Discrete Mathematics (SC612)
Insem Exam 2
Autumn 2022-23

22nd October 2022

This exam consists of 5 questions, printed on two pages back-to-back, on a single paper. Check that your paper is complete.
Total marks 50; each question is worth 10 marks
Duration: 90 minutes

1. (a) Suppose the probability that the first success in a series of independent and identical bernoulli trials comes at the 7th trial is: $\frac{2}{2187}$. What is the probability of success in an individual trial?
   (b) For the same probability as in part (a), what is the probability of exactly 3 successes from 4 independent and identical trials?

2. What is the Landau number of 50? Remember the Landau number is the order of the largest order element in the permutation group on an n element set. Here, $n = 50$.

3. Order the numbers 250, 280, 350, 400 in ascending order, in terms of number of integers less than that number and relatively prime to it.

4. Consider the list $1, 2, 3, 4, 5, 6, 7, 8, 9$. How many ways are there to arrange this list (including the original arrangement) such that no element is more than two positions away from its position in the original list. Example the number 7, which is originally on position 7, can be at either 5, 6, 7, 8, 9.

5. Construct a **surjective function** $f : D \to C$ where $D = \{e, a, b, c\}$ forms a group G_1 under opeartor $*_1$ and $C = \{e', a'\}$ forms a group G_2 under opeartor $*_2$. The group tables are given below. The identity elements of the two groups are, respectively, e and e'. The function should be such that $\forall x, y \in D$, $f(x *_1 y) = f(x) *_2 f(y)$.

$*_1$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

$*_2$	e'	a'
e'	e'	a'
a'	a'	e'

20 2, 8, 5, 10