# Capital Allocation under Fundamental Review of Trading Book

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# Fundamental Review of Trading Book (FRTB)

Basel Committee
on Banking Supervision

STANDARDS

Minimum capital
requirements for
market risk

January 2016

# Basel 2, 2.5 and FRTB

## Basel 2 and 2.5

- 10 days P&L of different risk positions are aggregrated

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  Liquidity is not taken into account
- Value-at-Risk (VaR)
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FRTB sets out revised standards for minimum capital requirements for market risk

- Incorporate the risk of market illiquidity
- An Expected Shortfall (ES) measure
- Constrain the capital-reducing effects of hedging

# Structure and implementation

- Standardized approach (SA), Internal models approach (IMA),
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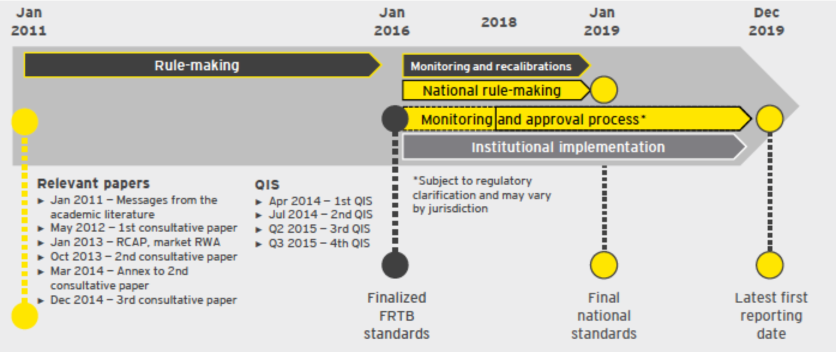
Implementation timeline (Picture from EY, Basel III revisions published in December 2017 affect the implementation date of FRTB)



# Updated timeline

| Standard | Original implementation date | Revised implementation date |
|---|---|---|
| Revised leverage ratio framework and G-SIB buffer | 1 January 2022 | 1 January 2023 |
| Revised standardised approach for credit risk | 1 January 2022 | 1 January 2023 |
| Revised IRB approach for credit risk | 1 January 2022 | 1 January 2023 |
| Revised operational risk framework | 1 January 2022 | 1 January 2023 |
| Revised CVA framework | 1 January 2022 | 1 January 2023 |
| Revised market risk framework | 1 January 2022 | 1 January 2023 |
| Output floor | 1 January 2022; transitional arrangements to 1 January 2027 | 1 January 2023; transitional arrangements to 1 January 2028 |
| Revised Pillar 3 disclosure framework | 1 January 2022 | 1 January 2023 |

IRB = internal ratings-based approach; CVA = credit valuation adjustment.

# Impact of FRTB

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FRTB IMA is more computationally demanding than the current framework

# Outline

- FRTB ES and its properties
- Capital allocation
- Two allocation methods under FTRB
- Simulation analysis

# Risk factor and liquidity horizon bucketing

P&L of a risk position is attributed to

$$\{\mathrm{RF}_i : 1 \leq i \leq 5\} = \{\mathrm{CM}, \mathrm{CR}, \mathrm{EQ}, \mathrm{FX}, \mathrm{IR}\}$$

$$\{\mathrm{LH}_j : 1 \leq j \leq 5\} = \{10, 20, 40, 60, 120\}$$

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BCBS (2016) 181(k)

| Risk factor category | $n$ |
|---|---|
| Interest rate: specified currencies - EUR, USD, GBP, AUD, JPY, SEK, CAD and domestic currency of a bank | 10 |
| Interest rate: – unspecified currencies | 20 |
| Interest rate: volatility | 60 |
| Interest rate: other types | 60 |
| Credit spread: sovereign (IG) | 20 |
| Credit spread: sovereign (HY) | 40 |
| Credit spread: corporate (IG) | 40 |
| Credit spread: corporate (HY) | 60 |
| Credit spread: volatility | 120 |

| Risk factor category | $n$ |
|---|---|
| Equity price (small cap): volatility | 60 |
| Equity: other types | 60 |
| FX rate: specified currency pairs[37] | 10 |
| FX rate: currency pairs | 20 |
| FX: volatility | 40 |
| FX: other types | 40 |
| Energy and carbon emissions trading price | 20 |
| Precious metals and non-ferrous metals price | 20 |
| Other commodities price | 60 |

