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Capital Structure Decisions and the Optimal Design of Corporate Market Debt Programs

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Abstract

This paper provides a joint quantitative analysis of *capital* structure decisions (debt versus equity) and *debt* structure decisions (fixed-rate debt versus floating-rate debt or inflation-linked debt) in a continuous-time setting. We show that optimizing the debt structure has an impact on capital structure decisions, and leads to increases in leverage ratios compared to a pure fixed-rate debt program. We also find that for realistic parameter values, jointly optimizing the debt and capital structures generates a significant increase in firm value with respect to a situation where only the capital structure is optimized.

Keywords: capital structure, debt structure, inflation risk, interest rate risk

1. Introduction

While asset allocation decisions are relatively well understood from the theoretical standpoint, with a range of prescriptions available in complex situations involving, among other things, a stochastic opportunity set or the presence of parameter uncertainty, our understanding of liability management decisions is comparatively much more limited. Two separate strands of the corporate finance literature have in fact been primarily concerned with optimal liability structure for a firm: the dynamic capital structure literature, which has abstracted away from debt allocation decisions so as to better focus on optimal capital structure decisions in tractable quantitative settings, and the risk management literature, which has studied the desirability of hedging and the form of optimal hedging in isolation from the capital structure problem.

On the one hand, the early capital structure literature has stated that corporate financing policy and the choice of liability structure is irrelevant in the absence of contracting costs and taxes; this is the fundamental insight from the Modigliani-Miller theorem [43]. Hence, the introduction of frictions provides one natural possible justification for a non-trivial capital structure choice that is based on the trade-off between the tax

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benefit of debt and the bankruptcy costs of debt.[2] A first elegant quantitative analysis of this *trade-off theory of capital structure* was provided for by [33] in a model where perpetuity was the single class of debt.[3] In an attempt to account for more realistic debt characteristics, [36] extend [33] to examine the effect of (an exogenously assumed) finite debt maturity on bond prices, credit spreads, and optimal leverage. In all these (as well as other related) papers, however, corporate debt is exclusively represented by a fixed-coupon bond.[4] In other words, these papers discuss the equity versus debt decision, but are silent on the optimal allocation to various types of debt instruments.[5] Because most of these papers assume away interest rate risk (as well as inflation risk), there is in fact no role and no need for interest rate (or inflation) hedging in such models, and as a consequence no need for floating-rate (or inflation-linked) bonds in the menu of liability classes. Ju and Ou-Yang [29] provide one example of a dynamic capital structure model with stochastic interest rates, but they maintain the assumption of a single class of debt with a fixed coupon so as to focus on the derivation of the bond optimal maturity.[6] Obviously, the restrictive assumption of a single class of fixed-rate market debt is at odds with corporate practice, where various classes of debt instruments are issued, including notably floating-rate bonds and even sometimes inflation-linked bonds.[7] One of the objectives of this paper is precisely to extend the dynamic capital structure literature to realistic settings, involving interest rate and/or inflation risks, with a specific focus on a quantitative analysis of the associated market debt structure choices.

If the question of optimal design of a corporation debt program has been ignored by the dynamic capital structure literature, the risk management literature, on the other hand, has heavily discussed the relevance and the optimal form of hedging strategies, which allow firms to reduce the variability of their net future cash flows by entering appropriate derivatives contracts: Smith and Stulz [51] show that firms will in principle benefit from a reduction in this variability if the tax function is convex in the pre-tax value, and that hedging allows an issuer to avoid default in some states of the world, thereby increasing debt capacity and the expected value of the associated tax benefits. Other possible reasons for hedging are managerial risk aversion or concern for reputation [53] and the less frequent resort to external funding when external funds are more costly to raise than internal funds [21]. Because taxes and bankruptcy costs are also at the

[2]Other possible justifications for the relevance of the capital structure decision include the *costs of asymmetric information* (e.g., the pecking order theory of [48]) or *agency costs* such as those related to the asset substitution effect [28], the underinvestment problem [47] or the free cash flow situation [27]). Yet another interpretation is the market timing hypothesis [1] stating that the first-order determinant of a corporation capital structure is the relative mispricing of these instruments at the time the firm needs to finance investment.

[3]See also [5], who were the first to use the contingent-claims analysis approach of [3], [40], and [2] in the analysis of capital structure decisions.

[4]Related papers also include [39], [34], [18], [22], [44], and [42], among many others.

[5]One exception is [25], who examine the optimal mixture of bank and market debt, but assume a single class of (fixed-rate) bonds as far as market debt is concerned.

[6]The impact of interest rate risk on the pricing of corporate bonds is also discussed in [37], [7] or [11], but these papers do not analyze the implications for optimal capital and debt structure decisions.

[7]The list of companies having recently issued inflation-linked bonds includes Tesco (£60 million in December 2011), Royal Bank of Scotland (£20 million in November 2011), Merrill Lynch ($6 million in March 2010), or Morgan Stanley ($3 million in June 2011), among others.

heart of trade-off theories of the capital structure, it is natural to ask how the debt-versus-equity decisions interact with hedging decisions. Mathematical models have been developed to derive the optimal amount of hedging for a risk-averse manager [53] or the optimal number of forward contracts or the optimal form of exotic contracts for firms that maximize the expected value of their profits [8]. These papers, however, do not explicitly link their approaches to structural models of default, in which equities and bonds are viewed as contingent claims written on the firm's assets, and they do not allow default to occur prior to the date bonds are redeemed, unlike in Black and Cox [2] and subsequent papers.

Several empirical studies have attempted to identify which of the theoretical determinants of hedging are at work in practice, finding that tax convexities play in fact little role [41, 23, 10], while more indebted firms – which face larger expected distress costs – hedge to a greater extent [23]. Related findings can be found in Chava and Purnanandam [9], who empirically study the drivers of the split between floating-rate and fixed-rate debt: although they are primarily concerned with the role of managerial incentives, they also report that more financially distressed firms as well as smaller firms tend to favor fixed-rate debt over floating-rate debt, which is consistent with a hedging approach aiming to reduce expected default costs. Some of the predictions of our model can be matched with these empirical patterns, as we shall see in Sections 3 and 4. Another empirical study of the determinants of fixed-rate versus floating-rate debt is Faulkender [19], who, unlike Chava and Purnanandam [9], conclude that market-timing considerations on the difference between the costs of the two types of bonds matter more than the hedging motive when a firm issues new debt. Chernenko and Faulkender [10] find empirical evidence for both hedging and speculative motives in the use of derivatives by firms.

The main contribution of our paper is to tie together the capital structure literature and the corporate hedging literature by providing a joint quantitative analysis of capital structure and debt management choices in a unified framework. We show that debt management decisions can be formally analyzed in the context of a dynamic capital structure model, with a trade-off between the (bankruptcy) costs and (tax shield) benefits associated not only with leverage but also with debt structure decisions. To do so, we consider various forms of market debt instruments (fixed-rate bonds, floating-rate bonds and inflation-indexed bonds) in a relatively rich stochastic environment, involving in particular interest rate and inflation risks.[8] We find that optimal debt management has an impact on capital structure decisions, with an optimal debt structure leading to higher leverage ratios compared to a sub-optimal situation with fixed-rate bonds only issued by the firm. We also confirm that the optimal allocation to fixed- versus floating-rate bonds critically depends on the correlation between the interest rate process and the firm asset value process, while interest rate volatility and the interest rate risk premium have comparatively lower impact for reasonable parameter values. The main benefit of optimizing the debt structure is to allow firms to reduce the variability of their net cash flows and therefore to decrease the probability of default. We provide a quantitative

[8]By taking the perspective of the owners of the firm, who are concerned with equity-plus-debt value, we abstract away from agency and asymmetric information problems. This is in contrast to the related literature on optimal security design, e.g., [12], [38], or [55], which has focused on the presence of asymmetric information in stylized settings.

assessment of the impact of debt structure choices on the levered firm value, and we show that the increase in total value that follows from optimizing the debt policy can be of the same order of magnitude as the increase that follows from optimizing the level of debt. In other words, debt structure optimization can generate nearly as much welfare improvement as capital structure optimization. When inflation-linked bonds are introduced in the liability mix, we find that they should account for a sizable fraction of optimal bond issuance for realistic parameter values.[9]

Closely related to ours is a contribution by Morellec and Smith [45], who analyze the joint issuance of fixed-rate debt and the investment in contracts allowing to hedge innovations in the firm's cash-flows. Their paper addresses a richer set of agency issues, including managers-shareholders conflicts, while we solely focus on the shareholders-bondholders conflict, thus following the bulk of the literature on optimal capital structure decisions. This allows them to show that hedging can help control the underinvestment problem, but also control managers' ability to overinvest. On the other hand, they abstract away from taxes, a key determinant of optimal corporate issuance policies, and, more importantly, assume a constant interest rate, which prevents them from analyzing the interest rate hedging properties of floating-rate debt. We instead solve for the optimal combination of various forms of indexed and non-indexed debt. As such, our paper complements their work on debt and risk management by formally showing that a mixture of various forms of debt in general dominates a 100% fixed-rate or 100% floating-rate debt structure.

We can also relate our study to the literature on financial intermediation, focused on explaining the determinants of debt structure as a way of mitigating the issues of information asymmetry between lenders and borrowers. The theoretical works by Fama [17], Diamond [15], Diamond [14] and [4] investigate on the incentives that firms have to utilize bank loans (either obtained from a single lender or from a syndicate of lenders) rather than public bonds or private placements.[10] While the greatest majority of public bonds in the market is fixed-rate, bank loans can be fixed- or floating-rate contracts. However, in the last 20 years, the weighted-average maturity for all commercial and industry loans issued by U.S. banks is a quantity that ranged from a minimum of 241 days (Q2 1997) to a maximum of 843 days (Q1 2014).[11] The average loan maturity is comprised between 1 and 2 years most of the time and these contracts are typically rolled over several times when they expire. Over a sufficiently long time span, it turns out that bank loans, including for example term loans or revolvers, behave very similarly to floating-rate debt from the perspective of interest-rate exposure. Our analysis provides a point of view complementary to that of information asymmetry for the analysis of the optimal debt structure of a firm, as the objective of maximizing the firm value, which we

[9] Inflation-linked bonds car be regarded as a specific form of floating-rate bonds, with a reference index used for coupon and/or principal payments based on changes in price levels, as opposed to changes in some interest rate index level.

[10] Also related is the work by [13], focused on the choice of the debt maturity with respect to the credit rating of the borrower. Although our framework allows to study the optimal issuance policy among different types of debt and maturities, we focus on the case where the leverage ratio is optimal to the firm, which endogenizes the default barrier and, at least partially, the credit risk. This approach provides results that are difficult to compare to an analysis that considers credit risk as an exogenous variable.

[11] Source: Federal Reserve Economic data.

show to be mostly related to interest-rate hedging, might be a further element of choice between public bonds/private placements (typically fixed-rate contracts) and bank loans (typically behaving as floating-rate debt). Our findings also complement the results in the empirical literature of financial intermediation. James [26] finds that the issuance of bank loans corresponds to positive abnormal stock returns, whilst the issuance of private debt to refinance bank loans leads to abnormal negative stock returns. The explanation invoked is that bank loans reduce the information asymmetries associated with public debt offerings. We relate this empirical fact to our analysis, as the issuance of floating-rate debt, when the correlation between firm's assets and the interest rate is high enough, leads to a higher tax shield effect, which reflects on higher equity values. We also show that, even in the case of a negative correlation between firm's assets and the interest rate, a risk-shifting from equity holders to debtholders may occur, entailing an increase in equity value, despite a reduction in firm value.[12] More recently, the empirical studies by Lee and Mullineaux [32] and Sufi [54] focused on the structure of lending syndicates, which have become one of the largest sources of financing worldwide. The vast majority of syndicated loan contracts is floating-rate, with a fixed interest rate spread based on credit risk.[13] Our work contributes in explaining the choice of this source of financing, alternatively or together with fixed-rate public bond issues. Finally, Rauh and Sufi [49] argue that most firms have a heterogeneous debt structure, where the two main sources of debt are bonds and bank loans, as the use of multiple debt types is able to reduce incentive conflicts. Our analysis also supports the importance of the issuance of different types of debt, analyzing this fact from the point of view of a value-maximizing firm and explaining it through a hedging perspective.

The remainder of the paper is organized as follows. In Section 2 we introduce a dynamic capital structure model with default triggered at a random date, and we derive the prices of equities and corporate bonds. Section 3 presents a numerical analysis of the optimal floating- versus fixed-rate debt structure of the firm. We perform a quantitative assessment of the importance of the hedging motive behind debt management in Section 4. In Section 5, we introduce inflation risk in the economy and consider inflation-linked bonds as an additional class of market debt. We present our conclusions and suggestions for further research in Section 6. Finally, technical details on pricing formulas are relegated to an appendix.

2. Debt Management Decisions in a Dynamic Capital Structure Model

We make standard assumptions regarding the nature of uncertainty. Let $[0, T]$, with $T > 0$, denote the (finite) time span of the economy. Uncertainty in the economy is described through a probability space $(\Omega, \mathcal{A}, \mathbb{P})$. The set of available information at time t is referred to as $\mathscr{F}_t$, and the filtration $(\mathscr{F}_t)_{t\in[0,T]}$ is denoted by $\mathscr{F}$. We assume that that

[12] This is consistent also with the empirical findings by Chernenko and Faulkender [10], who argue that firms where the CFO has a higher delta exposure to the firm's equity tend to use a higher fraction of floating-rate debt.

[13] In the empirical samples used by Lee and Mullineaux [32] and Sufi [54], the average maturity of syndicated loans is between 3 and 4 years. In the context of our analysis, this makes even fixed-rate contracts, when periodically rolled over, mostly similar to floating-rate debt.

there exists an equivalent martingale measure (EMM) $\mathbb{Q}$, under which discounted asset prices are martingales.

2.1. Asset Value

The first state variable is the value of firm's operating assets before leverage, denoted as V_t. An important standard assumption is that the process V is not affected by capital structure choices. We reinforce this assumption by postulating that the firm value process is also independent of debt structure decisions. Following [11] and [29], we assume that it evolves under the risk-neutral measure as:[14]

$$\mathrm{d}V_t = V_t\left[(R_t - \delta)\,\mathrm{d}t + \sigma_V\,\mathrm{d}z_t^V\right], \tag{2.1}$$

where z^V is a standard Brownian motion, R_t is the (nominal) short-term interest rate and δ denotes a constant payout rate.

The second state variable is the short-term interest rate R, which follows an Ornstein-Uhlenbeck process [56]:

$$\mathrm{d}R_t = a(b - R_t)\,\mathrm{d}t + \sigma_R\,\mathrm{d}z_t^R, \tag{2.2}$$

where z^R is a Brownian motion correlated to z^V, with correlation coefficient ρ_{RV}. The long-term mean that appears in the above equation is the long-term mean under the EMM. If $\widehat{b}$ is the long-term mean under the physical measure $\mathbb{P}$, we have $b = \widehat{b} - \frac{\sigma_r\lambda_r}{a}$, and the dynamics of the short-term rate under $\mathbb{P}$ reads:

$$\mathrm{d}R_t = a(\widehat{b} - R_t)\,\mathrm{d}t + \sigma_R\,\mathrm{d}\widehat{z}_t^R,$$

where $\widehat{z}_t^R = \mathrm{d}z_t^R - \lambda_R\,\mathrm{d}t$ defines a $\mathbb{P}$-Brownian motion.

