

11.9.3.17

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Question: If the 4th, 10th and 16th terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

TABLE 1
GIVEN INFORMATION

Symbol	Value	Description
x	$x(0)r^4$	$x(4)$
y	$x(0)r^{10}$	$x(10)$
z	$x(0)r^{16}$	$x(16)$
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$
$x(0)$	$x^{\frac{5}{3}}y^{-\frac{2}{3}}$	First term
$x(n)$	$x(0)r^n u(n)$	General Term

Solution:

1) From Table 1,

$$x = x(4) = x(0)r^4 \quad (1)$$

$$y = x(10) = x(0)r^{10} \quad (2)$$

$$z = x(16) = x(0)r^{16} \quad (3)$$

Consider $\frac{x(10)}{x(4)}$ and $\frac{x(16)}{x(10)}$;

$$\frac{x(10)}{x(4)} = \frac{x(0)r^{10}}{x(0)r^4} = r^6 \quad (4)$$

$$\frac{x(16)}{x(10)} = \frac{x(0)r^{16}}{x(0)r^{10}} = r^6 \quad (5)$$

Since, $\frac{x(10)}{x(4)} = \frac{x(16)}{x(10)}$;

$$x(4), x(10), x(16) \text{ are in G.P.} \quad (6)$$

$$\therefore x, y, z \text{ are in G.P.} \quad (7)$$

2) To extend the domain of n to -ve integers, the step function $u(n)$ can be used.

$$\therefore x(n) = x(0)r^n u(n) \quad \forall n \in \mathbb{Z} \quad (8)$$

3) $x(0)$ and r can be expressed in terms of x , y , and z in the following manner.

$$x = x(0)r^4 \quad (9)$$

$$\frac{y}{x} = r^6 \quad (10)$$

$$\Rightarrow r = \sqrt[6]{\frac{y}{x}} = \left(\frac{y}{x}\right)^{\frac{1}{6}} \quad (11)$$

$$x(0) = \frac{x}{r^4} \quad (12)$$

$$x(0) = x \left(\frac{x}{y}\right)^{\frac{4}{6}} \quad (13)$$

$$\therefore x(0) = x^{\frac{5}{3}}y^{-\frac{2}{3}} \quad (14)$$

$$\text{and } r = \left(\frac{y}{x}\right)^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \quad (15)$$

4) Z-transform: $x(n) \xleftrightarrow{Z} X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (16)$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad \forall |z| > |r| \quad (17)$$

Substituting r and $x(0)$,

$$X(z) = \frac{x^{\frac{5}{3}}y^{-\frac{2}{3}}}{1 - \left(\frac{y}{x}\right)^{\frac{1}{6}}z^{-1}} \quad (18)$$

5) Example

Let $x(0) = \frac{1}{256}$ and $r = 2$

$$x = x(4) = x(0)r^4 = \frac{1}{256}(2)^4 = \frac{1}{16}$$

$$x = \frac{1}{16} \quad (19)$$

$$y = x(10) = x(0)r^{10} = \frac{1}{256}(2)^{10} = 4 \quad (20)$$

$$y = 4$$

$$z = x(16) = x(0)r^{16} = \frac{1}{256}(2)^{16} = 256 \quad (21)$$

$$z = 256$$

Fig. 1. Stem Plot of $x(n)$ v/s n

