Limite

Exemplu: $x = \frac{2n+1}{3n+1} \quad => \lim x = \frac{2}{3}$

Cazuri de NEDETERMINARE: $\begin{cases} 0 * \pm\infty \\ +\infty + (-\infty) \\ 1^{\infty} \\ 0^{0} \\ \infty^{0} \\ \frac{0}{0} \\ \frac{\pm\infty}{\pm\infty} \end{cases}$

1) Limita unui polinom:

$$\lim(a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0) = \lim a_k n^k$$

n la puterea cea mai mare

2) Limita unui cat de polinom:

$$\lim \frac{(a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0)}{(b_l n^l + b_{l-1} n^{l-1} + \cdots + b_1 n + b_0)} = \lim \frac{a_k n^k}{b_l n^l}$$

$$= \lim \frac{a_k}{b_l} * n^{k-l} \begin{cases} 0 \quad , k < l \\ \frac{a_k}{b_l} \quad , k = l \\ \frac{a_k}{b_l} (\pm\infty)^{k-l} , k > l \end{cases}$$

3) Limita unei puteri:

$$\lim a^n = \begin{cases} \infty, \quad a > 1 \\ 1, \quad a = 1 \\ 0, \quad a \in (-1,1) \\ \nexists, \quad a = -1 \\ \nexists, \quad a < -1 \end{cases}$$

Scoti factor pe a^n, a cel mai mare

Daca avem $a^n + 7^n$ atunci luam pe cazuri $\begin{cases} a > 7 \\ a < 7 \\ a = 7 \end{cases}$

4) Limitele unor sume/produse:

Cu formulele:

$$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

5) Limitele unor siruri cu radicali:

Daca avem:

⬇

$\lim\left(\sqrt{a} - b\right)$ → rationalizam

⬇

$\lim\left(\sqrt{a} + b\right)$ → scoatem factor comun

!!! $\lim\left(\sqrt[3]{n^3 + \cdots} - \sqrt[2]{n^2 + \cdots}\right) = \lim\left(\sqrt[3]{n^3 + \cdots} - n\right) + \lim\left(n - \sqrt[2]{n^2 + \cdots}\right)$

6) Limite remarcabile:

$$\lim_{n\to\infty} \frac{\sin x_n}{x_n} = 1 \;\; , x_n \to \textit{tinde la } 0$$

$$\lim_{n\to\infty} \frac{\arcsin x_n}{x_n} = 1 \;\; , x_n \to \textit{tinde la } 0$$

$$\lim_{n\to\infty} \frac{\operatorname{tg} x_n}{x_n} = 1 \;\; , x_n \to \textit{tinde la } 0$$

$$\lim_{n\to\infty} \frac{\operatorname{arctg} x_n}{x_n} = 1 \;\; , x_n \to \textit{tinde la } 0$$

Numarul e:

$$\lim_{n\to\infty} (1+x_n)^{\frac{1}{x_n}} = e \quad , \qquad (1+x_n)^{\frac{1}{x_n}} \to \textit{tinde la } 1^{\infty}$$

$$\lim_{n\to\infty} \frac{a^{x_n-1}}{x_n} = \ln a \quad \textit{sau} \quad \lim_{n\to\infty} \frac{a^{\frac{1}{x_n}-1}}{\frac{1}{x_n}} = \ln a$$

$$\lim_{n\to\infty} \frac{\ln(1+x_n)}{x_n} = 1 \qquad\qquad \lim_{n\to\infty} \frac{\ln y_n}{y_n - 1} = 0$$

$$\lim_{n\to\infty} \frac{n^k}{a^k} = 0 \;\; , \;\; a > 1 \quad k \in \mathbb{N}$$

$(\ln 1 = 0)$

7) Criteriul Clestelui:

La o suma:

- Incardam fiecare termen al sumei intre cel mai mare si cel mai mic termen al sumei:

Cel mai mic (T1) < T1 < Cel mai mare (Tn)

Cel mai mic < T2 < Cel mai mare

...

Cel mai mic < Tn < Cel mai mare

- Facem suma

- Obtinem:

Cel mai mic *n < suma < Cel mai mare *n

Tinde la ...

!!! Partea intreaga o incadram intre:

$$x - 1 < [x] \leq x$$

8) Cesaro-Stolz:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} \quad Doar\ daca\ b_n \to> 0, nemarg.superior$$

9) Cauchy-d'Alembert (criteriul radicalului):

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{a_n + 1}{a_n} \qquad a_n > 0$$

10) Criteriul raportului:

$$\lim_{n\to\infty} \frac{a_n + 1}{a_n} = l \qquad \begin{array}{l} l > 1 \ => \ \lim = \infty \\ l \in [0,1] => \ \lim = 0 \end{array}$$

11) L'Hospital:

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} \overset{\frac{0}{0}}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$$

Cazuri de nedeterminare:

- $0 * \infty = f * g = \frac{f}{\frac{1}{g}} \ sau \ \frac{g}{\frac{1}{f}} + l'hospital$
- $1^{\infty}, 0^{0}, \infty^{0} = f^{g} = e^{g \ln f} + \lim g \ln f$
- $\infty - \infty = f - g = \begin{matrix} Calcule\ daca\ merg \\ sau \\ f\left(1 - \frac{g}{f}\right) + \lim \frac{g}{f} \end{matrix}$

!!!

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln n \Rightarrow \lim_{n \to \infty} a_n = c = 0{,}47$$

Constanta lui Euler

Limite de functii

$$\lim\big(f(x) + g(x)\big) = \lim f(x) + \lim\ g(x)$$

$$\lim\big(f(x) * g(x)\big) = \lim f(x) * \lim\ g(x)$$

$$\lim\left(\frac{f(x)}{g(x)}\right) = \frac{\lim f(x)}{\lim\ g(x)}$$

$$\lim\big(f(x)^{g(x)}\big) = \lim f(x)^{\lim\ g(x)}$$

$$\lim |f(x)| = |\lim f(x)|$$

- Daca un sir are limita:
  - $finita \Rightarrow sirul\ este\ \boldsymbol{CONVERGENT}$
  - $infinita \Rightarrow sirul\ este\ \boldsymbol{DIVERGENT}$