We can rewrite the risk-neutral dynamics (2.1) and (2.2) using a two-dimensional Brownian motion $\boldsymbol{z}$:

$$\mathrm{d}V_t = V_t[(R_t - \delta)\,\mathrm{d}t + \boldsymbol{\sigma}_V'\,\mathrm{d}\boldsymbol{z}_t], \tag{2.3}$$
$$\mathrm{d}R_t = a(b - R_t)\,\mathrm{d}t + \boldsymbol{\sigma}_R'\,\mathrm{d}\boldsymbol{z}_t. \tag{2.4}$$

In the Vasicek model, the price at time t of a default-free zero-coupon bond that pays \$1 at date T, denoted as $B(t,T)$, evolves as:

$$\mathrm{d}B(t,T) = B(t,T)[R_t\,\mathrm{d}t + \boldsymbol{\sigma}_B(t,T)'\,\mathrm{d}\boldsymbol{z}_t], \tag{2.5}$$

where the volatility vector $\boldsymbol{\sigma}_B$ is a deterministic function of time-to-maturity:

$$\boldsymbol{\sigma}_B(T-t) = \sigma_B(T-t)\frac{\boldsymbol{\sigma}_R}{\sigma_R}, \quad \sigma_B(T-t) = -\frac{1 - e^{-a(T-t)}}{a}\sigma_R. \tag{2.6}$$

[14] [22] suggest an alternative approach in which the state variable is earnings before interest and taxes and show that if drifts, volatilities and interest rates are constant, the unlevered asset value defined as the present value of a perpetual claim on EBIT is proportional to the current EBIT. This does not hold in the presence of stochastic interest rates.

2.2. Promised Liability Payments

Throughout this section and Section 3, we assume that the firm can issue two types of debt instruments represented as pure discount bonds with the same maturity T (the analysis will be extended in Section 5 to include inflation-linked bonds in the liability mix):

- *fixed-rate bonds* have a face value of 1 and they promise an interest payment at date T equal to $\exp(TR_{0,T}) - 1$, so that the full promised payment is $\exp(TR_{0,T})$. The price of this promised payoff at date t is $B(t,T)\exp(TR_{0,T})$, where $R_{0,T}$ denotes the zero-coupon rate at time 0 with maturity date T, that is $R_{0,T} = -\frac{1}{T}\ln B(0,T)$;
- *floating-rate bonds* have also a face value of 1, but their promised interest payment is $\exp\left(\int_0^T R_t\,\mathrm{d}t\right) - 1$. The integral within the exponential represents the cumulative short-term interest rate over the period $[0,T]$. Hence, the price of the promised payoff is $\exp\left(\int_0^t R_s\,\mathrm{d}s\right)$.

The promised payment on fixed-rate bonds is known in advance, while the promised payment on floating-rate bonds is stochastic as seen from the initial date, due to uncertainty about future short-term rate values. If the fixed-rate and floating-rate bonds were not subject to default risk, both bonds would be worth 1 at date 0. But because of default risk, their actual payoffs will be in general different from the promised ones, so their initial value will be less than 1.

At the initial date, the firm chooses the number n_1 of fixed-rate bonds and the number n_2 of floating-rate bonds to issue. The optimal choice of n_1 and n_2, which aims at maximizing the total value of the firm, is precisely one of the main focuses of this paper. We assume that no debt renegotiation, no anticipated redemption and no further debt issuance takes place after date 0. Hence, the debt structure is of the "issue-and-wait" form, which is the equivalent in liability management of "buy-and-hold" strategies in asset allocation. While an extension to an intertemporal optimization setting would be desirable in principle, allowing for a richer set of strategies would raise implementation issues. In particular, a strategy that would involve continuous issuance and purchase of bonds so as to maintain a target allocation to fixed-rate and floating-rate debt would likely generate prohibitive costs. It should also be noted that an optimal policy derived at date 0 may not be optimal at later dates, since in general the objective of maximizing the firm value does not enjoy the same time-consistency property as the objective of maximizing expected utility in portfolio choice problems.[15] To avoid this concern, we assume that the firm can credibly pre-commit to the strategy, so that neither equity nor bondholders anticipate any deviation from the debt mix that has been chosen at date 0.

The present value of the promised payment is:

$$L_t = n_1 e^{TR_{0,T}} B(t,T) + n_2 e^{\int_0^t R_s\,\mathrm{d}s}. \tag{2.7}$$

[15] An example of model where time-consistency holds is [33]. In his model, this property stems from infinite debt maturity and constant interest rate, two assumptions that imply stationarity and from which we deviate.

At date 0, it equals the face value of the debt, n_1+n_2. But it differs from the market value of debt, because the latter incorporates information about default. The debt structure is defined by the proportion of fixed-rate bonds chosen by the firm, denoted by $\theta = n_1/(n_1+n_2)$, which is constant for "issue-and-wait" strategies. On the other hand, the proportion of each class expressed as market value of bonds of that class over total market value of debt is not constant over time, because market prices of bonds fluctuate (see details in Section 2.3). In what follows, we represent the capital and debt structures with the pair (L_0, θ), where $L_0 = n_1 + n_2$ is the nominal quantity of debt issued, which defines the leverage and the capital structure, and $\theta = n_1/L_0$ is the proportion of fixed-rate debt issued, which defines the debt structure.

2.3. Default Time

Because of the presence of default risk, the total promised liability value process L is different from (almost surely higher than) the market value of debt, denoted as D. Following [2], we assume that bankruptcy is triggered at the first hitting time of a barrier by the process V. Various kinds of barriers have been considered in the literature, with the final choice being most often imposed by tractability constraints. [2] assume that the default boundary is a deterministic function of time, while [37] consider a constant exogenous barrier. In [33] and [36], the barrier is also constant, but is endogenously derived so as to maximize equity value. The assumptions made in these papers lead to analytical expressions for equity and bond prices, except in [37], who provide a numerical approximation for these prices. Such expressions allow to explicitly compute the optimal capital structure in [33] and [36]. Stochastic barriers are introduced in [7] and [29], together with stochastic interest rates. Given that our model also incorporates stochastic interest rates, we follow the latter papers by assuming that the default boundary is equal to the present value of the promised payment. Formally, the default instant is:[16]

$$\tau = \inf\left\{t \geq 0 \;;\; V_t e^{-\delta(T-t)} \leq L_t\right\}. \tag{2.8}$$

If the firm issues fixed-rate debt only, which is the standard assumption in the related literature, then L is proportional to the value of the default-free zero-coupon, as in [7] and [29]. Hence, our definition of the default time extends the one adopted in papers that consider a single class of debt. In our framework, the debt structure decision impacts the default barrier, since L depends on θ.

To motivate the choice of the default condition, it is useful to remark that the quantity $\left(V_t e^{-\delta(T-t)} - L_t\right)$ is the present value of the payoff $(V_T - L_T)$:

$$V_t e^{-\delta(T-t)} - L_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s \, \mathrm{d}s}(V_T - L_T)\right].$$

Thus, having $V_t e^{-\delta(T-t)} < L_t$ implies that there is a positive probability that $V_T < L_T$. Default is triggered as soon as the probability for the firm's assets to be too low to cover the promised redemption payoff is sufficiently large in the sense that $V_t e^{-\delta(T-t)} < L_t$.

[16]Considering a flow-based notion of default would be an alternative (see e.g. Kim et al. [31]). In fact, both notions would be equivalent in an EBIT-based model, where the unlevered value of the firm, V, is a multiple of the EBIT process (see Goldstein et al. [22]).

When the firm has not gone bankrupt by date T, we have $V_T \geq L_T$, so the corporate issuer is solvent, and the limited liability of equity holders is respected (they are never required to make a positive payment). Overall, the default condition implies that debt is protected by a positive net worth requirement, where net worth is defined as the excess of unlevered asset value over the present value of debt principal. It has been shown (see [33]) that a positive net-worth covenant prevents agency conflicts between shareholders and debtholders after debt has been issued, because none of them has an interest to increase the risk of the assets in place.

2.4. Bankruptcy Costs and Tax Shield

Upon default, the firm is liquidated, and a constant and exogenous fraction α of the asset value is lost on bankruptcy costs, so that the amount distributed to debtholders as a whole is $(1-\alpha)\,V_\tau$. We assume equal seniority between holders of fixed-rate debt and holders of floating-rate debt, so that they both receive a fraction of the remaining asset value proportional to the principal of the debt they hold. This is consistent with the standard practice, where a proportional priority rule (PPR) based on the face value of the claims is applied in the case where all claimants have equal seniority. As a result, holders of fixed-rate debt receive $\theta(1-\alpha)V_\tau$ at date τ, while holders of floating-rate debt receive $(1-\theta)(1-\alpha)V_\tau$. If the firm has not defaulted before date T, bondholders receive the promised payments. Hence the market value of outstanding corporate bonds is:

$$D_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^\tau R_s\,\mathrm{d}s}(1-\alpha)V_\tau \mathbb{1}_{\{\tau\leq T\}}\right]\mathbb{1}_{\{t\leq\tau\}} + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s\,\mathrm{d}s}L_T \mathbb{1}_{\{\tau>T\}}\right]\mathbb{1}_{\{t\leq\tau\}}, \quad (2.9)$$

while the market values of outstanding fixed-rate and floating-rate bonds are respectively:

$$D_t^1 = \theta\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^\tau R_s\,\mathrm{d}s}(1-\alpha)V_\tau \mathbb{1}_{\{\tau\leq T\}}\right]\mathbb{1}_{\{t\leq\tau\}} + n_1\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s\,\mathrm{d}s}e^{TR_{0,T}} \mathbb{1}_{\{\tau>T\}}\right]\mathbb{1}_{\{t\leq\tau\}},$$

$$D_t^2 = (1-\theta)\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^\tau R_s\,\mathrm{d}s}(1-\alpha)V_\tau \mathbb{1}_{\{\tau\leq T\}}\right]\mathbb{1}_{\{t\leq\tau\}} + n_2\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s\,\mathrm{d}s}e^{\int_0^T R_s\,\mathrm{d}s} \mathbb{1}_{\{\tau>T\}}\right]\mathbb{1}_{\{t\leq\tau\}}.$$

Bankruptcy costs are paid at time τ and are equal to a fraction α of the remaining asset value at that time, with a present value given by:

$$BC_0 = \alpha\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^\tau R_u\,\mathrm{d}u}V_\tau \mathbb{1}_{\{\tau\leq T\}}\right]. \quad (2.10)$$

In addition to liquidation costs, the other friction that makes the capital structure have an impact on firm value is taxes. By issuing debt, the firm takes advantage of a tax shield equal to a fraction of the interest payment. Because the debt instruments are zero-coupon bonds, interests are paid at date T, and the tax shield is of the form $k(L_T - L_0)$. It is perceived only if the firm redeems its debt in full, that is if it has not defaulted prior to T. In case of early default, no interest is paid and the tax shield is lost.[17] For brevity, we refer to the present value of the tax advantage to debt as the tax

[17] An alternative model for the tax shield would consist in recognizing that a corporation may capture interest expense as a deduction as it accrues, as opposed to at terminal date only, with a tax on cancellation of debt income in the event of default. As a result, we expect our model, when restricted to a single class of fixed-rate debt ($\theta = 1$), to lead to somewhat lower leverage ratios compared to similar models that assume coupon-paying bonds. This intuition is confirmed in the numerical results presented in Section 3.

shield itself:

$$TS_0 = k\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T R_u\,\mathrm{d}u}(L_T - L_0)\mathbb{1}_{\{\tau>T\}}\right]. \tag{2.11}$$

The total value of the firm equals unlevered asset value plus the present value of the tax shield minus the present value of bankruptcy costs:

$$v_0 = V_0 + TS_0 - BC_0, \tag{2.12}$$

and equity value is obtained as the difference between total value and market value of debt:[18]

$$E_0 = v_0 - D_0. \tag{2.13}$$

Capital and debt structure decisions are made at time 0 by initial owners of the firm. We extend existing literature by assuming that their objective is to maximize the total value of the firm not only with respect to the face value L_0 (capital structure decision), but also with respect to the variable θ (debt structure decision). This is written as:

$$\max_{L_0,\theta} v_0. \tag{2.14}$$

and we denote by L_0^* and θ^* the optimal values of L_0 and θ.

2.5. Pricing the Defaultable Claims

In order to find optimal debt and capital structures, we must compute the total value v_0, which requires the pricing of claims on firm's assets with a default barrier. Given the presence of two stochastic state variables (V and r), this is a difficult problem. It turns out that in the special case where the firm issues only one class of debt, *quasi*-analytical expressions can be obtained, which we present before we turn to the general case.

2.5.1. Pricing with a Single Class of Debt

If n_1 or n_2 is zero, then the process L is log-normal, and standard techniques can be applied to obtain a quasi-analytical expression for the prices of the various claims, as shown in the following proposition.

Proposition 1. *Let $\mathbb{Q}^T$ be the T-forward probability measure, defined as:*

$$\frac{d\mathbb{Q}^T}{d\mathbb{Q}} = \frac{1}{B(0,T)} e^{-\int_0^T R_s\,ds},$$

and let F and F^T be the cumulative distribution functions of τ under the probability measures $\mathbb{Q}$ and $\mathbb{Q}^T$.

[18] It can be checked that equity value is the present value of all cash-flows accruing to equity holders, as it should be. These cash-flows are: the cash-flows generated by firm's activities up to default time if default occurs prior to time T, or up to debt maturity if default has not occurred before; the unlevered asset value of the firm at time T only if default has not occurred before; the tax shield at time T net of debt repayment, only if default has not occurred; nothing at time T if default has occurred before.

- *If the firm issues only fixed-rate debt ($n_2 = 0$), then:*

$$\begin{aligned} BC_0 &= \alpha L_0 \left[F^T(T) + \delta e^{\delta T} \int_0^T F^T(t) e^{-\delta t}\, dt \right], \\ D_0 &= \frac{1-\alpha}{\alpha} BC_0 + L_0 \mathbb{Q}^T(\tau > T), \\ TS_0 &= k L_0 \left[1 - B(0,T)\right] \mathbb{Q}^T(\tau > T). \end{aligned}$$

- *If it issues only floating-rate debt ($n_1 = 0$), then:*

$$\begin{aligned} BC_0 &= \alpha L_0 \left[F(T) + \delta e^{\delta T} \int_0^T F(t) e^{-\delta t}\, dt \right], \\ D_0 &= \frac{1-\alpha}{\alpha} BC_0 + L_0 \mathbb{Q}(\tau > T), \\ TS_0 &= k L_0 \left[\mathbb{Q}(\tau > T) - B(0,T) \mathbb{Q}^T(\tau > T) \right]. \end{aligned}$$

Proof. See Appendix Appendix A. ■

It follows from these expressions that in order to compute the total value of the firm and the values of equity and debt, we need the cumulative distribution function (in short, cdf) of τ under $\mathbb{Q}^T$ if the firm issues only fixed-rate debt, and the cdfs of τ under both $\mathbb{Q}^T$ and $\mathbb{Q}$ when the firm issues floating-rate debt. The computation of $\mathbb{Q}^T(\tau \le t)$ when the firm has only fixed-rate bonds can be done analytically, because in that case the distance-to-default process $\left(V_t e^{\delta(t-T)}/L_t\right)_t$ follows a martingale under $\mathbb{Q}^T$ (see details in Appendix Appendix B). The situation is slightly more complicated when the firm has only floating-rate bonds, because the distance-to-default process does not follow a martingale under $\mathbb{Q}^T$, although it does so under $\mathbb{Q}$. But it still follows a Gaussian process under $\mathbb{Q}^T$, which allows to use the approximation technique described in [11] to obtain an approximate expression for $\mathbb{Q}^T(\tau > t)$. The following proposition provides the corresponding expressions.

Proposition 2.

