

11.9.3.17

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Question: If the 4th, 10th and 16th terms of a G.P. are x, y, and z, respectively. Prove that x, y, z are in G.P.

TABLE 0
GIVEN INFORMATION

Symbol	Value	Description
x	ar^3	$x(4)$
y	ar^9	$x(10)$
z	ar^{15}	$x(16)$
a	$x^{\frac{3}{2}}y^{-\frac{1}{2}}$	$x(1)$
r	$y^{\frac{1}{6}}x^{-\frac{1}{6}}$	$\frac{x(n)}{x(n-1)}$
$x(n)$	$y^{n-1}x^{2-n}u(n)$	General Term

Solution:

The nth term of a G.P. is $a_n = a_1 r^{n-1}$. Given that x, y, z are the 4th, 10th and 16th terms of a G.P., From the Table ,

$$\begin{aligned} x &= a_4 = ar^{4-1} = ar^3 \\ y &= a_{10} = ar^{10-1} = ar^9 \\ z &= a_{16} = ar^{16-1} = ar^{15} \end{aligned}$$

Consider $\frac{y}{x}$ and $\frac{z}{y}$;

$$\frac{y}{x} = \frac{ar^9}{ar^3} = r^6 \quad (1)$$

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} = r^6 \quad (2)$$

Since, $\frac{y}{x} = \frac{z}{y}$;

x, y, z are in G.P.

For this G.P, with x, y, z, as the first three terms, the general term $x(n)$ can be defined as:

$$\text{Common Ratio} = \frac{y}{x}$$

$$x(n) = x\left(\frac{y}{x}\right)^{n-1} \quad (3)$$

$$\text{also, } x(n) = x \cdot \left(\frac{z}{y}\right)^{n-1} \quad (4)$$

$$\therefore x(n) = \frac{y^{n-1}}{x^{n-2}} \quad \forall n \geq 1$$

To extend the domain of n to -ve integers, the step function $u(n)$ can be used.

$$\therefore x(n) = \frac{y^{n-1}}{x^{n-2}} u(n) \quad \forall n \in \mathbb{Z}$$

a and r can be expressed in terms of x, y, and z in the following manner.

$$x = ar^3$$

$$\frac{y}{x} = r^6$$

$$\Rightarrow r = \sqrt[6]{\frac{y}{x}} = \left(\frac{y}{x}\right)^{\frac{1}{6}} \quad (5)$$

$$a = \frac{x}{r^3}$$

$$a = x\left(\frac{x}{y}\right)^{\frac{3}{6}}$$

$$\therefore a = x^{\frac{3}{2}}y^{-\frac{1}{2}} \quad (6)$$

$$\text{and } r = \left(\frac{y}{x}\right)^{\frac{1}{6}} = y^{\frac{1}{6}}x^{-\frac{1}{6}} \quad (7)$$