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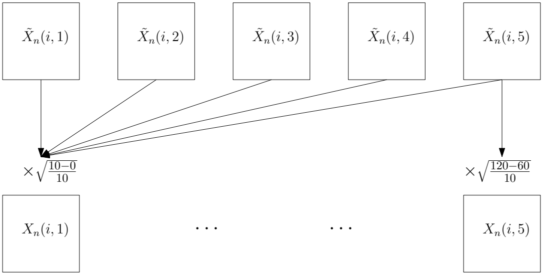
Liquidity horizon adjusted loss:

$$X_n(i, j) = \sqrt{\frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10}} \sum_{k=j}^{5} \tilde{X}_n(i, k), \quad 1 \le i, j \le 5$$



# Risk profile

We record the liquidity horizon bucketing by a $5 \times 5$ matrix:

$$X_n = \{X_n(i,j)\}_{1 \leq i,j \leq 5}$$

and call the matrix the risk profile of position n.

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The risk profile of a portfolio is

$$X = \sum_n X_n,$$

where the sum is the matrix sum.

Netting among different components (buckets) is not allowed.

# FRTB ES

The FRTB expected shortfall for portfolio loss attributed to $\mathrm{RF}_i$ is

$$\mathrm{ES}(X(i)) = \sqrt{\sum_{j=1}^{5} \mathrm{ES}(X(i,j))^2},$$

where $\mathrm{ES}(X(i,j))$ is the expected shortfall of $X(i,j)$ calculated at the 97.5% quantile.

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Example: Consider a portfolio with only one risk position whose is loss is concentrated on $\mathrm{RF}_i$ with $\mathrm{LH}_5 = 120$.

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Example: Consider a portfolio with only one risk position whose is loss is concentrated on $\mathrm{RF}_i$ with $\mathrm{LH}_5 = 120$.

$$\tilde{X}(i,j) = 0, \quad j = 1, \ldots, 4, \quad \tilde{X}(i,5) \sim N(0, \sigma^2)$$

Then the ES over 120 days is $\sqrt{120/10}\,\sigma \mathrm{ES}(N(0,1))$.

On the other hand, $X(i,j) = \sqrt{\frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10}}\tilde{X}(i,5)$, $1 \le j \le 5$. Then

$$\mathrm{ES}(X(i)) = \sqrt{\sum_{j=1}^{5} \frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10} \mathrm{ES}(\tilde{X}(i,5))^2} = \sqrt{\frac{120}{10}}\sigma \mathrm{ES}(N(0,1)).$$

# Stress period scaling

$\mathrm{ES}^{\mathrm{F,C}}(X(i))$: current 12-month, full set of risk factors

$\mathrm{ES}^{\mathrm{R,C}}(X(i))$: current 12-month, reduced set of risk factors

$\mathrm{ES}^{\mathrm{R,S}}(X(i))$: stress period, reduced set of risk factors

Restriction: $\mathrm{ES}^{\mathrm{R,C}}(X(i)) \geq 75\%\,\mathrm{ES}^{\mathrm{F,C}}(X(i))$.

