Problem 3

given $\lambda_p = 3dB$, $w_c = w_p = 2\pi \times 1000 = 2000\pi$

$\lambda_s = 10dB$; $w_s = 2 \times \pi \times 350 = 700\pi$ rad/sec

$T = 1/f = 1/5000 = 2 \times 10^{-4}$ sec

→ Filter is butterworth because characteristics are monotonic in both passband and stopband.

$$\lambda_p = \frac{2}{T} \tan \frac{w_p T}{2} = \frac{2}{2 \times 10^{-4}} \tan \frac{(2000\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^{+4} \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\lambda_s = \frac{2}{T} \tan \frac{w_s T}{2} = \frac{2}{2 \times 10^{-4}} \tan \frac{(200\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^{4} \tan(0.01\pi) = 2235 \text{ rad/sec}$$

order of filter.

$$N = \frac{\log \sqrt{\frac{10^{0.1\lambda_s} - 1}{10^{0.1\lambda_p} - 1}}}{\log \frac{\lambda_s}{\lambda_p}} = \frac{\log \sqrt{\frac{10^{0.1(10)} - 1}{10^{0.1(3)} - 1}}}{\log \frac{7265}{2235}}$$

$$= \frac{\log 3}{\log 3.25}$$

$$= \frac{0.4771}{0.5118} = 0.932,$$

we take $N = 1$.

P. 11

First Order Butterworth filter for $\lambda c = 1$ rad/sec

is $\lambda(s) = \frac{1}{1+s}$

$\lambda c = \lambda p = .7265$ rad/sec

$s = \frac{\lambda c}{s}$

$s \rightarrow \frac{.7265}{s}$

Transfer function of highpass filter

$$H(s) = \frac{1}{s+1} \Big|_{s = \frac{.7265}{s}}$$

$$= \frac{s}{s+.7265}$$

Using Bilinear Transformation

$$H(z) = H(s) \Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+.7265} \Big|_{s = \frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + .7265}$$

$$H(z) = \frac{0.5792\,(1-z^{-1})}{1-0.1584z^{-1}}$$