- *Assume that the firm issues only fixed-rate debt ($n_2 = 0$). Then:*

$$\begin{aligned} F^T(t) &= \mathbb{Q}^T(\tau \le t) \\ &= \mathcal{N}\left(\frac{\ln \frac{L_0}{V_0 e^{-\delta T}} + \frac{1}{2}\phi_1(t)}{\sqrt{\phi_1(t)}} \right) + \frac{V_0 e^{-\delta T}}{L_0} \mathcal{N}\left(\frac{\ln \frac{L_0}{V_0 e^{-\delta T}} - \frac{1}{2}\phi_1(t)}{\sqrt{\phi_1(t)}} \right), \end{aligned}$$

where $\phi_1(t) = \int_0^t \|\boldsymbol{\sigma}_V - \boldsymbol{\sigma}_B(T-s)\|^2\, ds$ and $\mathcal{N}$ denotes the cdf of the standard normal distribution.

- *Assume that the firm issues only floating-rate debt ($n_1 = 0$). Then:*

$$F(t) = \mathbb{Q}(\tau \le t) = \mathcal{N}\left(\frac{\ln \frac{L_0}{V_0 e^{-\delta T}} + \frac{1}{2}\sigma_V^2 t}{\sigma_V \sqrt{t}} \right) + \frac{V_0 e^{-\delta T}}{L_0} \mathcal{N}\left(\frac{\ln \frac{L_0}{V_0 e^{-\delta T}} - \frac{1}{2}\sigma_V^2 t}{\sigma_V \sqrt{t}} \right).$$

The survival probability $\mathbb{Q}^T(\tau > T)$ can be approximated by:

$$\widehat{\mathbb{Q}}^T(\tau > T) = 1 - \sum_{u=0}^{n_T-1} \widehat{g}\left((2u+1)\frac{\Delta t}{2}\right)\Delta t, \tag{2.15}$$

where n_T is a large integer, $\Delta t = T/n_T$, $t_i = i\Delta t$ for $i = 0, \ldots, n_T$ and the quantity $\widehat{g}\left((2u+1)\frac{\Delta t}{2}\right)$ is recursively computed through:

$$G_1(t_i) = \sum_{u=0}^{i-1} G_2\left(t_i, (2u+1)\frac{\Delta t}{2}\right)\widehat{g}\left((2u+1)\frac{\Delta t}{2}\right)\Delta t, \quad i = 1, \ldots, n_T,$$

$$G_1(t) = \mathscr{N}\left(\frac{\ln \frac{L_0}{e^{-\delta T}V_0} - \frac{1}{2}[\phi_2(t) - \phi_1(t)]}{\sqrt{\phi_1(t)}}\right), \quad 0 \le t \le T,$$

$$G_2(t,s) = \mathscr{N}\left(\frac{-\frac{1}{2}[\phi_2(t) - \phi_2(s) - \phi_1(t) + \phi_1(s)]}{\sqrt{\phi_1(t) - \phi_1(s)}}\right), \quad 0 \le s < t \le T,$$

$$\phi_2(t) = \int_0^t \sigma_B(T-s)^2\, ds, \quad 0 \le t \le T.$$

Proof. See Appendix Appendix B. ■

While being not completely analytical, the above expressions allow for an approximated computation of the prices with a simple numerical scheme that does not require Monte-Carlo simulations. We use this approximation in Section 3 to compare the total value achieved with fixed-rate debt only and that achieved with floating-rate debt only.

From the expressions written in Proposition 2, it can be verified that the probability for the firm to default by date t under the T-forward probability measure is an increasing function of $\phi_1(t)$ when only fixed-rate debt is available. Since $\phi_1(t)$ is the variance of $\ln[V_t/L_t]$, it follows that the probability of default within a given timeframe is lower for firms whose value process is better aligned with their liabilities. But the expressions in Proposition 1 imply that the present values of the bankruptcy costs and tax savings are respectively increasing and decreasing in the default probability. Hence, firms with a lower volatility of their assets relative to their liabilities face lower expected distress costs and a higher tax shield. A similar result holds when only floating-rate bonds are issued, by the approximated expressions for the T-neutral default probabilities. Thus, our model based on a stochastic default date yields predictions similar to those of the model in Smith and Stulz [51], where taxes are modeled as a convex function of pre-tax value, leading to an expected after-tax value that is decreasing in the volatility of earnings, and hedging allows the firm to avoid default in some states of the world, thereby decreasing expected default costs.

2.5.2. Pricing with Two Classes of Debt

For a more general debt structure, with both fixed-rate and floating-rate bonds, there exist no such quasi-closed forms expressions, for L is the sum of two log-normal processes (see (2.7)), rather than a log-normal process itself. As a result, the distribution of the default time (2.8) is not standard and the techniques used in the related literature to find the distribution of the default time under $\mathbb{Q}$, and more importantly, the joint distribution of default time, asset value and short-term rate, cannot be applied. In this context, we obtain the prices of the claims by Monte-Carlo simulations. We simulate 500,000 paths for asset value and interest rate using the distribution under $\mathbb{Q}$ and a monthly time step. We then compute the corresponding payoff to debtholders and bankruptcy costs at default date or at maturity date, and the tax shield at maturity date. Finally, we discount these quantities at the risk-free rate, and average the 500,000 outcomes, to obtain estimates for D_0, BC_0 and TS_0. The total value of the firm is obtained as (2.12), and the value of equity is obtained by taking the difference $v_0 - D_0$.

3. Numerical and Comparative Statics Analysis

We now turn to the computation of the optimal debt policy, which includes the determination of the face value, L_0, and the proportion of fixed-rate bonds, θ. We first describe the parameter values, and we then compare the total values achieved with fixed-rate or floating-rate debt only. Finally, we derive the value-maximizing mixture.

3.1. Base Case Parameter Values

We first define a set of base case parameters, related to the continuous-time processes for the short-term interest rate and the asset value. We estimate the parameters of the default-free short-term rate by using data on the US yield curve. Series of nominal zero-coupon yields with maturities ranging from 1 year to 30 years, constructed after the method of Gürkaynak et al. [24], are available on the website of the Federal Reserve. Our dataset consists of monthly observations of the yields of maturities ranging from one to ten years, covering the period Aug. 1971 – Aug. 2012. The yield of a default-free nominal zero-coupon bond implied by the Vasicek model is equal to:

$$y_B(t,T) = \frac{D_R(T-t)}{T-t} R_t - \frac{E_B(T-t)}{T-t}, \tag{3.1}$$

where:

$$D_R(\tau) = \frac{1-e^{-a\tau}}{a},$$
$$E_B(\tau) = \left(b - \frac{\sigma_R \lambda_R}{a}\right)[D_R(\tau) - \tau] + \frac{\sigma_R^2}{2a^2}\left[\tau - 2\frac{1-e^{-a\tau}}{a} + \frac{1-e^{-2a\tau}}{2a}\right].$$

We follow the same estimation methodology employed by Duffee [16] and Sangvinatsos and Wachter [50], in which it is assumed that a number of bonds equal to the number of state variables is perfectly observed.[19] In our term structure model there is only one state variable (the short-term rate R_t) and we decide to impose that the observed 3-year yield is equal to the model-implied value, as given by (3.1), while the other observed yields are equal to model-implied yields plus a residual η_t^τ:

$$\widehat{y}_B(t,\tau) = \frac{D_R(T-t)}{T-t} R_t - \frac{E_B(T-t)}{T-t} + \eta_t^\tau.$$

[19] Alternatively, we could have also assumed that all yields are observed with errors, following a methodology such as that in Brennan and Xia [6]. We have verified that the two methodologies provide estimates which are very similar to our purposes.

We also assume that the specific terms are homoskedastic with diagonal covariance matrix, but their variance, $(\sigma_\eta^\tau)^2$, is allowed to vary across maturities. The estimation is performed by maximizing the likelihood of the process R sampled at discrete dates. The estimated value for the long-run mean of the short-term nominal rate, $\widehat{b}$, is 0.0353, whereas the estimate for the market price of risk, λ_R, is -0.3340.[20] The estimated volatility of short-term rate, σ_R, is equal to 1.68%, which is slightly lower than the values of 2% used by [7] and [46], and than the 3.16% of [37].

Table C.1 about here.

Our choice of base case parameter values for the asset value process and for the correlation ρ_{RV} is inspired by the literature on the pricing of corporate debt with stochastic interest rates. Table C.1 lists the base case parameters used in [37], [36], [7], [11] and [29]. The asset return volatility, σ_V, is set to 20%. This value is the same as the one used in the five papers mentioned above, and is also close to the value of 23% that [35] reports for firms issuing AAA-rated debt.The choice of a base case value for the correlation parameter ρ_{RV} is more subtle, because the sensitivity of firm's cash flows to interest rates depends on the type of activities undertaken by the firm. For instance, [57] estimates the betas of firm's cash flows with respect to short-term rates in different sectors of the economy and finds positive as well as negative values. Non-depository financial institutions, and personal services firms exhibit negative betas, while mining activities and petroleum refining firms have positive betas. [19] also finds empirical evidence for both signs of the correlation. In the base case, we take $\rho_{RV} = -0.25$ as is done for example by [37] and [7], and we subsequently perform a comparative static analysis involving different values in an attempt to capture the various types of firms. Our base case parameter values are summarized in Table C.2.

Table C.2 about here.

3.2. Preliminary Analysis: Fixed-Rate Debt vs Floating-Rate Debt

We first perform a simple analysis showing how the choice of debt structure can have an impact on the firm value v_0 and how the optimal choice may depend on the specific characteristics of the firm and the economic environment. For three different values of the correlation between firm's assets and short-term rate, we calculate the total value in two extreme cases where the firm issues either fixed-rate debt only or floating-rate debt only. We fix the initial value of debt L_0 to 0.2, which is a reasonable arbitrary value, and we set all the other parameters to their base case values, as reported in Table C.2.[21]

As shown in Table C.3, for $\rho_{RV} = -0.5$ we obtain $v_0(\theta = 1) = 1.0212$ and $v_0(\theta = 0) = 1.0165$: the total value is therefore higher when the firm issues fixed-rate debt only. This can be explained considering that, when ρ_{RV} is negative enough, the firm value

[20]Note that a negative value is needed here for the instantaneous expected excess return of the bond over the risk-free rate under the physical measure $\mathbb{P}$ to be positive (this expected excess return is equal to $\sigma_B(t,T)\lambda_R$, and $\sigma_B(t,T)$ is negative).

[21]In unreported results, we have checked the robustness of our findings with respect to changes in L_0 and changes in parameters describing interest rate and unlevered firm's assets processes.

at date T is negatively correlated with the promised payoff of the floating-rate bond. Hence, if the firm issues floating-rate bonds, the payment due to bondholders tends to be high when assets are low, which increases the probability of default, which in turn has a negative effect on total firm value. On the other hand, when $\rho_{RV} = 0.5$, we have $v_0(\theta = 1) = 0.9943$ and $v_0(\theta = 0) = 1.0106$, so the total value is higher if the firm issues floating-rate bonds. Indeed, when ρ_{RV} is positive, V_T is positively correlated with the payoff of floating-rate bonds. Hence, the volatility of the asset value *net of promised payoff to bondholders* is lower with floating-rate bonds than with fixed-rate securities. When $\rho_{RV} = 0$, we obtain $v_0(\theta = 1) = 1.0077$ and $v_0(\theta = 0) = 1.0137$. Although the correlation between innovations to the firm value process and the short-term rate is null, V_T is positively correlated with the promised payoff of floating-rate debt, because the risk-neutral drift of the process V_t is precisely the short rate, R_t. Overall, these examples illustrate the relevance of debt structure choices and suggest that issuing 100% of fixed-rate bonds does not necessarily lead to maximizing firm value.

Besides looking at the total firm value, it is worth studying the effects on equity and debt values. Interestingly, irrespective of the value of the correlation ρ_{RV}, the equity value is always higher when floating-rate debt is issued if compared to the case where fixed-rate debt is issued. In light of the effects on the firm value v_0, this is intuitive for the case where $\rho_{RV} = 0.5$ or $\rho_{RV} = 0$, as in this case the issuance of floating-rate debt allows to reduce the probability of default, thanks to the positive correlation between V_T and L_T, increasing the tax shield effect and reducing bankruptcy costs, thus increasing both the equity value E_0 and the debt value D_0. For the case where $\rho_{RV} = -0.5$, instead, the probability of default is higher when floating-rate debt is issued, resulting in a reduced tax shield and higher bankruptcy costs, which entail a lower initial debt value D_0. However, when floating-rate debt is issued, the increased volatility of $V_T - L_T$ makes the optionality embedded in firm's equity worth more than in the case of fixed-rate debt, making the equity holders better off, despite a lower total firm value v_0. This highlights the opportunity for equity holders to benefit from a risk-shifting behavior, consisting in the issuance of a fraction of floating-rate debt which is higher than the optimal quantity maximizing the firm's value v_0.

The result just observed allows to reinterpret the empirical findings by James [26] under a different and complementary perspective. In particular, he finds an average positive abnormal stock return corresponding to bank loan announcements, and a nonpositive average abnormal returns for straight debt issues. Furthermore, he finds an average negative stock price response for private placements and straight debt issues used to repay bank loans, giving a possible interpretation to these phenomena in terms of the ability of banks of filling the informational gap between lenders and borrowing firms. We have already argued that, while public debt is typically fixed-rate and with a typically long maturity,[22] bank loans can be floating-rate contracts or, even when they are fixed-rate, they have a much shorter maturity[23] and are periodically renewed, making them similar to floating-rate debt in terms of interest rate exposure. Our model, which disregards the

[22] In the sample used by James [26], 17.96 years on average.

[23] In the sample used by James [26], 5.6 years on average. Federal Reserve data show that this quantity is around 1-2 years in the last couple of decades.

problem of informational asymmetry, predicts a raise of equity value corresponding to the issuance of floating-rate debt, either for a firm value-enhancing reason, which is dominant for positive values of ρ_{RV}, or because of a risk-shifting reason, which is dominant for negative values of ρ_{RV}. This result would also provide an explanation for two phenomena that James [26] deems to be difficult to interpret. On the one hand, our model is able to explain the nonpositive average abnormal stock return corresponding to the announcement of private debt placements, which are mostly long-term fixed-rate contracts, and for which a positive stock price reaction would be expected if their ability to resolve informational asymmetries were the only determinant of the values of corporate claims. On the other hand, our model also provides an explanation for the statistically significant positive difference between abnormal stock returns registered respectively for short-term and long-term straight debt placements, being the first ones closer to floating-rate debt instruments and the second ones to purely fixed-rate instruments.

3.3. Optimal Capital and Debt Structure

We now turn to a formal optimization exercise so as to find the optimal debt structure, and we study the impacts of various parameters characterizing the firm value process and the short-term rate on the optimal debt structure. For each set of parameter values, we compute the face value of debt (L_0^*) and the proportion of fixed-rate bonds (θ^*) that maximize the total value. The optimization is carried out numerically subject to the constraint $0 \leq \theta \leq 1$ so that the firm does not hold long positions in its own bonds. The second optimization that we run is the maximization of total value subject to the constraint of issuing fixed-rate bonds only, which is the debt structure that is typically considered as a default assumption in dynamic capital structure models. Mathematically, this program reads:

$$\max_{L_0} v_0 \qquad \text{s.t. } \theta = 1. \tag{3.2}$$

We denote the corresponding face value with $\widehat{L}_0$ and the maximum total value with $\widehat{v}_0$. To compare the respective impacts of debt management and capital structure optimization on the total value, we report the quantity $(v_0^* - \widehat{v}_0)/(\widehat{v}_0 - V_0)$, which measures the marginal contribution of debt structure optimization over capital structure optimization. Another quantity of interest is the leverage ratio D_0/v_0, which defines the capital structure.

