11.9.3.17

EE23BTECH11017 - Eachempati Mihir Divyansh*

Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

TABLE 1
GIVEN INFORMATION

Symbol	Value	Description
x	ar^4	$x(4)$
y	ar^{10}	$x(10)$
z	ar^{16}	$x(16)$
a	$x^{\frac{5}{3}}y^{-\frac{2}{3}}$	x(0)
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$
$x(n)$	$y^{n-1}x^{2-n}u(n)$	General Term

Solution:

The n^{th} term of a G.P. is $a_n = ar^n$. Given that $x,\ y,\ z$ are the 4^{th}, 10^{th} and 16^{th} terms of a G.P., From the Table ,

$$x = a_4 = ar^4$$
$$y = a_{10} = ar^{10}$$
$$z = a_{16} = ar^{16}$$

Consider $\frac{y}{x}$ and $\frac{z}{y}$;

$$\frac{y}{x} = \frac{ar^{10}}{ar^4} = r^6 \tag{1}$$
$$\frac{z}{y} = \frac{ar^{16}}{ar^{10}} = r^6 \tag{2}$$

Since, $\frac{y}{x} = \frac{z}{y}$;

$$x,\ y,\ z \text{ are in G.P.}$$

For this G.P, with $x,\ y,\ z$, as the first three terms, the general term $x(n)$ can be defined as:

$$\text{Common Ratio} = \frac{y}{x}$$
$$x(n) = x(\frac{y}{x})^{n-1} \tag{3}$$
$$also,\ x(n) = x \cdot (\frac{z}{y})^{n-1} \tag{4}$$
$$\therefore x(n) = \frac{y^{n-1}}{x^{n-2}}\ \forall\ n \geq 0$$

To extend the domain of n to -ve integers, the step function $u(n)$ can be used.

$$\therefore x(n) = \frac{y^{n-1}}{x^{n-2}}u(n)\ \forall\ n \in Z$$

The initial term $x(0)$ is :

$$x(0) = x(n)/r^n \tag{5}$$
$$= \left(y^{n-1}x^{2-n}u(n)\right)\left(\frac{y}{x}\right)^{-n}$$
$$x(0) = \frac{x^2}{y} \tag{6}$$

a and r can be expressed in terms of x, y, and z in the following manner.

$$x = ar^4$$
$$\frac{y}{x} = r^6$$
$$= \sqrt[6]{\frac{y}{x}} = (\frac{y}{x})^{\frac{1}{6}} \tag{7}$$
$$a = \frac{x}{r^4}$$
$$a = x(\frac{x}{y})^{\frac{4}{6}}$$
$$\therefore a = x^{\frac{5}{3}}y^{-\frac{2}{3}} \tag{8}$$
$$and\ r = (\frac{y}{x})^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \tag{9}$$