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AI24BTECH11006 - Bugada Roopansha

40) Let X be an arbitrary random variable that takes values in $[0, 1, \ldots, 10]$. The minimum and maximum possible values of the variance of X are
   a) 0 and 30
   b) 1 and 30
   c) 0 and 25
   d) 1 and 25
41) Let M be the space of all 4×3 matrices with entries in the finite field of three elements. Then the number of matrices of rank three in M is
   a) $(3^4 - 3)(3^4 - 3^2)(3^4 - 3^3)$
   b) $(3^4 - 1)(3^4 - 2)(3^4 - 3)$
   c) $(3^4 - 1)(3^4 - 3)(3^4 - 3^2)$
   d) $3^4(3^4 - 1)(3^4 - 2)$
42) Let V be a vector space of dimension $m \geq 2$. Let $T : V \to V$ be a linear transformation such that $T^{n+1} = 0$ and $T^n \neq 0$ for some $n \geq 1$. Then which of the following is necessarily **TRUE**?
   a) $\text{Rank}(T^n) \leq \text{Nullity}(T^n)$
   b) $\text{trace}(T) \neq 0$
   c) T is diagonalizable
   d) $n = m$
43) Let X be a convex region in the plane bounded by straight lines. Let X have 7 vertices. Suppose $f(x, y) = ax + by + c$ has maximum value M and minimum value N on X and $N<M$. Let $S = P : P$ is a vertex of X and $N<f(P)<M$. If S has n elements, then which of the following statements is **TRUE**?
   a) n cannot be 5
   b) n can be 2
   c) n cannot be 3
   d) n can be 4
44) Which of the following statements are **TRUE**?
   P: If $f \in L^1(\mathbb{R})$, then f is continuous.
   Q: If $f \in L^1(\mathbb{R})$ and $\lim_{|x|\to\infty} f(x)$ exists, then the limit is zero.
   R: If $f \in L^1(\mathbb{R})$, then f is bounded.
   S: If $f \in L^1(\mathbb{R})$ is uniformly continuous, then $\lim_{|x|\to\infty} f(x)$ exists and equals zero.
   a) Q and S only
   b) P and R only
   c) P and Q only
   d) R and S only
45) Let u be a real valued harmonic function on $\mathbb{C}$. Let $g : \mathbb{R}^2 \to \mathbb{R}$ be defined by
   $$g(x, y) = \int_0^{2\pi} u\left(e^{i\theta}(x + iy)\right) \sin\theta \, d\theta.$$
   Which of the following statements is **TRUE**?
   a) g is a harmonic polynomial
   b) g is a polynomial but not harmonic
   c) g is harmonic but not a polynomial
   d) g is neither harmonic nor a polynomial
46) Let $S = z \in \mathbb{C} : |z| = 1$ with the induced topology from $\mathbb{C}$ and let $f : [0, 2] \to S$ be defined as $f(t) = e^{2\pi it}$. Then, which of the following is **TRUE**?
   a) K is closed in $[0, 2] \Rightarrow f(K)$ is closed in S
   b) U is open in $[0, 2] \Rightarrow f(U)$ is open in S
   c) $f(X)$ is closed in $S \Rightarrow X$ is closed in $[0, 2]$
   d) $f(Y)$ is open in $S \Rightarrow Y$ is open in $[0, 2]$
47) Assume that all the zeros of the polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ have negative real parts. If $u(t)$ is any solution to the ordinary differential equation
   $$a_n \frac{d^n u}{dt^n} + a_{n-1}\frac{d^{n-1}u}{dt^{n-1}} + \cdots + a_1 \frac{du}{dt} + a_0 u = 0,$$
   then $\lim_{t\to\infty} u(t)$ is equal to
   a) 0
   b) 1
   c) ∞
   d) $n - 1$

Common Data for Questions 48 **and** 49**:**
Let c_{00} be the vector space of all complex sequences having finitely many non-zero terms. Equip c_{00} with the inner product $x, y =$

$\sum_{n=1}^{\infty} x_n y_n$ for all $x = (x_n)$ and $y = (y_n)$ in c_{00}. Define $f : c_{00} \to \mathbb{C}$ by $f(x) = \sum_{n=1}^{\infty} \frac{x_n}{n}$. Let N be the kernel of f.

48) Which of the following is **FALSE**?
   a) f is a continuous linear functional
   b) $||f|| \leq \frac{\pi}{\sqrt{6}}$
   c) There does not exist any $y \in c_{00}$ such that $f(x) = x, y$ for all $x \in c_{00}$
   d) $N^{\perp} \neq 0$

49) Which of the following is **FALSE**?
   a) $c_{00} \neq N$
   b) N is closed
   c) c_{00} is not a complete inner product space
   d) $c_{00} = N \oplus N^{\perp}$

Common Data for Questions 50 **and** 51**:**
Let $X_1, X_2, \ldots, X_n$ be an i.i.d random sample from an exponential distribution with mean μ. In other words, they have density

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-x/\mu} & \text{if } x>0 \\ 0 & \text{otherwise.} \end{cases}$$

50) Which of the following is **NOT** an unbiased estimate of μ?
   a) X_1
   b) $\frac{1}{n-1}(X_1 + X_2 + \cdots + X_n)$
   c) $n \min(X_1, X_2, \ldots, X_n)$
   d) $\frac{1}{n} \max(X_1, X_2, \ldots, X_n)$

51) Consider the problem of estimating μ. The error $m \cdot s \cdot e\ (meansquareerror)$ of the estimate $T(X) = \frac{X_1 + X_2 + \cdots + X_n}{n+1}$ is
   a) μ^2
   b) $\frac{\mu^2}{n+1}$
   c) $\frac{\mu^2}{(n+1)^2}$
   d) $\frac{n^2\mu^2}{(n+1)^2}$

Linked Answer Questions

Statement for Linked Answer Questions 52 **and** 53**:**
Let $X = ((x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1) \cup ([-1, 1] \times \{0\}) \cup (\{0\} \times [-1, 1])$. Let $n_0 = \max\{k : k<\infty,$ there are k distinct points $p_1, \ldots, p_k \in X$ such that $X \setminus \{p_1, \ldots, p_k\}$ is connected$\}$

52) The value of n_0 is ...