Question-Wise Details

Q. 1

Question 1

Which of the following operations is not associative when applied to a chain of literals? Associative means $(a * b) * c = a * (b * c)$ where a, b, c are elements and $*$ is the operation.

Response:

OPTIONS	RESPONSE	ANSWER
$\vee$		
$\Rightarrow$	✔	✔
$\wedge$		
$\oplus$		

Q. 2

Question 2

The number of rooted binary trees on 5 vertices is

Response:

OPTIONS	RESPONSE	ANSWER
60		
42	✔	✔
135		
120		

Q. 3

Question 3

Consider the complete binary tree on 7 nodes, as a binary search tree with the seven distinct keys {1, 2, 3, 4, 5, 6, 7}.

How many insert orders are ther such that this specific tree will be formed, using the standard insert order.

Response:

OPTIONS	RESPONSE	ANSWER
120		
16		
24		
80	✔	✔

Q. 4

Question 4

Which of the following languages, over $\Sigma = \{a, b\}$, cannot be recognised by a finite state automaton?

Response:

OPTIONS	RESPONSE	ANSWER
The language of words where none of the letters appear more than four times in a row.		
The language of words where each contiguous block of the same letter is at least 4 in length		
The language of words that do not contain a specific substring	✔	
The language of words where the number of *a*'s is less than the number of *b*'s		✔

Q. 5

Question 5

In the **Principle of Inclusion & Exclusion** applied to n sets, the number of terms in the expression is:

Response:

OPTIONS	RESPONSE	ANSWER
n^2		
$\binom{n}{2}$		
2^n	✓	✓
n		

Q. 6

Question 6

What is the smallest number of states needed in a finite state automaton over the alphabet $\Sigma = \{a, b, c\}$, to accept the language of all words that end with the letter a?

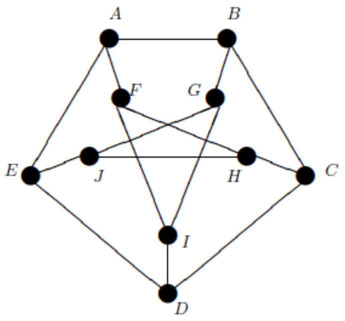
Response:

OPTIONS	RESPONSE	ANSWER
2		✓
3		
1		
4	✓	

Q. 7

Question 7

Consider the Petersen graph drawn below.



The minimum number of edges that need to be deleted from this graph such that the resultant graph is Eulerian is:

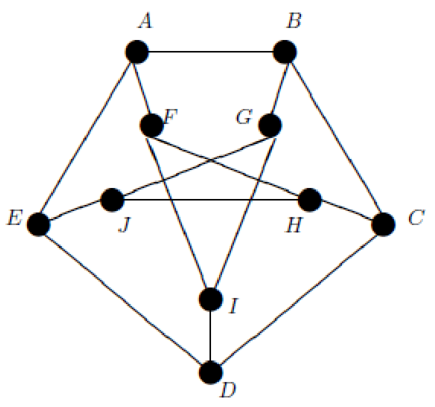
Response:

OPTIONS	RESPONSE	ANSWER
3	✔	
5		
4		
6		✔

Q. 8

Question 8

Consider the Petersen graph drawn below.



Response:

OPTIONS	RESPONSE	ANSWER
The longest cycle in this graph is 6 edges and the longest path is 8 edges		
The longest cycle in this graph is 9 edges and the longest path is 9 edges		✓
The longest cycle in this graph is 8 edges and the longest path is 9 edges		
The longest cycle in this graph is 10 edges and the longest path is 9 edges	✓	

Q. 9

Question 9

What is the smallest number of states needed in a finite state automaton over the alphabet $\Sigma = \{a, b, c\}$, to accept the language of all words that begin with the letter c and end with the letter a?

