2016-MA-'1-13'

AI24BTECH11006 - Bugada Roopansha

1) Let $\{X, Y, Z\}$ be a basis of $\mathbb{R}^3$. Consider the following statements P and Q:
   a) P: $\{X+Y, Y+Z, X-Z\}$ is a basis of $\mathbb{R}^3$.
   b) Q: $\{X+Y+Z, X+2Y-Z, X-3Z\}$ is a basis of $\mathbb{R}^3$.

   Which of the above statements hold TRUE?
   a) both P and Q
   b) only Q
   c) only P
   d) Neither P nor Q
2) Consider the following statements P and Q:

   P: If $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$, then M is singular.

   Q: Let S be a diagonalizable matrix. If T is a matrix such that $S+5T = I$, then T is diagonalizable.

   Which of the above statements hold TRUE?
   a) both P and Q
   b) only Q
   c) only P
   d) Neither P nor Q
3) Consider the following statements P and Q:

   P: If M is an $n \times n$ complex matrix, then $R(M) = (N(M^*))^{\perp}+$.

   Q: There exists a unitary matrix with an eigenvalue λ such that $|\lambda| < 1$.

   Which of the above statements hold TRUE?
   a) both P and Q
   b) only Q
   c) only P
   d) Neither P nor Q
4) Consider a real vector space V of dimension n and a non-zero linear transformation $T: V \to V$. If dimension $(T(V)) < n$ and $T^2 = \lambda T$, for some $\lambda \in \mathbb{R} \setminus \{0\}$, then which of the following statements is TRUE?
   a) determinant $(T) = |2|$
   b) There exists a non-trivial subspace V_1 of V such that $T(X) = 0$ for all $X \in V$
   c) T is invertible
   d) 2 is the only eigenvalue of T
5) Let $S = (0,1) \cup [2,3]$ and $f: S \to \mathbb{R}$ be a strictly increasing function such that $f(S)$ is connected. Which of the following statements is TRUE?
   a) f has exactly one discontinuity
   b) f has exactly two discontinuities
   c) f has infinitely many discontinuities
   d) f is continuous
6) Let $a_1 = 1$ and $a_n = a_{n-1} + 4$, $n \geq 2$. Then,
$$\lim_{n\to\infty} \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_{n-1} a_n}$$
   is equal to$\cdots$
7) $\max\{x+y : (x,y) \in B(0,1)\}$ is equal to $\cdots$
8) Let $a, b, c, d \in \mathbb{R}$ such that $c^2 + d^2 \neq 0$. Then, the Cauchy problem
$$au_x + bu_y = e^{x+y}, x, y \in \mathbb{R},$$
$$u(x,y) = 0 \text{ on } cx + dy = 0$$
   has a unique solution if
   a) $ac + bd \neq 0$
   b) $ad - bc \neq 0$
   c) $ac - bd \neq 0$
   d) $ad + bc \neq 0$
9) Let $u(x,t)$ be the d'Alembert's solution of the initial value problem for the wave equation
$$u_{tt} - c^2 u_{xx} = 0,$$
$$u(x,0) = f(x), u_t(x,0) = g(x),$$
   where c is a positive real number and f, g are smooth odd functions. Then, $u(0,1)$ is equal to $\cdots$
10) Let the probability density function of a random variable X be
$$f(x) = \begin{cases} c(2x-1) & 0 < x \leq 1, \\ \frac{1}{x} & 1 < x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$
   Then, the value of c is equal to $\cdots$

11) Let V be the set of all solutions of the equation $y\prime\prime + ay\prime + by = 0$ satisfying $y(0) = y(1)$, where a, b are positive real numbers. Then, dimension (V) is equal to
12) Let $y\prime\prime + p(x)\,y\prime + q(x)\,y = 0$, $x \in (-\infty, \infty)$, where $p(x)$ and $q(x)$ are continuous functions. If $y_1(x) = \sin(x) - 2\cos(x)$ and $y_2(x) = 2\sin(x) + \cos(x)$ are two linearly independent solutions of the above equation, then $|4p(0) + 2q(1)|$ is equal to
13) Let $P(x)$ be the Legendre polynomial of degree n and $I = \int_{-1}^{1} x^k P(x)\, dx$, where k is a non-negative integer. Consider the following statements P and Q:
    - P: $I = 0$ if $k<n$.
    - Q: $I = 0$ if $n + k$ is an odd integer.

    Which of the above statements hold TRUE?

    a) both P and Q
    b) only Q
    c) only P
    d) Neither P nor Q