Habitability

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We consider a terrestrial planet habitable if it can support liquid water on its surface. A rough assumption we make to meet this criterion is that the average surface temperature should lie between 220K and 320K. This report aims to outline a strategy to identify a region (often referred to as the Goldilocks zone), measured as a range of possible radii from a central star (here, the Sun), that allows for this surface temperature interval.

The system we will construct only operates under the influence of one energy source, solar irradiance. Similarly, the only sink considered is the planet's outgoing radiation. To simplify our calculations, the surface will be considered similar to Earth's, allowing us to utilize experimentally-observed values in this procedure. Regarding the acceptable surface temperature range - we expect a runaway greenhouse effect to take hold if temperature passes the 320K limit, while a breach of the lower limit could result in a prolonged snowball state. The primary input quantity is solar luminosity, treated as a constant in this system (although, this value changes across the star's life cycle).

By the inverse square law applied to radiation,

$$\phi \propto \frac{1}{R^2} \tag{1}$$

where ϕ is solar irradiance and R is the radius we look to identify in this exercise.

Viewing ϕ as a measure of intensity, we can define it as a function of R in terms of power (luminosity) and surface area as follows:

$$\phi(R) = \frac{L_\odot}{4\pi R^2} \tag{2}$$

where $L_\odot$ is the solar luminosity.

Therefore, we can use this function to model the total incident radiation on the surface and set up an equation (assuming a stable energy budget) to evaluate ϕ at a chosen surface temperature from our acceptable range.

$$\frac{\phi}{4}(1-\alpha) = OLR\ (T_e,\ CO_2) \tag{3}$$

where α is the average surface albedo and $OLR\ (T_e,\ CO_2)$ acts as a function (`OLRT` from the second notebook) to evaluate outgoing longwave radiation with average surface temperature and atmospheric CO_2 mass as parameters. The approach detailed in this report will evaluate the range of habitable radii using a selection of CO_2 masses (1, 10, 1000, 10000 ppm) to better understand the relationship between the two quantities.

We will begin our investigation by evaluating the maximum permissible distance from the Sun where the surface temperature is 220 K. We can take on an arbitrary CO_2 mass from the selection provided above, and repeat the process for the other possibilities. Restructuring (3),

$$\phi = \frac{4 \cdot OLR\ (T_e,\ CO_2)}{1 - \alpha} \tag{4}$$

α can be found using the following analytical solution:

$$\alpha(T) = \alpha_i - (\alpha_i - \alpha_o)\frac{(T\_T_i)^2}{(T_i - T_o)^2} \tag{5}$$

where α_i is the reflectivity of ice, α_o is the reflectivity of liquid water, T_i is the surface temperature of ice-covered land and T_o is the surface temperature of ice-free land. All of the 4 can be reasonably estimated observationally and are treated as constants.

We now substitute a $\alpha(220K)$ approximation in (4) and solve for ϕ. In turn, this value for solar irradiance can be used to find R using (2). Therefore, we have found an upper boundary for the Goldilocks zone. This procedure can be repeated using $T_e = 320K$ to find the zone's lower limit. All intermediate radii belong to the Goldilocks zone and form our range of habitability.

The final product of the outline describes the habitable zone as a function of atmospheric CO_2 mass. Its influence on the boundary radii can be explored empirically using the range specified earlier.