm-specific (idiosyncratic) and general market (systematic) information affect a firm's stock price. Under this presumption the stock price is viewed as a function of both pieces of information.

A two-factor linear model is the basic financial econometric model used to estimate a security's true value and from which damages estimates are computed. This model specifies the security's expected return as a linear function of the return on the whole market and the return on the industry sector. Specifically,

$$R_t = \alpha + \beta_1 R_{Mt} + \beta_2 R_{It} + e_t, \tag{1}$$

where R_t, R_{Mt}, and R_{It} are the time-t stock, market, and industry returns, respectively, and e_t is the residual value (assumed to be independent random variables with mean zero and constant variance). Using Equation 1 the expected return of the stock at time t is

$$E[R_t] = \alpha + \beta_1 R_{Mt} + \beta_2 R_{It}. \tag{2}$$

Regression analysis with historical data is performed to estimate α, β_1, and β_2. Suppose that τ_b is the start date of the fraud and τ_e is the date the misrepresentation is revealed, so that $[\tau_b, \tau_e]$ defines the class period. Using this econometric model, a value line representing the true stock value (i.e., the path the stock price would have followed in the absence of misrepresentation or omission during the class period) can easily be constructed [9]. This method constructs a series of daily returns R_{Ct} in the following way

[5] Economists typically refer to three forms of financial market efficiency — weak, semi-strong and strong. The reader is referred to [22] for further discussion of the Efficient Market Hypothesis.

- if no fraud-related information is disclosed, set the return equal to the actual return on the security;
- if fraud-related information is disclosed or leaked into the market, set the return for these days equal to the expected return calculated using Equation 2 and the estimates for α, β_1, β_2.

This series of daily returns is then used to construct the value line by

$$\tilde{S}_{t-1} = \tilde{S}_t/(1 + R_{Ct-1}), \tag{3}$$

where $\tilde{S}_t$ is the true value of stock at time t.

Alternative ways of constructing the value line are the constant percentage method and the constant dollar method. The constant percentage method assumes that the misrepresentation has a proportional effect on the stock price, i.e.,

$$\tilde{S}_t = \delta S_t, \tag{4}$$

for $t \in [\tau_b, \tau_e]$, where δ is the constant percentage change during the class period. Typically, δ is determined by the size of the stock price movement controlled for changes in the market and industry on the date the fraud is revealed. This is used with the model and method discussed above to construct the stock's value line. The constant dollar method assumes a fixed dollar change in stock price during the class period; and for reasons of brevity, it is not used in this paper.

Much work has been done on the methodology for estimating damages in securities fraud cases. A non-exhaustive list is [30], [25], [24], [9], [17], [47] [48] and [46] which provide variants on the methodology given here. What is missing in these works, however, is the impact of fraud on the value of other securities in the capital structure. In this paper our proposed method assesses the impact of fraud on the value of debts, given the easily measured and observed impact on the share price.

2. The First Passage Time Model with Bankruptcy Costs

In this paper, we measure damages for debts using the one-factor Longstaff-Schwartz [35] model, which is essentially the Black-Cox [5] model with a flat default boundary.[6] There is a large literature on structural models that focus on a variety of economic considerations, such as stochastic interest rates, exogenous default, endogenous default and stationary leverage ratios. The purpose of this paper is to propose a methodology for computing damages for secondary market securities class actions that uses a capital structure model. The First Passage Time model we selected to illustrate our methodology is simple to implement and allows for a variety of debt instruments (seniority, coupon, maturity) and bankruptcy costs. Other structural models that accommodate different features could be used in a straightforward manner. Here, we briefly discuss model assumptions and derive the closed-form valuation expressions for equity, debt, and bankruptcy costs.

[6]Predescu [42] use a similar modelling set up (without bankruptcy costs) to study the performance of structural models using CDS spreads.

2.1. Valuation of Debt

We assume a flat term structure in which the risk free rate r is constant. Let V^F be the total value of the assets of the firm. Assume that the assets follow the dynamic

$$dV_t^F = (r + \lambda^v \sigma^F) V_t^F dt + \sigma^F V_t^F d\hat{W}_t, \tag{5}$$

where λ^v, r, and σ^F are constant and $\hat{W}$ is a Brownian motion under the real world measure $\mathcal{P}$. Assume that the Modigliani-Miller Theorem [38] holds: the firm value is independent of the capital structure of the firm. We assume perfect, frictionless markets in which no arbitrage opportunity exists. We assume that default is exogenous and that there is a threshold value K; when the firm value V^F reaches the constant boundary K, the firm enters financial distress and simultaneously defaults on all of its obligations. The time-t price of T-maturity risky debt with face value one is given by

$$P(V_t^F, \sigma^F, r, w, K, t, T) = e^{-r(T-t)} E_{\mathcal{Q}}[1 - w I_{\tau \le T}] = e^{-r(T-t)}(1 - w\mathcal{Q}(\tau \le T)), \tag{6}$$

where w is the loss given default, τ is the first passage time of the firm value V^F to the boundary K, I_A is an indicator function of the event A and $\mathcal{Q}(\tau \le T)$ is the probability of the event $[\tau \le T]$ under the risk-neutral measure. The time-t conditional distribution of the first passage time, τ, is

$$\begin{aligned} \mathcal{Q}(\tau \le T) = {} & \Phi\Big(\frac{-\log(V^F/K) - r(T-t) + 0.5\sigma^{F^2}(T-t)}{\sigma^F \sqrt{T-t}}\Big) \\ & + \exp\Big(\frac{-2\log(V^F/K)(r - 0.5\sigma^{F^2})}{\sigma^{F^2}}\Big)\Phi\Big(\frac{-\log(V^F/K) + r(T-t) - 0.5\sigma^{F^2}(T-t)}{\sigma^F \sqrt{T-t}}\Big), \end{aligned} \tag{7}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.[7]

The time-t value of a T-maturity zero coupon bond with face value F is $F \times P(V_t^F, \sigma^F, r, w, K, t, T)$. For debt with coupon payments, we evaluate it using the "portfolio of zeros" approach [35], i.e., treat each coupon payment as a "mini" zero-coupon bond and compute the coupon bond value as the sum of the mini zero-coupon bond values. As noted in Eom, Helwege and Huang [19] this approach fails to take into account the dependence of the mini zero-coupon bonds. Incorporating the dependence of these coupon payments could improve model performance. Consider a risky coupon bond with face value F and N_C remaining coupon payments of C. Let TC_i be the ith-coupon payment date from a given time t, and TC_{N_C} be the last coupon payment date, which is T. The value of the risky coupon bond B_t at time t is the value of the corresponding portfolio of zeros, i.e.,

$$B_t = C \sum_{i=1}^{N_C - 1} P(V_t^F, \sigma^F, r, w, K, t, TC_i) + (F + C) P(V_t^F, \sigma^F, r, w, K, t, T). \tag{8}$$

For a firm with N^D different bonds outstanding, the present value of the firm's total liabilities D_t is the sum of all the bond prices, which is given by

$$D_t = \sum_{j=1}^{N^D} B(V_t^F, \sigma^F, r, w^j, K, t, T^j, C^j, F^j), \tag{9}$$

[7] Please refer to [26] for the derivation of the first passage time distribution.

where w^j, T^j and C^j are the parameter inputs corresponding to the jth-bond and B is given by equation (8). Note that the loss given default w^j may be different across bonds and that w^j is the same for the principle F^j and the coupon payment C^j (see [27]). Research on recovery rates of debt has been studied by many authors (see [2] and [40]). Ou, Chiu, and Metz [40] found the value weighted average loss given default of senior secured debt w^s to be 50.9% while the loss given default of a junior subordinated debt w^j to be 82.9%. For the case with only one type of debt outstanding, we set w as 48.67% following [27] and [19].

Let $PV(D)$ be the present value of the debts' face value discounted at the risk free rate. That is,

$$PV(D) = \sum_{j=1}^{N^D} \Big((C^j + F^j) e^{-r(T^j - t)} + C^j \sum_{i=1}^{N_C^j - 1} e^{-r(TC_i^j - t)} \Big), \tag{10}$$

and let $\bar{T}$ be the weighted average maturity date of all debts, namely,

$$\bar{T} - t = \sum_{j=1}^{N^D} \Big((T^j - t) \frac{(C^j + F^j) e^{-r(T^j - t)}}{PV(D)} + \sum_{i=1}^{N_C^j - 1} (TC_i^j - t) \frac{C^j e^{-r(TC_i^j - t)}}{PV(D)} \Big). \tag{11}$$

The default boundary K can be set equal to the time-$\bar{T}$ value of all future liability payments. For example, consider a firm with two bonds, a and b, outstanding. The face values are F^a and F^b and the maturities are T^a and T^b, respectively. The default boundary, K, of this firm is set equal to $F^a e^{r(\bar{T} - T^a)} + F^b e^{r(\bar{T} - T^b)}$.

