VS postopek za izračun vrednosti polinomov več spremenljivk

Janez Radeščček, Miha Avsec

Fakulteta za matematiko in fiziko

2019

Polinom v Bernstein-Bezierjevi obliki (BB)

Naj bo T trikotnik, potem polinom v baricentričnih koordinatah (r, s, t) lahko zapišemo kot

$$p(r, s, t) = \sum_{i=0}^{d} \sum_{j=0}^{i} b_{d-i,i-j,j} B_{d-i,i-j,j}^{d},$$

kjer je

$$B_{i,j,k}^{d}(r, s, t) = \frac{d!}{i!j!k!} r^i s^j t^k$$

Bernsteinov polinom stopnje d.

De Casteljau

De Casteljaujev algoritem

```
for k=1:d
   for i=0:d-k
      for j=0:i
```

$$b^k_{d-i-k,i-j,j} = r * b^{k-1}_{d-i-k+1,i-j,j} + s * b^{k-1}_{d-i-k,i-j+1,j} + r * b^{k-1}_{d-i-k,i-j,j+1}$$

$$p(r,s,t) = b^d_{0,0,0}$$

Algoritem potrebuje $d(d+1)(d+2)/2$ množenj.

Modificirana Bernstein-Bezierjeva oblika polinoma (MBB)

Polinom v Bernsteinovi obliki lahko zapišemo kot

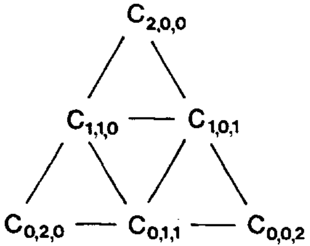
$$p(r,s,t) = \sum_{i=0}^{d} \sum_{j=0}^{i} c_{d-i,i-j,j} r^{d-i} s^{i-j} t^{j},$$

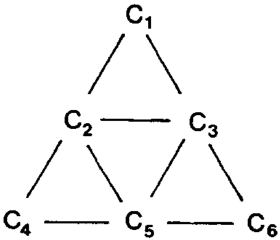
kjer za $c_{d-i,i-j,j}$ vzamemo

$$c_{d-i,i-j,j} = \frac{d!}{(d-i)!(i-j)!j!} b_{d-i,i-j,j}, \quad j = 0, \ldots, i; i = 0, \ldots, d.$$

Modificirana Bernstein-Bezierjeva oblika polinoma

Razdelitev domenskega trikotnika v primeru, ko je $d = 2$





Modificirana Bernstein-Bezierjeva oblika polinoma

Razvoj po spremenljivki r:

$$
\begin{aligned}
p(r,s,t) &= \sum_{i=0}^{2}\sum_{j=0}^{i} c_{2-i,i-j,j} r^{2-i} s^{i-j} t^{j} \\
&= r^2 \sum_{j=0}^{0} c_{2,-j,j} s^{-j} t^{j} + r \sum_{j=0}^{1} c_{1,1-j,j} s^{1-j} t^{j} + \sum_{j=0}^{2} c_{0,2-j,j} s^{2-j} t^{j} \\
&= r^2 (c_1) + r(c_2 s + c_3 t) + (c_4 s^2 + c_5 st + c_6 t^2) \\
&= r^2 (c_1 + \frac{s}{r} c_2 + \frac{t}{r} c_3 + \frac{s^2}{r^2} c_4 + \frac{st}{r^2} c_5 + \frac{t^2}{r^2} c_6) \\
&= r^2 (\frac{s}{r}(c_2 + \frac{s}{r} c_4 + \frac{t}{r} c_5) + \frac{t}{r}(\frac{t}{r} c_6 + c_3) + c_1)
\end{aligned}
$$

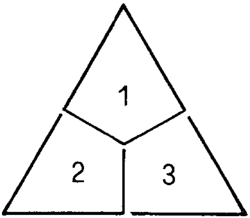
Modificiran Bernstein-Bezierjev algoritem

VS algoritem

```
sr = s/r,          tr= s/r
A = c_{0,d,0};
for i = 1:d
    B = c_{0,d-i,i}
    for j = i:-1:1
        B = B*tr + c_{i-j+1,d-i,j-1};
    A = A *sr +B;
p(r,s,t) = Ar^d
```

Algoritem potrebuje $(d^2 + 5d)/2$ množenj

Izbira spremenljivke



Posamezne regije so določena na sledeč način

1. $r \geq s, r \geq t$
2. $s > r, s \geq t$
3. $t > r, t > s$

Taylor

Zapis polinoma v Taylorjevi obliki

$$p(u, v) = \sum_{i=0}^{d} \sum_{j=0}^{d-i} a_{i,j} u^i v^j$$

Taylorjev algoritem

Taylorjev algoritem

```
p = a_{0,d}
for i = 1:d
    A = a_{i,d-i}
    for j = 1:i
        A = A * u + a_{i-j,d-i}
    end
    p = p * v + A
end
```

Algoritem potrebuje $(d^2 + 3d)/2$ množenj.

Primerjava metod

d	2	3	4	5	6	7	8	9
dCas	12	30	60	105	168	256	360	495
VSC	12	21	32	45	60	77	96	117
VS	9	14	20	27	35	44	54	65
Tay	5	9	14	20	27	35	44	54

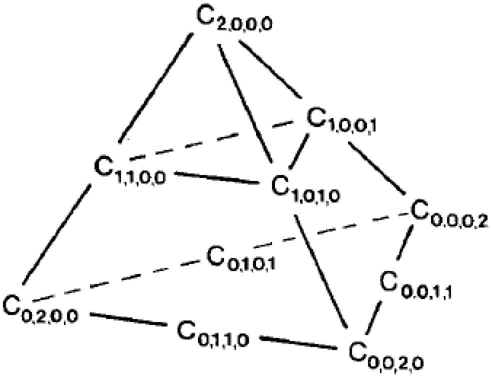
- dCas: De Casteljoujev algoritem
- VS: algoritem za polinom v MBB olbiki
- VSC: VS + pretvorba baz
- Tay: Taylorjev algoritem

Polinom v treh spremenljivkah

Naj bo T tetraeder v $\mathbb{R}^3$ in naj bodo (r, s, t, u) pripadajoče baricentrične koordinate točke P. Potem lahko polinom v točki P zapišemo kot

$$p(r, s, t, u) = \sum_{i=0}^{d} \sum_{j=0}^{i} \sum_{k=0}^{j} c_{d-i,i-j,j-k,k} r^{d-i} s^{i-j} t^{j-k} u^{k}.$$

Polinom v treh spremenljivkah



Algoritem za polinom v treh spremenljivkah

```
ru = r/u,        su= s/u          tu= t/u
A = c_{d,0,0,0};
for i = 1:d
    B = c_{d-i,i,0,0}
    for j = 1:i
        C = c_{d-i,i-j,j,0}
            for k = 1:j
                C = C * tu + c_{d-i,i-j,j-k,k}
        B = B * ru + C
    A = A * ru + B;
p(r,s,t,u) = Au^d
```