11.9.3.17

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Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

Solution:

Symbol	Value	Description
x	$x(0)\,r^4$	$x(4)$
y	$x(0)\,r^{10}$	$x(10)$
z	$x(0)\,r^{16}$	$x(16)$
r	?	$\frac{x(n)}{x(n-1)}$
$x(0)$	?	First term
$x(n)$	$x(0)\,r^n u(n)$	General Term

TABLE 0
GIVEN INFORMATION

1) From Table ,

$$x = x(3) = x(0)\,r^3 \tag{1}$$
$$y = x(9) = x(0)\,r^9 \tag{2}$$
$$z = x(15) = x(0)\,r^{15} \tag{3}$$

Consider $\frac{x(9)}{x(3)}$ and $\frac{x(15)}{x(9)}$;

$$\frac{x(9)}{x(3)} = \frac{x(0)\,r^9}{x(0)\,r^3} = r^6 = \frac{x(15)}{x(9)} = \frac{x(0)\,r^{15}}{x(0)\,r^9} \tag{4}$$

From (4), $x(3)$, $x(9)$, $x(15)$ are in G.P.
$\therefore$ x, y, z are in G.P.

2) $x(0)$ and r can be expressed in terms of x, y, and z in the following manner.

$$\frac{y}{x} = r^6 \tag{5}$$
$$\implies r = \sqrt[6]{\frac{y}{x}} = \left(\frac{y}{x}\right)^{\frac{1}{6}} \tag{6}$$
$$\implies x(0) = \frac{x}{r^3} = x\left(\frac{x}{y}\right)^{\frac{3}{6}} \tag{7}$$
$$\therefore\ x(0) = x^{\frac{5}{3}}y^{-\frac{2}{3}} \text{ and } r = \left(\frac{y}{x}\right)^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \tag{8}$$

3) From (??) Z-transform of a G.P. is

$$X(z) = \frac{x(0)}{1 - rz^{-1}};\ |z| > |r| \tag{9}$$

Substituting r and $x(0)$ from (8),

$$X(z) = \frac{x^{\frac{5}{3}}y^{-\frac{2}{3}}}{1 - \left(\frac{y}{x}\right)^{\frac{1}{6}}z^{-1}} \tag{10}$$

4) Example Let $x(0) = 1$ and $r = 1.2$

$$x = x(3) = (1.2)^3 \tag{11}$$
$$y = x(9) = = (1.2)^9 \tag{12}$$
$$z = x(15) = (1.2)^{15} \tag{13}$$

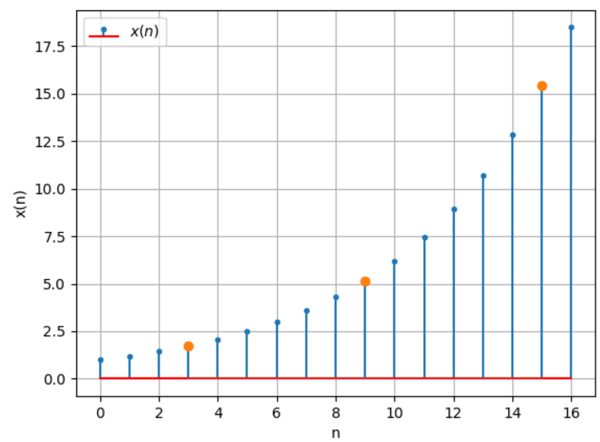

Fig. 4. Stem Plot of $x(n)$ vs n