11.9.3.17

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Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

TABLE 1
Given Information

Symbol	Value	Description
x	$x(0)\,r^3$	$x(3)$
y	$x(0)\,r^9$	$x(9)$
z	$x(0)\,r^{15}$	$x(15)$
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$
$x(0)$	$x^{\frac{3}{2}}y^{-\frac{1}{2}}$	First term
$x(n)$	$x(0)\,r^n u(n)$	General Term

Solution:

1) From Table 1,

$$x = x(3) = x(0)\,r^3 \tag{1}$$
$$y = x(9) = x(0)\,r^9 \tag{2}$$
$$z = x(15) = x(0)\,r^{15} \tag{3}$$

Consider $\frac{x(9)}{x(3)}$ and $\frac{x(15)}{x(9)}$;

$$\implies \frac{x(9)}{x(3)} = \frac{x(0)\,r^9}{x(0)\,r^3} = r^6 \tag{4}$$
$$\implies \frac{x(15)}{x(9)} = \frac{x(0)\,r^{15}}{x(0)\,r^9} = r^6 \tag{5}$$

From (4) and (5), $x(3)$, $x(9)$, $x(15)$ are in G.P.
$\therefore$ x, y, z are in G.P.

2) $x(0)$ and r can be expressed in terms of x, y, and z in the following manner.

$$\implies \frac{y}{x} = r^6 \tag{6}$$
$$\implies r = \sqrt{6}{\frac{y}{x}} = \left(\frac{y}{x}\right)^{\frac{1}{6}} \tag{7}$$
$$\implies x = x(0)\,r^3 \tag{8}$$
$$\implies x(0) = \frac{x}{r^3} \tag{9}$$
$$= x\left(\frac{x}{y}\right)^{\frac{3}{6}} \tag{10}$$
$$\therefore\ x(0) = x^{\frac{3}{2}}y^{-\frac{1}{2}} \text{ and } r = \left(\frac{y}{x}\right)^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \tag{11}$$

3) Z-transform: $x(n) \overset{Z}{\longleftrightarrow} X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\,z^{-n} \tag{12}$$
$$X(z) = \frac{x(0)}{1 - rz^{-1}}; |z| > |r| \tag{13}$$

Substituting r and $x(0)$ from (11),

$$X(z) = \frac{x^{\frac{3}{2}}y^{-\frac{1}{2}}}{1 - \left(\frac{y}{x}\right)^{\frac{1}{6}}z^{-1}} \tag{14}$$

4) Example

$$\text{Let } x(0) = \frac{1}{256} \text{ and } r = 2 \tag{15}$$
$$x = x(3) = x(0)\,r^3 = \frac{1}{256}(2)^3 = \frac{1}{32} \tag{16}$$
$$\implies x = \frac{1}{32} \tag{17}$$
$$y = x(9) = x(0)\,r^9 = \frac{1}{256}(2)^9 = 2 \tag{18}$$
$$\implies y = 2 \tag{19}$$
$$z = x(15) = x(0)\,r^{15} = \frac{1}{256}(2)^{15} = 128 \tag{20}$$
$$\implies z = 128 \tag{21}$$

Fig. 1. Stem Plot of $x(n)$ v/s n