We first perform a comparative static analysis of the optimal debt and capital structure with respect to the correlation ρ_{RV} between the short-term rate and the firm's unlevered value process, a subject that has been discussed the literature on risk management, but mostly on qualitative grounds. Results are displayed in Table C.4: Panel A corresponds to the joint optimization over L_0 and θ and Panel B to the optimization over L_0 subject to $\theta = 1$. We let the coefficient vary in the range $[-0.95, 0.95]$.

Table C.4 about here.

We observe that the optimal allocation to floating-rate debt, $1 - \theta^*$, is increasing in the correlation between changes in asset value and changes in interest rates; it grows from 0% for $\rho_{RV} = -0.5$ to 35.4% for $\rho_{RV} = -0.25$, then to 100% for $\rho_{RV} = 0.25$. This result is in line with the conclusions from the preliminary analysis, where it was shown that if ρ_{RV} is sufficiently high, then total value is higher when the firm issues floating-rate bonds as opposed to fixed-rate bonds. The key result here is that for some values of the correlation, approximately between -0.5 and 0, the optimal debt structure involves *both* types of bonds. In order to check whether the optimization of debt structure allows for a reduction in the volatility of the net asset value, we also compare the volatilities of V_T and $V_T - L_T$. The former quantity is the volatility of net assets that would be achieved by issuing fixed-rate instruments only. As a matter of fact, we observe that the volatility of $V_T - L_T$ after optimization is always lower than or equal to the volatility of V_T. Equality holds only when it is optimal for the firm not to issue any floating-rate bonds. This means that the joint optimization of L_0 and θ leads to a decrease in the uncertainty on the cash flows net of promised debt payment. This result is consistent with a hedging perspective: a firm whose cash flows covary positively with interest rates can take advantage from this natural hedge against rising rates by issuing more floating-rate debt.

A comparison between Panels A and B in Table C.4 shows that debt management decisions can have a substantial impact on capital structure decisions. We find that for firms that should optimally issue a positive fraction of floating-rate bonds, the optimal leverage ratio is always higher when debt is optimized than when it consists only of nominal bonds. For instance, it grows from 12.0% to 14.8% for $\rho_{RV} = 0.25$, and from 9.9% to 15.3% for $\rho_{RV} = 0.75$. Hence, by optimizing their debt structure, firms can maximize the advantages of debt, and therefore issue more debt. Another indicator pointing to the same conclusion is the face value of debt, which is higher with the joint optimization. These results are entirely consistent with the empirical finding reported by Graham and Rogers [23] that hedging increases debt capacity. Interestingly, by examining the various components of the total value displayed in the table, it is seen that optimizing the debt structure in addition to the capital structure increases the tax shield with respect to the situation where only the capital structure is selected, as long as the optimal debt mix does not consist of 100% of fixed-rate debt. This increase is at least partly accounted for by the increase in the amount of debt, since the firm benefits from tax deductions on interests. Again, this is consistent with Graham and Rogers [23], who note that while the convexity of tax schemes does not by itself motivate the use of hedging in practice, the tax benefits from the higher debt issuance are sizable. A similar pattern is found in the other comparative static analysis that we conduct below (see the next tables).

The empirical studies of Graham and Rogers [23] and Chava and Purnanandam [9] have also found that reducing bankruptcy costs appears to be a motive for hedging, given that firms facing greater default risk tend to hedge more. Our model does not imply that bankruptcy costs are lower when the firm optimizes its debt structure than when it does not: in fact, it predicts the opposite effect, though the increase is not as large as for the tax shield, which eventually leads to a higher total value. In fact, two competing effects are at work here, since debt optimization allows the firm to issue more debt – which mechanically increases expected default costs –, but it also results in a decrease in the volatility of the net asset $V_T - L_T$ – which, as explained in Section 2, decreases these costs. Since the face value of debt is endogenously determined in our model, it is impossible to say whether it is because the firm issues more debt that it hedges more through the issuance of a greater share of floating-rate debt, so as to eventually reduce these bankruptcy costs. We only note that this is one possible interpretation of our result, the other view being that the firm takes the risk to issue more debt because the lower volatility of the net asset value compensates the incurred increase in default costs. In fact, both effects are likely, since as explained by Graham and Rogers [23], there are two-way interactions between the extent of hedging and the size of debt in practice.

The impact of debt structure optimization on the total value depends on the correlation. It is small for large negative correlations but it represents 29% or more of the impact of capital structure optimization when ρ_{RV} is greater than 0.25. Intuitively, optimal management of debt structure matters more for firms whose cash-flows are more correlated with interest rate changes. Correlation values above 0.25 are not implausible given the values considered in the literature. For instance, in a quantitative analysis of credit spreads with fixed-rate debt only, Collin-Dufresne and Goldstein [11] let ρ_{RV} vary from -0.20 to 0.20, and they show that the credit spread of the ten-year bond increases from about 50 to about 75 basis points between these bounds. Ju and Ou-Yang [29] compute the optimal maturity of debt for ρ_{RV} equal to -0.30, 0 and 0.30.

We highlight that in our analysis, when referring to floating-rate debt, in practice we refer also to financing contracts having a fixed interest rate that is frequently updated. An example could be bank loans or syndicated loans with a short maturity, and an interest rate that is renegotiated every time that the contract expires and is rolled over.

3.4. Other Comparative Static Analysis

We now examine the impact of interest rate volatility. In Table C.5, we let the volatility of the short-term rate vary from 0.5% to 5%, a range that contains all values reported by papers that calibrated the Vasicek model. We find that the optimal share of fixed-rate debt is decreasing in the volatility. This result may seem counterintuitive: if σ_R increases, then the promised payment of floating-rate bonds is more uncertain, which should make this class of debt less attractive for the firm. But as we have seen in Section 3.3, the relevant risk indicator of a given mixture of bonds is not the volatility of L_T, but instead the volatility of the net asset value, $V_T - L_T$. For low values of σ_R (less than 1%), there is little uncertainty in the short-term rate, so that the uncertainty in V_T is mostly driven by the Brownian term $\sigma_V \, \mathrm{d}z_t^V$ in (2.1). Because we have assumed a negative correlation between this diffusion term and the innovation to the short rate, $\sigma_R \, \mathrm{d}z_t^R$, the correlation between V_T and the promised payoff of floating-rate debt is negative. This negative exposure to interest rate risk could be hedged with a long position in floating-rate debt, which we have not allowed. As a consequence, the volatility of $V_T - L_T$ is lower if the firm issues fixed-rate debt, and the optimal strategy consists of issuing only fixed-rate bonds. On the other hand, when the volatility exceeds 1%, the contribution of the drift to the uncertainty in V_T is no longer negligible compared to that of the diffusion term. Hence, the volatility of the net asset is lower if the firm issues some fraction of floating-rate bonds. In this sense, floating-rate debt provides a hedge against future changes in interest rates: the promised payoff of floating-rate bonds decreases when asset is low, and it increases precisely at times the firm can afford higher payments. When σ_R reaches high levels, such as 5%, the optimal debt mixture is even dominated by floating-rate debt, with the proportion of fixed-rate bonds being only 15.2%.

Table C.5 about here.

We then assess the impact of debt maturity (denoted by T) on the optimal debt structure. In Table C.6, we let the maturity vary from one to fifteen year(s). For maturities less than or equal to five years, the optimal issuance policy is to issue nearly 100% of fixed-rate bonds, while for longer maturities the optimal debt structure shows a significant amount of floating-rate bonds, up to 55% for a maturity equal to fifteen years. The mechanism behind this result is the same as in Table C.5. For short maturities, the uncertainty in the short-term rate does not contribute significantly to the variance of V_T, which is largely driven by the Brownian term. Because this term has negative correlation with the Brownian term of the short-term rate, it follows that V_T has negative correlation with the promised payoff of floating-rate bonds. Eventually, the firm issues only fixed-rate bonds. But when the maturity exceeds five years, the uncertainty in the short-term rate generates a positive correlation between V_T and the cumulative short-term rate, despite the negative correlation between the innovations to V and R. The volatility of the net assets, $V_T - L_T$, gets therefore lower when the firm issues some fraction of floating-rate bonds, which is apparent in the optimal debt structure.

Table C.6 about here.

Finally, we assess the impact of the interest rate price of risk, λ_R. This parameter enters the drift of a zero-coupon bond under the physical measure as follows:

$$\frac{\mathrm{d}B(t,T)}{B(t,T)} = [R_t - \sigma_B(t,T)\lambda_R]\ \mathrm{d}t - \sigma_B(t,T)\,\mathrm{d}\widehat{z}_t^R.$$

This parameter provides a measure of the excess cost of fixed-rate debt over floating-rate debt for the issuer in the following sense: the higher the absolute value of λ_R (i.e., the more negative it is), the steeper the default-free term structure. In a market environment with a large absolute λ_R, long-term bonds have much larger expected returns than short-term bonds, so the ex-ante cost of issuing fixed-rate debt for the firm is much larger than the cost of issuing floating-rate debt.

To test the impact of λ_R on debt management decisions, we let it vary from -1.05 to 0 in Table C.7. To fix the ideas, $\lambda_R = -1.05$ implies that a ten-year zero-coupon bond has a large expected excess return over the risk-free rate of 12.9% per year. With $\lambda_R = 0$, the term premium is zero. Although these two situations are very different, the optimal debt structure exhibits low sensitivity to λ_R: the optimal proportion of fixed-rate bonds varies from 55.4% for $\lambda_R = -0.6$ to 61.5% for $\lambda_R = 0$, which implies a much narrower range of values than in Table C.4. We also note that θ^* is not a monotonic function of λ_R: this means that the slope of the term structure has no directional effect on the optimal issuance of fixed-rate debt, and suggests that cost minimization – which would call for a higher share of floating-rate debt when the cost of fixed-rate debt increases – is not a dominant force in the choice of the optimal debt structure.

The most visible effects of a higher absolute λ_R in Table C.7 are an increase in the optimal amount of debt issued, which translates into an increase in the optimal leverage ratio, and an increase in the tax shield, which follows from the increase in debt issuance and from an increase in the interest rate burden on fixed-rate debt: a higher interest rate risk premium implies higher interests to pay on fixed-rate bonds and has no effect on the interests on floating-rate bonds, so the potential tax savings increase. These effects are present in both panels, that is whether the debt structure is optimized or not.

The interest rate risk premium is related to the business cycle because it tends to be higher in bear markets, when investors require higher compensation to take risks. Hence, our model predicts a higher leverage ratio in bear markets, but does not predict a directional effect of the business cycle on the optimal debt structure.

Table C.7 about here.

4. A Formal Analysis of the Hedging Motive in Debt Management

In Section 3, we have shown that the optimal allocation to fixed- versus floating-rate bonds critically depends on the correlation between the short-term rate and firm value processes, while the price of interest rate risk has comparatively little impact. Overall these findings suggest that the hedging motive is a dominant force from the normative standpoint, a conjecture which we now would like to assess quantitatively. To do so, we analyze in this section the optimal debt structure decision form a different perspective, focusing on risk minimization, as opposed to firm value maximization. We can think of this perspective as being the one favored by an infinitely risk-averse manager of the firm, as opposed to the perspective of well-diversified risk-neutral shareholders. Our goal here is to provide a formal comparison of the debt structures obtained by minimizing risk with those obtained by maximizing firm value.

4.1. A Generic Debt Management Problem

Consider the following two programs focusing on risk minimization:

$$\min_{\theta} \mathbb{V}[L_T], \tag{4.1}$$

$$\min_{\theta} \mathbb{V}\left[V_T - L_T\right]. \tag{4.2}$$

The variances are taken under the pricing measure $\mathbb{Q}$. The first program, (4.1), seeks to minimize the uncertainty of promised debt payment. Obviously, it has a trivial solution: since the promised payment of fixed-rate debt is not random, a focus on risk minimization unambiguously leads to issuing fixed-rate bonds only, which corresponds to $\theta = 1$. On the other hand, the manager in charge of debt structure decisions might recognize that what matters is not so much the variability of the promised payment as the volatility of the firm cash flow net of debt payment. This concern is captured by Program (4.2), where risk is measured as the variance of the "surplus" $V_T - L_T$ of assets with respect to liabilities, as opposed to the variance of L_T. Intuitively, we expect this "asset-liability management" program to provide a good approximation of the hedging motive behind debt management. To see that (4.2) in general differs from (4.1), let us consider the following identity:

$$\mathbb{V}[V_T - L_T] = \mathbb{V}[V_T] + \mathbb{V}[L_T] - 2\text{Cov}[V_T, L_T]. \tag{4.3}$$

Hence, increasing the proportion of floating-rate debt (i.e., decreasing θ) would lead to an increase in the $\mathbb{V}[L_T]$ term, but it could also lead to a decrease or an increase in the $\mathrm{Cov}[V_T, L_T]$ term depending on the sign of the correlation between V and R. The overall effect is thus ambiguous. Such a trade-off between increase in risk and increase in covariance results in a non-trivial solution to the program. Of course, a perfect hedge cannot be achieved in this incomplete market setting: as long as there is imperfect correlation between asset risk and interest rate risk, the equality $V_T = L_T$ cannot hold almost surely.

Program (4.2) is *ex-ante* essentially different from Program (2.14), which is the one that is relevant to the owners of the firm. In order to assess the loss of total value induced by the use of the risk-minimizing strategy, we set the face value of debt to L_0^*, and we solve (4.2) with respect to θ, which gives the risk-minimizing strategy $\dot{\theta}_{\mathrm{hed}}$. We then compute the total value achieved with this issuance policy, and compare it with the total value achieved by simultaneously optimizing over L_0 and θ. Table C.8 presents the results for different values of ρ_{RV}, which is the parameter that has been found to have the largest impact on debt structure decisions (see Section 3). It appears that the total value achieved with the risk-minimizing strategy is extremely close to the maximum total value that would be reached with the optimal capital and debt structures: the two values differ by less than 1%.[24] This finding confirms that the hedging motive drives most of optimal debt allocation decisions.

Table C.8 about here.

4.2. Regression of $\boldsymbol{\theta}^*$ *on* $\dot{\boldsymbol{\theta}}_{hed}$

In an attempt to quantify the importance of the hedging motive within optimal debt structure decisions, we now run a regression of θ^* on $\dot{\theta}_{\mathrm{hed}}$. The model that we estimate is:

$$\theta_i^* = a_0 + a_1 \dot{\theta}_{\mathrm{hed},i} + \varepsilon_i, \tag{4.4}$$

where index i refers to the i^{th} observation for the shares of fixed-rate debt. The observations have been generated by letting parameters ρ_{RV}, λ_R and σ_R vary across the values considered in the previous tables, while keeping the other parameters fixed at their base case values. This results in 616 sets of parameters, and we compute θ^* and $\dot{\theta}_{\mathrm{hed}}$ for each of these sets. We thus obtain a cross-section of firms (different ρ_{RV}) and of market conditions (different λ_R and σ_R). We then repeat the estimation procedure for three values of the volatility of the firm unlevered asset value, namely $\sigma_V = 10$, 20 and 30%.[25]

Table C.9 reports the estimates for a_0 and a_1, as well as the R-square of each regression, computed as:

$$R^2 = 1 - \frac{\sum_{i=1}^N (\theta_i^* - \widehat{a}_0 - \widehat{a}_1 \dot{\theta}_{\mathrm{hed},i})^2}{\sum_{i=1}^N (\theta_i^* - \overline{\theta^*})^2},$$

where $\overline{\theta^*}$ denotes the mean of θ_i^* over all observations.

[24]In unreported results, we have checked that the same conclusion obtains if one lets another parameter, such as interest rate volatility, vary.

[25]Because of the large number of combinations of the parameters involved in this exercise, we estimate the prices of the claims through Monte Carlo simulation by generating 100,000 paths only, rather than 500,000 as in the previous sections.

Table C.9 about here.

