Ec. Trigonometrice – Formule

I. Fundamentale:

$\left.\begin{array}{l}\text{sin x = a}\\ \text{cos x = a}\end{array}\right\} a \in [-1,1]$

tg x = a

ctg x = a

sin(x) = a $=>$ x=$(-1)^k \arcsin(a) + k\pi$

cos(x) = a $=>$ x=$\pm \arccos(a) + 2k\pi$

tg(x) = a $=>$ x=$\text{arctg}(a) + k\pi$

ctg(x) = a $=>$ x=$\text{arcctg}(a) + k$

!!! - sin(x) = sin(-x)

- cos(x) = cos($\pi \pm x$)

arcsin(-x) = -arcsinx

arccos(-x) = π-arccosx

arctg(-x) = -arctgx

arcctg(-x) = π-arcctgx

Ecuatia cos intr-un triunghi:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Ecuatia sin intr-un triunghi:

$$\sin A = \frac{a}{2R} \quad R - raza\ cercului\ format\ de\ mijloacele\ laturilor$$

II. De forma: sin f(x)=sin g(x), cos f(x)=cos g(x), tg f(x)=tg g(x), …

sin(x) = sin(a) $=>$ x=$(-1)^k$a $+ k\pi$

cos(x) = cos(a) $=>$ x=$\pm$ a $+ 2k\pi$

tg(x) = tg(a) $=>$ x= a $+ k\pi$

ctg(x) = ctg(a) $=>$ x= a $+ k\pi$

III. Care se reduc la **gradul 2** sau **3**:

cos 2x $= cos^2x - sin^2x$

$= 2cos^2x - 1$

$=1 - 2sin^2x$

sin 2x $= 2 \sin x\ cos\ x$

sin 3x $= 3 \sin x - 4sin^3x$

cos 3x $= 4cos^3x - 3 \cos x$

$tg\, 2x = \frac{2tg\, x}{1-tg^2 x}$

$tg\,(x \pm y) = \frac{tg\, x \pm tg\, y}{1 \pm tg\, x\, tg\, y}$

$ctg\, x = \frac{1}{tg\, x}$

$tg\left(\frac{\pi}{2} + y\right) = -\frac{1}{tg\, y}$

IV. Omogene:

$a\, sin^2 x + b \sin x \cos x + c\, cos^2 x = 0 \qquad : cos^2 x$

$a\, tg^2 x + b\, tg\, x + c = 0$

...

V. Aproape omogene:

$$a\, sin^2 x + b \sin x \cos x + c\, cos^2 x = d$$

$$d * 1 = d * (sin^2 x + cos^2 x)$$

⇨ ECUATIE omogena

VI. Liniare:

M1: $a \sin x + b \cos x = c \qquad : \sqrt{a^2 + b^2}$

M2: $\sin x = \frac{2\, tg\, \frac{x}{2}}{1 + tg^2 \frac{x}{2}} \qquad \cos x = \frac{1 - tg^2 \frac{x}{2}}{1 + tg^2 \frac{x}{2}}$

!!! La M2 verificam la final daca $x = (2k+1)\pi$ este solutie

$\sin(2k+1)\pi = 0$

$\cos(2k+1)\pi = -1$

VII. Simetrice:

$a(\sin x + \cos x) + b \sin x \cos x + c = 0$

$\sin x + \cos x = t \quad /^2$

...

VIII. Ce contin: $sin^{2n}x + cos^{2n}x$

$$sin^4x + cos^4x = 1 - 2(\frac{\sin 2x}{2})^2$$

$$sin^6x + cos^6x = 1 - 3(\frac{\sin 2x}{2})^2$$

IX. Ce contin **patrate** de sin si cos:

$$sin^2x = \frac{1-\cos 2x}{2} \qquad cos^2x = \frac{\cos 2x+1}{2}$$

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\cos a - \cos b == 2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\sin a - \sin b = 2\sin\frac{a-b}{2}\cos\frac{a+b}{2}$$

X. Ce contin **produse** de sin si cos:

$$\sin a * \cos b = \frac{\sin(a+b)+\sin(a-b)}{2}$$

$$\cos a * \cos b = \frac{\cos(a+b)+\cos(a-b)}{2}$$

$$\sin a * \sin b = \frac{\cos(a-b)-\cos(a+b)}{2}$$

$$\sin(a+b) = \sin a\cos b + \sin b\cos a \qquad \sin x = \cos(\frac{\pi}{2} - x)$$

$$\sin(a-b) = \sin a\cos b - \sin b\cos a \qquad \cos x = \sin(\frac{\pi}{2} - x)$$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$ $\qquad$ $sin^2x + cos^2x = 1$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$ $\qquad$ $ctg\ x = \frac{1}{tg\ x}$

VIII. Suma de sin:

$$S = \sin x + \sin 2x + \sin 3x + \cdots + \sin nx$$

- Inmultim cu sin de jumatate de ratie $(\frac{x}{2})$

$$\sin\frac{x}{2}S = \sin\frac{x}{2}\sin x + \cdots$$

- Scriem cu formula de inmultire de sin
- …

- Arcsin / sin :

$\arcsin(-x) = -\arcsin(x)$

0	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
0	$\frac{\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$

$$\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$$

$$\arcsin: [-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$\sin(\arcsin x) = x$ $\qquad$ $\arcsin(\sin x) = x$

- Arccos / cos :

$\arccos(-x) = \pi - \arccos(x)$

0	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\boldsymbol{\frac{\pi}{2}}$	0	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

$$\cos:[0,\pi] \to [-1,1]$$

$$\arccos: [-1,1] \to [0,\pi]$$

$\cos(\arccos x) = x$ $\qquad$ $\arccos(\cos x) = x$

$arctg(\infty) = \frac{\pi}{2}$ $\qquad$ $arctg(-\infty) = -\frac{\pi}{2}$

$arcctg(\infty) = 0$ $\qquad$ $arcctg(-\infty) = \pi$

- Teorema medianei:

$$m_A{}^2 = \frac{2(b^2 + c^2) - a^2}{4}$$

Forma trigonometrica

$$z = r(\cos t + i \sin t)$$

Forma algebrica → Forma trigonometrica

$z = x + iy$ $\qquad$ $r = \sqrt{x^2 + y^2}$

$$t = arctg\frac{y}{x} + k\pi\ , k = \begin{cases} 0, r \in C1 \\ 1, r \in C2, C3 \\ 2, r \in C4 \end{cases}$$

Operatii cu forma trigonometrica:

1. Inmultire:

$z_1 = r_1(\cos t_1 + i \sin t_1)$

$z_2 = r_2(\cos t_2 + i \sin t_2)$

$z_1 * z_2 = r_1 * r_2(\cos t_1 * \cos t_2 - \sin t_1 \sin t_2) + i(\cos t_1 * \cos t_2 + \sin t_1 \sin t_2)$

2. Inversul:

$z' = \frac{1}{z} = \frac{1}{r}(\cos(-t) + i \sin(-t))$

3. Impartire:

$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(t_1 - t_2) + i \sin(t_1 - t_2))$

4. Ridicare la putere:

$z^n = r^n(\cos(n * t) + i \sin(n * t))$