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FRTB ES capital charge BSBC (2016) 181 (d) :

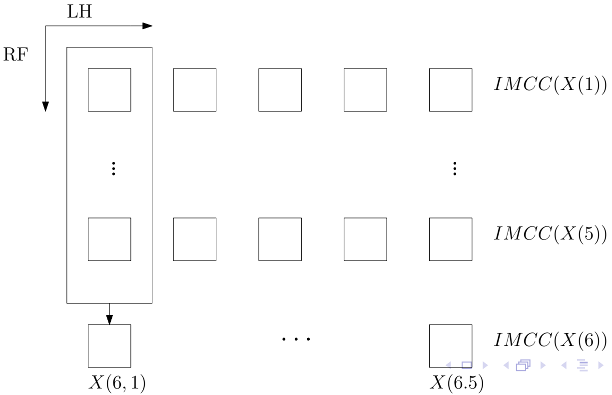
$$\mathrm{IMCC}(X(i)) = \frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))}\mathrm{ES}^{\mathrm{F,C}}(X(i)), \quad 1 \leq i \leq 5.$$

# Unconstrained portfolio

$$X(6,j) = \sum_{i=1}^{5} X(i,j).$$

We add $X(6,\cdot)$ as the 6-th row of $5 \times 5$ matrix, and call it extended risk profile.

IMCC($X(6)$) is calculated similarly as before.



# Capital charge for modellable risk factors

IMCC: BCBS (2016) 189:

The aggregate capital charge for modellable risk factors is

$$\mathrm{IMCC}(X) = \rho\,\mathrm{IMCC}(X(6)) + (1-\rho)\sum_{i=1}^{5}\mathrm{IMCC}(X(i)),$$

where $\rho = 0.5$.

# Capital allocation

Consider a portfolio of $N$ risk positions with losses $L_1, \ldots, L_N$.

The total loss is $L = \sum_{n=1}^{N} L_n$.

$\rho$ is a risk measure.

An allocation is a map

$$(L_1, \ldots, L_N) \mapsto \rho(L_n \mid L), \qquad \text{for each } n,$$

such that

$$\sum_{n=1}^{N} \rho(L_n \mid L) = \rho(L).$$

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Banks need allocations to calculate return on risk-adjusted capital (RORAC):

$$\frac{-\mathbb{E}[L_n]}{\rho(L_n \,|\, L)}.$$

RORAC evaluates the capital efficiency of each position.

# Euler allocation principle

Let $v_1, \dots, v_N$ be a sequence of numbers and $L^v = \sum_{i=1}^{N} v_n L_n$.

Per-unit Euler allocation is

$$\rho(L_n \,|\, L)(v) := \frac{\partial}{\partial v_n} \rho(L^v).$$

Setting all $v_n = 1$, we denote the allocation to $L_n$ as $\rho(L_n \,|\, L)$.

if $\rho$ is homogeneous of degree 1, Euler's theorem for homogeneous functions implies

$$\rho(L^v) = \sum_n v_n \frac{\partial}{\partial v_n} \rho(L^v).$$

Setting $v = 1$, we have the full allocation property.

# Pros and Cons of Euler allocation

Tasche (1999) shows that

$$\frac{\partial}{\partial v_n}\Big(\frac{-\mathbb{E}[L^v]}{\rho(L^v)}\Big)\begin{cases} > 0, \text{ if } \frac{-\mathbb{E}[L_i]}{\rho(L_i \mid L)(v)} > \frac{-\mathbb{E}[L^v]}{\rho(L^v)} \\ < 0, \text{ if } \frac{-\mathbb{E}[L_i]}{\rho(L_i \mid L)(v)} < \frac{-\mathbb{E}[L^v]}{\rho(L^v)} \end{cases}$$

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However,

- Euler allocation is unstable.
- Euler allocation induces large negative allocations.

