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AI24BTECH11023 - Tarun Reddy Pakala

27) Let $T : \mathbb{C}^3 \to \mathbb{C}^3$ be defined by $T\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} z_1 - iz_2 \\ iz_1 + z_2 \\ z_1 + z_2 + iz_3 \end{pmatrix}$. Then, the adjoint T^* of T is given by $T^*\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} =$
    a) $\begin{pmatrix} z_1 + iz_2 \\ -iz_1 + z_2 \\ z_1 + z_2 - iz_3 \end{pmatrix}$
    b) $\begin{pmatrix} z_1 - iz_2 + z_3 \\ -iz_1 + z_2 + z_3 \\ iz_3 \end{pmatrix}$
    c) $\begin{pmatrix} z_1 - iz_2 + z_3 \\ iz_1 + z_2 + z_3 \\ -iz_3 \end{pmatrix}$
    d) $\begin{pmatrix} iz_1 + z_2 \\ z_1 - iz_2 \\ z_1 - z_2 - iz_3 \end{pmatrix}$
28) Let $f(z)$ be an entire function that $|f(z)| \leq K|z|, \forall z \in \mathbb{C}$, for some $K > 0$. If $f(1) = i$, the value of $f(i)$ is
    a) 1
    b) -1
    c) i
    d) $-i$
29) Let y be the solution of the initial value problem

$$\frac{d^2y}{dx^2} + y = 6\cos 2x,\ y(0) = 3,\ y'(0) = 1.$$

    Let the Laplace transform of y be $F(s)$. Then, the value of $F(1)$ is
    a) $\frac{17}{5}$
    b) $\frac{13}{5}$
    c) $\frac{11}{5}$
    d) $\frac{9}{5}$
30) For $0 \leq x \leq 1$, let

$$f_n(x) = \begin{cases} \frac{n}{1+n}, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$$

    and $f(x) = \lim_{n\to\infty} f_n(x)$. Then, on the interval [0,1]
    a) f is measurable and Riemann integrable
    b) f is measurable and Lebesgue integrable
    c) f is not measurable
    d) f is not Lebesgue integrable

31) If x, y and z are positive real numbers, then the minimum value of

$$x^2 + 8y^2 + 27z^2 \text{ where } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

is

a) 108
b) 216
c) 405
d) 1048

32) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be defined by

$$T(x, y, z, w) = (x + y + 5w, x + 2y + w, -z + 2w, 5x + y + 2z).$$

The dimension of the eigenspace of T is

a) 1
b) 2
c) 3
d) 4

33) Let y be a polynomial solution of the differential equation

$$(1 - x^2)y'' - 2xy' + 6y = 0.$$

If $y(1) = 2$, then the value of the integral $\int_{-1}^{1} y^2 dx$ is

a) $\frac{1}{5}$
b) $\frac{2}{5}$
c) $\frac{4}{5}$
d) $\frac{8}{5}$

34) The value of the integral

$$I = \int_{-1}^{1} exp(x^2)dx$$

using a rectangular rule is approximated as 2. Then, the approximation error $|I - 2|$ lies in the interval

a) $(2e, 3e]$
b) $(\frac{2}{3}, 2e]$
c) $(\frac{e}{8}, \frac{2}{3}]$
d) $(0, \frac{e}{8}]$

35) The integral surface for the Cauchy problem

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1,$$

which passes through the circle $z = 0,\ x^2 + y^2 = 1$ is

a) $x^2 + y^2 + 2z^2 + 2zx - 2yz - 1 = 0$
b) $x^2 + y^2 + 2z^2 + 2zx + 2yz - 1 = 0$
c) $x^2 + y^2 + 2z^2 - 2zx - 2yz - 1 = 0$
d) $x^2 + y^2 + 2z^2 + 2zx + 2yz + 1 = 0$

36) The vertical displacement $u(x, t)$ of an infinitely long elastic string is governed by the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2},\ -\infty < x < \infty,\ t > 0,$$

$$u(x, 0) = -x \text{ and } \frac{\partial u}{\partial t}(x, 0) = 0.$$

The value of $u(x, t)$ at $x = 2$ and $t = 2$ is equal to

a) 2
b) 4
c) -2
d) -4

37) We have to assign four jobs I, II, III, IV to four workers A, B, C and D. The time taken by different workers (in hours) in completing different jobs is given below:

		I	II	III	IV
Workers	A	5	3	2	8
	B	7	9	2	6
	C	6	4	5	7
	D	5	7	7	8

The optimal assignment is as follows:
Job III to worker A; Job IV to worker B; Job II to worker C and Job I to worker D and hence the time taken by different workers in completing different jobs is now changed as:

		I	II	III	IV
Workers	A	5	3	2	5
	B	7	9	2	3
	C	4	2	3	2
	D	5	7	7	5

Then the minimum time (in hours) taken by the workers to complete all the jobs is

a) 10
b) 12
c) 15
d) 17

38) The following table shows the information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in rupees) from each warehouse to each market.

		Market				
		M_1	M_2	M_3	M_4	Supply
Warehouse	W_1	6	3	5	4	22
	W_2	5	9	2	7	15
	W_3	5	7	8	6	8
	Requirement	7	12	17	9	

The present transportation schedule is as follows:
W_1 to M_2: 12 units; W_1 to M_3: 1 unit; $W_1$to M_4: 9 units; W_2 to M_3: 15 units; W_3 to M_1: 7 units and W_3 to M_3: 1 unit. Then the minimum total transportation cost (in rupees) is

a) 150
b) 149
c) 148
d) 147

39) If $\mathbb{Z}[i]$ is the ring of Gaussian integers, the quotient $\mathbb{Z}[i]/(3 - i)$ is isomorphic to

a) $\mathbb{Z}$
b) $\mathbb{Z}/3\mathbb{Z}$
c) $\mathbb{Z}/4\mathbb{Z}$
d) $\mathbb{Z}/10\mathbb{Z}$