gate 6

EE24Btech11041 - Mohit

1) Let

$$f(x) = \frac{x^2}{x^2 + (1-nx)^2}, x \in [0,1], n = 1,2,3,\dots. \tag{1}$$

Then, which of the following statements is TRUE? (MA 2022)

a) $\{f_n\}$ is not equicontinous in $[0,1]$
b) $\{f_n\}$ is uniformly convergent on $[0,1]$
c) $\{f_n\}$ is equicontinous on $[0,1]$
d) $\{f_n\}$ is uniformly bounded and has a subsquence coverging uniformly in $[0,1]$

2) Let $(\mathbb{Q}, d)$ be the metric space with $d(x,y) = |x-y|$. Let $E = \{p \in \mathbb{Q} : 2 < p^2 < 3\}$. Then, the set E is (MA 2022)

a) closed but not compact
b) not closed but not compact
c) compact
d) neither closed nor compact

3) Let $T : L^2[-1,1] \to L^2[-1,1]$ be defined by $Tf = \tilde{f}$, where $\tilde{f}(x) = f(-x)$ almost everywhere. If M is the kernel of $I-T$, then the distance between the function $\phi(t) = e^t$ and M is (MA 2022)

a) $\frac{1}{2}\sqrt{(e^2 - e^{-2} + 4)}$
b) $\frac{1}{2}\sqrt{(e^2 - e^{-2} - 2)}$
c) $\frac{1}{2}\sqrt{(e^2 - 4)}$
d) $\frac{1}{2}\sqrt{(e^2 - e^{-2} - 4)}$

4) X, Y and Z be Banach spaces. Suppose that $T : X \to Y$ is linear and $S : Y \to Z$ is linear, bounded and injective. In addition, if $S \circ T : X \to Z$ is bounded, then, which of the following statements is TRUE? (MA 2022)

a) T is surjective
b) T is bounded but not continuous
c) T is bounded
d) T is not bounded

5) The first derivative of a function $f \in C^\infty(-3,3)$ is approximated by an interpolating polynomial of degree 2, using the data

$$(-1, f(-1)), (0, f(0)) \text{ and } (2, f(2)). \tag{2}$$

It is found that

$$f'(0) \approx \frac{2}{3}f(-1) + \alpha f(0) + \beta f(2). \tag{3}$$

Then, the value of $\frac{1}{\alpha\beta}$ is (MA 2022)

a) 3
b) 6
c) 9
d) 12

6) The work done by the force $F = (x+y)\hat{\imath} - \left(x^2 + y^2\right)\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in **OX** and **OY** directions, respectively, along the upper half of the circle $x^2 + y^2 = 1$ from $(1,0)$ to $(-1,0)$ in the $xy-$plane is (MA 2022)

a) $-\pi$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) π

7) Let $u(x,t)$ be the solution of the wave equation (MA 2022)

$$\frac{\partial^2 u}{\partial^2} - \frac{\partial^2 u}{\partial x^2} = 0, 0 < x < \pi, t > 0, \tag{4}$$

with the initial conditions

$$u(x,0) = \sin x + \sin 2x + \sin 3x, \frac{\partial u}{\partial t}(x,0) = 0, 0 < x < \pi \tag{5}$$

and the boundary comditions $u(0,t) = u(n,t) = 0, t \geq 0$.Then, the value of $u\left(\frac{\pi}{2}, \pi\right)$ is

a) $-\frac{1}{2}$ b) 0 c) $\frac{1}{2}$ d) 1

8) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by

$$T((1,2)) = (1,0) \text{ and } T((2,1)) = (1,1) \tag{6}$$

For $p, q \in \mathbb{R}^2$, let $T^{-1}((p,q)) = (x,y)$.
Which of the following statements is TRUE? (MA 2022)

a) $x = p - q; y = 2p - q$
b) $x = p + q; y = 2p - q$
c) $x = p + q; y = 2p + q$
d) $x = p - q; y = 2p + q$

9) Let $y = (\alpha, -1)^T, \alpha \in \mathbb{R}$ be a feasible solution for the dual problem of the linear programming problem

$$\text{Maximize} : 5x_1 + 12x_2 \tag{7}$$
$$\text{subject to} : x_1 + 2x_2 + x_3 \leq 10 \tag{8}$$
$$2x_1 - x_2 + 3x_3 = 8 \tag{9}$$
$$x_1, x_2, x_3 \geq 0. \tag{10}$$

Which of the following statements is TRUE? (MA 2022)

a) $\alpha < 3$ b) $3 \leq \alpha < 5.5$ c) $5.5 \leq \alpha < 7$ d) $\alpha \geq 7$

10) Let K denote the subset of $\mathbb{C}$ consisting of elements algebraic over $\mathbb{Q}$. Then, which of the following statements are TRUE? (MA 2022)

a) No element of $\mathbb{C} \backslash K$ is algebraic over $\mathbb{Q}$
b) K is an algebraically closed field
c) For any bijective ring homomorphism $f : \mathbb{C} \to \mathbb{C}$, we have $f(K) = K$
d) There is no bijection between K and $\mathbb{Q}$

11) Let T be a Mobius transformation such that $T(0) = \alpha, T(\alpha) = 0$ and $T(\infty) = -\alpha$, where $\alpha = \frac{-1+i}{\sqrt{2}}$ Let L denote the straight line passing through the origin with slope -1, and let C denote the circle of unit radius centred at the origin. Then, which of the following statements are TRUE? (MA 2022)

a) T maps L to a straight line
b) T maps L to a circle
c) T^{-1} maps C to a straight line
d) T^{-1} maps C to a circle

12) Let $a > 0$. Define $D_a : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ by $(D_a f)(x) = \frac{1}{\sqrt{a}} f\left(\frac{x}{a}\right)$,almost everywhere, for $f \in L^2(\mathbb{R})$. Then, which of the following statements are TRUE? (MA 2022)

a) D_a is a linear isometry

b) D_a is a bijection
c) $D_a \circ D_b = D_{a+b}, b > 0$
d) D_a is bounded from below

13) Let $\{\phi_0, \phi_1, \phi_2, \dots\}$ be an orthonormal set in $L^2[-1,1]$ such that $\phi_n = C_n P_n$, where C_n is a constant and P_n is the Legendre polynomial of degree n, for each $n \in \mathbb{N} \cup \{0\}$. Then, which of the following statements are TRUE? (MA 2022)

a) $\phi_6(1) = 1$ b) $\phi_7(-1) = 1$ c) $\phi_7(1) = \sqrt{\frac{15}{2}}$ d) $\phi_7(-1) = \sqrt{\frac{13}{2}}$