11.9.3.17

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Question: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

Solution:

The nth term of a G.P. is $a_n = a_1 r^{n-1}$. Given that x, y, z are the 4^{th}, 10^{th} and 16^{th} terms of a G.P.,

$$x = a_4 = ar^{4-1} = ar^3 \tag{1}$$

$$y = a_{10} = ar^{10-1} = ar^9 \tag{2}$$

$$z = a_{16} = ar^{16-1} = ar^{15} \tag{3}$$

Consider $\frac{y}{x}$ and $\frac{z}{y}$;

$$\frac{y}{x} = \frac{ar^9}{ar^3} \tag{4}$$

$$\frac{y}{x} = r^6 \tag{5}$$

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \tag{6}$$

$$\frac{z}{y} = r^6 \tag{7}$$

Since, $\frac{y}{x} = \frac{z}{y}$;

$$x,\ y,\ z \text{ are in G.P.}$$

For this G.P, with x, y, z, as the first three terms, the general term $x(n)$ can be defined as:

$$\text{Common Ratio} = \frac{y}{x}$$

$$x(n) = x \cdot (\frac{y}{x})^{n-1} \tag{8}$$

$$also,\ x(n) = x \cdot (\frac{z}{y})^{n-1} \tag{9}$$

$$\therefore\ x(n) = \frac{y^{n-1}}{x^{n-2}}\ \forall\ n \geq 1$$

To extend the domain of n to -ve integers, the step function $u(n)$ can be used.

$$\therefore\ x(n) = \frac{y^{n-1}}{x^{n-2}} \cdot u(n)\ \forall\ n \in Z$$

TABLE 0

Given Information

Symbol	Value	Description
x	ar^3	x(4)
y	ar^9	x(10)
z	ar^{15}	x(16)
a	$x^{\frac{3}{2}}y^{-\frac{1}{2}}$	x(1)
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$

a and r can be expressed in terms of x, y, and z in the following manner.

$$x = ar^3$$

$$\frac{y}{x} = r^6$$

$$\Rightarrow r = \sqrt[6]{\frac{y}{x}} = (\frac{y}{x})^{\frac{1}{6}} \tag{10}$$

$$a = \frac{x}{r^3}$$

$$a = x \cdot (\frac{x}{y})^{\frac{3}{6}}$$

$$\therefore a = x^{\frac{3}{2}}y^{-\frac{1}{2}} \tag{11}$$

$$and\ r = (\frac{y}{x})^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \tag{12}$$