IT461 Assignment 4: Financial option pricing and Martingales

(1) Simulate a five period binomial model with $u = 1.5, d = 0.8$. The initial stock price is 1. Compute by simulation the price of a European call option with strike price 3. The risk free interest rate is $r = \frac{1}{4}$. Verify with the theoretical value.

(2) Write a program to generate the stock price of a hypothetical company XYZ. The stock is modeled as a geometric Brownian motion with average rate of return $\mu = 18\%$ per year and volatility $\sigma = 30\%$ per year. Generate the data for a period of 240 days with the initial stock price being Rs 100/-

(3) The Black-Scholes-Merton formula for the price of a call option at time t and when the current stock price is x is given by

$$c(x,t) = xN(d_+(T-t,x)) - Ke^{-r(T-t)}N(d_-(T-t,x))$$

where T is the expiry time of the option, K is the strike price, $N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{\frac{-y^2}{2}}dy$, $d_\pm(\tau,x) = \frac{\log(\frac{x}{K})+(r\pm\frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$. Compute the value of a European call option when the stock price is 30, strike price is 29, the risk free interest rate is 5%, the volatility is 25% per annum and the time remaining to expiry is 4 months. Empirically verify by running a simulation program.

(4) An urn contains w white and b black balls. An experiment is conducted where a ball is uniformly sampled from the urn and the selected ball is put back along with one more ball of the same color. This is repeated. Let M_n be the proportion of white balls at time n. Write a program to simulate this with $w = 5$ and $b = 7$. Numerically estimate the value of M_{100}.