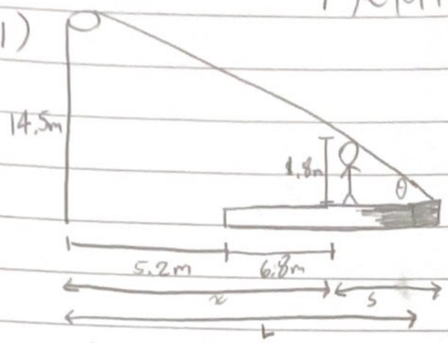
Related Rates Ex 3

1)



$x = 5.2 + 6.8$
$= 12$

① $\tan\theta \;\; \frac{14.5}{x+5} = \frac{1.8}{s}$

$14.5s = 1.8(x+s)$

$14.5s = 1.8x + 1.8s$

$12.7s = 1.8x$

② $12.7\frac{ds}{dt} = 1.8\frac{dx}{dt}$ ← $\frac{dx}{dt} = 0.9\,m/s$

$12.7\frac{ds}{dt} = 1.8 \times 0.9$

$\frac{ds}{dt} = 0.128\ m/s$

③ $\frac{14.5}{L} = \frac{1.8}{s}$

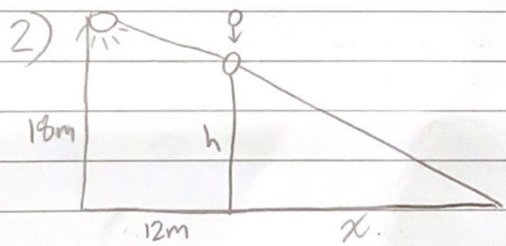
$14.5s = 1.8L$

$14.5\frac{ds}{dt} = 1.8\frac{dL}{dt}$

$14.5(0.128) = 1.8\frac{dL}{dt}$

$\frac{dL}{dt} = 1.028\ m/s$

2)



$h = 18 - 4.9t^2 \qquad \frac{dx}{dt} = ?$

$\frac{x}{h} = \frac{x+12}{18}$

$18x = xh + 12h$

$x(18-h) = 12h$

$x = \frac{12h}{18-h}$

$x = \frac{12(18-4.9t^2)}{18-(18-4.9t^2)}$

$x = \frac{216 - 58.8t^2}{4.9t^2}$

$x = 44.08t^{-2} - 12$

$\frac{dx}{dt} = -88.16\,t^{-3}$

Plug in 1.5 seconds for t

$\frac{dx}{dt} = -88.16(1.5)^{-3}$

$= -26.12\ m/s$

∴ the balls shadow is moving at a speed of 26.12 m/s [towards the ball]

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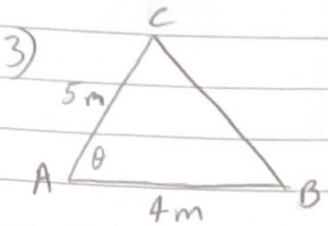
3)



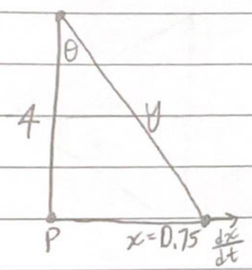
$\frac{d\theta}{dt} = 0.06$ rad/s

$A = 0.5 \times b \times h$
$A = 0.5 \times 4 \times 5\sin\theta$
$A = 10\sin\theta$
$\frac{dA}{dt} = 10\cos\theta \frac{d\theta}{dt}$

$\theta = \frac{\pi}{3}$ and $\frac{d\theta}{dt} = 0.06$ rad/s
$\frac{dA}{dt} = 10\cos\frac{\pi}{3}(0.06)$
$\frac{dA}{dt} = 0.3$ m²/s

$\therefore$ the area is increasing at 0.3 m²/s

4)



$\frac{d\theta}{dt} = 3$ rev/min $= 6\pi$ rad/min
$\frac{dx}{dt} = ?$

② $y = \sqrt{4^2 + 0.75^2}$
$= \frac{\sqrt{265}}{4}$

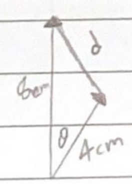
③ Plug in our $\sec^2\theta$ and $\frac{d\theta}{dt}$ into equation ①
$4 \cdot \frac{265}{256} \cdot 6\pi = \frac{dx}{dt}$
$\frac{dx}{dt} = 78.05$ Km/min

① $\tan\theta = \frac{x}{4}$
$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{4}\frac{dx}{dt}$
$4\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$\sec\theta = \frac{\frac{\sqrt{265}}{4}}{4}$
$\sec^2\theta = \frac{265}{256}$

$\therefore$ the light is traveling at 78.05 km/min along the shoreline

5)



$d = \sqrt{4^2 + 8^2 - 2(4)(8)\cos\theta}$
$d = \sqrt{80 - 64\cos\theta}$
$d = \sqrt{16(5 - 4\cos\theta)}$
$d = 4\sqrt{5 - 4\cos\theta}$
$d' = 4 \cdot \frac{1}{2}(5 - 4\cos\theta)^{-\frac{1}{2}}(4\sin\theta)(\theta)'$
$d' = \frac{8\sin\theta}{\sqrt{5 - 4\cos\theta}}\theta'$

minus because minute hand makes angle smaller

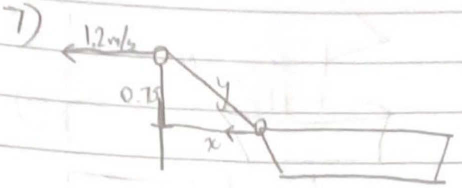
$\theta' = \theta'_h - \theta'_m$
$= \frac{2\pi}{12} - 2\pi$
$= -\frac{11\pi}{6}$ rad/hr

