Савельев Михаил, 3.8

№302

$$f(x)=\frac{1}{\beta^{\alpha+1}\Gamma(\alpha+1)}x^{\alpha}e^{-x/\beta} \quad (\alpha>-1;\beta>0),\ x\geq 0$$

$$f(x)=0,\ x<0$$

а)

$$M(x)=\int_0^\infty x f(x)\,dx=\int_0^\infty\frac{x^{\alpha+1}e^{-x/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)}dx\ \ominus$$

$$\Gamma(n)=\int_0^\infty x^{n-1}e^{-x}dx \qquad \Gamma(n)=(n-1)!$$

$$\ominus\int_0^\infty\frac{x^{\alpha+1}e^{-x/\beta}}{\beta^{\alpha+1}\alpha!}dx=\frac{1}{\alpha!}\int_0^\infty\left(\frac{x}{\beta}\right)^{\alpha+1}e^{-x/\beta}dx=\begin{bmatrix}y=x/\beta\\ dy=1/\beta\,dx\end{bmatrix}=$$

$$=\frac{\beta}{\alpha!}\int_0^\infty y^{\alpha+1}e^{-y}dy=\frac{\beta}{\alpha!}\Gamma(\alpha+2)=\frac{\beta}{\alpha!}(\alpha+1)!=\underline{(\alpha+1)\beta}$$

б)

$$D(x)=\int_0^\infty x^2 f(x)\,dx-\beta^2(\alpha+1)^2=\int_0^\infty\frac{x^{\alpha+2}e^{-x/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)}dx-\beta^2(\alpha+1)^2=$$

$$=\begin{bmatrix}y=x/\beta\\ dy=\frac{1}{\beta}dx\end{bmatrix}=\frac{\beta^2}{\alpha!}\int_0^\infty y^{\alpha+2}e^{-y}dy-\beta^2(\alpha+1)^2=$$

$$=\beta^2\left((\alpha+1)(\alpha+2)-(\alpha+1)^2\right)=\underline{\beta^2(\alpha+1)}$$