Derivatives Exercise 7

2) $g(x) = \operatorname{arccot} 2x^3$

$g'(x) = -\frac{1}{1+(2x^3)^2} \cdot 6x^2$

$= -\frac{1}{1+4x^6} \cdot 6x^2$

$= -\frac{6x^2}{1+4x^6}$

4) $f(y) = y \cdot \arcsin y$

$f'(y) = (y)(\arcsin y)' + (y)'(\arcsin y)$

$= (y)\left(\frac{1}{\sqrt{1-y^2}}\right) + (1)(\arcsin y)$

$= \frac{y}{\sqrt{1-y^2}} + \arcsin y$

6) $h(x) = -2e^{\frac{1}{2}\arctan x}$

$h'(x) = (-2)(e^{\frac{1}{2}\arctan x})' + (-2)'(e^{\frac{1}{2}\arctan x})$

$= (-2)(e^{\frac{1}{2}\arctan x})(\frac{1}{2}\arctan x)' + (0)(e^{\frac{1}{2}\arctan x})$

$= (-2)(e^{\frac{1}{2}\arctan x})\left(\frac{1}{2} \cdot \frac{1}{1+x^2}\right)$

$= (-2)(e^{\frac{1}{2}\arctan x})\left(\frac{1}{2+2x^2}\right)$

$= -\frac{e^{\frac{1}{2}\arctan x}}{1+x^2}$

8) $y = (\arccos 3x)^4$

$y' = 4(\arccos 3x)^3 (\arccos 3x)'$

$= 4(\arccos 3x)^3 \left(-\frac{1}{\sqrt{1-(3x)^2}}\right)(3)$

$= 4(\arccos 3x)^3 \left(-\frac{3}{\sqrt{1-9x^2}}\right)$

$= \frac{12\sqrt{(1+3x)(1-3x)} \cdot \arccos(3x)^3}{(1+3x)(1-3x)}$

Derivatives Exercise 7

10) $y = \operatorname{arccsc} x \cdot \operatorname{arcsec} x$

$y' = (\operatorname{arccsc} x)(\operatorname{arcsec} x)' + (\operatorname{arccsc} x)'(\operatorname{arcsec} x)$

$= (\operatorname{arccsc} x)\left(\dfrac{1}{|x|\sqrt{x^2-1}}\right) + \left(-\dfrac{1}{|x|\sqrt{x^2-1}}\right)(\operatorname{arcsec} x)$

$= \dfrac{\operatorname{arccsc} x}{|x|\sqrt{x^2-1}} + -\dfrac{\operatorname{arcsec} x}{|x|\sqrt{x^2-1}}$

$= \dfrac{\operatorname{arccsc} x - \operatorname{arcsec} x}{|x|\sqrt{x^2-1}}$

12) $w(z) = \operatorname{arcsec}\sqrt{z}$

$= \dfrac{1}{|\sqrt{z}|\sqrt{\sqrt{z}^2-1}} \cdot \frac{1}{2}z^{-\frac{1}{2}}$

$= \dfrac{1}{2z\sqrt{z-1}}$

14) $m(n) = 2n^3 \operatorname{arccsc}(2n-3)$

$= (2n^3)(\operatorname{arccsc}(2n-3))' + (2n^3)'(\operatorname{arccsc}(2n-3))$

$= (2n^3)\left(-\dfrac{1}{|2n-3|\sqrt{(2n-3)^2-1}}\right)(2) + (6n^2)(\operatorname{arccsc}(2n-3))$

$= -\dfrac{4n^3}{|2n-3|\sqrt{(2n-3)^2-1}} + 6n^2 \operatorname{arccsc}(2n-3)$