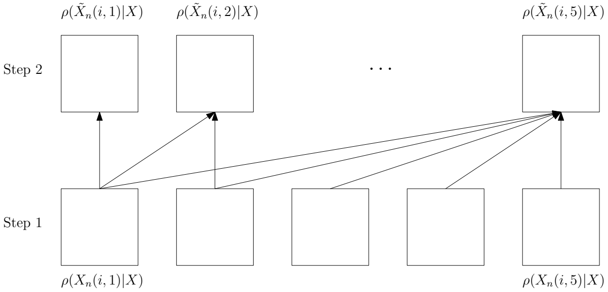
# Two steps allocation for FRTB

Given liquidity horizon adjusted risk profiles $\{X_n\}_{1\le n\le N}$,

Step 1: allocate to each $X_n(i,j)$

$$\rho(X_n(i,j)\,|\,X).$$

Step 2: allocate to each $\tilde{X}_n(i,k)$



# Euler allocation under FRTB

Let $v = \{v_1, \ldots, v_n\}$ be real numbers

Let $X^{v,j}(i) = \sum_n X_n^{v_n,j}(i)$, where

$$X_n^{v_n,j}(i) = \big(X_n(i,1), \cdots, X_n(i,j-1), v_n X_n(i,j), X_n(i,j+1), \cdots, X_n(i,5)\big).$$

For each $\mathrm{RF}_i$, we define the Euler allocation for FRTB ES as

$$\mathrm{ES}(X_n(i,j) \,|\, X(i)) := \frac{\partial}{\partial v_n} \mathrm{ES}(X^{v,j}(i))\Big|_{v=1},$$

where $v = 1$ means all $v_n = 1$.

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## Lemma

$$ES(X_n(i,j) \,|\, X(i)) = \frac{ES(X(i,j))}{ES(X(i))} \frac{\partial}{\partial v_n} ES\big(X^v(i,j)\big)\Big|_{v=1},$$

*where* $X^v(i,j) = \sum_n v_n X_n(i,j)$.

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$$\frac{\partial}{\partial v_n} \mathrm{ES}\big(X^v(i,j)\big)\Big|_{v=1} = \mathbb{E}\big[X_n(i,j) \,|\, X(i,j) \geq \mathrm{VaR}(X(i,j))\big].$$

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- Euler allocation of IMCC

$$\mathrm{IMCC}^E(X_n(i,j) \,|\, X) := 0.5 \, \frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))} \, \mathrm{ES}^{\mathrm{F,C}}\big(X_n(i,j) \,|\, X(i)\big).$$

It is a full allocation.

# Negative allocations

Hedging among different RFs or LHs does not lead to negative allocations.

Example:

Consider two loss $Y$ with $\mathrm{RF}_i$ and $Z$ with $\mathrm{RF}_k$, $i \neq k$, $Y, Z \sim N(0, \sigma)$, and $Y + Z = 0$.

Euler of regular ES:

$$\mathrm{ES}^R(Y|Y+Z) + \mathrm{ES}^R(Z|Y+Z) = \mathrm{ES}^R(Y+Z) = 0.$$

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Euler of FRTB ES:
Let $X$ be the risk profile containing $X$ and $Y$. $X(i) = Y$, $X(k) = Z$, then

$$\mathrm{ES}(Y|X(i)) = \mathrm{ES}(Y) > 0, \quad \mathrm{ES}(Z|X(k)) = \mathrm{ES}(Z) > 0.$$

Even though $Y + Z = 0$, $IMCC(X) > 0$.

# Constrained Aumann-Shapley allocation

Motivated by Li, Naldi, Nisen, and Shi (2016), who combine Shapley and Aumann-Shapley allocations.

LH permutation matrix

$$\mathcal{L} := \begin{bmatrix} 10 & 20 & 40 & 60 & 120 \\ 10 & 20 & 40 & 120 & 60 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 120 & 60 & 40 & 20 & 10 \end{bmatrix}_{5! \times 5} .$$

Let $\mathcal{L}^{-1}(r, j)$ be the column of $\mathcal{L}$ in which $\text{LH}_j$ locates. e.g. $\mathcal{L}^{-1}(2, 5) = 4$.

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Let $v = \{v_1, \ldots, v_N\}$,

$$X^{v,r,j}(i) = \sum_n X_n^{v,r,j}(i),$$

where $X^{v,r,j}(i)$ is a row depending on when $\mathrm{LH}_j$ appears in $r$.

