NCERT 10.5.2.7

EE23BTECH11017 - Eachempati Mihir Divyansh*

Question

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Given Information

Symbol	Value	Description
$x\left(0\right)$	-32	First term
$x\left(10\right)$	38	11th term
$x\left(15\right)$	73	16th term
d	7	Common Difference
$x\left(n\right)$	$x(0)+nd$	$(n+1)$th term

Table: Given Information

Solution: Part 1

From Table 1

$$x(0) + 10d = 38 \tag{1}$$
$$x(0) + 15d = 73 \tag{2}$$

From equations 1 and 2, the augmented matrix is:

$$\begin{pmatrix} 1 & 10 & 38 \\ 1 & 15 & 73 \end{pmatrix} \tag{3}$$

Finding $x(0)$ and d

$$\begin{pmatrix} 1 & 10 & 38 \\ 1 & 15 & 73 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 10 & 38 \\ 0 & 5 & 35 \end{pmatrix} \tag{4}$$

$$\xleftrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & -32 \\ 0 & 5 & 35 \end{pmatrix} \tag{5}$$

$$\xleftrightarrow{R_2 \to \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & -32 \\ 0 & 1 & 7 \end{pmatrix} \tag{6}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} -32 \\ 7 \end{pmatrix} \tag{7}$$

$x(n)$ and it's Z-transform

The general term $x(n)$ is given by

$$x\left(n\right) = \left(-32 + 7n\right) u\left(n\right) \tag{8}$$

$$\tag{9}$$

The Z-Transform of $x\left(n\right)$ is given by

$$X\left(z\right) = \frac{-32}{1 - z^{-1}} + \frac{7z^{-1}}{\left(1 - z^{-1}\right)^2} \tag{10}$$

The 31st term of this A.P. is

$$x\left(30\right) = 178 \tag{11}$$

C Code

```c
#include<stdio.h>

int main(){
    FILE *ptr=fopen("series.dat", "w"); //opening the file to
    ↪  store values
    int x_0=-32, d=7; //Giving the parameters of the sequence

    //printing the terms of the sequence
    for(int i=0; i<32; i++) fprintf(ptr, "%d ", (x_0+7*i));

    //last term outseide the for loop to avoid space at EOF.
    fprintf(ptr, "%d", x_0+7*32);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
n_1=np.arange(0, 33)
n_2=np.array([10, 15, 30])
#reading the values from the dat file
y1=np.loadtxt("series.dat", delimiter=" ", max_rows=1)
#choosing the values to mark
y2=y1[n_2]
#plotting the graph
plt.stem(n_1, y1, markerfmt='.', linefmt='-', basefmt='r',
↪  label=r'$x(n)$')
plt.stem(n_2, y2, markerfmt='o', linefmt='-')
plt.xlabel('n')
plt.ylabel('x(n)')
plt.grid(True)
plt.legend()
plt.savefig('../figs/fig1.png')
```

Plot

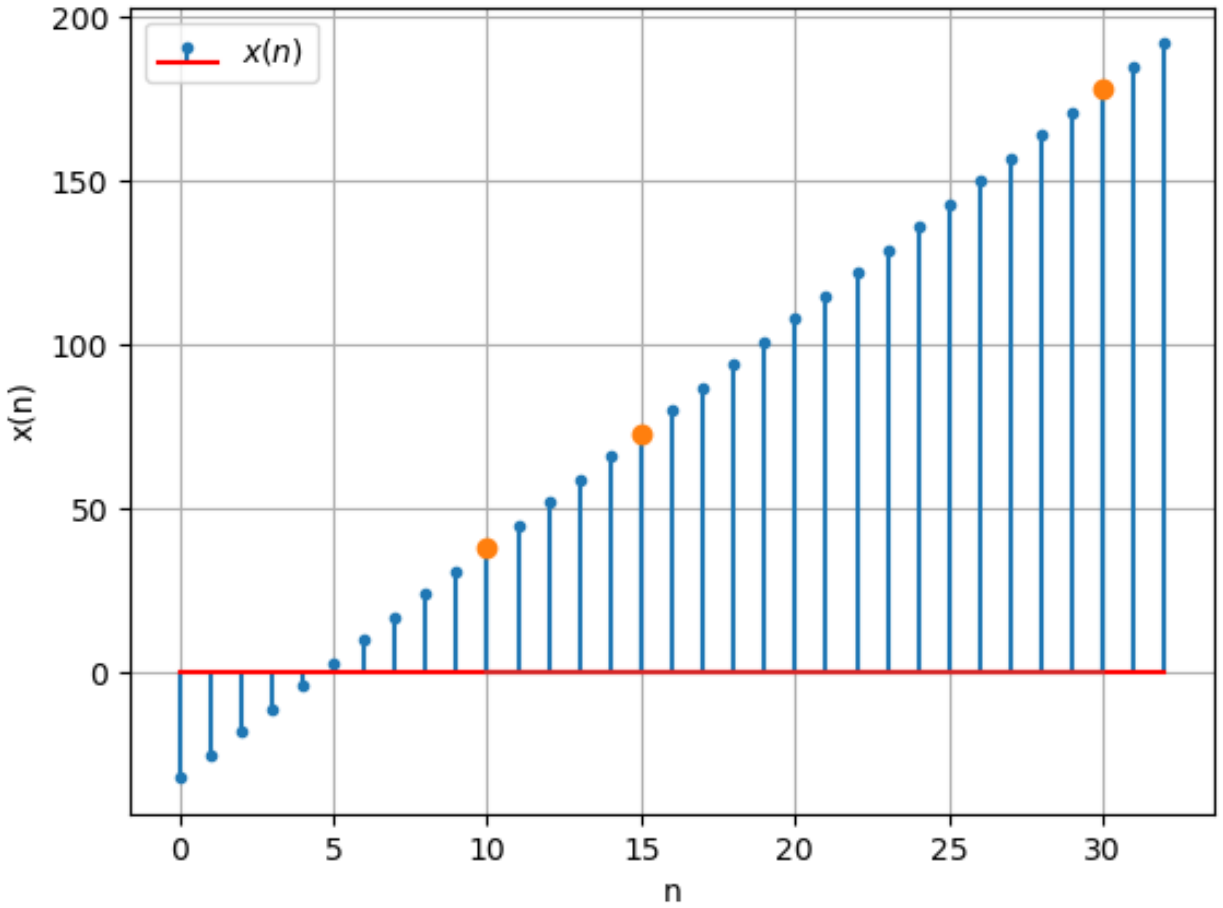

Figure: Stem plot of $x(n)$ v/s n