For $\sigma_V = 10\%$ or 20%, the R-square is greater than 80%, which shows that θ^* is close to an affine function of $\dot{\theta}_{\text{hed}}$. For $\sigma_V = 30\%$, the variations of $\dot{\theta}$ still explain 68% of the variance of θ^*. These results thus confirm that hedging considerations, that aim at reducing the variance of the net asset value $V_T - L_T$, explain most of optimal debt structure decisions across firms and market conditions. Furthermore, the values for θ^* and $\dot{\theta}_{\text{hed}}$ vary in the same direction, as all the estimates for $\widehat{a}_1$ are strictly positive and higher than 0.79. Nevertheless, the analysis confirms that the two optimization problems are different, as θ^* and $\dot{\theta}_{\text{hed}}$ are not equal: indeed, the t-statistics for the tests $a_0 = 0$ and $a_1 = 1$ are all larger than 4, which leads to reject the null assumptions of these tests. In the same table, in order to quantify the relative distance in terms of firm value between the firm-value maximization program and the risk-minimization program, we report the average relative increase in firm value for two different situations. The first average value relates to the case where one switches from the risk minimization program (see 4.2) to the value-maximization program (see (2.14)). This increase is always around 2%, showing that the difference between the two programs is small. The second average value relates to the case where the firm switches from fixed-rate bonds only to the optimal mixture of fixed-rate and floating-rate bonds. This second average ranges from 35% to 83%, which is much higher than the first one. The comparison between the two values confirms that the first effect can be regarded as economically small.

The proximity between the total value achieved with the pure hedging strategy and the maximum possible total value notwithstanding the difference between the two strategies shows that total value is a relatively flat function of the allocation in the vicinity of its optimum. This is also confirmed by the slow convergence of the optimizer towards the value-maximizing allocation. Eventually, it appears that a debt structure dictated by pure hedging motives (minimizing the volatility of the net assets) does almost as well in terms of total value as the value-maximizing structure, though it corresponds to a different allocation between fixed-rate and floating-rate debt.

5. Introducing Inflation-Linked Bonds

We have found so far that debt management decisions influence the optimal capital structure, so that debt and capital structure choices should be analyzed simultaneously in the context of a unified framework. We also find that issuing a single class of fixed-rate debt, as typically assumed in the capital structure literature, is sub-optimal in a stochastic interest rate environment and can induce a significant loss in firm value. One possible approach for enhancing the hedging properties of the debt portfolio with respect to the asset portfolio consists of further expanding the liability mix by introducing inflation-linked bonds, which is what we turn to now.[26]

[26]In principle, a firm could be tempted to issue bonds with interest payment indexed on a specific risk factor that affects the present value of the operating cash-flows generated by the firm, e.g., issue bonds indexed on oil prices for an oil company (see for example Morellec and Smith [45] for a related analysis). This would allow for a substantial increase in the correlation between assets and liabilities, and the analysis in Section 3 suggests that significant increases in the total value could be expected.

Inflation-linked bonds were first issued in the United Kingdom in 1981, where they represented 27.3% of the total outstanding debt in 2016. Similar securities have been issued since then in other large economies, but they are still a minor part of the whole pool of public bonds: at the end of 2016, only 8.9% of US debt, 12.3% of French debt and 12.7% of Italian debt were linked to inflation.[27] In the US, it has never been greater than 11% over the past two decades. On the corporate side, inflation-indexed bonds have been issued by companies in the regulated electricity and water sectors, notably in the US and also in the UK, where future regulated rate increases are linked to inflation indices such as the retail price index (RPI). There has also been a recent interest amongst corporations, in particular utility or financial-services companies, for such bonds.[28] The limited size of the market for public inflation-indexed debt, as well as the currently low inflation expectations, which drive down the demand for protection against rising prices, perhaps explain why indexed corporate debt is still marginal. On the other hand, intuition suggests that if a firm's earnings tend to grow with inflation, then having some inflation-linked issuance can be a natural hedge. The formal model that we introduce in this section quantitatively shows that if a firm can issue bonds whose redemption value is indexed on the price level, it will enjoy an increase in its value by issuing some of them.

5.1. Inflation Risk

As in Brennan and Xia [6], we model the stochastic price index as:

$$\frac{\mathrm{d}\Phi_t}{\Phi_t} = (\pi_t - \sigma_\Phi \lambda_\Phi)\,\mathrm{d}t + \sigma_\Phi\,\mathrm{d}z_t^\Phi. \tag{5.1}$$

$\frac{\mathrm{d}\Phi_t}{\Phi_t}$ is the realized inflation rate over $[t, t+\,\mathrm{d}t]$, while π_t represents the instantaneous expected inflation rate, and λ_Φ is the price of inflation risk. Following Brennan and Xia [6], we assume that both the expected inflation (π_t) and the real interest rate (r_t) follow Ornstein-Uhlenbeck processes under the pricing measure:

$$\begin{aligned} \mathrm{d}\pi_t &= \kappa\,(\beta - \pi_t)\;\mathrm{d}t + \sigma_\pi\,\mathrm{d}z_t^\pi \\ \mathrm{d}r_t &= a\,(b - r_t)\;\mathrm{d}t + \sigma_r\,\mathrm{d}z_t^r. \end{aligned} \tag{5.2}$$

The physical dynamics of these two quantities are still Ornstein-Uhlenbeck processes, with long-term means $\widehat{b} = b + \frac{\sigma_r \lambda_r}{a}$ and $\widehat{\beta} = \beta + \frac{\sigma_\pi \lambda \pi}{\kappa}$. We denote with $\widehat{z^\Phi}$, $\widehat{z^r}$ and $\widehat{z^\pi}$ the Brownian motions under $\mathbb{P}$. The unlevered asset value process of the firm follows the same process as in Section 2:

$$\frac{\mathrm{d}V_t}{V_t} = (R_t - \delta)\,\mathrm{d}t + \sigma_V\,\mathrm{d}z_t^V, \tag{5.3}$$

where R_t still denotes the *nominal* short-term interest rate at date t. As shown by Brennan and Xia [6], the nominal rate in this framework is given by a modified version of the Fisher's relation:

$$R_t = r_t + \pi_t - \sigma_\Phi \lambda_\Phi. \tag{5.4}$$

While we restrict in this paper our attention to existing bonds, the setup that we develop could be used to assess the usefulness of issuing such broader forms of contingent bonds.

[27] Data from the respective national central banks.

[28] Inflation-linked debt is also commonly issued across several sectors in Israel.

This equality is used in the pricing of default-free nominal and inflation-indexed bonds (see derivation in Appendix Appendix C). We denote by $I(t,T)$ the price of a default-free indexed zero-coupon bond paying Φ_T at time T, and with $\rho_{0,T}$ the equivalent real zero-coupon rate:

$$\rho_{0,T} = -\frac{1}{T}\ln I(0,T).$$

We now assume that the firm can issue inflation-indexed bonds in addition to fixed-rate and floating-rate bonds. Each corporate inflation-indexed bond promises the payment $e^{T\rho_{0,T}}\Phi_T$ at date T. The firm now issues n_1 fixed-rate bonds, n_2 floating-rate bonds and n_3 indexed bonds at date 0. The composition of debt is then unchanged until the maturity date, so that the present value of the promised payment is:

$$L_t = n_1 e^{TR_{0,T}} B(t,T) + n_2 e^{\int_0^t R_u \,\mathrm{d}u} + n_3 e^{T\rho_{0,T}} I(t,T).$$

The face value of debt at the initial date is L_0, and the debt structure is fully described by the proportions of fixed-rate and inflation-indexed bonds:

$$\theta_1 = \frac{n_1}{n_1+n_2+n_3}, \quad \theta_2 = \frac{n_2}{n_1+n_2+n_3}.$$

As in Section 2, we assume that in case of default prior to time T, a fraction α of the remaining unlevered firm value is lost to third parts and that the recovery payment, $(1-\alpha)V_\tau$, is shared amongst bondholders with equal priority. As a result, the market values of corporate fixed-rate bonds and floating-rate bonds are the same as in Section 2, and that of corporate indexed bonds is:

$$D_t^3 = \theta_2 \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^\tau R_s \,\mathrm{d}s}(1-\alpha)V_\tau \mathbb{1}_{\{\tau\le T\}}\right]\mathbb{1}_{\{t\le\tau\}} + n_3 \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s \,\mathrm{d}s} e^{T\rho_{0,T}}\Phi_T \mathbb{1}_{\{\tau>T\}}\right]\mathbb{1}_{\{t\le\tau\}}.$$

5.2. Base Case Parameter Values

The model now involves three state variables: the real rate and the expected inflation rate, that drive the nominal term structure, and the price index. Following Brennan and Xia [6] and Sangvinatsos and Wachter [50], we use likelihood maximization to estimate the various parameters. As in Section 2, we use monthly US data from August 1971 to August 2012. Nominal yields are those used in Section 3. In order to recover the real rate and the expected inflation rate from the yields, we need to assume that two observed yields are equal to model-implied ones (see Appendix Appendix C for expressions of the zero-coupon yields in the model). We take these yields to be the two-year and the five-year ones. The two factors can then be obtained by solving the following linear system, for $\tau = 2$ or 5 years

$$y_B(t,\tau) = \frac{D_r(\tau)}{\tau} r_t + \frac{D_\pi(\tau)}{\tau}\pi_t - \frac{E_B(\tau)}{\tau},$$

where expressions for the functions D_r, D_π and E_B are given in Appendix Appendix C. The other observed yields are equal to model-implied yields plus a residual:

$$\widehat{y}_B(t,\tau) = \frac{D_r(\tau)}{\tau} r_t + \frac{D_\pi(\tau)}{\tau}\pi_t - \frac{E_B(\tau)}{\tau} + \eta_t^\tau.$$

The error terms η^τ are normally distributed with mean zero, no autocorrelation, and constant variance $\left(\sigma_\eta^\tau\right)^2$. The price index is represented by the Consumer Price Index for all urban consumers (all items), available on the website of the Federal Reserve. It should be noted that the price of inflation risk cannot be identified from data on nominal yields and realized inflation only. One option would be to set it to zero in the calibration, but doing this we would fail at recognizing that inflation risk is correlated with real interest rate risk and expected inflation risk, and thus carries part of the risk premia that are associated with these two risks. Therefore, we take a different option by setting the *specific* price of inflation risk equal to zero. Formally, this constraint is equivalent to:

$$\lambda_\Phi = \frac{\rho_{r\Phi}\lambda_r + \rho_{\pi\Phi}\lambda_\pi - \rho_{r\pi}\left(\rho_{r\Phi}\lambda_\pi + \rho_{\pi\Phi}\lambda_r\right)}{1-\rho_{r\pi}^2}.$$

TABLE C.10 ABOUT HERE.

Maximum likelihood estimates for the parameters of the real rate, the expected inflation rate and the price index processes are reported in Table C.10. We also set the base case values for ρ_{rV}, $\rho_{\pi V}$ and $\rho_{\Phi V}$ equal to zero.

5.3. Impact of Correlation Between Firm Value and Realized Inflation

Present values of bankruptcy costs and the tax shield are defined as in (2.10) and (2.11). The total value is then obtained as $v_0 = V_0 + TS_0 - BC_0$, and the value of equity as $E_0 = v_0 - D_0$. The objective of the initial owners of the firm is to maximize the total value with respect to the capital and debt structures:

$$\max_{L_0,\theta_1,\theta_2} v_0. \tag{5.5}$$

TABLE C.11 ABOUT HERE.

We perform a comparative static analysis to assess the impact of the correlation between asset value and realized inflation, $\rho_{\Phi V}$. The results reported in Panel A of Table C.11 show that the optimal share of inflation-linked bonds is increasing in the correlation $\rho_{\Phi V}$; it is zero when $\rho_{\Phi V}$ is less than 0, and grows to 37.0% for $\rho_{\Phi V} = 0.25$ and to 66.0% for $\rho_{\Phi V} = 0.75$. The mechanism at work here is the same as in Section 3.3, where it was shown that the optimal share of floating-rate debt is increasing in the correlation between the firm value and the nominal rate. Here, if $\rho_{\Phi V}$ is sufficiently high, the firm has a natural hedge against realized inflation risk, and it can align inflation risk exposure on the asset and liability sides by issuing indexed bonds. This issuance strategy allows the firm to reduce the probability of default (even though the face value L_0 is higher than when only fixed-rate debt is issued, as can be seen in the table), which decreases bankruptcy costs and increases the tax shield.

Panel B of Table C.11 shows the optimal capital and debt structures when the firm can issue only fixed-rate and floating-rate bonds. The debt structure is unsurprisingly very different from the one obtained in Panel A, as in this case the allocation to fixed-rate debt is still small, but the remaining portion of debt is entirely allocated to floating-rate bonds. Interestingly, however, the last two rows of the table show that the marginal gains that follow from debt structure optimization, in the two cases discussed, are sizable, but not strongly different from each other for reasonable values of $\rho_{\Phi V}$. This result is likely to be caused by the fact that the instantaneous tracking error of inflation-linked bonds with respect to floating-rate bonds is, for our estimates of the model parameters, rather small. In particular, this is caused by the fact that the speed of mean reversion of the real rate process, a, is large, and therefore the sensitivity of inflation-linked bonds to variations of the real rate, measured by the duration $D_r(\tau)$, tends to be small (2.2 years for a 10-year bond), as well as the fact that the volatility of the unexpected inflation, σ_Φ, is also rather small (1.37%).[29] Similar conclusions can be drawn noticing that the decrease of the volatility of the net asset value $V_T - L_T$ is negligible moving from Panel B to Panel A. These results are compatible with the fact that the firm value, as highlighted in Section 4.2, is a quantity that may be relatively insensitive to the control variables $(\theta_1, \theta_2, \theta_3)$, at least in some directions, in the neighborhood of the optimum. An issuance policy that substitutes inflation-indexed bonds with floating-rate bonds, in fact, seems not to be very harmful in terms of firm value according to our model. We confirm, however, that the welfare improvements obtained optimizing the debt structure, with respect to the case where only fixed-rate debt is issued, are important and range from 10.4% for $\rho_{\Phi V} = -0.75$ to 15.3% for $\rho_{\Phi V} = 0.75$.

Overall, these results illustrate that a firm whose earnings are positively correlated with the price index would increase its value by issuing inflation-indexed bonds, which should account for a sizable fraction of optimal bond issuance for reasonable parameter values. However, the size of the corporate inflation-linked bond market is nowadays tiny. This is most likely to be linked with the scarce demand for inflation-indexed instruments in general, even in the sovereign bond market, especially in periods with low inflation expectations. Furthermore, we have shown that the economic incentives to issue corporate inflation-linked bonds are rather small for firms that are already issuing an optimal amount of fixed-rate and floating-rate debt.

6. Conclusions and Suggestions for Further Research

Our understanding of liability management decisions barely extends beyond the *capital structure decision* (equity versus debt allocation), and when it addresses the *debt structure decision* (e.g., fixed- versus floating-rate debt allocation), it mostly relies on qualitative insights. The main contribution of this paper is to provide a joint quantitative analysis of capital and debt structure decisions within a standard continuous-time model in the presence of interest rate and inflation risks. Our main findings are that debt management decisions have an impact on capital structure decisions, and the increases in firm value that can be expected from optimizing the debt structure can represent a significant fraction of the increases that follow from capital structure optimization. We confirm that corporations, whose revenues are positively correlated with the price index, would benefit from issuing inflation-linked bonds, which is a segment of the bond market issuance that is so far mostly occupied by sovereign states. We also provide additional explanation to stylized facts observed in the empirical literature of financial intermediation, such as the positive average abnormal stock returns corresponding to the announcement

[29] Our parameter estimates are compatible with those found in the literature of dynamic term structure models, as for example in Brennan and Xia [6].

of bank loans, which is similar to floating-rate debt from the interest rate hedging perspective, and, conversely, the nonpositive average abnormal stock returns corresponding to the announcement of straight public debt or private placements.