For example,

$$X_n^{v,2,5}(i) = \big(X_n(i,1), X_n(i,2), X_n(i,3), 0, v_n X_n(i,5)\big).$$

# Constrained Aumann-Shapley allocation

We define the Constrained Aumann-Shapley allocation(CAS) in the permutation $r$ as

$$\mathrm{CAS}(r, X_n(i,j)) := \int_0^1 \frac{\partial}{\partial v_n} \mathrm{ES}(X^{v,r,j}(i))\Big|_{v=q} dq,$$

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## Lemma

$$CAS(r, X_n(i,j)) = \eta(r,i,j) \frac{\partial}{\partial v_n} ES\big(X^v(i,j)\big)\Big|_{v=1},$$

*where*

$$\eta(r,i,j) = \frac{\sqrt{\sum_{1 \le s \le \mathcal{L}^{-1}(r,j)} ES\big(X(i,\mathcal{L}(r,s))\big)^2} - \sqrt{\sum_{1 \le s < \mathcal{L}^{-1}(r,j)} ES\big(X(i,\mathcal{L}(r,s))\big)^2}}{ES\big(X(i,j)\big)}.$$

- CAS is a scaled version of Euler
- Losses with the same LH need to be added to the same portfolio at the same time, to ensure computational efficiency

# CAS allocation of IMCC

We define the CAS allocation for IMCC as

$$\mathrm{IMCC}^{C}(X_n(i,j)\,|\,X) := 0.5\,\frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))}\,\frac{1}{5!}\sum_{r=1}^{5!}\mathrm{CAS}^{\mathrm{F,C}}(r, X_n(i,j)).$$

It is a full allocation.

# Stress scaling adjustment

In the previous allocation, the $X_n(i,j)$ induced risk contribution is not considered in the stress scaling factor $\frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))}$.

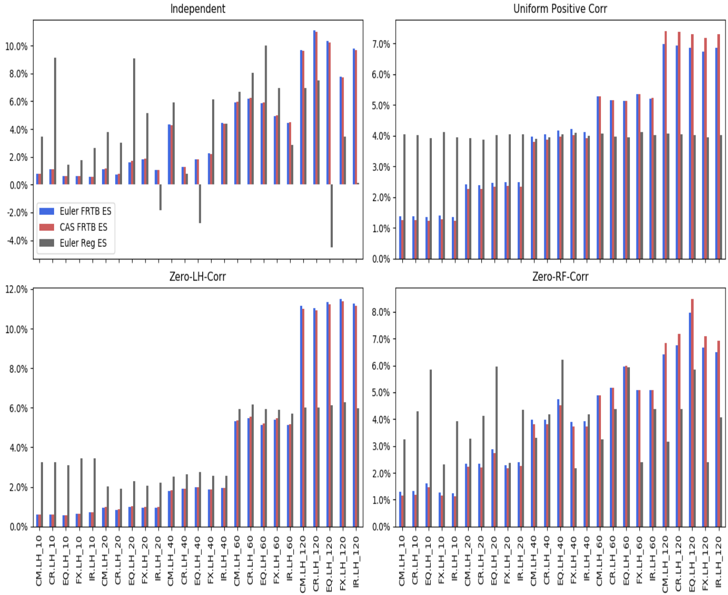
We define the Euler allocation with stress scaling adjustment as

$$\mathrm{IMCC}^{\mathrm{E,S}}(X_n(i,j)\,|\,X(i)) := 0.5\frac{\partial}{\partial v_n}\left[\frac{\mathrm{ES}^{\mathrm{R,S}}(X^{v,j}(i))}{\mathrm{ES}^{\mathrm{R,C}}(X^{v,j}(i))}\mathrm{ES}^{\mathrm{F,C}}(X^{v,j}(i))\right]\Big|_{v=1}.$$

