

Complex Numbers Test

$$\begin{aligned} 1) \frac{z_1}{z_2} &= \frac{a+bi}{c+di} \\ &= \frac{\sqrt{a^2+b^2}(\operatorname{cis} \theta)}{\sqrt{c^2+d^2}(\operatorname{cis} \theta)} \end{aligned}$$

$$\begin{aligned} \frac{\overline{z_1}}{\overline{z_2}} &= \frac{a-bi}{c-di} \\ &= \frac{\sqrt{a^2+b^2}(\operatorname{cis} \theta)}{\sqrt{c^2+d^2}(\operatorname{cis} \theta)} \end{aligned}$$

$$\begin{aligned} \therefore L S &= R S \\ \therefore \frac{z_1}{z_2} &= \frac{\overline{z_1}}{\overline{z_2}} \end{aligned}$$

2a) $-1+i$

$$\begin{aligned} b) (-1-i)(3-5i) \\ &= -3+5i-3i-5 \\ &= -8+2i \end{aligned}$$

$$\begin{aligned} c) \frac{3-5i}{(1-i)(i)} \\ &= \frac{3-5i}{1-i} \times \frac{1+i}{1+i} \\ &= \frac{(3-5i)(1+i)}{1^2+0^2} \\ &= \frac{8-2i}{1} \\ &= 8-2i \end{aligned}$$

$$\begin{aligned} d) 16 \operatorname{cis} \pi \\ &= 16(\cos \pi + i \sin \pi) \\ &= 16 \cos \pi + 16i \sin \pi \\ &= 16(-1) + 16i(0) \\ &= -16 \end{aligned}$$

$$\begin{aligned}
 e) (3-5i)^2 \\
 &= (3-5i)(3-5i) \\
 &= 9 - 15i - 15i \\
 &= 9 - 25 \\
 &= -16
 \end{aligned}$$

$$\begin{aligned}
 f) (16 \text{cis } \pi)(3-5i) \\
 &= (-16)(3-5i) \\
 &= -48 + 80i
 \end{aligned}$$

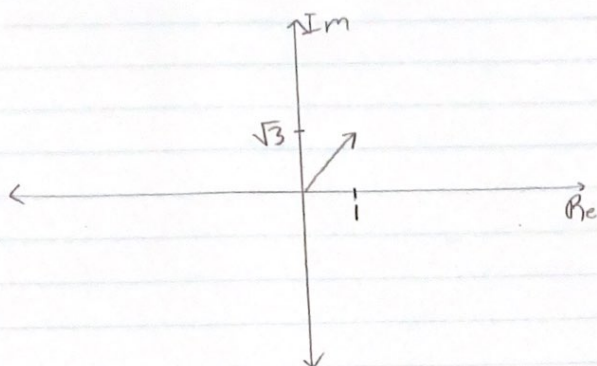
$$\begin{aligned}
 g) \frac{3-5i}{16 \text{cis } \pi} \\
 &= \frac{3-5i}{-16} \\
 &= -\frac{3}{16} + \frac{5i}{16}
 \end{aligned}$$

$$\begin{aligned}
 h) k &= 3-5i \\
 |k| &= \sqrt{3^2 + (-5)^2} \\
 |k| &= \sqrt{34}
 \end{aligned}$$

$$\begin{aligned}
 i) z &= -1-i \\
 \arg(z) &= \tan^{-1}\left(\frac{-1}{-1}\right) \\
 &= \frac{\pi}{4} \\
 &\hookrightarrow \frac{5\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 h) k &= 3-5i \\
 r &= \sqrt{3^2 + (-5)^2} \\
 &= \sqrt{34} \\
 \theta &= \tan^{-1}\left(-\frac{5}{3}\right) \\
 \theta &=
 \end{aligned}$$

3)



$$4a) r = \sqrt{(-5\sqrt{3})^2 + (-5)^2}$$

$$= 10$$

$$\theta = \tan^{-1}\left(\frac{-5}{-5\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \frac{7\pi}{6}$$

$$10 \operatorname{cis} \frac{7\pi}{6}$$

$$b) r = \sqrt{7^2 + (-7)^2}$$

$$= 7\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-7}{7}\right)$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -\frac{\pi}{4}$$

$$7\sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$$

$$5a) 12\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 12\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= -6\sqrt{2} + 6i\sqrt{2}$$

$$b) 4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$

$$= 4(0 - i)$$

$$= 4(-i)$$

$$= -4i$$

$$6a) 5 \operatorname{cis} 60^\circ$$

$$b) 7 \operatorname{cis} 225^\circ$$