Integrale

$$F'(x) = f(x) \qquad\qquad F(x) = \int f(x)$$

$F - primitiva\ lui\ f\ (antiderivata)$

$f - derivata\ lui\ F$

!!! *O functie sa admita primitive => functia sa fie continua pe D (sa nu aibe discontinuitatii de speta 1)*

1) Integrare prin parti:

$$\int f'(x) * g(x)dx = f(x) * g(x) - \int f(x) * (g(x))'dx$$

$$(f * g)' = f'(x) * g(x) + f(x) * g'(x)$$

$$\left(\frac{f}{g}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

2) Formule de recurenta:

I =

Trebuie sa ajungem sa scriem I_n in functie de $I_{n-1}, I_{n+2}, \ldots$

3) Integrare prin schimbare de variabila:

Notam cu u = f(x) astfel incat (f(x))' sa ne apara in ecuatie si scriem I(x) in fuctie de u I(u)

x = t |'

dx = dt

4) Integrarea functiilor rationale oarecare:

Ex: $\frac{3x^2+x+1}{2x+1} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{2x+1}$

Le aflam pe cele cu puterea ce mai mare: $(x-2)^2$ *si* $(2x+1)$

$B = \frac{3x^2+x+1}{2x+1}$ *si inlocuim cu x din ecuatia* $(x-2)^2 = 0 \;\; => \;\; x = 2$

...

5) Impartirea polinoamelor:

Ex: $f = 2x^5 - 3x^4 + 7x^3 - x + 2$

$g = x^2 - x - 1$

f/g:

1. Ne uitam la puterea cea mai mare a lui x

$$\begin{array}{r|l}
2x^5 - 3x^4 + 7x^3 - x + 2 & x^2 - x - 1 \\ \cline{2-2}
-2x^5 + 2x^4 + 2x^3 & 2x^3 - x^2 + 8x + 7 \\ \hline
-x^4 + 9x^3 - x - 2 & \\
\ldots & \\ \hline
14x + 9 &
\end{array}$$

2. Alegem cu ce inmultim

Nr. crt.	Funcția	Mulțimea primitivelor (integrala nedefinită)
1.	$f:\mathbb{R}\to\mathbb{R},\ f(x)=x^n,\ n\in\mathbb{N}$	$\int x^n dx=\frac{x^{n+1}}{n+1}+\mathscr{C}$
2.	$f:I\to\mathbb{R},\ f(x)=x^r,\ I\subset(0,+\infty),$ $r\in\mathbb{R}\setminus\{-1\}$	$\int x^r dx=\frac{x^{r+1}}{r+1}+\mathscr{C}$
3.	$f:\mathbb{R}\to\mathbb{R},\ f(x)=a^x,\ a>0,\ a\neq 1$	$\int a^x dx=\frac{a^x}{\ln a}+\mathscr{C}$
4.	$f:I\to\mathbb{R},\ f(x)=\frac{1}{x},\ I\subset\mathbb{R}^*$	$\int\frac{1}{x}dx=\ln\lvert x\rvert+\mathscr{C}$
5.	$f:I\to\mathbb{R},\ f(x)=\frac{1}{x^2-a^2},$ $I\subset\mathbb{R}\setminus\{\pm a\},\ a\neq 0$	$\int\frac{1}{x^2-a^2}dx=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+\mathscr{C}$
6.	$f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{1}{x^2+a^2},\ a\neq 0$	$\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+\mathscr{C}$
7.	$f:\mathbb{R}\to\mathbb{R},\ f(x)=\sin x$	$\int\sin x\,dx=-\cos x+\mathscr{C}$
8.	$f:\mathbb{R}\to\mathbb{R},\ f(x)=\cos x$	$\int\cos x\,dx=\sin x+\mathscr{C}$
9.	$f:I\to\mathbb{R},\ f(x)=\operatorname{tg}x,$ $I\subset\mathbb{R}\setminus\left\{(2k+1)\frac{\pi}{2}\ \middle\vert\ k\in\mathbb{Z}\right\}$	$\int\operatorname{tg}x\,dx=-\ln\lvert\cos x\rvert+\mathscr{C}$
10.	$f:I\to\mathbb{R},\ f(x)=\operatorname{ctg}x,$ $I\subset\mathbb{R}\setminus\{k\pi\mid k\in\mathbb{Z}\}$	$\int\operatorname{ctg}x\,dx=\ln\lvert\sin x\rvert+\mathscr{C}$
11.	$f:I\to\mathbb{R},\ f(x)=\frac{1}{\cos^2 x},$ $I\subset\mathbb{R}\setminus\left\{(2k+1)\frac{\pi}{2}\ \middle\vert\ k\in\mathbb{Z}\right\}$	$\int\frac{1}{\cos^2 x}dx=\operatorname{tg}x+\mathscr{C}$
12.	$f:I\to\mathbb{R},\ f(x)=\frac{1}{\sin^2 x},$ $I\subset\mathbb{R}\setminus\{k\pi\mid k\in\mathbb{Z}\}$	$\int\frac{1}{\sin^2 x}dx=-\operatorname{ctg}x+\mathscr{C}$
13.	$f:I\to\mathbb{R},\ f(x)=\frac{1}{\sqrt{a^2-x^2}},$ $I\subset(-a,a),\ a>0$	$\int\frac{1}{\sqrt{a^2-x^2}}dx=\arcsin\frac{x}{a}+\mathscr{C}$
14.	$f:I\to\mathbb{R},f(x)=\frac{1}{\sqrt{x^2-a^2}},$ $I\subset(-\infty,-a)$ sau $I\subset(a,+\infty)$	$\int\frac{1}{\sqrt{x^2-a^2}}dx=\ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert+\mathscr{C}$
15.	$f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{1}{\sqrt{x^2+a^2}},$ $a\neq 0$	$\int\frac{1}{\sqrt{x^2+a^2}}dx=\ln\left(x+\sqrt{x^2+a^2}\right)+\mathscr{C}$