Discrete Mathematics (SC612)
Insem Exam 1
Autumn 2022-23

16^{th}September 2022

This exam consists of 5 questions, printed on two pages back-to-back, on a single paper. Check that your paper is complete.
Total marks 50; each question is worth 10 marks
Duration: 90 minutes

1. Suppose you are given $|\overline{A}| = 70$, $|\overline{B}| = 90$,$|\overline{C}| = 110$, where the complement is with respect to a universal set U of which A, B, C are subsets.

   (a) Write down the **possible** subset relationships among A, B, C.

   (b) What is the minimum number of elements in the universal set U?

   (c) What is the maximum number of elements in the universal set U, such that the sets A, B, C are pairwise disjoint.

2. Let ψ be an arbitrary boolean function on three propositional variables (there are 256 of them).

   (a) Find a boolean function ϕ such that $\psi \oplus \phi = \psi$. The function ϕ should be independent of ψ. That is, the same ϕ works for all choices of ψ.

   (b) For an arbitrary choice of function, ψ, find a corresponding function ψ' such that $\psi \oplus \psi'$ is equal to the answer obtained in part (a).

   (c) Find a boolean function α, such that for all possible choices of ψ, $\psi \vee \alpha = \psi$. The function α should be independent of ψ. That is, the same α works for all choices of ψ

3. Consider the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   (a) We define a relation R on S, where two numbers are related, if and only if their sum is a prime number (the sum need not belong to the set S). How many ordered pairs does R have?

   (b) We define a relation R on S, where two numbers are related, if and only if their product is a prime number (the product need not belong to the set S). How many ordered pairs does R have?

4. Suppose there are two players A and B in separate locations who each toss a coin, and cannot see the other's outcome. Suppose A guesses B's outcome and B guesses A's outcome using the following strategies:

   - A will guess for B the same outcome as A got
   - B will guess for A the opposite outcome as B got.

   Consider the following two propositions:

   - p: A's guess is correct
   - q: B's guess is correct

   (a) Write down the truth table for all possible configurations of p and q (there are four rows), and in the formula column, write 0 if this outcome is not possible in the game (including the strategies used by the two players), and 1 if this outcome is possible in the game.

   (b) Write this truth table in terms of a syntactic forumla using standard logical connectives, p and q.

5. Draw the Hasse diagram for each of the following:

   (a) An upper lattice on four element

   (b) A lower lattice on four elements

   (c) A lattice on four elements