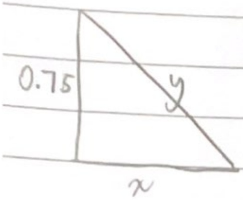
$d' = \frac{8\sin\frac{\pi}{6}}{\sqrt{5 - 4\cos\frac{\pi}{6}}} \times -\frac{11\pi}{6}$
$= -18.5896$ cm/hr
$= -0.31$ cm/min

$\therefore$ the angle is shrinking at a rate of 0.31 cm/min

Related Rates Ex 3

7)





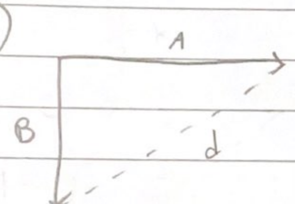
$$x^2 + 0.75^2 = y^2$$
$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$
$$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$$
$$\frac{dx}{dt} = -1.2\left(\frac{y}{x}\right)$$

$$y = \sqrt{3.5^2 + 0.75^2}$$
$$= \frac{\sqrt{205}}{4}$$

$$\frac{dx}{dt} = -1.2\left(\frac{\frac{\sqrt{205}}{4}}{3.5}\right)$$
$$= -\frac{3\sqrt{205}}{35}\ m/s$$
$$\approx -1.23\ m/s$$

$\therefore$ the boat is approaching the dock at 1.23m/s

6)



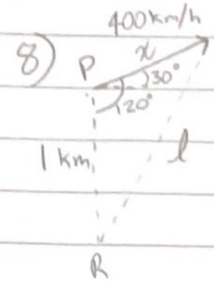
$$\frac{dA}{dt} = 70\ km/h$$
$$\frac{dB}{dt} = 80\ km/hr$$
d when 2.5 km

$$d = \sqrt{2.5^2 + 3.2^2}$$
$$= 4.06$$

$$d^2 = A^2 + B^2$$
$$2d\frac{dd}{dt} = 2A\frac{dA}{dt} + 2B\frac{dB}{dt}$$
$$2(4.06)\frac{dd}{dt} = 2(2.5)(70) + 2(3.2)(80)$$
$$8.12\frac{dd}{dt} = 162$$
$$\frac{dd}{dt} = 19.95\ km/hr$$

$\therefore$ distance is changing at 19.95 km/hr

Related Rates Ex 3.

8) 

$\frac{dx}{dt} = 400$ km/h

$\frac{dl}{dt} = ?$

$x = 400 \times \frac{2}{60} = \frac{40}{3}$ km

$l = \sqrt{1^2 + \frac{40}{3}^2 - 2(1)(\frac{40}{3})\cos 120^\circ}$

≈ 13.86 km

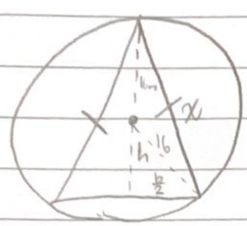
$$l^2 = x^2 + 1^2 - 2x\cos 120^\circ$$

$$2l\frac{dl}{dt} = 2x\frac{dx}{dt} + 0 + 1\frac{dx}{dt}$$

$$\frac{dl}{dt} = \frac{2x\frac{dx}{dt} + 1\frac{dx}{dt}}{2l}$$

$$= \frac{2(\frac{40}{3})(400) + 1(400)}{2(13.86)}$$

$$= 399.23 \text{ km/h}$$

9) 

$$16^2 = h^2 + \frac{b^2}{4} \rightarrow \frac{b}{2} = \sqrt{256 - h^2}$$

$$S = \frac{1}{2}b(h+16)$$

$$S = (h+16)\sqrt{256-h^2}$$

$$\frac{ds}{dt} = -\frac{2(h-8)(h+16)}{(256-h^2)^{\frac{1}{2}}} = -\frac{2(8-8)(8+16)}{(256-8^2)^{\frac{1}{2}}} = 0 \text{ m}^2/\text{s}$$

$\frac{dh}{dt} = 2.5$ m/s

$h(t_0) = 8$m

$\frac{ds}{dt} = ?$

$\frac{dP}{dt} = ?$

$$P = 2x + b$$

$$P = 2\sqrt{\frac{b^2}{4} + (h+16)^2} + b$$

$$P = 2\sqrt{256 - h^2 + h^2 + 32h + 256} + 2\sqrt{256-h^2}$$

$h = 24 - 16$

$= 8$m

$$P = 2\sqrt{512 + 32h} + 2\sqrt{256 - h^2}$$

$$\frac{dP}{dt} = -\frac{2h}{(256-h^2)^{\frac{1}{2}}} + \frac{32}{(512+32h)^{\frac{1}{2}}}$$

$$= -\frac{2(8)}{(256-8^2)^{\frac{1}{2}}} + \frac{32}{(512+32(8))^{\frac{1}{2}}}$$

$$= 0 \text{ m/s}$$

$\therefore$ Area is increasing at $0\text{m}^2/\text{s}$ and perimeter is increasing at 0m/s so neither are actually changing.