2.2. *Barrier Option Framework for Equity Value*

The equity value is given as the value of a European down-and-out call option written on firm value V^F. Similar approaches to valuing equity can be found explicitly in [7] and [42] and implicitly in [28]. The price of a European down-and-out call option on the firm value C_{DO} with barrier K_B and strike price K_S is $C_{DO}(V_t^F, \sigma^F, r, K_B, K_S, t, T)$, whose exact formula is given in Appendix A.[8] Under the First Passage Time model framework, a bond holder would suffer no loss provided the firm value V_t^F never reaches the default boundary K prior to maturity T. In the case of a single zero-coupon bond, the default boundary K is at least equal to the face value of the bond in order to guarantee sufficient asset value to pay off the debt at T. For a general debt structure the time-t equity value EQ_t is given by

$$EQ_t = V_t^F \Phi(d_1) - K e^{-r(\bar{T} - t)} \Phi(d_2) - V_t^F (K/V_t^F)^{2\lambda} \Phi(d_3) + K e^{-r(\bar{T} - t)} (K/V_t^F)^{2\lambda - 2} \Phi(d_4), \tag{12}$$

where

$$\lambda = (r + {\sigma^F}^2/2)/{\sigma^F}^2 \tag{13}$$

$$d_1 = \big(\log(V_t^F/K) + (r + {\sigma^F}^2/2)(\bar{T} - t)\big)/(\sigma^F \sqrt{\bar{T} - t}), \tag{14}$$

$$d_2 = d_1 - \sigma^F \sqrt{\bar{T} - t}, \tag{15}$$

$$d_3 = \big(\log(K/V_t^F) + (r + {\sigma^F}^2/2)(\bar{T} - t)\big)/(\sigma^F \sqrt{\bar{T} - t}), \tag{16}$$

[8] Barrier option valuation can be found in [44] and [37].

and

$$d_4 = d_3 - \sigma^F \sqrt{\bar{T} - t}. \tag{17}$$

Given the number of outstanding shares, N, of the firm, the stock price of the firm is the equity value divided by N. In Proposition 1 we show that EQ_t defined by equation (12) is a one-to-one function of V_t^F (see Appendix B). This is used in the connection between equity value and firm value providing a more efficient implementation (Section 3.1) and explicit justification for use of the maximum likelihood method (Section 3.2).

2.3. Bankruptcy Costs

In the case of a single zero-coupon bond outstanding, when the firm value V^F reaches the default boundary K at time $\tau < T$, the equity value becomes zero, and hence the realized bankruptcy cost BC_τ is the difference between the default boundary K and the value of recovered risk-free bond $e^{-r(T-\tau)}(1-w)F$.[9] Before default happens, firm value is the sum of the debt and equity values and the expected present value of bankruptcy cost. The expected present value of bankruptcy costs at time t is

$$BC_t = V_t^F - D_t - EQ_t, \tag{18}$$

where D_t is the time-t debt value. BC_t defined in Equation (18) is a decreasing, convex function of the firm value V_t^F. It has the same properties as the bankruptcy cost defined in [31] and [33], i.e., BC_t satisfies the boundary conditions[10]

$$\text{at } V_t^F = K,\ BC_t = K - e^{-r(T-t)}(1-w)F, \text{ and} \tag{19}$$

$$\text{as } V_t^F \to \infty,\ BC_t \to 0. \tag{20}$$

It is easy to see that the boundary condition (19) holds. Boundary condition (20) also holds: when the firm value V_t^F becomes very large, the debt value D_t approaches to $e^{-r(T-t)}F$, which is the value of risk-free debt with the same maturity and face value, and the equity value EQ_t approaches to $V_t^F - Ke^{-r(T-t)}$, which is the upper bound for a European call option price under the Black-Scholes framework, and hence the present value of the bankruptcy costs BC_t approaches to zero, because $K = F$.[11] A numerical example in the case of a single zero-coupon bond is illustrated in Figure 1.

Figure 1: Firm Value and Capital Structure - Firm value V_t^F versus equity value EQ_t, debt value D_t and the sum of EQ_t and D_t using the model in Section 2. The bankruptcy costs BC_t are the difference between V_t^F and $D_t + EQ_t$. Parameters are: $\sigma^F = 21\%$, $r = 5\%$, $T - t = 5$, $K = 50$, and $w = 48.67\%$ (parameter values r and w are choosen following Huang and Huang[27]).

In equation (12) we assume that the barrier K_B and the strike price K_S of the down-and-out call option are both equal to the time-$\bar{T}$ value of future liability payments. It is possible to relax

[9] In the general debt structure case the recovered risk-free debt has value $(1-\bar{w})PV(D)$, where $\bar{w}$ is the resulting loss given default for all outstanding debt given that bonds with different seniority have different recovery rates.

[10] In the general debt structure case the first condition becomes $BC_t = K - (1-\bar{w})PV(D)$ when $V_t^F = K$.

[11] In the general debt structure case, the bankruptcy costs BC_t also satisfies the second boundary condition. When the firm value V_t^F goes to infinity, the probability of default becomes zero, and hence the debt value D_t approaches to $PV(D)$. At the same time, the value of equity EQ_t, which is given by equation (12), approaches $V_t^F - Ke^{-r(\bar{T}-t)}$ as V_t^F goes to infinity. By the definition of K, $Ke^{-r(\bar{T}-t)}$ equals to $PV(D)$. Hence BC_t defined by equation (18) approaches $V_t^F - PV(D) - (V_t^F - PV(D))$, which is zero.

this assumption in the case when a firm issues multiple bonds. Setting the default boundary K equal to the time-$\bar{T}$ value of future liability payments results in high bankruptcy costs when the loss given default w is high (see discussion in [27]): in our case, the 48.67% loss given default w implies that the bankruptcy costs are around 50%, with the precise value depending on the default time. However, empirical studies show that bankruptcy costs are usually lower than 50%, for example, Davydenko, Strebulaev, and Zhao[13] found the average bankruptcy costs to be 21.7% of the market value of the firm's assets. To lower the bankruptcy costs, BC, one can set the strike price K_S of the down-and-out call equal to the time-$\bar{T}$ value of future liability payments, and set the barrier K_B to be the default boundary that is less than K_S. Under this setup, the value of equity is given by equation (A.1) and the bankruptcy costs, BC_t, defined by equation (18) still satisfy the boundary conditions (19) and (20) with K replaced by K_B. Discussion of how to set the default boundary K can be found in [10], [19], [42], [32], [28], [27], and [12]. In the following discussion of this paper, we consider the case in which both K_B and K_S are equal to the time-$\bar{T}$ value of future liability payments.

3. Connection between Equity Value and Firm Value

In this section we discuss the relation between the equity value and firm value for the capital structure model discussed in Section 2. Qualitatively, it is well known that debt values increases with equity value, due to a decrease in the likelihood of default. Thus as equity value increases, debt values increases, and hence leveraged firm value increases. Conversely, debt and leveraged firm values decrease with decreases in equity value.

Moreover, it is easily seen that the equity volatility and the firm value volatility cannot be constant at the same time. A firm is financed by a combination of equity, debt and other securities in its capital structure. Hence firm value volatility is a combination of the volatility of equity, debt and other capital structure components. If a firm's equity value changes, for example, the equity to firm value ratio changes and hence so does the contribution of equity volatility to firm value volatility.

In this section, we provide two methods to connect the observable equity value EQ_t and its volatility σ^E to the unobservable firm value V_t^F and its volatility σ^F. We discuss the traditional method and present a technique to improve computational performance in Section 3.1. We briefly discuss the maximum likelihood estimation (MLE) method in Section 3.2 with the mathematical details given in Appendix D.

3.1. The Traditional Method

Given the equity value EQ_t and its volatility σ^E, the firm value V_t^F and σ^F can be found by solving a non-linear system as discussed in [43] and [29]. We refer to this method as the traditional method. Under the risk neutral measure, the firm value dynamics are

$$dV_t^F = rV_t^F dt + \sigma^F V_t^F dW_t, \tag{21}$$

where W is a Brownian motion. Since EQ_t is a function of V_t^F, it can be shown by Itô's Lemma that the equity volatility is

$$\sigma^E = \sigma^F \frac{\partial EQ_t}{\partial V_t^F} \frac{V_t^F}{EQ_t}, \tag{22}$$

where $\partial EQ_t/\partial V_t^F$ is given in equation (B.3).

Combining equation (22) with the valuation expression of equity value (see equation (12)) constructs a system of two non-linear equations. Thus given EQ_t and σ^E, the value of V_t^F and σ^F can be found by simultaneously solving the equations,

$$EQ_t = V_t^F \Phi(d_1) - Ke^{-r(\bar{T}-t)}\Phi(d_2) - V_t^F (K/V_t^F)^{2\lambda}\Phi(d_3) + Ke^{-r(\bar{T}-t)}(K/V_t^F)^{2\lambda-2}\Phi(d_4), \tag{23}$$

$$\sigma^E = \frac{\sigma^F V_t^F}{EQ_t}\Big[\Phi(d_1) + (2r/\sigma^{F^2})(K/V_t^F)^{2\lambda}\Phi(d_3) + (1 - 2r/\sigma^{F^2})(K/V_t^F)^{2\lambda-1}e^{-r(\bar{T}-t)}\Phi(d_4)\Big]. \tag{24}$$

Proposition 1. *If parameters r, σ^F, K_S, K_B, t, and T are given and satisfy $r \geq 0$, $\sigma^F > 0$, $K_S \geq 0$, $K_B \geq 0$, and $T > t$, respectively, the value of equity EQ_t given by equation (12), (A.1) or (A.5) is a one-to-one function of the firm value V_t^F, for $V_t^F \in [K_B, \infty)$.*

Proof of the proposition is shown in Appendix B. The proposition explicitly shows that the equity value EQ_t in (23) is a one-to-one function of V_t^F. In other words, given parameters r, K, T, t, and EQ_t, for any firm volatility $\sigma^F \in (0, \infty)$, there is only one firm value V_t^F satisfying equation (23). For any given equity value EQ_t, we can define a function v_{EQ_t} such that $V_t^F = v_{EQ_t}(\sigma^F)$ satisfies equation (23). Since the firm value V_t^F is completely determined by the firm volatility σ^F, instead of simultaneously solving the non-linear system (23) and (24), we only need to solve one equation

$$\sigma^E = \sigma^F \frac{\partial EQ_t}{\partial V_t^F}\Big(\sigma^F, v_{EQ_t}(\sigma^F)\Big)\frac{v_{EQ_t}(\sigma^F)}{EQ_t} \tag{25}$$

for σ^F.