Our analysis could be extended in a number of directions. In particular, it would be useful to include other types of instruments, such as convertible bonds, preferred shares and other equity-linked structures in the liability mix, in addition to fixed-rate and floating-rate bonds. On a different note, it should be emphasized that our model, following previous literature, considers the liability allocation problem from the standpoint of the original owners of the firm, assumed to be risk-neutral with respect to the (diversifiable) source of uncertainty impacting the firm value. In practice, however, the managers of the firm, as opposed to the owners of the firm, are in charge of making corporate risk-management and liability allocation decisions. Several works have documented the role of conflicts of interests and managerial incentives in the design of corporate debt structure programs (see [51] and [52], as well as [9] for an empirical analysis). In this respect, we have provided evidence in Section 3.2 that a simplified model is able to fit some basic stylized facts, but incorporating these aspects beyond the generic model discussed in Section 4 would also be desirable. These questions are left for further research.

Appendix

Appendix A. Proof of Proposition 1

By definition of default time (see (2.8)), the present value of bankruptcy costs (see (2.10)) is:

$$BC_0 = \alpha \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^{\tau\wedge T} R_u \, \mathrm{d}u} e^{\delta(T-\tau\wedge T)} L_{\tau\wedge T} \mathbb{1}_{\{\tau\leq T\}} \right].$$

Note that the random variable $e^{\delta(T-\tau\wedge T)}\mathbb{1}_{\{\tau\leq T\}}$ is $\mathscr{F}_{\tau\wedge T}$-measurable and that the quantity $\left(e^{-\int_0^t R_u \, \mathrm{d}u} L_t\right)_t$ follows a $\mathbb{Q}$-martingale. From Theorem 3.22 in [30], we thus have that $\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T R_u \, \mathrm{d}u} L_T | \mathscr{F}_{\tau\wedge T}\right] = e^{-\int_0^{\tau\wedge T} R_u \, \mathrm{d}u} L_{\tau\wedge T}$. Hence:

$$BC_0 = \alpha \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^{T} R_u \, \mathrm{d}u} e^{\delta(T-\tau)} L_{T} \mathbb{1}_{\{\tau\leq T\}} \right].$$

When the firm issues only fixed-rate debt, we have $L_T = L_0 e^{T R_{0,T}}$, so that:

$$\begin{aligned} BC_0 &= \alpha L_0 \mathbb{E}^{\mathbb{Q}^T} \left[e^{\delta(T-\tau)} \mathbb{1}_{\{\tau\leq T\}} \right] \\ &= \alpha L_0 \int_0^T e^{\delta(T-t)} f^T(t) \, \mathrm{d}t. \end{aligned}$$

On the other hand, when it issues only floating-rate debt, we have $L_T = L_0 e^{\int_0^T r_s \, \mathrm{d}s}$, hence:

$$\begin{aligned} BC_0 &= \alpha L_0 \mathbb{E}^{\mathbb{Q}} \left[e^{\delta(T-\tau)} \mathbb{1}_{\{\tau\leq T\}} \right] \\ &= \alpha L_0 \int_0^T e^{\delta(T-t)} f(t) \, \mathrm{d}t. \end{aligned}$$

Integrating by parts, we obtain the expressions for BC_0 given in the proposition.

Considering the value of debt (see (2.9)), we have:

$$\begin{aligned} D_0 = (1-\alpha)\mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^T R_u \, \mathrm{d}u} e^{\delta(T-\tau)} \mathbb{1}_{\{\tau\leq T\}} \right] &+ n_1 \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^T R_u \, \mathrm{d}u} e^{T R_{0,T}} \mathbb{1}_{\{\tau> T\}} \right] \\ &+ n_2 \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^T R_u \, \mathrm{d}u} e^{\int_0^T R_u \, \mathrm{d}u} \mathbb{1}_{\{\tau> T\}} \right], \end{aligned}$$

which can be rewritten as:

$$D_0 = \frac{1-\alpha}{\alpha} BC_0 + n_1 \mathbb{Q}^T(\tau > T) + n_2 \mathbb{Q}(\tau > T).$$

Finally, the present value of the tax shield (see (2.11)) can be rewritten as:

$$\begin{aligned} TS_0 &= k \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^T R_u \, \mathrm{d}u} \left(n_1 e^{T R_{0,T}} + n_2 e^{\int_0^T R_u \, \mathrm{d}u} - L_0 \right) \mathbb{1}_{\{\tau> T\}} \right] \\ &= k n_1 \mathbb{Q}^T(\tau > T) + k n_2 \mathbb{Q}(\tau > T) - k L_0 B(0,T) \mathbb{Q}^T(\tau > T). \end{aligned}$$

Appendix B. Proof of Proposition 2

Appendix B.1. Derivation of $\mathbb{Q}^{\boldsymbol{T}}(\boldsymbol{\tau} \leq \boldsymbol{t})$ *When* $\boldsymbol{n_2} = \boldsymbol{0}$

By Girsanov's theorem, the process $\boldsymbol{z}^T$ defined by $\boldsymbol{z}_0^T = 0$ and $\mathrm{d}\boldsymbol{z}_t^T = \mathrm{d}\boldsymbol{z}_t - \boldsymbol{\sigma}_B(T - t)\,\mathrm{d}t$ is a Brownian motion under $\mathbb{Q}^T$. For brevity, we omit the argument of $\boldsymbol{\sigma}_B$ in what follows. Using (2.3) and (2.5), we have:

$$\frac{V_t}{B(t,T)} = \frac{V_0}{B(0,T)} \exp\left[-\int_0^t \frac{\|\boldsymbol{\sigma}_V - \boldsymbol{\sigma}_B\|^2}{2}\,\mathrm{d}u + \int_0^t [\boldsymbol{\sigma}_V - \boldsymbol{\sigma}_B]'\,\mathrm{d}\boldsymbol{z}_u^T - \delta t\right].$$

By Theorem 4.6 (page 174) in Karatzas and Shreve [30], any continuous local martingale can be written as a time-changed Brownian motion. More precisely, there exists a $\mathbb{Q}^T$-Brownian motion W^T such that:

$$W_{\phi_1(t)}^T = \int_0^t [\boldsymbol{\sigma}_V - \boldsymbol{\sigma}_B]'\,\mathrm{d}\boldsymbol{z}_u^T,$$

with $\phi_1(t) = \int_0^t \|\boldsymbol{\sigma}_V - \boldsymbol{\sigma}_B\|^2\,\mathrm{d}u$. The definition of the default time can thus be rewritten as:

$$\tau = \inf\left\{t\ ;\ W_{\phi_1(t)}^T - \frac{1}{2}\phi_1(t) \leq \ln\frac{L_0}{V_0 e^{-\delta T}}\right\},$$

and the cumulative default probability as:

$$\begin{aligned}\mathbb{Q}^T(\tau \leq t) &= \mathbb{Q}^T\left(\min_{0\leq s\leq t}\left[W_{\phi_1(s)}^T - \frac{1}{2}\phi_1(s)\right] \leq \ln\frac{L_0}{V_0 e^{-\delta T}}\right)\\ &= \mathbb{Q}^T\left(\min_{0\leq s\leq \phi_1(t)}\left[W_s^T - \frac{1}{2}s\right] \leq \ln\frac{L_0}{V_0 e^{-\delta T}}\right).\end{aligned}$$

The distribution of the running minimum of a drifted Brownian motion can be found in [30]. It leads to the expression for $\mathbb{Q}^T(\tau \leq t)$ written in the first item of the proposition.

Appendix B.2. Derivation of $\mathbb{Q}(\boldsymbol{\tau} \leq \boldsymbol{T})$ *When* $\boldsymbol{n_1} = \boldsymbol{0}$

For the case where $n_1 = 0$, we proceed in a similar way, but no change of probability measure is needed this time. Indeed, we have $L_t = L_0 e^{\int_0^t R_u\,\mathrm{d}u}$, so that:

$$\frac{V_t}{L_t} = \frac{V_0}{L_0}\exp\left[-\int_0^t \frac{\sigma_V^2}{2}\,\mathrm{d}u + \int_0^t \boldsymbol{\sigma}_V'\,\mathrm{d}\boldsymbol{z}_u - \delta t\right].$$

We introduce a $\mathbb{Q}$-Brownian motion W such that $W_{\sigma_V^2 t} = \int_0^t \boldsymbol{\sigma}_V' \boldsymbol{z}_u$. Then the probability of default before date t can be expressed as:

$$\mathbb{Q}(\tau \leq t) = \mathbb{Q}\left(\min_{0\leq s\leq \sigma_V^2 t}\left[W_s - \frac{1}{2}s\right] \leq \ln\frac{Le^{\delta T}}{V_0}\right).$$

Again, this probability can be computed by using the results of [30] on the distribution of the minimum of a drifted Brownian motion, which leads to the expression given in the proposition.

Appendix B.3. Approximation of $\mathbb{Q}^{\boldsymbol{T}}(\boldsymbol{\tau} > \boldsymbol{T})$ *When* $\boldsymbol{n_1} = \mathbf{0}$

Let X denote the logarithm of the distance-to-default:

$$X_t = \ln \frac{e^{-\delta(T-t)} V_t}{L_t}.$$

Under the T-forward probability measure, we have that:

$$X_t = -\delta T + \ln \frac{V_0}{L_0} + \int_0^t [\boldsymbol{\sigma}_V - \theta \boldsymbol{\sigma}_B(s,T)]' \,\mathrm{d}\boldsymbol{z}_s^T + \frac{1}{2}[\phi_2(t) - \phi_1(t)],$$

where $\boldsymbol{z}^T$ is a $\mathbb{Q}^T$-Brownian motion. Hence X is continuous and Gaussian, and has the Markov property. Default is triggered when X crosses the barrier 0 from above. We also denote with $\pi(X_t, t|X_0, 0)$ the transition density from X_0 at time 0 to X_t at time t, and with $g(t|X_0, 0)$ the probability density that τ is equal to t when the process starts from X_0 at time 0, *i.e.* $g(t|X_0,0) = \frac{1}{\mathrm{d}t}\mathbb{Q}^T(t \le \tau < t + \mathrm{d}t|X_0, 0)$. The survival probability can be written as:

$$\mathbb{Q}^T(\tau > T) = 1 - \int_0^T g(t|X_0, 0)\,\mathrm{d}t.$$

Hence we need to estimate $g(t|X_0, 0)$ for $t \le T$. The following equation, due to [20], holds for $X_t < 0 < X_0$:

$$\pi(X_t, t|X_0, 0) = \int_0^t \pi(X_t, t|X_s = 0, s) g(s|X_0, 0)\,\mathrm{d}s.$$

It states that in order to be at the level X_t under the barrier at time t, the process must have crossed the barrier at some date s between 0 and t. Integrating both sides by $\int_{-\infty}^0 \mathrm{d}X_t$, we obtain that:

$$\mathbb{Q}^T(X_t < 0|X_0, 0) = \int_0^t g(s|X_0, 0)\,\mathbb{Q}^T(X_t < 0|X_s = 0, s)\,\mathrm{d}s. \tag{B.1}$$

We note that, for $s \le t$:

$$X_t = X_s + \frac{1}{2}[\phi_2(t) - \phi_2(s) + \phi_1(t) - \phi_1(s)] + \int_s^t [\boldsymbol{\sigma}_V - \boldsymbol{\sigma}_B(T-u)]' \,\mathrm{d}\boldsymbol{z}_u^T,$$

hence the conditional moments, for $s \ge t$:

$$\mathbb{E}^{\mathbb{Q}^T}[X_t|X_s] = X_s + \frac{1}{2}[\phi_2(t) - \phi_2(s) - \phi_1(t) + \phi_1(s)], \quad \mathbb{V}^{\mathbb{Q}^T}[X_t|X_s] = \phi_1(t) - \phi_1(s).$$

We thus obtain:

$$\mathbb{Q}^T(X_t < 0|X_0, 0) = \mathscr{N}\left(\frac{-X_0 - \frac{1}{2}[\phi_2(t) - \phi_1(t)]}{\sqrt{\phi_1(t)}}\right) \stackrel{\text{def}}{=} G_1(t),$$

$$\mathbb{Q}^T(X_t < 0|X_s = 0, s) = \mathscr{N}\left(\frac{-\frac{1}{2}[\phi_2(t) - \phi_2(s) - \phi_1(t) + \phi_1(s)]}{\sqrt{\phi_1(t) - \phi_1(s)}}\right) \stackrel{\text{def}}{=} G_2(t, s).$$

We now turn to the discretization of (B.1). We choose an integer n_T, and we define the time step $\Delta t = T/n_T$ as well as n_T time points $t_i = i\Delta t$ for $i = 0, \ldots, n_T$. The discrete version of (B.1) with integrand estimated at midpoints of the interval (as in [11]) can be written as:

$$G_1(t_i) = \sum_{u=0}^{i-1} G_2\left(t_i, (2u+1)\frac{\Delta t}{2}\right) \widehat{g}\left((2u+1)\frac{\Delta t}{2}\right) \Delta t, \quad i = 1, \ldots, n_T,$$

where $\widehat{g}(s)$ is our estimator of $g(s|X_0, 0)$. It can be checked numerically that the resulting estimator of $\mathbb{Q}^T(\tau > T)$ is not significantly sensitive to the choice of midpoints rather than endpoints in the discretization, and that a satisfying convergence is obtained for a discretization step $\Delta t = 0.1$.

Appendix C. Prices of Risk-Free Bonds in the Presence of Inflation Risk

The prices of the nominal zero-coupon bond paying 1 in nominal terms at time T and of the indexed zero-coupon bond paying Φ_T are given by:

$$B(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s\,\mathrm{d}s}\right] \quad \text{and} \quad I(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T R_s\,\mathrm{d}s}\Phi_T\right].$$

From the properties of Ornstein-Uhlenbeck processes, we have that:

$$\int_t^T r_u\,\mathrm{d}u = b(T-t) - \frac{1}{a}\left(1 - e^{-a(T-t)}\right)(b - r_t) + \frac{1}{a}\int_t^T \left(1 - e^{-a(T-u)}\right)\sigma_r\,\mathrm{d}z_t^r,$$
$$\int_t^T \pi_u\,\mathrm{d}u = \beta(T-t) - \frac{1}{\kappa}\left(1 - e^{-\kappa(T-t)}\right)(\beta - \pi_t) + \frac{1}{\kappa}\int_t^T \left(1 - e^{-\kappa(T-u)}\right)\sigma_\pi\,\mathrm{d}z_u^\pi.$$

After some algebra we arrive at:

$$B(t,T) = \exp\left[-D_r(T-t)r_t - D_\pi(T-t)\pi_t + E_B(T-t)\right],$$

with:

$$\begin{aligned} D_r(u) &= \frac{1-e^{-au}}{a}, \quad D_\pi(u) = \frac{1-e^{-\kappa u}}{\kappa}, \\ E_B(u) &= -b\left(u - \frac{1-e^{-au}}{a}\right) - \beta\left(u - \frac{1-e^{-\kappa u}}{\kappa}\right) + \sigma_\Phi\Lambda_\Phi u \\ &\quad + \frac{\sigma_r\sigma_\pi\rho_{r\pi}}{a\kappa}\left(u - \frac{1-e^{-au}}{a} - \frac{1-e^{-\kappa u}}{\kappa} + \frac{1-e^{-(a+\kappa)u}}{a+\kappa}\right) \\ &\quad + \frac{\sigma_r^2}{2a^2}\left(u + \frac{1-e^{-2au}}{2a} - 2\frac{1-e^{-au}}{a}\right) \\ &\quad + \frac{\sigma_\pi^2}{2\kappa^2}\left(u + \frac{1-e^{-2\kappa u}}{2\kappa} - 2\frac{1-e^{-\kappa u}}{\kappa}\right). \end{aligned}$$

We also have:

$$\Phi_T = \Phi_t e^{\int_t^T\left(\pi_u - \frac{\sigma_\Phi^2}{2}\right)\mathrm{d}u + \int_t^T \sigma_\Phi\,\mathrm{d}z_u^\Phi}.$$

After some algebra, we obtain:

$$I(t,T) = \Phi_t \exp\left[-\frac{1-e^{-a(T-t)}}{a} r_t - \left(b + \frac{\sigma_r \sigma_\Phi \rho_{r\Phi}}{a}\right)\left[T - t - \frac{1-e^{-a(T-t)}}{a}\right] + \frac{\sigma_r^2}{2a^2}\left[T - t + \frac{1-e^{-2a(T-t)}}{2a} - 2\frac{1-e^{-a(T-t)}}{a}\right]\right].$$

Table C.1: Values of firm-related parameters in the literature.

