VS postopek za izračun vrednosti polinomov več spremenljivk

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Polinom v Bezierjevi obliki

Naj bo T trikotnik, potem polinom v baricentričnih koordinatah (r, s, t) lahko zapišemo kot

$$p(r,s,t) = \sum_{i=0}^{d} \sum_{j=0}^{i} b_{d-i,i-j,j} B^d_{d-i,i-j,j},$$

kjer je

$$B^d_{i,j,k}(r,s,t) = \frac{d!}{i!j!k!} r^i s^j t^k$$

Bernsteinov polinom stopnje d.

de Casteljau

```
for k=1:d
   for i=0:d-k
      for j=0:i
```

$$b^k_{d-i-k,i-j,j} = r * b^{k-1}_{d-i-k+1,i-j,j} + s * b^{k-1}_{d-i-k,i-j+1,j} + r * b^{k-1}_{d-i-k,i-j,j+1}$$

$$p(r,s,t) = b^d_{0,0,0}$$

Modificirana Bernstein-Bezierjeva oblika polinoma

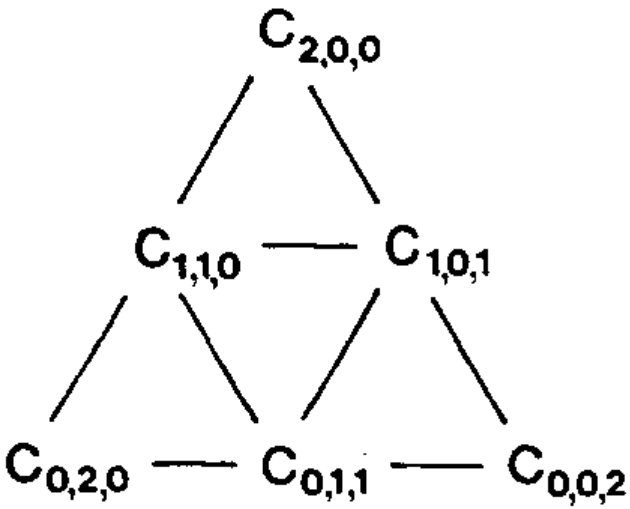
$$p(r,s,t) = \sum_{i=0}^{d} \sum_{j=0}^{i} c_{d-i,i-j,j} r^{d-i} s^{i-j} t^{j},$$

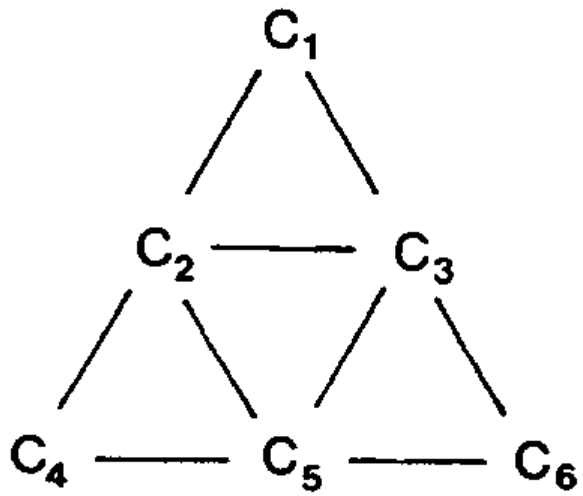
kjer za $c_{d-i,i-j,j}$ vzamemo

$$c_{d-i,i-j,j} = \frac{d!}{(d-i)!(i-j)!j!} b_{d-i,i-j,j}, \quad j = 0,\ldots,i; i = 0,\ldots,d.$$

Modificirana Bernstein-Bezierjeva oblika polinoma

Razdelitev domenskega trikotnika v primeru, ko je $d = 2$





Modificiran Bernstein-Bezierjev algoritem

```
sr = s/r,          tr= s/r
A = c_{0,n,0};
for i = 1:n
    B = c_{0,n-i,i}
    for j = i:-1:1
        B = B*tr + c_{i-j+1,n-i,j-1};
    end
    A = A *sr +B;
end
p(r,s,t) = A r^n
```

