$f(x_1, x_2)$ - unknown, nonlinear but static function, with two input variables

$\boxed{\text{EXERCISE}}$: To create a polynomial approximator g of the function f, with a configurable degree m. For example, for the first few values of m, the approximator has the form:

$$m = 1, \quad \hat{g}(x) = [1, x_1, x_2] \cdot \Theta = \Theta_1 + \Theta_2 x_1 + \Theta_3 x_2$$

$$m = 2, \quad \hat{g}(x) = [1, x_1, x_2, x_1^2, x_2^2, x_1 x_2] \cdot \Theta = \Theta_1 + \Theta_2 x_1 + \Theta_3 x_2 + \Theta_4 x_1^2 + \Theta_5 x_2^2 + \Theta_6 x_1 x_2$$

$$m = 3, \quad \hat{g}(x) = [\underbrace{1, x_1, x_2, x_1^2, x_2^2, x_1^3, x_2^3, x_1 x_2, x_1^2 x_2, x_1 x_2^2}_{\text{regressors vectors}}] \cdot \Theta$$

where the first two polynomials have been made explicit for clarity. So we need to create an algorithm that generates the regressors vector, for any degree m.

$\boxed{\text{SOLUTION}}$:

Using the commutative law of addition we can rearrange the terms in a suitable order, such that the implementation of the algorithm to be simpler. So the goal is to find such an arrangement.

degree 1: $\left[\ 1 \quad \underbrace{x_1^1} \quad \underbrace{x_2^1}\ \right]$

degree 2: $\left[\ 1 \quad \underbrace{x_1^1 \quad x_1^2} \quad \underbrace{x_2^1 \quad x_2^2} \quad \underbrace{x_1^1 x_2^1}_{2}\ \right]$

degree 3: $\left[\ 1 \quad \underbrace{x_1^1 \quad x_1^2 \quad x_1^3} \quad \underbrace{x_2^1 \quad x_2^2 \quad x_2^3} \quad \underbrace{x_1^1 x_2^1}_{2} \quad \underbrace{x_1^1 x_2^2 \quad x_1^2 x_2^1}_{3}\ \right]$

degree 4: $\left[\ 1 \quad \underbrace{x_1^1 \quad x_1^2 \quad x_1^3 \quad x_1^4} \quad \underbrace{x_2^1 \quad x_2^2 \quad x_2^3 \quad x_2^4}\right.$

$\left. \underbrace{x_1^1 x_2^1}_{2} \quad \underbrace{x_1^1 x_2^2 \quad x_1^2 x_2^1}_{3} \quad \underbrace{x_1^1 x_2^3 \quad x_1^2 x_2^2 \quad x_1^3 x_2^1}_{4}\ \right]$

$4 \longrightarrow$ sumOfPowers

and so on ...

So we have:

- always the constant term 1,
- a for loop, for every x_1 to power from 1 to m,
- a for loop, for every x_2 to power from 1 to m,
- a for loop, for every $x_1 x_2$ with the sum of powers from 2 to m.