11.9.3.17

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Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

Solution:

The n^{th} term of a G.P. is $a_n = a_1 r^{n-1}$. Given that $x,\ y,\ z$ are the 4^{th}, 10^{th} and 16^{th} terms of a G.P.,

$$x = a_4 = ar^{4-1} = ar^3$$
$$y = a_{10} = ar^{10-1} = ar^9$$
$$z = a_{16} = ar^{16-1} = ar^{15}$$

Consider $\frac{y}{x}$ and $\frac{z}{y}$;

$$\frac{y}{x} = \frac{ar^9}{ar^3} \tag{1}$$
$$\frac{y}{x} = r^6 \tag{2}$$

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \tag{3}$$
$$\frac{y}{x} = r^6 \tag{4}$$

Since, $\frac{y}{x} = \frac{z}{y}$;

$$x,\ y,\ z \text{ are in G.P.}$$

For this G.P, with $x = ar^3,\ y = ar^9,\ z = ar^{15}$, the general term $x(n)$ can be defined as:

$$x(n) = x \cdot (\frac{y}{x})^{n-1} \tag{5}$$
$$x(n) = (ar^3) \cdot (\frac{ar^9}{ar^3})^{n-1} \tag{6}$$
$$x(n) = ar^3 \cdot (r^6)^{n-1} \tag{7}$$
$$x(n) = ar^{6n-3} \tag{8}$$