Theorem 1. *Given equity value EQ_t and its volatility σ^E, solving the non-linear system (23) and (24) for V_t^F and σ^F is equivalent to solving the following equation for σ^F:*

$$\sigma^E = \frac{\sigma^F v_{EQ_t}(\sigma^F)}{EQ_t}\Big(\Phi(\hat{d}_1) + (2r/\sigma^{F^2})(K/v_{EQ_t}(\sigma^F))^{2\lambda}\Phi(\hat{d}_3) + (1 - 2r/\sigma^{F^2})(K/v_{EQ_t}(\sigma^F))^{2\lambda-1}e^{-r(\bar{T}-t)}\Phi(\hat{d}_4)\Big), \tag{26}$$

where

$$\hat{d}_1 = (\log(v_{EQ_t}(\sigma^F)/K) + (r + \sigma^{F^2}/2)(T-t))/(\sigma^F\sqrt{T-t}), \tag{27}$$

$$\hat{d}_3 = (\log(K/v_{EQ_t}(\sigma^F)) + (r + \sigma^{F^2}/2)(T-t))/(\sigma^F\sqrt{T-t}), \tag{28}$$

$$\hat{d}_4 = \hat{d}_3 - \sigma^F\sqrt{T-t}, \tag{29}$$

and the function v_{EQ_t} from $(0, \infty)$ to $[K, \infty)$ is defined for any given EQ_t, such that $V_t^F = v_{EQ_t}(\sigma^F)$ satisfies equation (23). The firm value V_t^F is $V_t^F = v_{EQ_t}(\sigma^F)$, where σ^F comes from solving (26).

The proof Theorem 1 is given in Appendix C. The practical implication of Theorem 1 lies in the fact that when solving the non-linear system simultaneously, it is possible to get different solutions depending on the initial guess. Hence it can be necessary to manually select the most reasonable solution from multiple solutions (see [34]). By reducing the non-linear system into one equation, the issue of multiple solutions can be avoided. Even though Theorem 1 is derived under the model in Section 2, it is easily extended to other structural models admitting a one-to-one relationship between equity and firm value such as the Merton model [36].[12]

[12] For some capital structure models that include warrants (e.g. [11]) there is not a one-to-one relationship between firm and equity values.

3.2. Maximum Likelihood Estimation (MLE)

The traditional method of solving for V_t^F and σ^F has a straightforward implementation and is widely applied in academic studies. This section provides a popular alternative, the MLE method, to estimate the firm value and firm volatility from observable equity information. The method has also been extended to incorporate market information about default probabilities from credit default swaps [42].

Recent studies (see [14], [15], [20], and [16]) show that the traditional method is misspecified: structural models imply that the equity volatility σ^E is a function of V_t^F and σ^F, but typically σ^E is estimated assuming that it is constant. Using simulation, Ericsson and Reneby[20] show that the MLE method proposed by Duan[14] is better than the traditional method in estimating the firm value V_t^F and firm volatility σ^F. An empirical study by Li and Wong[34] shows that the MLE method substantially improves the performance of structural models in pricing corporate bonds.

However, when using the MLE method, the firm volatility is assumed constant during the estimation period. If the leverage ratio of the firm changes during this period, i.e., bonds are expired/issued, it might be necessary to divide the time period into sub-periods and estimate the firm volatility for each sub-period. This issue does not arise for the traditional method because the firm volatility changes daily and is calculated based on the daily equity volatility. In addition, the MLE method assumes that the market price of risk, λ^V, is constant over the estimation period (a specific and restrictive assumption). On the other hand, the traditional method does not require such a restrictive assumption. The results in Huang and Zhou [28] shows that the traditional method still works when used in combination with Generalized Method of Moments estimation.

Let $EQ^{TS} \equiv \{EQ_i^{ts} : i = 1, 2, \ldots, n\}$ be a daily time series of equity value, where i is the time index. Using MLE, the firm volatility σ^F can be estimated from EQ^{TS}. Given the maximum likelihood estimator of firm volatility $\hat{\sigma}^F$, the maximum likelihood estimator of firm value $\hat{V}_t^F$ is just $v_{EQ_t}(\hat{\sigma}^F)$. The MLE method requires that the equity value be a one-to-one function of firm value V_t^F given the other parameter values (see equation (D.5)). Proposition 1 provides the justification for using the MLE method with the structural model in Section 2. Details of the implementation can be found in Appendix D.

4. Misrepresentation and Debt Value

In this section we study the relation between fraud and debt value based on the connection between the equity value and firm value discussed in the previous section. When the underlying stock price is inflated/deflated by misrepresentation or omission, the debt value will also be inflated/deflated. We propose a methodology to quantify the effect of misrepresentation on debt value and use it to construct the damages ribbon for debt. The damages ribbon is the difference between the debt value computed using the observed share price and and the debt value computed using the value line for the share price (see Section 1.1). An important advantage of using the econometric model (see Section1.1) to compute stock value line is that it measures the effect of fraud related information on the stock price and filters out the price movement due to non-fraud related information. As such, the fraud impact is measured using observed information from the efficient equity market. Our methodology takes the fraud impact signal observed from the equity market and, using a capital structure model, translates this to the impact of fraud on debt value thus providing damages assessments that are consistent across all capital structure components. In Section 4.1, we discuss the effect of fraud on debt value with numerical examples. We demonstrate the proposed method in a case study by constructing the bond damages ribbon for the recent Agnico Eagle Mines securities class action case in Section 4.2.

4.1. *Effect of Fraud on Debt Value*

In this section we show how fraud affects the debt value based on the model of Section 2. When implementing the traditional method to measure the effect of fraud, we fix the firm value volatility and allow the stock price volatility to vary; this is consistent with the fundamental principle of the MLE method. We define the relative price

$$\delta = \frac{\tilde{S}_t}{S_t} \tag{30}$$

and use δ as a measure of the fraud size. Values of δ greater than 1 indicate that the fraud depressed the share price (not common) and values of δ less than one correspond to fraud inflating the share price. If $\delta = 1$ then $\tilde{S}_t = S_t$ and there is no misrepresentation.

Figure 2: Effect of Fraud on Debt Value (FPT) - Fraud size, δ, versus junior/senior zero coupon debt in the left/right panel using the First Passage Time model in Section 2. The junior and senior debts have the same maturity date. The quasi-leverage L is defined as quasi market value of debt to the quasi asset value, i.e., $L = (e^{-r(T-t)}K)/(EQ_t + e^{-r(T-t)}K)$. The initial equity value is 100, we use the quasi-leverage ratio to compute the corresponding debt face value K. The face value for junior debt and senior debt are the same, i.e., $K^j = K^s = K/2$. Following the empirical results in Schaefer and Strebulaev[45], we set the leverage ratio L and equity volatility σ^E as 0.10, 0.32, and 0.50 and 25%, 31%, and 42% for AAA, A, and BB credit rating firms, respectively. The rest of the parameter values are $r = 5\%$, $T - t = 5$, $w^j = 82.9\%$, and $w^s = 50.9\%$ (the loss given default w follows [40]).

Figure 3: Effect of Fraud on Debt Value (SD) - Fraud size, δ, versus junior/senior zero coupon debt in the left/right panel using the Subordinated Debt model. The junior and senior debts have the same maturity date. The quasi-leverage L is defined as quasi market value of debt to the quasi asset value, i.e., $L = (e^{-r(T-t)}F)/(EQ_t + e^{-r(T-t)}F)$. The initial equity value is 100, we use the quasi-leverage ratio to compute the corresponding debt face value F. The face value for junior debt F^j and senior debt F^s are the same, i.e., $F^j = F^s = F/2$. Following the empirical results in Schaefer and Strebulaev[45], we set the leverage ratio L and equity volatility σ^E as 0.10, 0.32, and 0.50 and 25%, 31%, and 42% for AAA, A, and BB credit rating firms, respectively. The rest of the parameter values are $r = 5\%$, and $T - t = 5$.

We plot the fraud size δ versus the value of junior and senior debt (normalized to face value) using the First Passage Time (FPT) model and the Subordinated Debt (SD) model (see Appendix E) in Figures 2 and 3, respectively. In both models, we set the initial equity value as 100, and use the quasi-leverage ratio to compute the corresponding default boundary/face value. The quasi-leverage ratio L is defined as the quasi market value of debt to the quasi asset value, i.e., $L = (e^{-r(T-t)}K)/(EQ_t + e^{-r(T-t)}K)$ and $L = (e^{-r(T-t)}F)/(EQ_t + e^{-r(T-t)}F)$ for the FPT and the SD models, respectively.

Following the empirical study of Schaefer and Strebulaev[45], we set the leverage ratio and equity volatility as 0.10, 0.32, and 0.50 and 25%, 31%, and 42% for AAA, A, and BB credit-rated firms, respectively. The constant risk-free rate is 5%, the time to maturity of the debts is 5 years and the losses given default are $w^j = 82.9\%$ and $w^s = 50.9\%$, respectively. Given the equity value and corresponding equity volatility, the firm value and firm volatility are computed using the traditional method. The true value of firm is computed using the non-linear system (given in the traditional method) by using the true equity value and the firm volatility. The effect of fraud on debt value can be computed by taking the difference in debt values when $\delta = 1$ and $\delta \neq 1$.

