NCERT 11.9.3.17

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Question

If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that $x,\ y,\ z$ are in G.P.

Given Informatn

Symbol	Value	Description
x	$x(0)\,r^3$	$x(4)$
y	$x(0)\,r^9$	$x(10)$
z	$x(0)\,r^{15}$	$x(16)$
r	?	$\frac{x(n)}{x(n-1)}$
$x(0)$	?	First term
$x(n+1)$	$x(0)\,r^n u(n)$	$(n+1)^{th}$ term

Table: Given Information

Solution: Part 1

3 numbers x, y and z are in G.P. if

$$\frac{y}{x} = \frac{z}{y} \tag{1}$$

From Table 1

$$\frac{y}{x} = \frac{x(0)\,r^9}{x(0)\,r^3} = r^6 = \frac{x(0)\,r^{15}}{x(0)\,r^9} = \frac{z}{y} \tag{2}$$

By (1) and (2), x, y and z are in G.P.

Expressing $x(0)$ and r in terms of x and y

From (2),

$$r^6 = \frac{y}{x} \implies r = \sqrt[6]{\frac{y}{x}} = \left(\frac{y}{x}\right)^{\frac{1}{6}} \tag{3}$$

$$x\left(0\right) = \frac{x}{r^3} \implies x\left(0\right) = \left(\frac{x^3}{y}\right)^{\frac{1}{2}} \tag{4}$$

Z-Transform of $x\left(0\right)$

$$X\left(z\right) = \frac{x\left(0\right)}{1 - rz^{-1}} \tag{5}$$

$$= \frac{\left(\frac{x^3}{y}\right)^{\frac{1}{2}}}{1 - \left(\frac{y}{x}\right)^{\frac{1}{6}} z^{-1}} \tag{6}$$

Exaample

Let $x(0) = 1$ and $r = 1.2$

$$x = x(3) = (1.2)^3 \tag{7}$$

$$y = x(9) = (1.2)^9 \tag{8}$$

$$z = x(15) = (1.2)^{15} \tag{9}$$

C Code

```c
#include<stdio.h>
#include<math.h>

int main(){
    FILE *ptr;
    ptr= fopen("series.dat", "w");
    float x_0=1.0;
    float r= 1.2;
    for(int i=0; i<17; i++){
        fprintf(ptr, "%f ", x_0*pow(r,i));
    }
    fprintf(ptr, "\b ");
    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

n_1=np.arange(0, 17)
n_2=np.array([3, 9, 15])
y1=np.loadtxt("series.dat", delimiter=" ", max_rows=1)
y2=y1[n_2]
plt.stem(n_1, y1, markerfmt='.', linefmt='-', basefmt='r',
↪  label=r'$x(n)$')
plt.stem(n_2, y2, markerfmt='o', linefmt='-')

plt.xlabel('n')
plt.ylabel('x(n)')
plt.grid(True)
plt.legend()
plt.savefig('../figs/A_1.png')
```

Plot

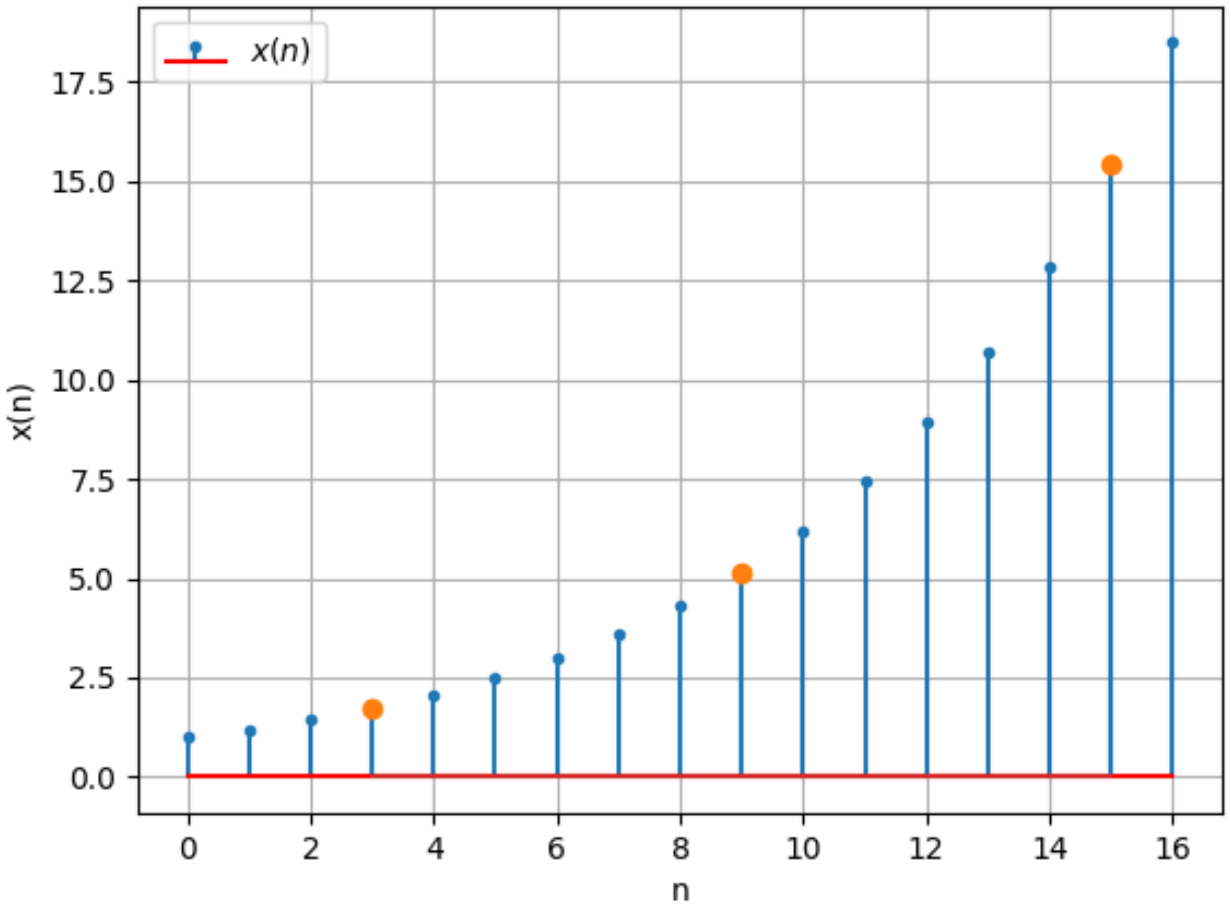

Figure: Stem plot of $x(n)$ v/s n