	Longstaff and Schwartz (1995)	Leland and Toft (1997)	Briys and de Varenne (1997)	Ju and Ou-Yang (2006)	Collin-Dufresne and Goldstein (2001)
ρ_{RV}	-0.25	–	-0.25	0	-0.2
δ	0	0.07	0	0.05	0.03
σ_V	0.2	0.2	0.2	0.2	0.2
T	$[1, 20]$	$[0, 20]$	$[1, 20]$	–	$[4, 20]$
k	–	0.35	–	0.35	–
α	–	0.5	0.2	0.5	0.56
R_0	–	0.05	0.05	0.07	0.06

The firm's unlevered asset value evolves as in (2.1). δ is the payout rate, σ_V is the volatility of the process and ρ_{RV} is the correlation between changes in the asset value and changes in the short-term interest rate. T is the initial maturity of debt; k is the tax rate and α is the rate of bankruptcy costs. R_0 denotes the initial short-term rate.

Table C.2: Base case parameter values.

Short-term rate: $\mathrm{d}R_t = a(\widehat{b} - R_t)\,\mathrm{d}t + \sigma_R\,\mathrm{d}\widehat{z}_t^R$.	
a	0.0668 (0.0015)
$\widehat{b}$	0.0353 (0.0392)
λ_R	-0.3340 (0.1564)
σ_R	0.0168 (0.0006)
R_0	0.05
Unlevered asset value: $\frac{\mathrm{d}V_t}{V_t} = [R_t - \delta]\,\mathrm{d}t + \sigma_V\,\mathrm{d}z_t^V$.	
δ	0.05
σ_V	0.2
ρ_{RV}	-0.25
V_0	$1
Other parameters.	
T (debt maturity, in years)	10
k (tax rate)	0.35
α (rate of bankruptcy costs)	0.5

This table summarizes the base case parameter values used in the numerical analysis. Parameters for the short-term rate process have been estimated from monthly US zero-coupon yields over the period Aug. 1971 – Aug. 2012. Standard errors are in brackets. Other parameters are set with reference to Table C.1.

Table C.3: Effect of the issuance of fixed- vs floating-rate debt for different values of the correlation between the interest rate and the unlevered firm's assets.

	$\rho_{RV} = -0.5$		$\rho_{RV} = 0$		$\rho_{RV} = 0.5$	
	Fixed-rate	Floating-rate	Fixed-rate	Floating-rate	Fixed-rate	Floating-rate
v_0	1.0212	1.0165	1.0077	1.0137	0.9943	1.0106
E_0	0.8286	0.8304	0.8214	0.8249	0.8131	0.8186
D_0	0.1925	0.1861	0.1863	0.1888	0.1811	0.1920
TS_0	0.0309	0.0292	0.0280	0.0292	0.0253	0.0292
BC_0	0.0097	0.0128	0.0203	0.0155	0.0310	0.0187
$\mathrm{Vol}\,[V_T - L_T]$	0.7161	0.7292	0.9477	0.9202	1.1987	1.1400

This table shows the total firm value v_0, the equity value E_0, the debt value D_0, the tax shield TS_0, the bankruptcy costs BC_0, and the volatility of the difference between the final value of assets V_T and the final value of the total liabilities L_T, for different values of the correlation between the innovations of the interest rate and the unlevered firm's assets, ρ_{RV}. The initial value of debt, L_0, is set to 0.2, and the firm issues either fixed-rate debt only, or floating-rate debt only. All the other parameters are set at their base case values (see Table C.2).

Table C.4: Impact of the correlation between short-term interest rate and asset value on optimal capital and debt structures.

Panel A: Optimal debt and capital structures									
ρ_{RV}	-0.95	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.95
Face value L_0	0.228	0.203	0.166	0.159	0.153	0.154	0.155	0.160	0.157
Fixed-rate debt θ	1.000	1.000	0.898	0.264	0.004	0.000	0.000	0.000	0.000
Floating-rate debt $1-\theta$	0.000	0.000	0.102	0.736	0.996	1.000	1.000	1.000	1.000
Equity value E_0	0.808	0.829	0.859	0.865	0.870	0.869	0.869	0.865	0.869
Debt value D_0	0.224	0.199	0.162	0.156	0.150	0.151	0.151	0.156	0.153
Total value v_0	1.032	1.027	1.022	1.020	1.020	1.020	1.020	1.021	1.021
Tax shield TS_0	0.037	0.033	0.027	0.026	0.025	0.025	0.025	0.026	0.026
Bankruptcy cost BC_0	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.005	0.005
Volatility of V_T	0.514	0.604	0.761	0.830	0.948	1.070	1.198	1.334	1.448
Volatility of $V_T - L_T$	0.514	0.604	0.761	0.826	0.926	1.035	1.152	1.276	1.384
Correlation of V_T and L_T	–	–	0.027	0.147	0.323	0.475	0.610	0.732	0.822
Leverage ratio D_0/v_0	0.217	0.193	0.159	0.153	0.147	0.148	0.148	0.153	0.150
Panel B: Optimal capital structure with fixed-rate debt									
ρ_{RV}	-0.95	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.95
Face value L_0	0.228	0.203	0.169	0.158	0.142	0.125	0.115	0.103	0.098
Fixed-rate debt θ	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Floating-rate debt $1-\theta$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Equity value E_0	0.808	0.829	0.857	0.866	0.879	0.893	0.902	0.912	0.917
Debt value D_0	0.224	0.199	0.165	0.154	0.138	0.122	0.112	0.101	0.096
Total value v_0	1.032	1.027	1.022	1.020	1.017	1.015	1.014	1.013	1.012
Tax shield TS_0	0.037	0.033	0.027	0.025	0.023	0.020	0.018	0.016	0.015
Bankruptcy cost BC_0	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004
Volatility of V_T	0.514	0.604	0.761	0.830	0.948	1.070	1.198	1.334	1.448
Volatility of $V_T - L_T$	0.514	0.604	0.761	0.830	0.948	1.070	1.198	1.334	1.448
Leverage ratio D_0/v_0	0.217	0.193	0.170	0.151	0.136	0.120	0.110	0.099	0.094
Relative importance of debt versus capital structure decisions									
ρ_{RV}	-0.95	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.95
Relative increase in total value (%)	0.000	0.000	0.004	2.327	12.676	29.221	47.175	64.231	77.710

Panel A describes the optimal capital and debt structures when the firm solves Program (2.14), and Panel B shows the optimal capital structure when the firm issues only fixed-rate bonds, solving Program (3.2). L_T is the promised payment to bondholders and V_T is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_0^* - \widehat{v_0}) / (\widehat{v_0} - V_0)$, where $\widehat{v_0}$ is the maximum total value achieved with fixed-rate debt only and v_0^* is the maximum total value achieved with fixed-rate and floating-rate bonds. Apart from ρ_{RV}, all parameters are fixed at their base case values (see Table C.2).

Table C.5: Impact of interest rate volatility on optimal capital and debt structures.

Panel A: Optimal debt and capital structures							
σ_R	0.005	0.0075	0.01	0.02	0.03	0.04	0.05
Face value L_0	0.151	0.154	0.157	0.160	0.162	0.165	0.166
Fixed-rate debt θ	0.999	0.993	0.968	0.555	0.363	0.246	0.152
Floating-rate debt $1-\theta$	0.001	0.007	0.032	0.445	0.637	0.754	0.848
Equity value E_0	0.868	0.867	0.865	0.865	0.865	0.864	0.865
Debt value D_0	0.148	0.150	0.154	0.156	0.158	0.160	0.161
Total value v_0	1.016	1.017	1.018	1.021	1.023	1.025	1.026
Tax shield TS_0	0.021	0.022	0.023	0.026	0.029	0.031	0.032
Bankruptcy cost BC_0	0.004	0.004	0.004	0.004	0.005	0.005	0.005
Volatility of V_T	0.699	0.718	0.741	0.887	1.147	1.577	2.268
Volatility of $V_T - L_T$	0.699	0.718	0.741	0.879	1.106	1.462	2.014
Correlation of V_T and L_T	0.000	0.000	0.006	0.211	0.386	0.520	0.615
Leverage ratio D_0/v_0	0.146	0.148	0.151	0.153	0.155	0.156	0.157
Panel B: Optimal capital structure with fixed-rate debt							
σ_R	0.005	0.0075	0.01	0.02	0.03	0.04	0.05
Face value L_0	0.152	0.155	0.157	0.153	0.144	0.126	0.103
Fixed-rate debt θ	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Floating-rate debt $1-\theta$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Equity value E_0	0.868	0.866	0.865	0.871	0.880	0.895	0.915
Debt value D_0	0.149	0.151	0.153	0.150	0.140	0.122	0.099
Total value v_0	1.016	1.017	1.018	1.020	1.020	1.017	1.014
Tax shield TS_0	0.021	0.022	0.023	0.025	0.025	0.023	0.019
Bankruptcy cost BC_0	0.004	0.004	0.004	0.004	0.005	0.005	0.004
Volatility of V_T	0.699	0.718	0.741	0.887	1.147	1.577	2.268
Volatility of $V_T - L_T$	0.699	0.718	0.741	0.887	1.147	1.577	2.268
Correlation of V_T and L_T	–	–	–	–	–	–	–
Leverage ratio D_0/v_0	0.146	0.149	0.151	0.147	0.137	0.120	0.098
Relative importance of debt versus capital structure decisions							
σ_R	0.005	0.0075	0.01	0.02	0.03	0.04	0.05
Relative increase in total value (%)	0.000	0.009	0.004	4.894	18.529	43.758	84.412

Panel A describes the optimal capital and debt structures when the firm solves Program (2.14), no long position in corporate bonds being allowed, and Panel B shows the optimal capital structure when the firm issues only fixed-rate bonds, solving Program (3.2). L_T is the promised payment to bondholders and V_T is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_0^* - \widehat{v}_0) / (\widehat{v}_0 - V_0)$, where $\widehat{v}_0$ is the maximum total value achieved with fixed-rate debt only and v_0^* is the maximum total value achieved with fixed-rate and floating-rate bonds. Apart from σ_R, all other parameters are fixed at their base case values (see Table C.2).

Table C.6: Impact of debt maturity on optimal capital and debt structures.

Panel A: Optimal debt and capital structures							
T (years)	1	2	3	5	7	10	15
Face value L_0	0.518	0.417	0.355	0.274	0.216	0.159	0.100
Fixed-rate debt θ	1.000	1.000	0.999	0.991	0.745	0.646	0.442
Floating-rate debt $1-\theta$	0.000	0.000	0.001	0.009	0.255	0.354	0.558
Equity value E_0	0.491	0.598	0.664	0.750	0.810	0.865	0.919
Debt value D_0	0.518	0.416	0.353	0.271	0.212	0.156	0.096
Total value v_0	1.009	1.014	1.017	1.021	1.022	1.020	1.015
Tax shield TS_0	0.009	0.015	0.019	0.024	0.026	0.026	0.021
Bankruptcy cost BC_0	0.000	0.001	0.002	0.003	0.004	0.004	0.004
Volatility of V_T	0.200	0.286	0.356	0.480	0.607	0.830	1.378
Volatility of $V_T - L_T$	0.200	0.286	0.356	0.480	0.607	0.826	1.361
Correlation of V_T and L_T	–	–	0.000	0.000	0.062	0.147	0.244
Leverage ratio D_0/v_0	0.513	0.410	0.347	0.265	0.208	0.153	0.095
Panel B: Optimal capital structure with fixed-rate debt							
T (years)	1	2	3	5	7	10	15
Face value L_0	0.518	0.417	0.357	0.272	0.213	0.158	0.094
Fixed-rate debt θ	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Floating-rate debt $1-\theta$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Equity value E_0	0.491	0.598	0.662	0.752	0.812	0.866	0.923
Debt value D_0	0.518	0.416	0.355	0.269	0.210	0.154	0.091
Total value v_0	1.009	1.014	1.017	1.021	1.022	1.020	1.014
Tax shield TS_0	0.009	0.015	0.019	0.024	0.026	0.025	0.020
Bankruptcy cost BC_0	0.000	0.001	0.002	0.003	0.004	0.005	0.004
Volatility of V_T	0.200	0.286	0.356	0.480	0.607	0.830	1.378
Volatility of $V_T - L_T$	0.200	0.286	0.356	0.480	0.607	0.830	1.378
Correlation of V_T and L_T	–	–	–	–	–	–	–
Leverage ratio D_0/v_0	0.513	0.410	0.349	0.263	0.205	0.151	0.090
Relative importance of debt versus capital structure decisions							
T (years)	1	2	3	5	7	10	15
Relative increase in total value (%)	0.000	0.000	0.000	0.000	0.557	2.327	9.790

Panel A describes the optimal capital and debt structures when the firm solves Program (2.14), no long position in corporate bonds being allowed, and Panel B shows the optimal capital structure when the firm issues only fixed-rate bonds, solving Program (3.2). L_T is the promised payment to bondholders and V_T is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_0^* - \widehat{v}_0)\,/\,(\widehat{v}_0 - V_0)$, where $\widehat{v}_0$ is the maximum total value achieved with fixed-rate debt only and v_0^* is the maximum total value achieved with fixed-rate and floating-rate bonds. Apart from T, all parameters are fixed at their base case values (see Table C.2).

Table C.7: Impact of the interest rate market price of risk on optimal capital and debt structures.