In the FPT model we see that junior debt value is more sensitive to fraud size than senior debt value, with this sensitivity increasing with leverage. With a low-leverage firm (L = 0.10), the likelihood of default is so low that the fraud sizes considered here have no effect on the debt values.

Using the SD model that includes both junior and senior debts, we find that the effect of fraud size on debt value increases with leverage and that junior debt value is more sensitive to fraud size than senior debt value (in accordance with our findings with the FPT model). The latter observation is a result of the absolute priority rule in allocating losses, with the junior debt serving as a loss-absorbing cushion for the senior debt. Comparing results between the two models, we see that debt values in the FPT model are more sensitive to fraud size than those from the SD model. This is due to a combination of factors including i) the bankruptcy costs; ii) violation of the absolute priority rule; and iii) and the possibility of default prior to bond maturity.

4.2. Calculating the Damages Ribbon for Debt

In the previous sections, we have shown that misrepresentation not only inflates/deflates the underlying share price but also inflates/deflates the debt value of the firm during the class period. In Section 4.2.1, we propose a method to construct the debt value line (equivalently the damages ribbon) based on the share price value line. We demonstrate the proposed method with a case study in Section 4.2.2

4.2.1. Methodology for Debt Damages Ribbon

Given the share price value line we propose a method for constructing the debt value line (equivalently the damages ribbon for debt). The true value of a bond on a date *i* during the class period can be calculated using the following steps:

1 Calculate the true value of equity $\widetilde{EQ}_i$ using the share price value line as described in Section 1.1;

2 Calculate the firm value V_i^F and firm value volatility σ_i^F using

  a the traditional method and the equity value EQ_i and equity volatility σ_i^E; or

  b using the MLE method and EQ^{TS}, which is the time series of equity value from a given sample window;

3 Compute the true firm value $\tilde{V}_i^F = v_{\widetilde{EQ}_i}(\sigma_i^F)$;

4 Calculate the bond values $B(V_i^F, \sigma_i^F)$ and $B(\tilde{V}_i^F, \sigma_i^F)$ based on the firm values V_i^F and $\tilde{V}_i^F$, respectively, and hence the date-*i* damages ribbon $R_i^B = B(V_i^F, \sigma_i^F) - B(\tilde{V}_i^F, \sigma_i^F)$.

When calculating the share price value line, the econometric model in Section 1.1 measures the effect of fraud related information on the stock price and the effect of non-fraud related information is filtered out by the model. Since the debt value line is calculated based on the share price value line, the debt value line preserves this advantage — the debt damages ribbon reflects the bond value change due to the fraud related information. Additionally, the debt value line/damages ribbon provides a methodology for assessing debt damages that is consistent with the standard method for computing equity damages. This is important in the assessment of damages because some jurisdictions (e.g., Ontario Securities Act Section 138.5(1)) require that damages are calculated taking into account the result of hedging and other risk limitation transactions. Moreover, it is straightforward to implement this methodology as it only requires observable information (e.g., equity value, treasury bond yield and corporate financial statements). A limitation of the methodology is that the structural model used may not be rich enough to sufficiently capture all relevant features of the capital structure (e.g., callable/convertible features of bonds) and the bankruptcy process — this limitation generally arises in any modelling exercise. Additionally other factors such as liquidity, call and conversion features and taxes are also the key determinants of corporate bond prices (see [18], [8], [4], and [27]). By using a structural model to compute the debt damages ribbon, the price change reflects the effect of fraud on bond value only due to changes in credit risk. This approach does not reflect how fraud may influence the other factors (if at all) that determine corporate bond prices. In the following section, the proposed methodology is demonstrated using a recent securities class action case.

4.2.2. The Agnico-Eagle Mines Ltd. Case Study

Agnico-Eagle Mines Ltd. (AEM) is a Canadian based mining company with operations in Canada, Finland, Mexico and the United States. In March 2012, secondary market securities class actions were filed in Ontario and Quebec against AEM and certain of AEM's current and former officers and directors. The actions allege that AEM failed to disclose a water inflow issue at its Goldex mine (in Quebec) during the class period — March 26, 2010 to October 18, 2011 (indicated by the vertical red lines in Figure 4). In Figure 4, the stock price drops significantly on the fraud disclosure date (October 18, 2011) indicating that the share price was inflated during the class period. The common share value line was calculated using the event study approach as described in Section 1.1. The S&P/TSX Composite Total Return Index and the S&P/TSX Gold Total Return Index are used as the market return R_{Mt} and the industry return R_{It}, respectively. Two years of daily observations from January 04, 2010 to December 30, 2011 are used to estimate the parameters in the econometric model. On October 18, 2011, there was a price change of \$10.59 of which \$7.03 was attributed to the revelation of fraud after controlling for market and industry factors.

Figure 4: Common Shares Value Line - The period between the two vertical lines is the class period. The blue line shows the historical stock price of AEM from January 2010 to December 2011. The value line of stock, which is represented by the red dash line, is constructed by the event study approach (see Section 1.1).

Figure 5: Notes Damages Ribbon - The graph shows the damages ribbon of notes during the period from April 7, 2010 to October 18, 2011.

During the class period the financial statements of AEM indicate that long-term liabilities include two types of long-term debts — bank credit facilities and notes (see [1]). A credit facility is a loan in the form of revolving credit, in which the customer is allowed to borrow/repay funds as needed. Short-term liabilities and the other long-term liabilities (e.g., deferred income and mining tax liabilities) are ignored in our analysis. AEM equity was comprised of 156.7 million common shares, 8.4 million employee stock options, and 8.6 million warrants on March 31, 2010, the start of the class period.[13] Here we make the simplification that AEM equity is comprised

[13] During the class period, the numbers of common shares and employee stock options changed, while the number of warrants was unchanged. The number of common shares increased from 156.7 million to 169.2 million.

solely of common shares, giving a capital structure for our model consisting of common shares and two types of long-term debt.

On June 22, 2010 (during the class period), AEM amended its credit facilities: the amount available increased to \$1.2 billion and the maturity date extended to June 22, 2014. Details of the interest rate paid on the credit facilities are not available. On April 7, 2010, the company closed a note offering with institutional investors in the U.S. and Canada for a private placement of \$600 million of guaranteed senior unsecured notes due in 2017, 2020 and 2022. The notes had a weighted average maturity of 9.84 years and weighted average yield of 6.59% at issuance. Proceeds from the notes were used to repay amounts owed under the company's then outstanding credit facilities. The credit facilities and the notes rank equally in seniority. Details of the coupon payments and face value of the notes proved difficult to obtain. For this case study, we treat the notes as a single coupon bond with the weighted average maturity and calculate its damages ribbon during the class period. As the outstanding balance of the credit facilities changes dramatically during the class period, we refrain from computing its damages ribbon.

In order to use our methodology, we consider AEM's two types of long-term debt as two coupon bonds. The first bond corresponds to the credit facilities and is assigned a face value of \$69.29 million — the weighted average amount drawn from the credit facilities during the class period. The maturity date of this bond is June 22, 2014. The second bond corresponds to the notes and is given a face value of \$600 million — the total face value of the issued notes. The maturity date of this bond is Feb 8, 2020, the weighted average maturity date of the three notes. Since details of the interest and coupon payments are not available, both bonds are assumed to pay annual coupons of 6.59% (the average yield of the notes), with coupon payments made at the end of each year.

For the other model parameters, the risk-free rate is taken as 3.72%, the average U.S. Treasury long-term composite rate[14] from April 7, 2010 to October 18, 2011. On April 7, 2010, the weighted average time to maturity is 7.4 years, and hence the weighted average maturity date of debts, $\bar{T}$, is September 02, 2017. The default boundary, K, is \$1070.56 million and is assumed constant during the class period (see Section 2.1 for details on calculating the default boundary). The loss given default, w, is 62.6%, which follows Ou, Chiu and Metz [40] who estimated the value-weighted recovery rate for senior unsecured bond as 37.4%. Given the time series of equity value[15] from April 7, 2010 to October 18, 2011, the firm value volatility estimate is $\hat{\sigma}^F = 35.24\%$ (using the MLE method). Using $\hat{\sigma}^F$, the time series of equity value, and the value line of equity value, the time series of firm value V_i^F (using the observed share price) and the true firm value $\tilde{V}_i^F$ (using the share price value line) are calculated. From these time series, bond prices and hence the notes damages ribbon are computed according to the methodology in Section 4.2.1. The notes damages ribbon is shown in Figure 5 and can be used to assess damages in a securities class action.

5. Conclusion

Using an extended Black and Cox capital structure modelling framework and a connection between the observable share price and firm value we connect the impact of an observable fraud-induced share price change on the debt value. Generally, debt for higher-leverage firms is more

[14] The long-term composite rate is the unweighted average of bid yields on all outstanding fixed-coupon bonds neither due nor callable in less than 10 years. Data can be found on the website of the U.S. Department of the Treasury.

[15] The daily equity value is the product of the daily close price and the daily current shares outstanding, which are obtained from Bloomberg.

sensitive to the fraud size than lower-leverage firms and junior debt is more affected by fraud size than senior debt.

This study proposes a methodology to compute damages in securities class actions for investors with debt positions in the fraud-committing company. This work is relevant not only for estimating potential damages, but in the fair allocation of damage awards across holders of the fraudulent firm's securities. The legal requirement that damages due to fraud must be computed net of any hedge or risk limitation transaction underscores the importance of the work presented here. For example, one can hedge a long bond position by shorting the shares of the bond issuer. This methodology allows one to compute fraud-induced share and debt value changes in a consistent manner.

