11.9.3.17

EE23BTECH11017 - Eachempati Mihir Divyansh*

Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

TABLE 1
Given Information

Symbol	Value	Description
x	$x(0)\,r^4$	$x(4)$
y	$x(0)\,r^{10}$	$x(10)$
z	$x(0)\,r^{16}$	$x(16)$
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$
$x(0)$	$x^{\frac{5}{3}}y^{-\frac{2}{3}}$	First term
$x(n)$	$x(0)\,r^n u(n)$	General Term

Solution:

1) From Table 1,

$$x = x(4) = x(0)\,r^4 \tag{1}$$
$$y = x(10) = x(0)\,r^{10} \tag{2}$$
$$z = x(16) = x(0)\,r^{16} \tag{3}$$

Consider $\dfrac{x(10)}{x(4)}$ and $\dfrac{x(16)}{x(10)}$;

$$\frac{x(10)}{x(4)} = \frac{x(0)\,r^{10}}{x(0)\,r^4} = r^6 \tag{4}$$
$$\frac{x(16)}{x(10)} = \frac{x(0)\,r^{16}}{x(0)\,r^{10}} = r^6 \tag{5}$$

Since, $\dfrac{x(10)}{x(4)} = \dfrac{x(16)}{x(10)}$;

$$x(4),\ x(10),\ x(16) \text{ are in G.P.} \tag{6}$$
$$\therefore x,\ y,\ z \text{ are in G.P.} \tag{7}$$

2) To extend the domain of n to -ve integers, the step function $u(n)$ can be used.

$$\therefore x(n) = x(0)\,r^n u(n) \ \ \forall\ n \in Z \tag{8}$$

3) $x(0)$ and r can be expressed in terms of x, y, and z in the following manner.

$$x = x(0)\,r^4 \tag{9}$$
$$\frac{y}{x} = r^6 \tag{10}$$
$$\Rightarrow r = \sqrt[6]{\frac{y}{x}} = \left(\frac{y}{x}\right)^{\frac{1}{6}} \tag{11}$$
$$x(0) = \frac{x}{r^4} \tag{12}$$
$$x(0) = x\left(\frac{x}{y}\right)^{\frac{4}{6}} \tag{13}$$
$$\therefore\ x(0) = x^{\frac{5}{3}}y^{-\frac{2}{3}} \tag{14}$$
$$\text{and } r = \left(\frac{y}{x}\right)^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \tag{15}$$

4) Z-transform: $x(n) \overset{Z}{\longleftrightarrow} X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\,z^{-n} \tag{16}$$
$$X(z) = \frac{x(0)}{1 - rz^{-1}} \ \forall\ |z| > |r| \tag{17}$$

Substituting r and $x(0)$,

$$X(z) = \frac{x^{\frac{5}{3}}y^{-\frac{2}{3}}}{1 - \left(\frac{y}{x}\right)^{\frac{1}{6}}z^{-1}} \tag{18}$$

5) Example

Let $x(0) = \dfrac{1}{256}$ and $r = 2$

$$x = x(4) = x(0)\,r^4 = \frac{1}{256}(2)^4 = \frac{1}{16}$$
$$x = \frac{1}{16} \tag{19}$$
$$y = x(10) = x(0)\,r^{10} = \frac{1}{256}(2)^{10} = 4 \tag{20}$$
$$y = 4$$
$$z = x(16) = x(0)\,r^{16} = \frac{1}{256}(2)^{16} = 256 \tag{21}$$
$$z = 256$$

Fig. 1. Stem Plot of $x(n)$ v/s nc