Probabilistic Robotics
UFMFNF-15-3 2018-19

Week 6: Extended Kalman filter worked example

(Chapter 7.4 of Thrun et al.)

Applying the vanilla EKF algorithm to localisation

Algorithm Extended_Kalman_filter$(\boxed{\mu_{t-1}, \Sigma_{t-1}}, u_t, z_t)$

$$\boxed{\bar{\mu}_t} = \boxed{g(\mu_{t-1}, u_t)}$$

$$\boxed{\bar{\Sigma}_t} = \underline{G_t} \Sigma_{t-1} \underline{G_t^T} + \underline{R_t}$$

$$K_t = \bar{\Sigma}_t \underline{H_t^T} (\underline{H_t} \bar{\Sigma}_t \underline{H_t^T} + \underline{Q_t})^{-1}$$

$$\boxed{\mu_t} = \bar{\mu}_t + K_t \boxed{(z_t - \boxed{h(\bar{\mu}_t)})}$$

$$\boxed{\Sigma_t} = (I - K_t \underline{H_t}) \bar{\Sigma}_t$$

$return\ \boxed{\mu_t, \Sigma_t}$

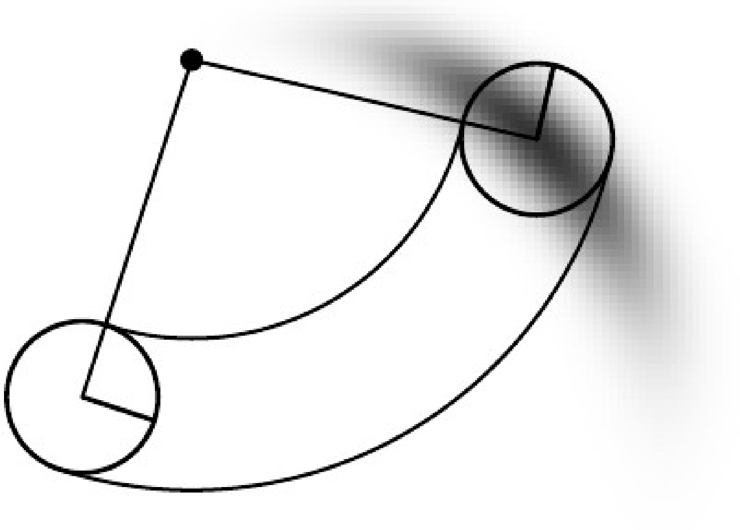
Prior
Prediction
Posterior
~~Transition/input matrix~~
~~Transformation matrix~~
JACOBIANS
Kalman gain
Motion noise
Measurement noise
Innovation
Motion model
Measurement model

Non-linear Motion model

Velocity model of motion

$$\begin{pmatrix} \hat{v}_t \\ \hat{w}_t \end{pmatrix} = \begin{pmatrix} v_t \\ w_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{\alpha 1 v_t^2 + \alpha 2 w_t^2} \\ \varepsilon_{\alpha 3 v_t^2 + \alpha 4 w_t^2} \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{w}} sin\theta + \frac{\hat{v}}{\hat{w}} sin(\theta + \hat{w}\Delta t) \\ \frac{\hat{v}}{\hat{w}} cos\theta - \frac{\hat{v}}{\hat{w}} cos(\theta + \hat{w}\Delta t) \\ \hat{w}\Delta t \end{pmatrix}$$



Linearization of motion model: prediction mean

pass through noise free motion model from μ_{t-1}

$$\underbrace{\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v}{w} sin\theta + \frac{v}{w} sin(\theta + w\Delta t) \\ \frac{v}{w} cos\theta - \frac{v}{w} cos(\theta + w\Delta t) \\ w\Delta t \end{pmatrix}}_{\boxed{g(\mu_{t-1}, u_t)}}$$

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Linearization of motion model: prediction covariance (i)

$$\overline{\Sigma}_t = \underline{G_t}\Sigma_{t-1}\underline{G_t^T} + R_t$$

Calculate derivative of noise free g(.) with respect to x_{t-1} at μ_{t-1}

$$\underline{G_t} = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} 1 & 0 & \frac{v}{w}(-cos\mu_{t-1,\theta} + cos(\mu_{t-1,\theta} + w_t\Delta t)) \\ 0 & 1 & \frac{v}{w}(-sin\mu_{t-1,\theta} + sin(\mu_{t-1,\theta} + w_t\Delta t)) \\ 0 & 0 & 1 \end{pmatrix}$$

Linearization of motion model: prediction covariance (ii)

$$\overline{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + \underline{R_t}$$

The covariance of the additive zero mean Gaussian noise Component is derived from covariance matrix M_t and transform Matrix V_t ...