Response:

OPTIONS	RESPONSE	ANSWER
2		
4		✓
3		
1	✓	

Q. 10

Question 10

The recurrence relation associated with the well known Binary Search algorithm is

$$T(n) = T(\frac{n}{2}) + cn$$

where c is a constant. The base case is $T(1) = k$, where k is a constant. The solution to this recurrence in asymptotic terms is:

Response:

OPTIONS	RESPONSE	ANSWER
$\Theta(n^2)$		
$\Theta(\log n)$	✔	✔
$\Theta(1)$		
$\Theta(n \log n)$		

Q. 11

Question 11

What is the smallest number of states needed in a finite state automaton over the alphabet $\Sigma = \{a, b, c\}$, to accept the language of all words that begin with the letter b?

Response:

OPTIONS	RESPONSE	ANSWER
3		✔
1	✔	
4		
2		

Q. 12

Question 12

The minimum number of ordered pairs in an equivalence relation on a set S, with $|S| = n$ is

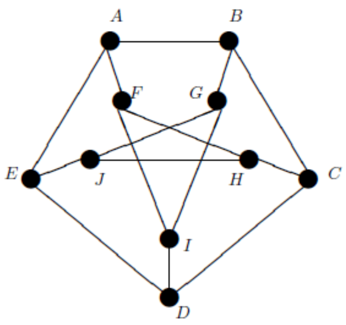
Response:

OPTIONS	RESPONSE	ANSWER
1		
n	✔	✔
0		
n^2		

Q. 13

Question 13

Consider the Petersen graph drawn below.



The minimum number of edges that need to be deleted from this graph such that the resultant graph is bipartite is:

Response:

OPTIONS	RESPONSE	ANSWER
4	✔	
3		✔
2		
5		

Q. 14

Question 14

Let $\Sigma = \{a, b, c\}$. The number of strings of length n over Σ that are palindromes (read the same from left-to-right and right-to-left) is:

Response:

OPTIONS	RESPONSE	ANSWER
$\frac{3n}{2}$		
n^3		
$3^{\lceil \frac{n}{2} \rceil}$	✔	✔
3^n		

Q. 15

Question 15

The recurrence relation associated with the well known insertion Sort algorithm is

$$T(n) = T(n-1) + cn$$

where c is a constant. The base case is $T(1) = k$, where k is a constant. The solution to this recurrence in asymptotic terms is:

Response:

OPTIONS	RESPONSE	ANSWER
$\Theta(1)$		
$\Theta(n^2)$	✔	✔
$\Theta(\log n)$		
$\Theta(n \log n)$		

Q. 16

Question 16

In **proof techniques** The contrapositive of $p \Rightarrow q$ is:

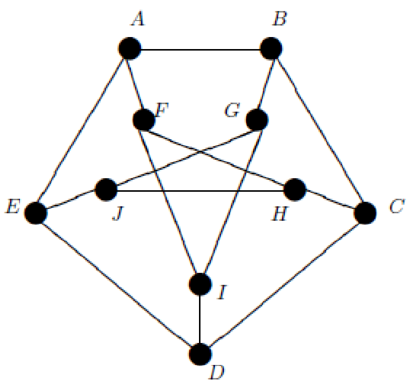
Response:

OPTIONS	RESPONSE	ANSWER
$\neg p \Rightarrow \neg q$		
$q \Rightarrow p$		
$\neg q \Rightarrow \neg p$	✓	✓
$p \wedge q$		

Q. 17

Question 17

Consider the Petersen graph drawn below.



Response:

OPTIONS	RESPONSE	ANSWER
The longest trail in this graph is closed and consists of 15 edges		
The longest trail in this graph is open and consists of 15 edges		
The longest trail in this graph is closed and consists of 11 edges	✓	
The longest trail in this graph is open and consists of 11 edges		✓

Q. 18

Question 18

The number of valid arrangements of 7 distinct keys in a min heap of size 7 is:

Response:

OPTIONS	RESPONSE	ANSWER
16		
24		
80	✔	✔
120		

Q. 19

Question 19

Suppose $\mathcal{F}$ denote the set of all bijective functions from a finite set S, to itself. What is the largest subset of $\mathcal{F}$ such that no two functions in the subset evaluate to the same value on the same argument? The set size is $|S| = n$

Response:

OPTIONS	RESPONSE	ANSWER
$\sqrt{n}$		
2		
n		✔
n^2	✔	

Q. 20

Question 20

The recurrence relation associated with the well known Merge Sort algorithm is

$$T(n) = 2T(\frac{n}{2}) + cn$$

where *c* is a constant. The base case is *T*(1) = *k*, where *k* is a constant. The solution to this recurrence in asymptotic terms is:

Response:

OPTIONS	RESPONSE	ANSWER
$\Theta(\log n)$		
$\Theta(n \log n)$	✓	✓
$\Theta(n^2)$		
$\Theta(1)$		

Question-Wise Details

Q. 1

Question 1

Consider the formula defined inductively as follows:

- **Base case:**
$$F_2 = p_1 \vee p_2$$
- **Induction:**
  - If n is odd:
$$F_n = F_{n-1} \wedge p_n$$
  - If n is even:
$$F_n = F_{n-1} \vee p_n$$

The number of satisfying assignments of F_{10} is:

Response:

OPTIONS	RESPONSE	ANSWER
171	✓	
43		
683		✓
11		

Q. 2

Question 2

Consider the formula defined inductively as follows:

- **Base case:**

$$F_2 = p_1 \Rightarrow p_2$$

- **Induction:**

$$F_n = F_{n-1} \Rightarrow p_n$$

The number of satisfying assignments of F_9 is:

Response:

OPTIONS	RESPONSE	ANSWER
341	✔	✔
683		
85		
171		

Q. 3

Question 3

Which pair of matrices given below do not commute with each other under multiplication (i.e. $A.B \neq B.A$)?

Response:

OPTIONS	RESPONSE	ANSWER
$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$		
$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	✔	✔
$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$		
$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		

Q. 4

Question 4

Suppose $\binom{n}{48} = \binom{n}{49}$. Then:

Response:

OPTIONS	RESPONSE	ANSWER
97	✔	✔
99		
98		
96		

Q. 5

Question 5

The number of satisfying assignments of the formula

$$\{(p_1 \vee (p_2 \wedge \neg p_3))\} \Rightarrow \{(\neg p_1 \wedge (p_2 \Rightarrow p_3)) \vee (p_2 \wedge p_4) \vee (\neg p_2 \wedge \neg p_4)\}$$

is

Response:

OPTIONS	RESPONSE	ANSWER
5		
11	✔	✔
3		
14		

Q. 6

Question 6

The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$ is

Response:

OPTIONS	RESPONSE	ANSWER
2	✔	✔
3		
4		
1		

Q. 7

Question 7

We know that a **regular language** is one that is accepted by a finite state automaton. Suppose L_1 and L_2 are two regular languages over the same finite alphabet Σ. Then which of the following is not necessarily a regular language

Response:

OPTIONS	RESPONSE	ANSWER
L where $L \subseteq L_1$	✔	✔
$L_1 \cup L_2$		
$\Sigma^* \setminus L_1$		
$L_1 \cap L_2$		

Q. 8

Question 8

Consider a labelled directed graph with the vertices placed around a circle with equal spacing. The vertices are labelled from 1 to 17 in clockwise circular order starting at an arbitrary vertex labelled 1. Each vertex has exactly one directed edge going out of it. The directed edge from the node of label k goes to the vertex exactly k positions from it in the clockwise direction.

We define a relation R on the vertices of this graph where $(u, v) \in R$ if anf only if there is a **directed** path in the graph from u to v. It turns out that this relation as defined, is an equivalence relation. The number of equivalence classes of R is:

Response:

OPTIONS	RESPONSE	ANSWER
3		✔
2	✔	
1		
4		

Q. 9

Question 9

The value of $7^{23456} \pmod{18}$ is

Response:

OPTIONS	RESPONSE	ANSWER
5		
7		
13	✔	✔
1		

Q. 10

Question 10

Suppose the average degree of a tree is $\frac{8}{5}$. Then its number of vertices is:

Response:

OPTIONS	RESPONSE	ANSWER
7		
6		
5	✔	✔
4		

Q. 11

Question 11

A bipartite graph has 9 vertices and 20 edges. The part sizes of this bipartite graph are:

Response:

OPTIONS	RESPONSE	ANSWER
3,6		
4,5	✓	✓
1,8		
2,7		

Q. 12

Question 12

Refer to the 10 element array shown in the picture. Suppose you are allowed to move the characters around, with the restriction that any character can either be at its initial position or one position to the left or right, No element can be at distance two or more from its original poition. How many con gurations are possible respecting this rule?

a	b	c	d	e	f	g	h	i	j

Response:

OPTIONS	RESPONSE	ANSWER
32		
89	✓	✓
1024		
20		

Q. 13

Question 13

The valid degree sequence of a simple graph, among those given below is:

Response:

OPTIONS	RESPONSE	ANSWER
7, 6, 5, 5, 4, 3, 2, 2	✔	✔
6, 6, 4, 4, 2, 2, 0, 0		
7, 7, 5, 5, 5, 3, 2, 2		
7, 6, 5, 5, 5, 3, 2, 2		

Q. 14

Question 14

Consider the formula defined inductively as follows:

- **Base case:**

$$F_2 = p_1 \oplus p_2$$

- **Induction:**

$$F_n = F_{n-1} \oplus p_n$$

The number of satisfying assignments of F_n is:

Response:

OPTIONS	RESPONSE	ANSWER
2^{n-1}		✔
$2^n - 1$	✔	
2^n		
2^{n-2}		

Q. 15

Question 15

Consider the formula defined inductively as follows:

- **Base case:**

$$F_2 = p_1 \Rightarrow p_2$$

- **Induction:**

$$F_n = p_n \Rightarrow F_{n-1}$$

The number of satisfying assignments of F_n is:

Response:

OPTIONS	RESPONSE	ANSWER
2^{n-2}		
$2^n - 1$		✔
2^n		
2^{n-1}	✔	

Q. 16

Question 16

The inverse of the matrix

$$\begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}$$

is:

Response:

OPTIONS	RESPONSE	ANSWER
$\begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix}$	✔	✔
$\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$		
$\begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}$		
$\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$		

Q. 17

Question 17

Consider a 4×4 matrix A with all distinct entries. Let us introduce two operations on this matrix.

- Reverse the entries of a row from left-to-right
- reverse the entries of a column from top-to-bottom

Beginning with any such matrix, and performing any sequence of these operations, each element can occupy one of exactly how many possible positions?

Response:

OPTIONS	RESPONSE	ANSWER
10		
4	✔	✔
8		
6		

Q. 18

Question 18

Consider the formula defined inductively as follows:

- **Base case:**

$$F_2 = p_1 \wedge p_2$$

- **Induction:**
  - If n is odd:

$$F_n = F_{n-1} \vee p_n$$

  - If n is even:

$$F_n = F_{n-1} \wedge p_n$$

The number of satisfying assignments of F_{10} is:

Response:

OPTIONS	RESPONSE	ANSWER
85		
341	✔	✔
5		
21		

Q. 19

Question 19

Which of the following cannot be the average degree of any tree

Response:

OPTIONS	RESPONSE	ANSWER
$\frac{5}{5}$	✔	
$\frac{8}{5}$		
$\frac{7}{5}$		✔
$\frac{9}{5}$		

Q. 20

Question 20

Consider a 4×4 matrix A with all distinct entries. Let us introduce two operations on this matrix.

- Reverse the entries of a row from left-to-right
- reverse the entries of a column from top-to-bottom

Starting with

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

which of the following matrices cannot be reached by a series of the two operations described above?

Response:

OPTIONS	RESPONSE	ANSWER
$\begin{bmatrix} 4 & 2 & 15 & 1 \\ 5 & 6 & 11 & 8 \\ 9 & 10 & 7 & 12 \\ 16 & 14 & 3 & 13 \end{bmatrix}$		
$\begin{bmatrix} 1 & 14 & 3 & 16 \\ 8 & 7 & 10 & 9 \\ 12 & 11 & 6 & 5 \\ 13 & 2 & 15 & 4 \end{bmatrix}$		
$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 10 & 9 \\ 8 & 7 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$		
$\begin{bmatrix} 11 & 13 & 3 & 6 \\ 5 & 1 & 7 & 8 \\ 12 & 10 & 4 & 9 \\ 2 & 14 & 15 & 16 \end{bmatrix}$	✔	✔