11.9.3.17

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Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

TABLE 1
Given Information

Symbol	Value	Description
x	$x(0)r^4$	$x(4)$
y	$x(0)r^{10}$	$x(10)$
z	$x(0)r^{16}$	$x(16)$
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$
$x(0)$	$x^{\frac{5}{3}}y^{-\frac{2}{3}}$	First term
$x(n)$	$x(0)r^n u(n)$	General Term

Solution:

From Table 1,

$$x = x(4) = x(0)r^4$$
$$y = x(10) = x(0)r^{10}$$
$$z = x(16) = x(0)r^{16}$$

Consider $\frac{x(10)}{x(4)}$ and $\frac{x(16)}{x(10)}$;

$$\frac{x(10)}{x(4)} = \frac{x(0)r^{10}}{x(0)r^4} = r^6 \tag{1}$$
$$\frac{x(16)}{x(10)} = \frac{x(0)r^{16}}{x(0)r^{10}} = r^6 \tag{2}$$

Since, $\frac{x(10)}{x(4)} = \frac{x(16)}{x(10)}$;

$x(4),\ x(10),\ x(16)$ are in G.P.

$\therefore x,\ y,\ z$ are in G.P.

To extend the domain of n to -ve integers, the step function $u(n)$ can be used.

$$\therefore x(n) = x(0)r^n u(n)\ \forall\ n \in Z$$

$x(0)$ r can be expressed in terms of x, y, and z in the following manner.

$$x = x(0)r^4$$
$$\frac{y}{x} = r^6$$
$$\Rightarrow r = \sqrt[6]{\frac{y}{x}} = (\frac{y}{x})^{\frac{1}{6}} \tag{3}$$
$$x(0) = \frac{x}{r^4}$$
$$x(0) = x(\frac{x}{y})^{\frac{4}{6}}$$
$$\therefore\ x(0) = x^{\frac{5}{3}}y^{-\frac{2}{3}} \tag{4}$$
$$and\ r = (\frac{y}{x})^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \tag{5}$$