In addition to our main findings, we explicitly discuss bankruptcy costs for the First Passage Time model. Furthermore, we are able to reduce a system of two non-linear equations, used to connect the unobservable firm value and firm value volatility to observable equity value and equity volatility, into one equation. This technique improves the ability to solve the non-linear system.

There are many different avenues for future research. Extending the modelling framework to more general capital structures (e.g., include preferred shares, warrants, and employee stock options) and to incorporate the callable/convertible features in the bonds are obvious directions to pursue. Using this extended modelling framework, an empirical analysis of the model performance would be required to see how well the model-predicted security value changes match the observed changes. Additionally, one could investigate if misrepresentation is a partial explanation of the credit spread puzzle as analyzed in Huang and Huang [27].

Appendix A. Formula of European Down-and-Out Call Options

Under this modelling framework, the value of a European down-and-out call option written on V^F with barrier K_B and strike price K_S $(K_B \leq K_S)$ is

$$\begin{aligned} C_{DO} =& V_t^F \Phi(x_1) - K_S e^{-r(T-t)} \Phi(x_1 - \sigma^F \sqrt{T-t}) - V_t^F (K_B/V_t^F)^{2\lambda} \Phi(y_1) \\ &+ K_S e^{-r(T-t)} (K_B/V_t^F)^{2\lambda-2} \Phi(y_1 - \sigma^F \sqrt{T-t}), \end{aligned} \tag{A.1}$$

where

$$\lambda = (r + {\sigma^F}^2/2)/{\sigma^F}^2, \tag{A.2}$$

$$x_1 = \left(\log(V_t^F/K_S) + (r + {\sigma^F}^2/2)(T-t) \right)/(\sigma^F \sqrt{T-t}), \tag{A.3}$$

and

$$y_1 = \left(\log(K_B^2/(V_t^F K_S)) + (r + {\sigma^F}^2/2)(T-t) \right)/(\sigma^F \sqrt{T-t}). \tag{A.4}$$

If $K_B \geq K_S$, the value is

$$\begin{aligned} C_{DO} =& V_t^F \Phi(x_2) - K_S e^{-r(T-t)} \Phi(x_2 - \sigma^F \sqrt{T-t}) - V_t^F (K_B/V_t^F)^{2\lambda} \Phi(y_2) \\ &+ K_S e^{-r(T-t)} (K_B/V_t^F)^{2\lambda-2} \Phi(y_2 - \sigma^F \sqrt{T-t}), \end{aligned} \tag{A.5}$$

where

$$x_2 = \left(\log(V_t^F/K_B) + (r + {\sigma^F}^2/2)(T-t) \right)/(\sigma^F \sqrt{T-t}), \tag{A.6}$$

and

$$y_2 = \left(\log(K_B/V_t^F) + (r + {\sigma^F}^2/2)(T-t) \right)/(\sigma^F \sqrt{T-t}). \tag{A.7}$$

Appendix B. Proof of Proposition 1

Proof. We only show the proof of the case where $K_B \leq K_S$ and EQ_t is given by equation (A.1). When $K_B \geq K_S$, the proof is similar to the case where $K_B \leq K_S$.

To show that EQ_t is a one-to-one function of V_t^F, we show that the following statements are true:

1 EQ_t approaches zero as the firm value V_t^F approaches the default boundary K_B.

2 EQ_t approaches infinity as the firm value approaches infinity.

3 EQ_t is an increasing function of V_t^F.

To simplify notation, we let $\bar{x}_1 = x_1 - \sigma^F \sqrt{T-t}$ and $\bar{y}_1 = y_1 - \sigma^F \sqrt{T-t}$.

To show statement one is true, note that as V_t^F approaches K_B, K_B/V_t^F and V_t^F/K_B approach one, and the differences between x_1 and y_1, and $\bar{x}_1$ and $\bar{y}_1$ approach zero, where x_1 and y_1 are given by equations (A.3) and (A.4), respectively. By equation (A.1), the limiting value of equity is

$$\lim_{V_t^F \to K_B} EQ_t = K_B \Phi(x_1) - K_S e^{-r(T-t)} \Phi(\bar{x}_1) - K_B 1^{2\lambda} \Phi(x_1) + K_S e^{-r(T-t)} 1^{2\lambda-2} \Phi(\bar{x}_1) = 0.$$

To show that the second statement is true, note that as V_t^F approaches infinity, x_1 and $\bar{x}_1$ approach infinity, and y_1 and $\bar{y}_1$ approach negative infinity. So $\Phi(x_1)$ and $\Phi(\bar{x}_1)$ approach one, and $\Phi(y_1)$ and $\Phi(\bar{y}_1)$ approach zero. From equation (A.1) the equity value is

$$\begin{aligned} EQ_t &= V_t^F \Phi(x_1) - K_S e^{-r(T-t)} \Phi(\bar{x}_1) - V_t^F (K_B/V_t^F)^{2\lambda} \Phi(y_1) + K_S e^{-r(T-t)} (K_B/V_t^F)^{2\lambda-2} \Phi(\bar{y}_1) \\ &\geq V_t^F \Phi(x_1) - K_S e^{-r(T-t)} \Phi(\bar{x}_1) - K_B^{2\lambda} {V_t^F}^{1-2\lambda} \Phi(y_1). \end{aligned}$$

By the definition of λ (see equation(13)) we have

$$1 - 2\lambda = -\frac{2r}{{\sigma^F}^2}, \tag{B.1}$$

which is negative, and so when V_t^F approaches infinity, ${V_t^F}^{1-2\lambda}$ approaches zero. Hence,

$$\begin{aligned} \lim_{V_t^F \to \infty} EQ_t &\geq \lim_{V_t^F \to \infty} [V_t^F \Phi(x_1) - K_S e^{-r(T-t)} \Phi(\bar{x}_1) - K_B^{2\lambda} {V_t^F}^{1-2\lambda} \Phi(y_1)] \\ &= \infty - K_S e^{-r(T-t)} - 0 = \infty. \end{aligned}$$

For statement three, we show that EQ_t is an increasing function of V_t^F by showing its derivative w.r.t. V_t^F is positive. Taking the partial derivative we have

$$\begin{aligned} \frac{\partial EQ_t}{\partial V_t^F} =& (V_t^F \Phi(x_1) - K_S e^{-r(T-t)} \Phi(\bar{x}_1))' - (V_t^F (K_B/V_t^F)^{2\lambda} \Phi(y_1) - K_S e^{-r(T-t)} (K_B/V_t^F)^{2\lambda-2} \Phi(\bar{y}_1))' \\ =& (V_t^F \Phi(x_1) - K_S e^{-r(T-t)} \Phi(\bar{x}_1))' - ((V_t^F (K_B/V_t^F)^{2\lambda})' \Phi(y_1) + V_t^F (K_B/V_t^F)^{2\lambda} \Phi'(y_1) \\ & - K_S e^{-r(T-t)} ((K_B/V_t^F)^{2\lambda-2})' \Phi(\bar{y}_1) - K_S e^{-r(T-t)} (K_B/V_t^F)^{2\lambda-2} \Phi'(\bar{y}_1)), \end{aligned} \tag{B.2}$$

where $(\cdot)'$ denotes differentiation with respect to V_t^F. The first term in equation (B.2) is the delta of the European call option written on firm value and struck at K_S in the Black-Scholes [6] model, and is equal to $\Phi(x_1)$. It can be shown through straightforward and tedious calculation that

$$(V_t^F (K_B/V_t^F)^{2\lambda}\Phi'(y_1))/(K_S e^{-r(T-t)}(K_B/V_t^F)^{2\lambda-2}\Phi'(\bar{y}_1)) = 1.$$

Noting that $V_t^F (K_B/V_t^F)^{2\lambda}\Phi'(y_1) = K_S e^{-r(T-t)}(K_B/V_t^F)^{2\lambda-2}\Phi'(\bar{y}_1)$ equation (B.2) becomes

$$\begin{aligned}\frac{\partial EQ_t}{\partial V_t^F} =&\Phi(x_1) - ((V_t^F (K_B/V_t^F)^{2\lambda})'\Phi(y_1) - K_S e^{-r(T-t)}((K_B/V_t^F)^{2\lambda-2})'\Phi(\bar{y}_1))\\ =&\Phi(x_1) - ((1-2\lambda)(K_B/V_t^F)^{2\lambda}\Phi(y_1) - (2-2\lambda)e^{-r(T-t)}(K_B/V_t^F)^{2\lambda-1}\Phi(\bar{y}_1)(K_S/K_B))\\ =&\Phi(x_1) + (2r/\sigma^{F2})(K_B/V_t^F)^{2\lambda}\Phi(y_1) + (1-2r/\sigma^{F2})(K_B/V_t^F)^{2\lambda-1}e^{-r(T-t)}\Phi(\bar{y}_1)(K_S/K_B)\end{aligned} \tag{B.3}$$

If $1-2r/\sigma^{F2} \geq 0$, the derivative in equation (B.3) positive, because all the terms are positive. So we only need to show that equation (B.3) is positive when $1-2r/\sigma^{F2} < 0$. Since $V_t^F \geq K_B$ and $K_S \geq K_B$, we have $x_1 \geq y_1$ by $(V_t^F/K_S)/(K_B^2/(V_t^F K_S)) \geq 1$. Hence $\Phi(x_1) \geq \Phi(y_1) \geq \Phi(\bar{y}_1)$. Combining this and the assumption of $1-2r/\sigma^{F2} < 0$ and $K_S/K_B \geq 1$, we have

$$\begin{aligned}\frac{\partial EQ_t}{\partial V_t^F} \geq&\Phi(\bar{y}_1) + (2r/\sigma^{F2})(K_B/V_t^F)^{2\lambda}\Phi(\bar{y}_1) + (1-2r/\sigma^{F2})(K_B/V_t^F)^{2\lambda-1}\Phi(\bar{y}_1)\\ =&\Phi(\bar{y}_1)(K_B/V_t^F)^{2\lambda}((V_t^F/K_B)^{2\lambda} + 2r/\sigma^{F2} + (1-2r/\sigma^{F2})(V_t^F/K_B))\end{aligned} \tag{B.4}$$