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 w_t^2 \end{pmatrix}$$

(Covariance matrix of noise in control space)

$$V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix} \frac{-sin\theta + sin(\theta + w_t \Delta t)}{w_t} & \frac{v_t(sin\theta - sin(\theta + w_t \Delta t))}{w_t^2} + \frac{v_t cos(\theta + w_t \Delta t)\Delta t}{w_t} \\ \frac{cos\theta - cos(\theta + w_t \Delta t)}{w_t} & -\frac{v_t(cos\theta - cos(\theta + w_t \Delta t))}{w_t^2} + \frac{v_t sin(\theta + w_t \Delta t)\Delta t}{w_t} \\ 0 & \delta t \end{pmatrix}$$

(Transformation from control space to state space)

$$\underline{R_t} = V_t M_t V_t^T$$

Prediction step complete

$$\boxed{\begin{aligned}\overline{\mu}_t\end{aligned}} = g(\mu_{t-1}, u_t)$$

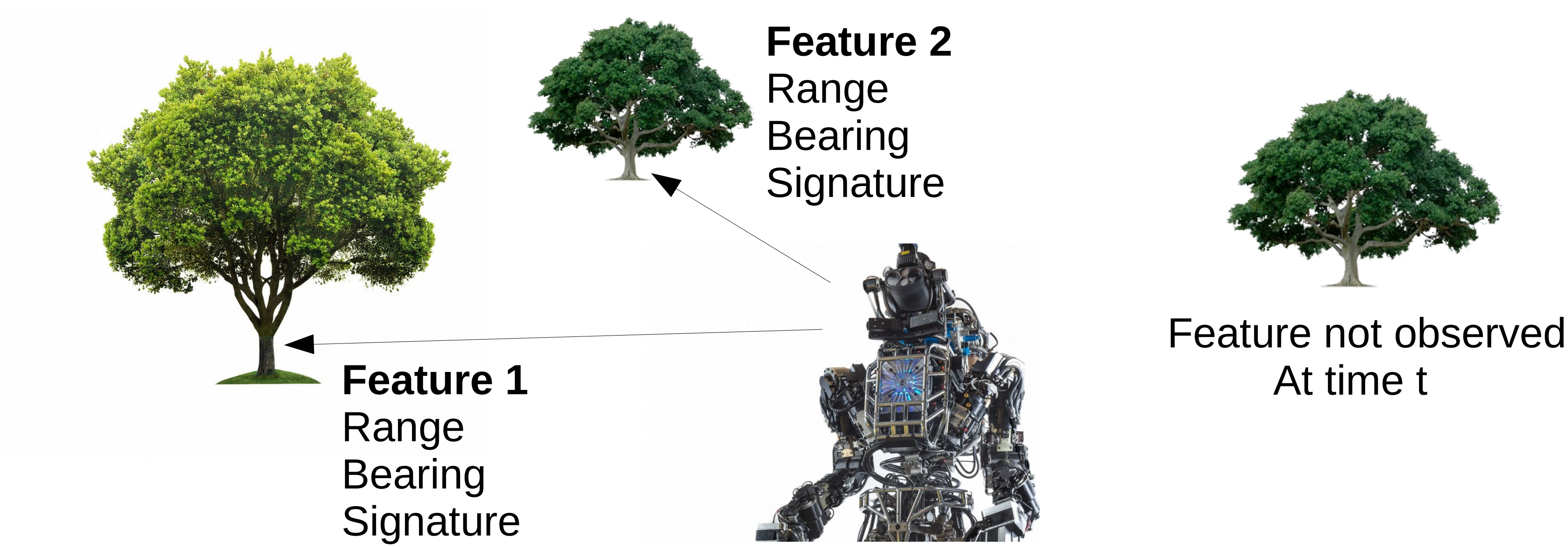
$$\boxed{\overline{\Sigma}_t} = G_t \Sigma_{t-1} G_t^T + R_t$$

Update step ...