## Lemma

$$\begin{aligned}
IMCC^{E,S}(X_n(i,j)\,|\,X(i)) = 0.5\Bigg[&\frac{ES^{R,S}(X(i))}{ES^{R,C}(X(i))}\,ES^{F,C}(X_n(i,j)\,|\,X(i)) \\
&+\frac{ES^{F,C}(X(i))}{ES^{R,C}(X(i))}\,ES^{R,S}(X_n(i,j)\,|\,X(i)) \\
&-\frac{ES^{R,S}(X(i))ES^{F,C}(X(i))}{ES^{R,C}(X(i))^2}\,ES^{R,C}(X_n(i,j)\,|\,X(i))\Bigg].
\end{aligned}$$

# Simulation analysis 1



# Simulation analysis 2

Three hedging structures:

1. Strong hedging between EQ and IR
2. Strong hedging between $LH_1$ and $LH_2$
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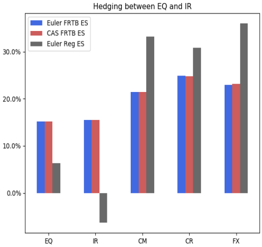
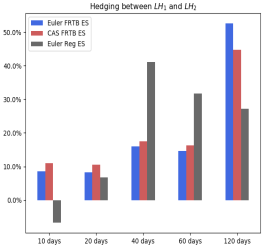
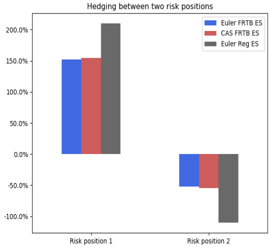


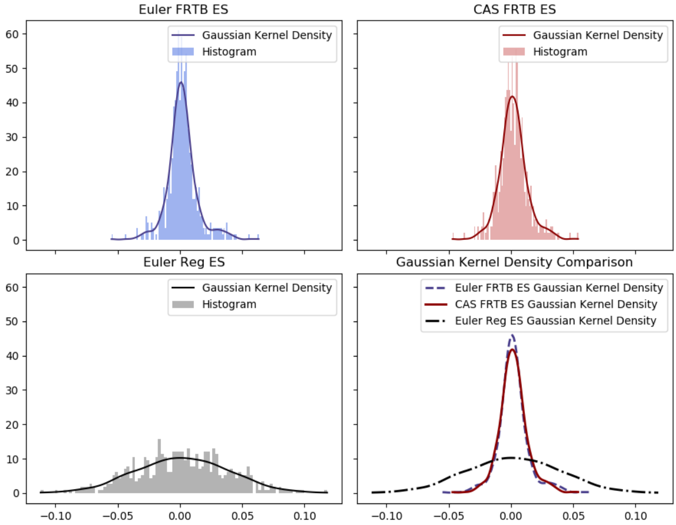




FRTB allocations produce

- no negative allocations for hedging among different bucket
- some negative allocations for hedging in the same bucket, but with smaller magnitude

# FRTB allocations are more stable

Histograms and fitted kernel densities for allocations in Case 3:



# Simulation analysis 3

Loss in EQ with 40-days LH and CM with 60 days LH have 9 times of volatility in the stress period than the normal period.

Two reduced set of risk factors

Set A : Include both RFs with large variations

Set B : Exclude both RFs with large variations

# Simulation analysis 3

Loss in EQ with 40-days LH and CM with 60 days LH have 9 times of volatility in the stress period than the normal period.

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| | Set A (Adj) | Set A (Without adj) | Set B (Adj) | Set B (Without adj) |
|---|---|---|---|---|
| CM.60 | 4.00% | 2.24% | 1.43% | 1.43% |
| EQ.40 | 5.04% | 3.26% | 2.11% | 2.11% |

Table: IMCC(Set A)=11.55 and IMCC(Set B)=3.14

The choice of reduced set of risk factors has large impact on allocations.

# Conclusion

Two allocation methods reduce FRTB allocations to Euler allocations

- Computational efficiency
- Easy to adapt to the current system

Simulation analysis shows

- Longer LH leads to more allocation
- Much less negative allocations
- More stable allocations
- Sensitive to the choice of reduced set of risk factors

## Thanks for your attention!