Expanding $(V_t^F/K_B)^{2\lambda}$ around the point 1 we get

$$(V_t^F/K_B)^{2\lambda} = 1^{2\lambda} + 2\lambda 1^{2\lambda-1}(V_t^F/K_B - 1) + R_2,$$

where R_2 is the reminder term given by

$$R_2 = \frac{2\lambda(2\lambda-1)\xi^{2\lambda-2}}{2!}(V_t^F/K_B - 1)^2,$$

with $\xi \in [1, V_t^F/K_B]$. Since $R_2 \geq 0$, we have

$$(V_t^F/K_B)^{2\lambda} \geq 1 + 2\lambda(V_t^F/K_B - 1),$$

and hence

$$\begin{aligned}\frac{\partial EQ_t}{\partial V_t^F} \geq&\Phi(\bar{y}_1)(K_B/V_t^F)^{2\lambda}(1 + 2\lambda(V_t^F/K_B - 1) + 2r/\sigma^{F2} + (1-2r/\sigma^{F2})(V_t^F/K_B))\\ =&\Phi(\bar{y}_1)(K_B/V_t^F)^{2\lambda}(1 + \frac{(2r+\sigma^{F2})(V_t^F - K_B)}{\sigma^{F2}K_B} + \frac{2rK_B}{\sigma^{F2}K_B} + \frac{(\sigma^{F2}-2r)V_t^F}{\sigma^{F2}K_B})\\ \geq&\Phi(\bar{y}_1)(K_B/V_t^F)^{2\lambda}(1 + \frac{2r(V_t^F - K_B) + 2rK_B + (\sigma^{F2}-2r)V_t^F}{\sigma^{F2}K_B})\\ =&\Phi(\bar{y}_1)(K_B/V_t^F)^{2\lambda}(1 + V_t^F/K_B) > 0.\end{aligned}$$

□

Appendix C. Proof of Theorem1

Proof. Given EQ_t and σ^E, if $\bar{\sigma}^F$ satisfies equation (26), the firm value $\bar{V}_t^F = v_{EQ_t}(\bar{\sigma}^F)$ and $\bar{\sigma}^F$ satisfy equation (23) by the definition of function v_{EQ_t}. Since equation (26) is derived by combining (23) and (24), $\bar{\sigma}^F$ and $\bar{V}_t^F$ must satisfy equation (24).

If $\bar{\sigma}^F$ and $\bar{V}_t^F$ satisfy the non-linear system (23) and (24), $\bar{V}_t^F = v_{EQ_t}(\bar{\sigma}^F)$ by the definition of function v_{EQ_t}. So $\bar{\sigma}^F$ satisfies equation (26). $\square$

Appendix D. The Maximum Likehood Method for FPT Model

In this section we provide the details of the MLE method introduced in [14], [15], [16] and [20]. Given the time series of equity value $EQ^{TS} \equiv \{EQ_i^{ts} : i = 1, 2, \ldots, n\}$, firm volatility σ^F can be estimated by MLE, and hence the firm value V_t^F can be computed using the function $V_t^F = v_{EQ_t}(\sigma^F)$ which is defined in Theorem 1.

Let $f(\cdot)$ be the conditional density of EQ_i^{ts} given EQ_{i-1}^{ts}, the log-likelihood function for vector EQ^{TS} is

$$L_{EQ}(EQ^{TS}; \sigma^F, \lambda^v) = \sum_{i=2}^{n} \log f(EQ_i^{ts}|EQ_{i-1}^{ts}; \sigma^F, \lambda^v), \tag{D.1}$$

where λ^v is the market price of risk given in the firm dynamic (see equation (5)). From the dynamic of V_t^F, it is easy to show that logarithm of V_t^F is normally distributed, and its conditional density function $g(\cdot)$ is given by

$$g(\log V_i^F | \log V_{i-1}^F; \sigma^F, \lambda^v) = \frac{1}{\sqrt{2\pi s_i^2}} \exp\Big(-\frac{(\log V_i^F - m_i)^2}{2s_i^2} \Big), \tag{D.2}$$

where

$$m_i = \log V_{i-1}^F + (r + \lambda^v \sigma^F - 0.5{\sigma^F}^2)\Delta t, \tag{D.3}$$

$$s_i = \sigma^F \sqrt{\Delta t}, \tag{D.4}$$

and $\Delta t = t_i - t_{i-1}$. By using the fact that V_t^F is a one-to-one and hence invertible function of EQ_t (see Proposition 1), the conditional density $f(\cdot)$ can be written as

$$f(EQ_i^{ts}|EQ_{i-1}^{ts}; \sigma^F, \lambda^v) = g(\log V_i^F | \log V_{i-1}^F; \sigma^F, \lambda^v)\Big|_{V_i^F = v_{EQ_i^{ts}}(\sigma^F)} \times \Big(\frac{\partial EQ_i}{\partial \log V_i^F}\Big|_{V_i^F = v_{EQ_i^{ts}}(\sigma^F)} \Big)^{-1}. \tag{D.5}$$

Substituting equations (B.3) and (D.5) into the log-likelihood function (D.1) gives

$$\begin{aligned} L_{EQ}(EQ^{TS}; \sigma^F, \lambda^v) &= \sum_{i=2}^{n} \Big(\log g(\log V_i^F | \log V_{i-1}^F; \sigma^F, \lambda^v)\Big|_{V_i^F = v_{EQ_i^{ts}}(\sigma^F)} - \log \frac{\partial EQ_i}{\partial \log V_i^F}\Big|_{V_i^F = v_{EQ_i^{ts}}(\sigma^F)} \Big) \\ &= \sum_{i=2}^{n} \Big(-\frac{1}{2}\log(2\pi s_i^2) - \frac{(\log V_i^F - m_i)^2}{2s_i^2} \\ &\quad - \log\big(V_i^F(\Phi(d_1) + (2r/{\sigma^F}^2)(K/V_i^F)^{2\lambda}\Phi(d_3) \\ &\quad + (1 - 2r/{\sigma^F}^2)(K/V_i^F)^{2\lambda - 1} e^{-r(T-t_i)}\Phi(d_4))\big)\Big)\Big|_{V_i^F = v_{EQ_i^{ts}}(\sigma^F)}. \end{aligned} \tag{D.6}$$

Taking into account the survivorship issue [16], the log-likelihood function for the First Passage Time model is

$$L_{EQ}^{FPT}(EQ^{TS};\sigma^F,\lambda^v) = L_{EQ}(EQ^{TS};\sigma^F,\lambda^v) + \log\Big(\Pr(\mathcal{D}_n | v_{EQ_0^{ts}}(\sigma^F), v_{EQ_1^{ts}}(\sigma^F),\cdots, v_{EQ_n^{ts}}(\sigma^F))\Big) - \log(\Pr(\mathcal{D}_n)), \tag{D.7}$$

where $\mathcal{D}_i$ is the event that the firm does not default up to time t_i and $\Pr(\cdot)$ is the probability under the physical measure. The expressions of second term and third term of (D.7) is given by[16]

$$\begin{aligned}&\log\Big(\Pr(\mathcal{D}_n | v_{EQ_0^{ts}}(\sigma^F), v_{EQ_1^{ts}}(\sigma^F),\cdots, v_{EQ_n^{ts}}(\sigma^F))\Big)\\ &= \sum_{i=2}^{n}\log\Big(1-\exp\big(\frac{2}{{\sigma^F}^2\Delta t}\log(\frac{v_{EQ_i^{ts}}(\sigma^F)}{K})\log(\frac{K}{v_{EQ_{i-1}^{ts}}(\sigma^F)})\big)\Big),\end{aligned} \tag{D.8}$$

and

$$\begin{aligned}\log(\Pr(\mathcal{D}_n)) = \log\Bigg(&\Phi\Big(\frac{(r+\lambda^v\sigma^F-\frac{{\sigma^F}^2}{2})n\Delta t - \log(\frac{K}{v_{EQ_0^{ts}}(\sigma^F)})}{\sqrt{n\Delta t}\sigma^F}\Big)\\ &-\exp\Big(\frac{2(r+\lambda^v\sigma^F-\frac{{\sigma^F}^2}{2})}{{\sigma^F}^2}\log\big(\frac{K}{v_{EQ_0^{ts}}(\sigma^F)}\big)\Big)\Phi\Big(\frac{(r+\lambda^v\sigma^F-\frac{{\sigma^F}^2}{2})n\Delta t + \log\big(\frac{K}{v_{EQ_0^{ts}}(\sigma^F)}\big)}{\sqrt{n\Delta t}\sigma^F}\Big)\Bigg).\end{aligned} \tag{D.9}$$

Maximizing the log-likelihood function (D.7) gives the estimators $\hat{\sigma}^F$ and $\hat{\lambda}^v$ from which we get the estimated firm value $\hat{V}_t^F = v_{EQ_t}(\hat{\sigma}^F)$.