Taylor

Zapis polinoma v Taylorjevi obliki

$$p(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{n-i} a_{i,j} u^i v^j$$

Taylorjev algoritem

```
p = a_{0,n}
for i = 1:d
    A = a_{i,n-i}
    for j = 1:i
        A = A * u + a_{i-j,n-i}
    end
    p = p * v + A
end
```

Primerjava metod

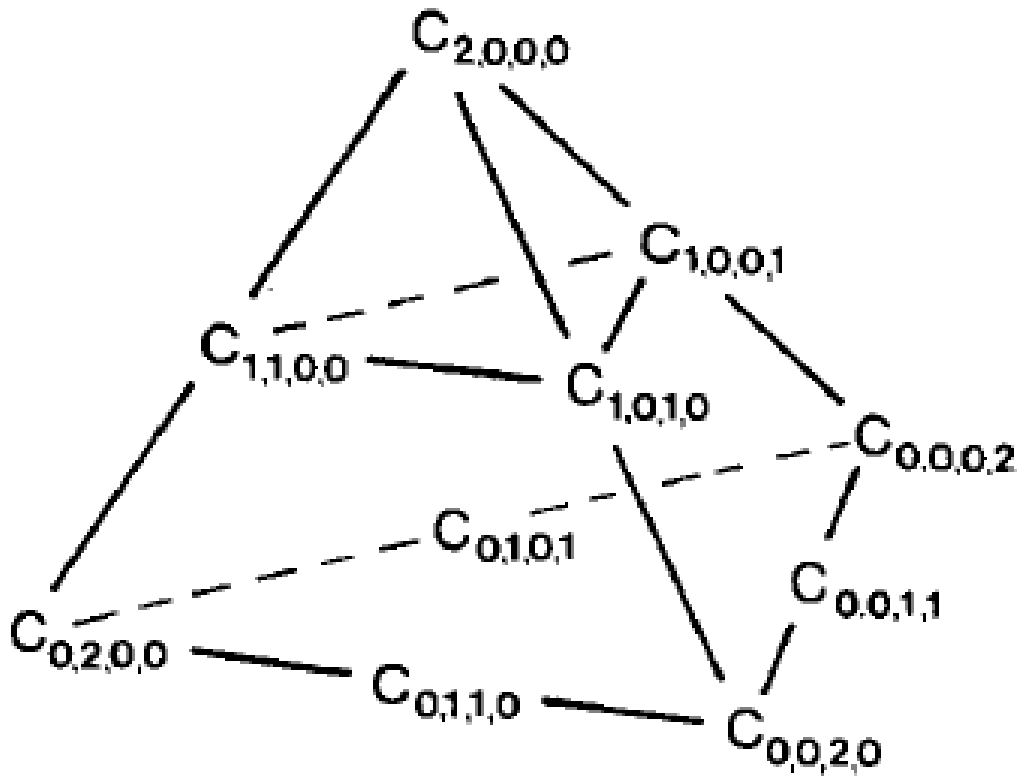
d	2	3	4	5	6	7	8	9
dCas	12	30	60	105	168	256	360	495
VSC	12	21	32	45	60	77	96	117
VS	9	14	20	27	35	44	54	65
Tay	5	9	14	20	27	35	44	54

Polinom v treh spremenljivkah

Naj bo T tetraeder v $\mathbb{R}^3$ in naj bodo (r, s, t, u) pripadajoče baricentrične koordinate točke P. Potem lahko polinom v točki P zapišemo kot

$$p(r,s,t,u) = \sum_{i=0}^{d} \sum_{j=0}^{i} \sum_{k=0}^{j} c_{d-i,i-j,j-k,k} r^{d-i} s^{i-j} t^{j-k} u^{k}.$$

Polinom v treh spremenljivkah



Algoritem za polinom v treh spremenljivkah