Panel A: Optimal debt and capital structures								
λ_R	-1.05	-0.9	-0.75	-0.6	-0.45	-0.3	-0.15	0
Face value L_0	0.173	0.167	0.167	0.164	0.160	0.158	0.153	0.145
Fixed-rate debt θ	0.605	0.588	0.600	0.554	0.649	0.588	0.580	0.615
Floating-rate debt $1-\theta$	0.395	0.412	0.400	0.446	0.351	0.412	0.420	0.385
Equity value E_0	0.863	0.866	0.865	0.865	0.866	0.865	0.867	0.871
Debt value D_0	0.168	0.163	0.162	0.160	0.156	0.154	0.150	0.142
Total value v_0	1.031	1.029	1.027	1.025	1.022	1.020	1.017	1.014
Tax shield TS_0	0.039	0.036	0.034	0.031	0.028	0.025	0.021	0.017
Bankruptcy cost BC_0	0.006	0.006	0.005	0.005	0.004	0.004	0.004	0.003
Volatility of V_T	1.351	1.220	1.102	0.995	0.898	0.811	0.732	0.661
Volatility of $V_T - L_T$	1.344	1.213	1.096	0.989	0.894	0.807	0.729	0.658
Correlation of V_T and L_T	0.147	0.147	0.147	0.147	0.147	0.147	0.147	0.147
Leverage ratio D_0/v_0	0.163	0.158	0.158	0.156	0.153	0.151	0.147	0.140
Panel B: Optimal capital structure with fixed-rate debt								
λ_R	-1.05	-0.9	-0.75	-0.6	-0.45	-0.3	-0.15	0
Face value L_0	0.168	0.165	0.164	0.163	0.158	0.154	0.149	0.144
Fixed-rate debt θ	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Floating-rate debt $1-\theta$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Equity value E_0	0.868	0.868	0.867	0.866	0.867	0.869	0.870	0.872
Debt value D_0	0.163	0.161	0.160	0.158	0.155	0.151	0.146	0.142
Total value v_0	1.031	1.029	1.027	1.024	1.022	1.019	1.016	1.013
Tax shield TS_0	0.037	0.035	0.033	0.030	0.027	0.024	0.021	0.017
Bankruptcy cost BC_0	0.006	0.006	0.005	0.005	0.005	0.004	0.003	0.003
Volatility of V_T	1.351	1.220	1.102	0.995	0.898	0.811	0.732	0.661
Volatility of $V_T - L_T$	1.351	1.220	1.102	0.995	0.898	0.811	0.732	0.661
Correlation of V_T and L_T	–	–	–	–	–	–	–	–
Leverage ratio D_0/v_0	0.158	0.156	0.155	0.155	0.151	0.148	0.144	0.140
Relative importance of debt versus capital structure decisions								
λ_R	-1.05	-0.9	-0.75	-0.6	-0.45	-0.3	-0.15	0
Relative increase in total value (%)	2.029	2.100	2.123	2.175	2.258	2.348	2.363	2.421

Panel A describes the optimal capital and debt structures when the firm solves Program (2.14), no long position in corporate bonds being allowed, and Panel B shows the optimal capital structure when the firm issues only fixed-rate bonds, solving Program (3.2). L_T is the promised payment to bondholders and V_T is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_0^* - \widehat{v}_0)\,/\,(\widehat{v}_0 - V_0)$, where $\widehat{v}_0$ is the maximum total value achieved with fixed-rate debt only and v_0^* is the maximum total value achieved with fixed-rate and floating-rate bonds. Apart from λ_R, all parameters are fixed at their base case values (see Table C.2).

Table C.8: Comparison of firm-value maximizing and risk-minimizing strategies.

Panel A: Optimal debt and capital structures									
ρ_{RV}	-0.95	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.95
Face value L_0	0.228	0.203	0.178	0.159	0.153	0.154	0.155	0.160	0.157
Fixed-rate debt θ	1.000	1.000	1.000	0.646	0.004	0.000	0.000	0.000	0.000
Floating-rate debt $1-\theta$	0.000	0.000	0.000	0.354	0.996	1.000	1.000	1.000	1.000
Equity value E_0	0.808	0.829	0.849	0.865	0.870	0.869	0.869	0.865	0.869
Debt value D_0	0.224	0.199	0.174	0.156	0.150	0.151	0.151	0.156	0.153
Total value v_0	1.032	1.027	1.023	1.020	1.020	1.020	1.020	1.021	1.021
Tax shield TS_0	0.037	0.033	0.029	0.026	0.025	0.025	0.025	0.026	0.026
Bankruptcy cost BC_0	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.005	0.005
Volatility of V_T	0.514	0.604	0.716	0.830	0.948	1.070	1.198	1.334	1.448
Volatility of $V_T - L_T$	0.514	0.604	0.716	0.826	0.926	1.035	1.152	1.276	1.384
Correlation of V_T and L_T	–	–	–	0.147	0.323	0.475	0.610	0.732	0.822
Leverage ratio D_0/v_0	0.217	0.193	0.170	0.153	0.147	0.148	0.148	0.153	0.150
Panel C: Optimal capital structure with $\theta = \dot{\theta}_{\mathbf{hed}}$									
ρ_{RV}	-0.95	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.95
Face value L_0	0.228	0.203	0.178	0.159	0.153	0.154	0.155	0.160	0.157
Fixed-rate debt θ	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
Floating-rate debt $1-\theta$	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
Equity value E_0	0.808	0.829	0.849	0.864	0.870	0.869	0.869	0.865	0.869
Debt value D_0	0.224	0.199	0.174	0.155	0.150	0.151	0.151	0.156	0.153
Total value v_0	1.032	1.027	1.023	1.019	1.020	1.020	1.020	1.021	1.021
Tax shield TS_0	0.037	0.033	0.029	0.025	0.025	0.025	0.025	0.026	0.026
Bankruptcy cost BC_0	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.005	0.005
Volatility of V_T	0.514	0.604	0.716	0.830	0.948	1.070	1.198	1.334	1.448
Volatility of $V_T - L_T$	0.514	0.604	0.716	0.822	0.926	1.035	1.152	1.276	1.384
Correlation of V_T and L_T	–	–	–	0.147	0.323	0.475	0.610	0.732	0.822
Leverage ratio D_0/v_0	0.217	0.193	0.170	0.152	0.147	0.148	0.148	0.153	0.150
Relative importance of firm value maximization with respect to risk minimization									
ρ_{RV}	-0.95	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.95
Relative increase in total value (%)	0.000	0.001	0.003	5.532	0.056	0.004	0.002	0.000	0.000

Panel A describes the optimal capital and debt structures when the firm solves Program (2.14), and Panel C shows the optimal capital structure when the firm issues the same initial amount of debt L_0 as in Panel A, and minimizes the volatility of the quantity $V_T - L_T$, as in Program (4.2). L_T is the promised payment to bondholders and V_T is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_0^* - \dot{v}_0) / (\dot{v}_0 - V_0)$, where $\dot{v}_0$ is the total firm value achieved minimizing the volatility of the quantity $V_T - L_T$, solving Program (4.2), and v_0^* is the maximum total value achieved with fixed-rate and floating-rate debt, solving Program (2.14). Apart from ρ_{RV}, all parameters are fixed at their base case values (see Table C.2).

Table C.9: Regression of θ_i^* on $\dot{\theta}_{\mathrm{hed},i}$ for different values of the volatility of firm's assets.

σ_V	0.10	0.20	0.30
Intercept ($\widehat{a}_0$)	0.0968	0.1346	0.1820
	(15.67)	(14.49)	(14.29)
Slope ($\widehat{a}_1$)	0.9344	0.8879	0.7978
	(4.50)	(6.34)	(9.28)
R^2	0.8697	0.8041	0.6858
mean $\left[\frac{v_0^* - \dot{v}_0}{\dot{v}_0 - V_0}\right]$	0.0177	0.0189	0.0206
mean $\left[\frac{v_0^* - \widehat{v}_0}{\widehat{v}_0 - V_0}\right]$	0.8309	0.5080	0.3515

This table presents the estimated coefficients $\widehat{a}_0$ and $\widehat{a}_1$, and the R-square of the regression equation (4.4). t-statistics are computed with respect to the null hypothesis $a_0 = 0$ and $a_1 = 1$ and are reported in brackets. The first average relative increase in total value reported is the arithmetic mean of the ratio $(v_0^* - \dot{v}_0) / (\dot{v}_0 - V_0)$, where $\dot{v}_0$ is the total firm value achieved minimizing the volatility of the quantity $V_T - L_T$, solving Program (4.2), and v_0^* is the maximum total value achieved with fixed-rate and floating-rate debt, solving Program (2.14). The second average relative increase in total value reported is the arithmetic mean of the ratio $(v_0^* - \widehat{v}_0) / (\widehat{v}_0 - V_0)$, where $\widehat{v}_0$ is the maximum total value achieved with fixed-rate debt only and v_0^* is the maximum total value achieved with fixed-rate, floating-rate and inflation-linked bonds. The 616 observations have been generated by letting ρ_{RV}, λ_R and σ_R vary over the values tested in Tables C.4, C.5 and C.7. σ_V denotes the volatility of firm's unlevered asset value, which is set to 10, 20 or 30%. Other parameters are fixed at their base case values (see Table C.2).

Table C.10: Base case parameter values in the presence of inflation risk.

Real short-term rate: $\mathrm{d}r_t = a(\widehat{b} - r_t)\,\mathrm{d}t + \sigma_r\,\mathrm{d}\widehat{z}_t^r$.	
a	0.4477 (0.0077)
$\widehat{b}$	0.0137 (0.0090)
σ_r	0.0262 (0.0006)
λ_r	−0.0952 (0.1564)
r_0	0.02
Expected inflation rate: $\mathrm{d}\pi_t = \kappa(\widehat{\beta} - \pi_t)\,\mathrm{d}t + \sigma_\pi\,\mathrm{d}\widehat{z_t^\pi}$.	
κ	0.0646 (0.0014)
$\widehat{\beta}$	0.0143 (0.0445)
σ_π	0.0185 (0.0006)
λ_π	−0.2543 (0.1545)
π_0	0.02
Price index: $\frac{\mathrm{d}\Phi_t}{\Phi_t} = \pi_t\,\mathrm{d}t + \sigma_\Phi\,\mathrm{d}\widehat{z_t^\Phi}$.	
σ_Φ	0.0137 (0.0004)
λ_Φ	−0.0867
Unlevered asset value: $\frac{\mathrm{d}V_t}{V_t} = (r_t + \pi_t - \sigma_\Phi\lambda_\Phi - \delta)\,\mathrm{d}t + \sigma_V\,\mathrm{d}z_t^V$.	
δ	0.05
σ_V	0.2
V_0	\$1
Correlations.	
$\rho_{\pi\Phi}$	0.0807 (0.0443)
$\rho_{r\pi}$	−0.5691 (0.0315)
$\rho_{r\Phi}$	0.0908 (0.0447)
ρ_{rV}	0
$\rho_{\pi V}$	0
$\rho_{\Phi V}$	0
Other parameters.	
T (debt maturity)	10 years
α (rate of bankruptcy costs)	0.5
k (tax rate)	0.35

This table summarizes the base case parameter values used in the model with inflation risk. Parameters of the real short-term rate, the expected inflation rate and the price index process have been estimated from US monthly data over the period Aug. 1971 – Aug. 2012. Standard errors are reported in brackets. Other parameters, including firm-related parameters, are set as shown in Table C.1.

Table C.11: Impact of correlation between asset value and realized inflation on optimal capital and debt structures.

Panel A: Optimal debt and capital structures with fixed-rate, floating-rate and inflation-indexed debt

$\rho_{\Phi V}$	-0.75	-0.5	-0.25	0	0.25	0.5	0.75
Face value L_0	0.149	0.149	0.150	0.150	0.151	0.150	0.153
Fixed-rate debt θ_1	0.038	0.065	0.046	0.017	0.009	0.030	0.085
Floating-rate debt θ_2	0.962	0.935	0.954	0.983	0.621	0.551	0.255
Inflation-linked debt $1-\theta_1-\theta_2$	0.000	0.000	0.000	0.000	0.370	0.418	0.660
Equity value E_0	0.872	0.872	0.871	0.870	0.869	0.870	0.868
Debt value D_0	0.146	0.145	0.146	0.147	0.148	0.147	0.150
Total value v_0	1.018	1.018	1.017	1.017	1.017	1.017	1.018
Tax shield TS_0	0.021	0.021	0.021	0.021	0.022	0.022	0.022
Bankruptcy cost BC_0	0.004	0.004	0.004	0.004	0.004	0.004	0.004
Volatility of V_T	0.839	0.839	0.838	0.838	0.838	0.839	0.840
Volatility of V_T-L_T	0.823	0.824	0.822	0.821	0.820	0.820	0.819
Correlation of V_T and L_T	0.301	0.301	0.301	0.301	0.307	0.324	0.351
Leverage ratio D_0/v_0	0.143	0.143	0.144	0.145	0.146	0.145	0.147

Panel B: Optimal debt and capital structures with fixed-rate and floating-rate debt

$\rho_{\Phi V}$	-0.75	-0.5	-0.25	0	0.25	0.5	0.75
Face value L_0	0.149	0.149	0.150	0.150	0.151	0.151	0.148
Fixed-rate debt θ_1	0.038	0.065	0.046	0.017	0.003	0.000	0.080
Floating-rate debt θ_2	0.962	0.935	0.954	0.983	0.997	1.000	0.920
Inflation-linked debt $1-\theta_1-\theta_2$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Equity value E_0	0.872	0.872	0.871	0.870	0.869	0.870	0.872
Debt value D_0	0.146	0.145	0.146	0.147	0.148	0.147	0.145
Total value v_0	1.018	1.018	1.017	1.017	1.017	1.017	1.017
Tax shield TS_0	0.021	0.021	0.021	0.021	0.022	0.021	0.021
Bankruptcy cost BC_0	0.004	0.004	0.004	0.004	0.004	0.004	0.004
Volatility of V_T	0.839	0.839	0.838	0.838	0.838	0.839	0.840
Volatility of V_T-L_T	0.823	0.824	0.822	0.821	0.821	0.822	0.825
Correlation of V_T and L_T	0.301	0.301	0.301	0.301	0.302	0.302	0.302
Leverage ratio D_0/v_0	0.143	0.143	0.144	0.145	0.145	0.145	0.143

Panel C: Optimal capital structure with fixed-rate debt only

$\rho_{\Phi V}$	-0.75	-0.5	-0.25	0	0.25	0.5	0.75
Face value L_0	0.140	0.136	0.138	0.138	0.138	0.138	0.135
Fixed-rate debt θ_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Floating-rate debt θ_2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Inflation-linked debt $1-\theta_1-\theta_2$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Equity value E_0	0.879	0.883	0.880	0.881	0.881	0.881	0.883
Debt value D_0	0.137	0.133	0.135	0.135	0.135	0.135	0.132
Total value v_0	1.016	1.016	1.016	1.016	1.016	1.015	1.015
Tax shield TS_0	0.020	0.019	0.020	0.020	0.020	0.020	0.019
Bankruptcy cost BC_0	0.004	0.003	0.004	0.004	0.004	0.004	0.003
Volatility of V_T	0.839	0.839	0.838	0.838	0.838	0.839	0.840
Volatility of V_T-L_T	0.839	0.839	0.838	0.838	0.838	0.839	0.840
Correlation of V_T and L_T	–	–	–	–	–	–	–
Leverage ratio D_0/v_0	0.135	0.131	0.133	0.133	0.133	0.133	0.130

Relative importance of debt versus capital structure decisions

$\rho_{\Phi V}$	-0.75	-0.5	-0.25	0	0.25	0.5	0.75
Panel A vs Panel C: relative increase in total value (%)	10.387	10.856	10.454	10.969	11.370	12.839	15.339
Panel B vs Panel C: relative increase in total value (%)	10.387	10.856	10.454	10.969	10.704	10.775	11.523

Panel A describes the optimal capital and debt structures when the firm solves Program (5.5), no long position in corporate bonds being allowed, Panel B shows the optimal capital and debt structures when the firm issues floating-rate and fixed-rate bonds, and Panel C shows the optimal capital structure when the firm issues only fixed-rate bonds. L_T is the promised payment to bondholders and V_T is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_0^* - \widehat{v}_0)\,/\,(\widehat{v}_0 - V_0)$, where $\widehat{v}_0$ is the maximum total value achieved with fixed-rate debt only and v_0^* is the maximum total value achieved with fixed-rate, floating-rate and inflation-linked bonds (Panel A vs Panel C), or with fixed-rate and floating-rate bonds (Panel B vs Panel C). Apart from $\rho_{\Phi V}$, all parameters are fixed at their base case values (see Table C.10).

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Highlights

- A formal model for the optimization of capital and debt structures is proposed;
- Firm value is significantly higher when the issuance policy is optimized;
- The optimal issuance policy is mainly driven by hedging considerations.