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53) Let $T : l_2 \to l_2$ be defined by
$$T((x_1, x_2, \dots, x_n, \dots)) = (x_2 - x_1, x_3 - x_2, \dots, x_{n+1} - x_n, \dots).$$
Then
  a) $\|T\| = 1$
  b) $\|T\| > 2$ but bounded
  c) $1 < \|T\| \leq 2$
  d) $\|T\|$ is bounded
54) Minimize $w = x + 2y$ subject to
$$2x + y \geq 3$$
$$x + y \geq 2$$
$$x \geq 0, \ y \geq 0.$$
Then, the minimum value of w is equal to ________
55) Maximize $w = 11x - z$ subject to
$$10x + y - z \leq 1$$
$$2x - 2y + z \leq 2$$
$$x, y, z \geq 0.$$
Then the maximum value of w is equal to ________
56) Let $X_1, X_2, X_3, \dots$ be a sequence of i.i.d. random variables with mean 1. If N is a geometric random variable with the probability mass function $P(N = k) = \frac{1}{2^k}$; $k = 1, 2, 3, \dots$ and its independent of the $X_i's$, then $E(X_1 + X_2 + \dots + X_N)$ is equal to ________
57) Let X_1 be an exponential random variable with mean 1 and X_2 a gamma random variable with mean 2 and variance 2. If X_1 and X_2 are independently distributed, then $P(X_1 < X_2)$ is equal to ________
58) Let $X_1, X_2, X_3, \dots$ be a sequence of i.i.d. uniform $(0, 1)$ random variables. Then, the value of
$$\lim_{n\to\infty} P(-\ln(1 - X_1) - \dots - \ln(1 - X_n) \geq n)$$
is equal to ________
59) Let X be a standard normal random variable. Then, $P(X < 0| \, |[X]| = 1)$ is equal to
  a) $\frac{\Phi(1)-\frac{1}{2}}{\Phi(2)-\frac{1}{2}}$
  b) $\frac{\Phi(1)+\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$
  c) $\frac{\Phi(1)-\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$
  d) $\frac{\Phi(1)+1}{\Phi(2)+1}$
60) Let $X_1, X_2, X_3, \dots X_n$ be a random sample from the probability density function
$$f(x) = \begin{cases} \theta\alpha e^{-\alpha x} + (1-\theta)\, 2\alpha e^{-2\alpha x}; & \text{if } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha > 0,\ 0 \le \theta \le 1$ are parameters. Consider the following testing problem:
$H_0 : \theta = 1, \alpha = 1$ versus $H_1 : \theta = 0, \alpha = 2$.

a) Uniformly Most Powerful test does NOT exist
b) Uniformly Most Powerful test is of the form $\sum_{i=1}^{n} X_i > c$, for some $0 < c < \infty$
c) Uniformly Most Powerful test is of the form $\sum_{i=1}^{n} X_i < c$, for some $0 < c < \infty$
d) Uniformly Most Powerful test is of the form $c_1 < \sum_{i=1}^{n} X_i < c_2$, for some $0 < c_1 < c_2 < \infty$

61) Let $X_1, X_2, X_3, \ldots$ be a sequence of i.i.d. $N(\mu, 1)$ random variables. Then,

$$\lim_{n\to\infty} \frac{\sqrt{\pi}}{2n} \sum_{i=1}^{n} E\left(|X_i - \mu|\right)$$

is equal to __________

62) Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from uniform $[1, \theta]$, for some $\theta > 1$. If $X_{(n)}$=Maximum $(X_1, X_2, X_3, \ldots, X_n)$, then the UMVUE of θ is

a) $\frac{n+1}{n} X_{(n)} + \frac{1}{n}$
b) $\frac{n+1}{n} X_{(n)} - \frac{1}{n}$
c) $\frac{n}{n+1} X_{(n)} + \frac{1}{n}$
d) $\frac{n}{n+1} X_{(n)} + \frac{n+1}{n}$

63) Let $x_1 = x_2 = x_3 = 1,\ x_4 = x_5 = x_6 = 2$ be a random sample from a Poisson random variable with mean θ, where $\theta \in \{1, 2\}$. Then, the maximum likelihood estimator of θ is equal to __________

64) The remainder when 98! is divided by 101 is equal to __________

65) Let G be a group whose presentation is

$$G = \{x, y \mid x^5 = y^2 = e,\ x^2 y = yx\}$$

Then G is isomorphic to

a) $\mathbb{Z}_5$
b) $\mathbb{Z}_{10}$
c) $\mathbb{Z}_2$
d) $\mathbb{Z}_{30}$