Measurement model

Landmark based model of measurement

$$f(z_t) = \{f_t^1, f_t^2, \ldots\} = \left\{ \begin{pmatrix} r_t^1 \\ \phi_t^1 \\ s_t^1 \end{pmatrix}, \begin{pmatrix} r_t^2 \\ \phi_t^2 \\ s_t^2 \end{pmatrix}, \ldots \right\}$$



Product of measurement likelihoods of features with known correspondence to landmarks in a map

Algorithm landmark_model_known_correspondence(f_t^i, c_t^i, x_t, m)

$$j = c_t^i$$

$$\hat{r} = \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2}$$

$$\hat{\phi} = arctan2(m_{j,y} - y, m_{j,x} - x) - \theta$$

$$q = prob(r_t^i - \hat{r}, \sigma_r).prob(\phi_t^i - \hat{\phi}, \sigma_\phi).prob(s_t^i - s_j, \sigma_s)$$

return q

$$p(f(z_t)|c_t, x_t, m) = \prod_i \boxed{p(r_t^i, \phi_t^i, s_t^i | c_t^i, x_t, m)}$$

Non-linear Measurement model

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{\text{Observed feature, i}} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ atan2(m_{j,y}-y, m_{j,x}-x)-\theta \\ s_j \end{pmatrix}}_{h(x_t, j, m)} + \underbrace{\begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}}_{\text{Zero mean Gaussian noise}}$$

(Landmark c_i in map as viewed from x_t)

Observed feature, i

Zero mean Gaussian noise

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

Jacobian of noise free measurement model

Transformation from state to measurement space must be linear...

Derivative of h with respect to robot location x_t computed at $\overline{\mu}_t$:

$$H_t^i = \frac{\partial h(\overline{\mu}_t, j, m)}{\partial x_t} = \begin{pmatrix} -\frac{m_{j,x}-\overline{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y}-\overline{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y}-\overline{\mu}_{t,y}}{q} & -\frac{m_{j,x}-\overline{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$q = (m_{j,x} - \overline{\mu}_{t,x})^2 + (m_{j,y} - \overline{\mu}_{t,y})^2$$

Note: Correspondences known so zeros for signature

EKF localisation: prediction

Algorithm EKF_localisation$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m)$

$$\theta = \mu_{t-1,\theta}$$

$$G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{w_t}cos\theta + \frac{v_t}{w_t}cos(\theta + w_t\Delta t) \\ 0 & 1 & -\frac{v_t}{w_t}sin\theta + \frac{v_t}{w_t}sin(\theta + w_t\Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_t = \begin{pmatrix} \frac{-sin\theta + sin(\theta + w_t\Delta t)}{w_t} & \frac{v_t(sin\theta - sin(\theta + w_t\Delta t))}{w_t^2} + \frac{v_t cos(\theta + w_t\Delta t)\Delta t}{w_t} \\ \frac{cos\theta - cos(\theta + w_t\Delta t)}{w_t} & -\frac{v_t(cos\theta - cos(\theta + w_t\Delta t))}{w_t^2} + \frac{v_t sin(\theta + w_t\Delta t)\Delta t}{w_t} \\ 0 & \Delta t \end{pmatrix}$$

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 w_t^2 \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{w_t}sin\theta + \frac{v_t}{w_t}sin(\theta + w\Delta t) \\ \frac{v_t}{w_t}cos\theta - \frac{v_t}{w_t}cos(\theta + w\Delta t) \\ w\Delta t \end{pmatrix}$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T$$

EKF localisation: update

$$\textit{for all observed features } z_t^i = (r_t^i, \phi_t^i, s_t^i)^T \textit{ do}$$

$$j = c_t^i$$

$$q = (m_{j,x} - \overline{\mu}_{t,x})^2 + (m_{j,y} - \overline{\mu}_{t,y})^2$$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ atan2(m_{j,y} - \overline{\mu}_{t,y}, m_{j,x} - \overline{\mu}_{t,x}) - \overline{\mu}_{t,\theta} \\ m_{j,s} \end{pmatrix}$$

$$H_t^i = \begin{pmatrix} -\frac{m_{j,x} - \overline{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y} - \overline{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \overline{\mu}_{t,y}}{q} & -\frac{m_{j,x} - \overline{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$K_t^i = \overline{\Sigma}_t [H_t^i]^T [H_t^i \overline{\Sigma}_t [H_t^i]^T + Q_t]^{-1}$$

$$\overline{\mu}_t = \overline{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$\overline{\Sigma}_t = (I - K_t^i H_t^i) \overline{\Sigma}_t$$

$$\textit{end for}$$

$$\mu_t = \overline{\mu}_t$$

$$\Sigma_t = \overline{\Sigma}_t$$

Questions?

How would you build this using Matlab?