Appendix E. The Subordinated Debt Model

In this section we briefly introduce the Subordinated Debt model under Merton's framework.[17] Subordinated debt has a lower priority than senior debt at the time of liquidation and the absolute priority rule holds. Let D^S and D^J denote the senior and subordinated debt values, respectively. Assume that both types of debt are zero-coupon and have the same maturity date T, which is also the liquidation date. The time-T payoffs for senior debt, subordinated debt, and equity on date T are given in Table E.1. F^S and F^J are the face values of senior and junior debt,

Table E.1: Time-T Payoff

	Pay-off at maturity date T
Equity	$\max\{V_T^F - F^S - F^J, 0\}$
JuniorDebt	$\min\{\max\{V_T^F - F^S, 0\}, F^J\}$
SeniorDebt	$\min\{V_T^F, F^S\}$

respectively. We can evaluate the time-t prices based on the payoffs given in Table E.1. The

[16] Derivation of these equations can be found in [16]

[17] Black and Cox [5] price subordinated bonds using a very similar modelling set up.

equity payoff at time T is the same as a European call option on V^F with strike $F^S + F^J$. Thus the time-t equity value can be written as

$$EQ_t = C(V_t^F, F^J + F^S, \sigma^F, r, T - t), \tag{E.1}$$

where $C(\cdot)$ is the Black-Scholes formula for a European call option. It is easily shown that the payoff of the sum of junior debt and equity is the same as a European call on V^F with strike price F^S, i.e.,

$$D_t^J + EQ_t = C(V_t^F, F^S, \sigma^F, r, T - t). \tag{E.2}$$

Hence the value of senior debt D_t^S is

$$\begin{aligned} D_t^S &= V_t^F - (EQ_t + D_t^J) \\ &= V_t^F - C(V_t^F, F^S, \sigma^F, r, T - t). \end{aligned} \tag{E.3}$$

The junior debt position can also be expressed as a call bull spread, which has value

$$\begin{aligned} D_t^J &= V_t^F - D_t^S - EQ_t \\ &= C(V_t^F, F^S, \sigma^F, r, T - t) - C(V_t^F, F^S + F^J, \sigma^F, r, T - t). \end{aligned} \tag{E.4}$$

In the Merton model, the equity value EQ_t and equity volatility σ^E are connected to the firm value V_t^F and firm value volatility σ^F though the following non-linear system:

$$\sigma^E = \sigma^F \frac{\Phi(k_1) V_t^F}{EQ_t}, \quad \text{and} \tag{E.5}$$

$$EQ_t = C(V_t^F, F, \sigma^F, r, T - t), \tag{E.6}$$

where

$$k_1 = \left(\log(V_t^F / (F^J + F^S)) + (r + {\sigma^F}^2/2)(T - t) \right) / (\sigma^F \sqrt{T - t}). \tag{E.7}$$

As with the FPT model, this system of equations can be simplified to a single equation. Note that one can also use the MLE method for this model.

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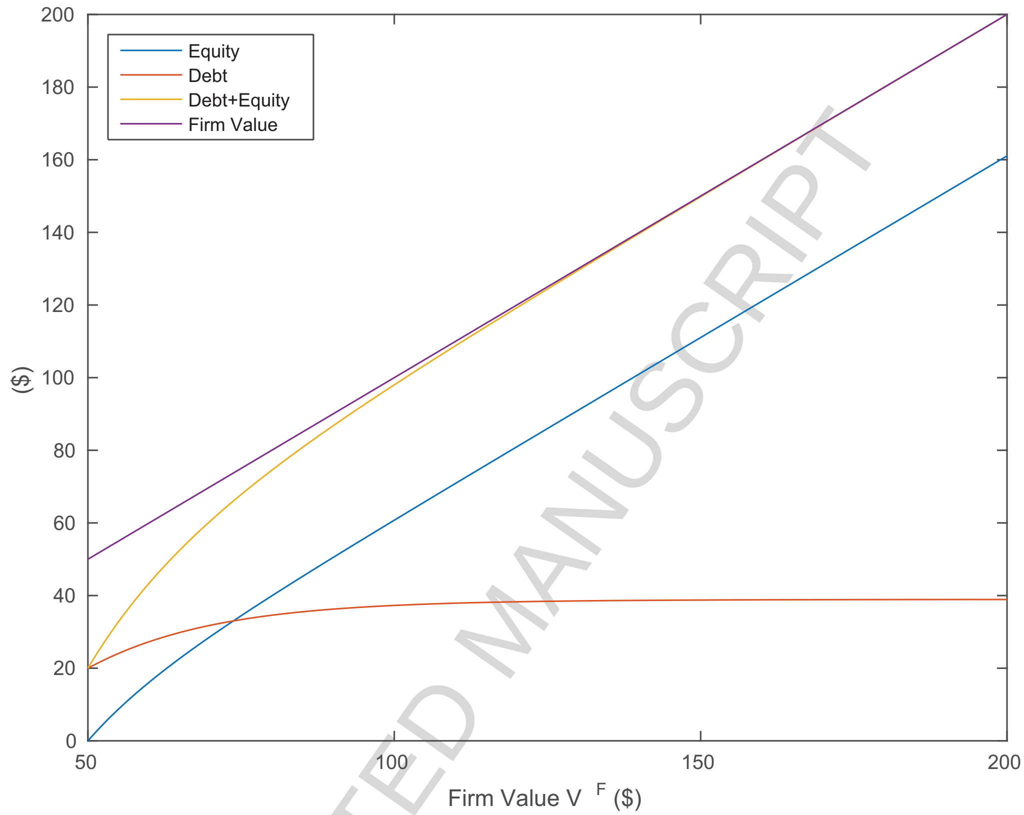

Figure 1: Firm Value and Capital Structure - Firm value V_t^F versus equity value EQ_t, debt value D_t and the sum of EQ_t and D_t using the model in Section 2. The bankruptcy costs BC_t are the difference between V_t^F and $D_t + EQ_t$. Parameters are: $\sigma^F = 21\%$, $r = 5\%$, $T - t = 5$, $K = 50$, and $w = 48.67\%$ (parameter values r and w are choosen following Huang and Huang[27]).

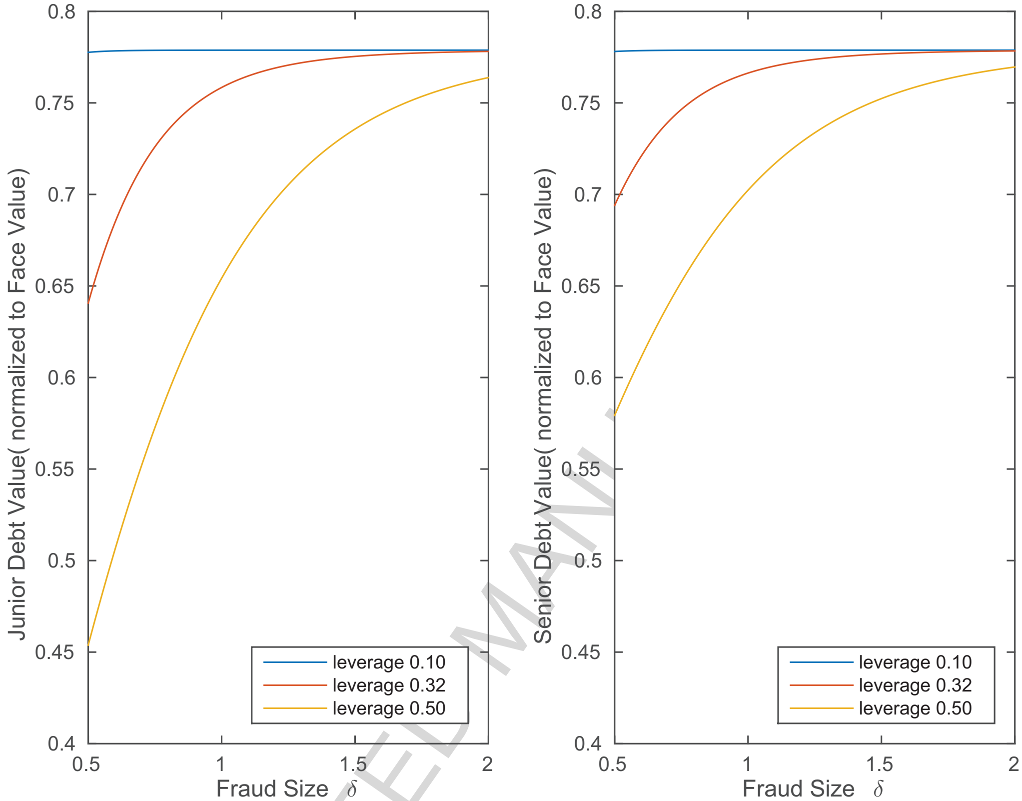

Figure 2: Effect of Fraud on Debt Value (FPT) - Fraud size, δ, versus junior/senior zero coupon debt in the left/right panel using the First Passage Time model in Section 2. The junior and senior debts have the same maturity date. The quasi-leverage L is defined as quasi market value of debt to the quasi asset value, i.e., $L = (e^{-r(T-t)}K)/(EQ_t + e^{-r(T-t)}K)$. The initial equity value is 100, we use the quasi-leverage ratio to compute the corresponding debt face value K. The face value for junior debt and senior debt are the same, i.e., $K^j = K^s = K/2$. Following the empirical results in Schaefer and Strebulaev[45], we set the leverage ratio L and equity volatility σ^E as 0.10, 0.32, and 0.50 and 25%, 31%, and 42% for AAA, A, and BB credit rating firms, respectively. The rest of the parameter values are $r = 5\%$, $T - t = 5$, $w^j = 82.9\%$, and $w^s = 50.9\%$ (the loss given default w follows [40]).

