

4) Which of the following is a quasi-linear partial differential equation?

- a)  $\frac{\partial^2 u}{\partial t^2} + u^2 = 0$
- b)  $\left(\frac{\partial u}{\partial t}\right)^2 + \frac{\partial u}{\partial x} = 0$
- c)  $\left(\frac{\partial u}{\partial t}\right)^2 - \left(\frac{\partial u}{\partial x}\right)^2 = 0$
- d)  $\left(\frac{\partial u}{\partial t}\right)^4 - \left(\frac{\partial u}{\partial x}\right)^3 = 0$

5) Let  $P(x)$  and  $Q(x)$  be the polynomials of degree 5, generated by Lagrange and Newton interpolation methods respectively, both passing through given six distinct points on the  $xy$ -plane. Which of the following is correct?

- a)  $P(x) \equiv Q(x)$
- b)  $P(x) - Q(x)$  is a polynomial of degree 1
- c)  $P(x) - Q(x)$  is a polynomial of degree 2
- d)  $P(x) - Q(x)$  is a polynomial of degree 3

6) The Laurent series of  $f(z) = 1/(z^3 - z^4)$  with center at  $z = 0$  in the region  $|z| > 1$  is

- a)  $\sum_{n=0}^{\infty} z^{n-3}$
- b)  $-\sum_{n=0}^{\infty} \frac{1}{z^{n+4}}$
- c)  $\sum_{n=0}^{\infty} z^n$
- d)  $\sum_{n=0}^{\infty} \frac{1}{z^n}$

7) The value of the surface integral  $\iint_{\Gamma} \mathbf{F} \cdot d\mathbf{s}$  over the sphere  $\Gamma$  given by  $x^2 + y^2 + z^2 = 1$  where  $\mathbf{F} = 4x\hat{i} - z\hat{k}$ , and  $n$  denotes the outward unit normal, is

- a)  $\pi$
- b)  $2\pi$
- c)  $3\pi$
- d)  $4\pi$

Q.8 - Q.11 carry two marks each

8) A diagnostic test for a certain disease is 90% accurate. That is, the probability of a person having (respectively, not having) the disease tested positive (respectively, negative) is 0.9. Fifty percent of the population has the disease. What is the probability that a randomly chosen person has the disease given that the person tested negative?

9) Let  $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ . Which of the following is correct?

- a) Rank of  $M$  is 1 and  $M$  is not diagonalizable
- b) Rank of  $M$  is 2 and  $M$  is diagonalizable
- c) 1 is the only eigenvalue and  $M$  is not diagonalizable
- d) 1 is the only eigenvalue and  $M$  is diagonalizable

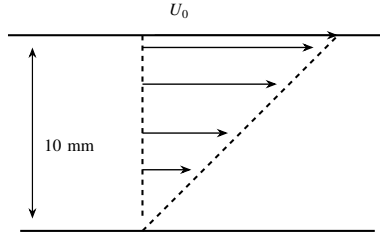
10) Let  $f(x) = 2x^3 - 3x^2 + 69$ ,  $-5 \leq x \leq 5$ . Find the point at which  $f$  attains the global maximum.

11) Calculate  $\int_{c_1} \mathbf{F} \cdot d\mathbf{r} - \int_{c_2} \mathbf{F} \cdot d\mathbf{r}$ , where  $c_1 : \mathbf{r}(t, t^2)$  and  $c_2 : \mathbf{r}(t, \sqrt{t})$ ,  $t$  varying from 0 to 1 and  $\mathbf{F} = xy\hat{j}$ .

## B.Fluid Mechanics

Q.1-Q.9 carry one mark each.

- 1) In the parallel-plate configuration shown, steady-flow of an incompressible Newtonian fluid is established by moving the top plate with a constant speed,  $U_0 = 1\text{ m/s}$ . If the force required on the top plate to support this motion is 0.5 per unit area (in  $\text{m}_2$ ) of the plate then the viscosity of the fluid between the plates is \_\_\_\_\_  $\text{N-s/m}^2$



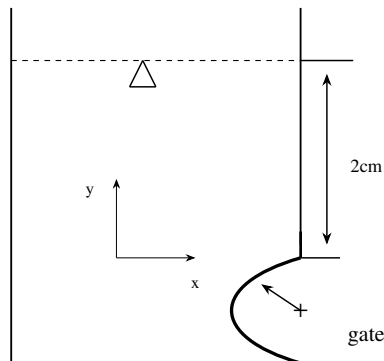
- 2) For a newly designed vehicle by some students, volume of fuel consumed per unit distance travelled ( $q_f \text{ in } \text{m}^3/\text{m}$ ) depends upon the viscosity ( $\mu$ ) and density ( $\rho$ ) of the fuel and, speed ( $U$ ) and size ( $L$ ) of the vehicle as

$$q_f = C \frac{\rho U^2 L}{\mu^3}$$

where  $C$  is a constant. The dimensions of the constant  $C$  are

- a)  $M^0 L^0 T^0$       b)  $M^2 L^{-1} T^{-1}$       c)  $M^2 L^{-5} T^{-1}$       d)  $M^{-2} L^1 T^1$

- 3) A semicircular gate of radius  $1\text{ m}$  is placed at the bottom of a water reservoir as shown in the figure below. The hydrostatic force per unit width of the cylindrical gate in  $y$ -direction is \_\_\_\_\_  $\text{kN}$ . The gravitational acceleration,  $g = 9.8\text{ m/s}^2$  and density of the water =  $1000\text{ kg/m}^3$ .



- 4) Velocity vector in  $\text{m/s}$  for a 2-D flow is given in Cartesian coordinate  $(x, y)$  as  $\vec{V} = \left( \frac{x^2}{4} \hat{i} - \frac{xy}{2} \hat{j} \right)$ . Symbols bear usual meaning. At a point in the flow, the  $x$ -component

and y-component of the acceleration vector are given as  $1m/s^2$  and  $-0.5m/s^2$ , respectively. The velocity magnitude at that points is \_\_\_\_\_  $m/s$ .

- 5) If  $\phi(x, y)$  is velocity potential and  $\psi(x, y)$  is stream function for a 2-D, steady, incompressible and irrotational flow, which of the followings is correct?

a)  $\left(\frac{dy}{dx}\right)_{\phi=const} = -\frac{1}{\left(\frac{dy}{dx}\right)_{\psi=const}}$

b)  $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

c)  $\left(\frac{dy}{dx}\right)_{\phi=const} = \frac{1}{\left(\frac{dy}{dx}\right)_{\psi=const}}$

d)  $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$