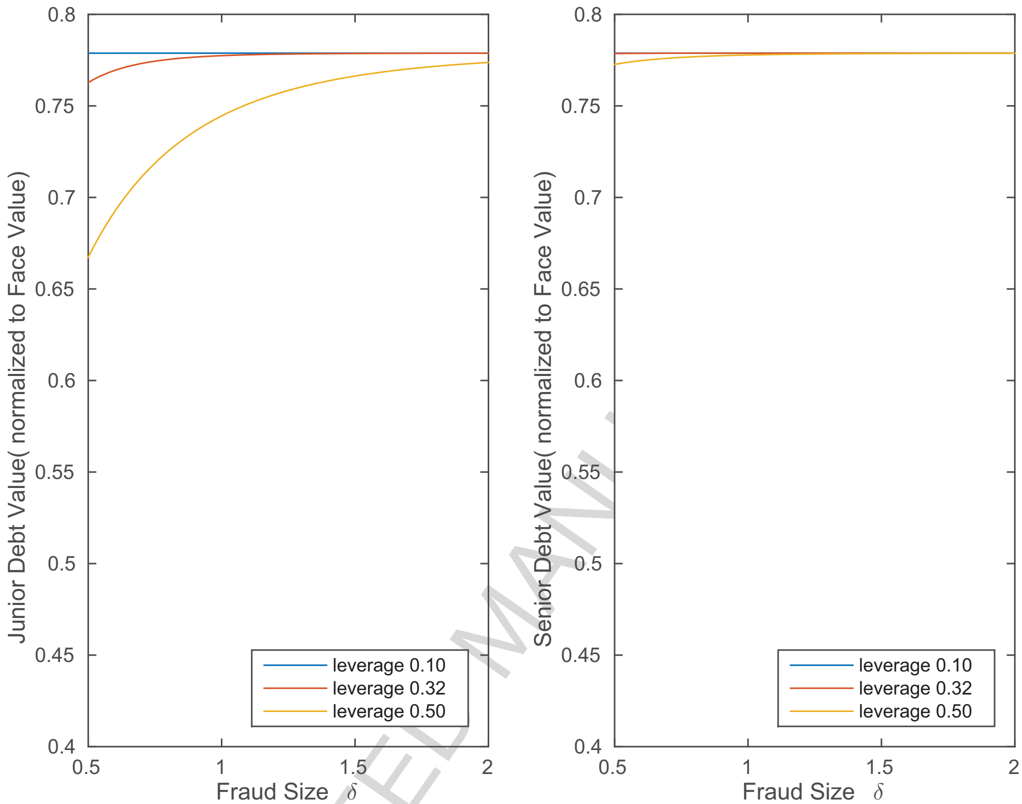

Figure 3: Effect of Fraud on Debt Value (SD) - Fraud size, δ, versus junior/senior zero coupon debt in the left/right panel using the Subordinated Debt model. The junior and senior debts have the same maturity date. The quasi-leverage L is defined as quasi market value of debt to the quasi asset value, i.e., $L = (e^{-r(T-t)}F)/(EQ_t + e^{-r(T-t)}F)$. The initial equity value is 100, we use the quasi-leverage ratio to compute the corresponding debt face value F. The face value for junior debt F^j and senior debt F^s are the same, i.e., $F^j = F^s = F/2$. Following the empirical results in Schaefer and Strebulaev[45], we set the leverage ratio L and equity volatility σ^E as 0.10, 0.32, and 0.50 and 25%, 31%, and 42% for AAA, A, and BB credit rating firms, respectively. The rest of the parameter values are $r = 5\%$, and $T - t = 5$.

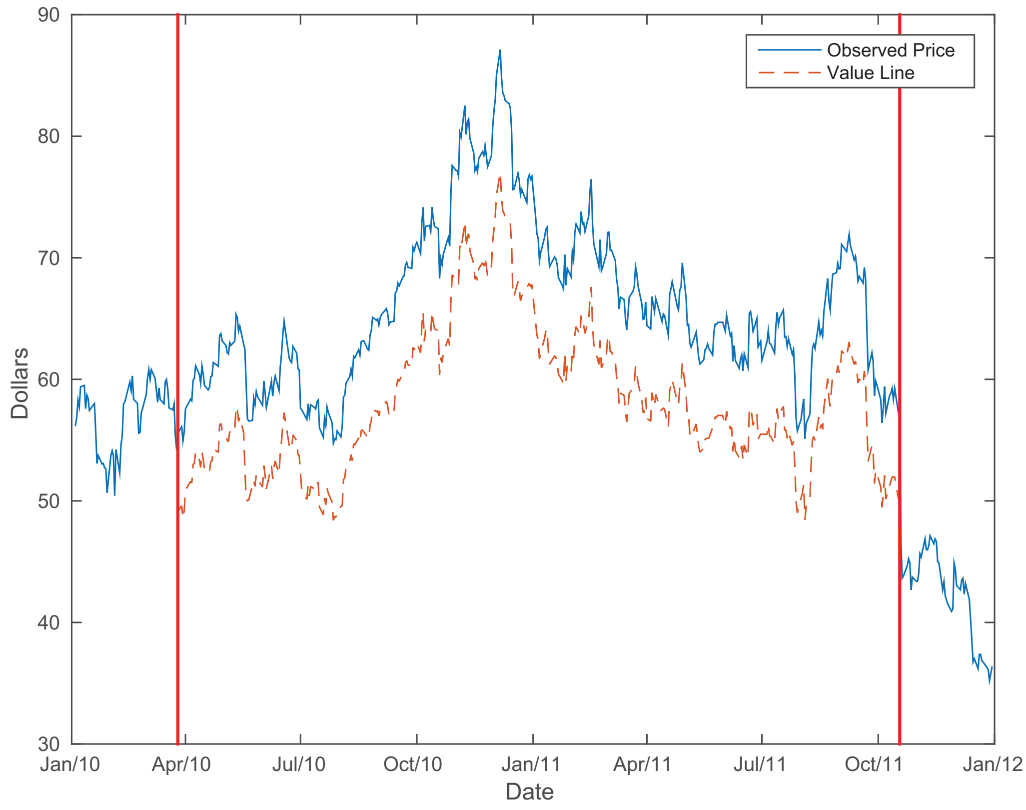

Figure 4: Common Shares Value Line - The period between the two vertical lines is the class period. The blue line shows the historical stock price of AEM from January 2010 to December 2011. The value line of stock, which is represented by the red dash line, is constructed by the event study approach (see Section 1.1).

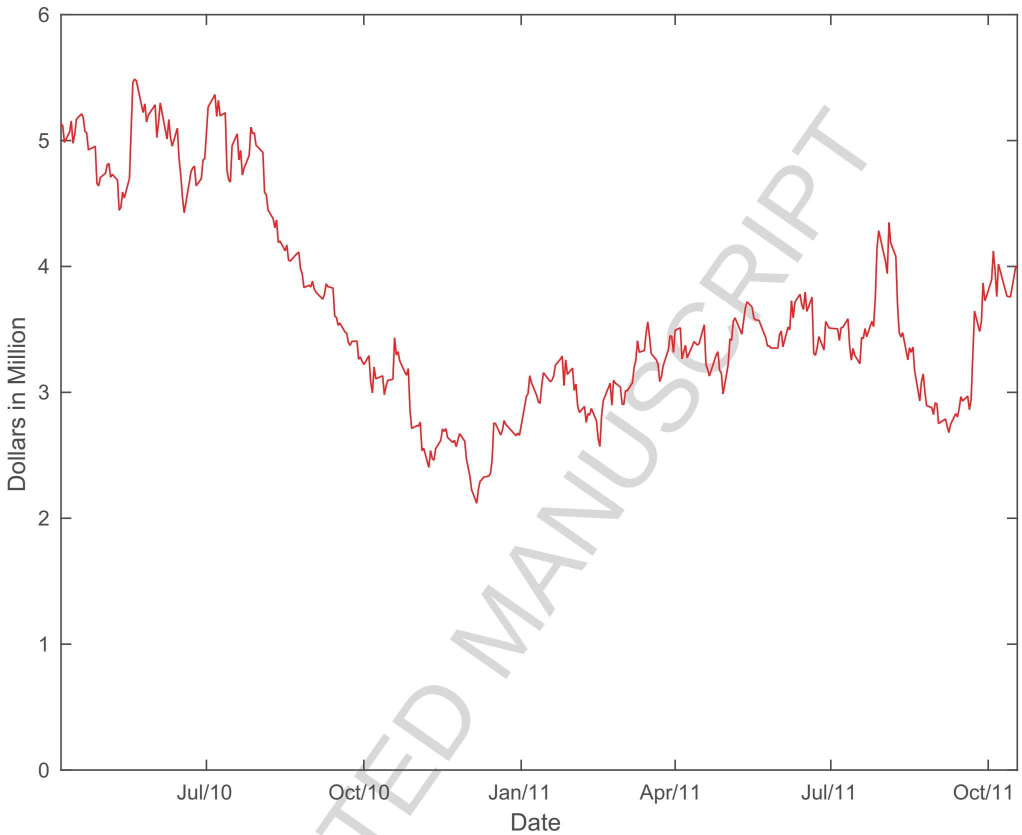

Figure 5: Notes Damages Ribbon - The graph shows the damages ribbon of notes during the period from April 7, 2010 to October 18, 2011.

Highlights

- When misrepresentation becomes public knowledge, the share price of a firm drops.
- Observable fraud-induced share price drop is translated to changes in debt values.
- Higher credit-risk debt values more sensitive to fraud than lower credit-risk debt.
- Findings relevant for damages assessment and awards in securities class actions.
- Bankruptcy costs explicitly discussed for Longstaff-Schwartz model.