

Trig Exercise 11

$$1) \frac{1 + \cos \theta}{1 + \sec \theta} = \cos \theta$$

$$\begin{aligned} LS &= \frac{1 + \cos \theta}{1 + \sec \theta} \\ &= \frac{1 + \cos \theta}{1 + \frac{1}{\cos \theta}} \\ &= \frac{\cos \theta (1 + \cos \theta)}{\cos \theta (1 + \frac{1}{\cos \theta})} \\ &= \frac{\cos \theta (1 + \cos \theta)}{\cos \theta + 1} \\ &= \cos \theta \end{aligned}$$

$$RS = \cos \theta$$

$$\begin{aligned} \therefore LS &= RS \\ \therefore \frac{1 + \cos \theta}{1 + \sec \theta} &= \cos \theta \end{aligned}$$

$$2) \sin^2 \theta \sec^2 \theta = \sec^2 \theta - 1$$

$$\begin{aligned} LS &= \sin^2 \theta \sec^2 \theta \\ &= (\sin \theta \sec \theta)^2 \\ &= \left(\sin \theta \frac{1}{\cos \theta} \right)^2 \\ &= \left(\frac{\sin \theta}{\cos \theta} \right)^2 \\ &= (\tan \theta)^2 \\ &= \tan^2 \theta \end{aligned}$$

$$\begin{aligned} RS &= \sec^2 \theta - 1 \\ &= \tan^2 \theta \end{aligned}$$

$$\begin{aligned} \therefore LS &= RS \\ \therefore \sin^2 \theta \sec^2 \theta &= \sec^2 \theta - 1 \end{aligned}$$

$$3) \csc^2 \theta - \cot^2 \theta = 1$$

$$\begin{aligned} LS &= \csc^2 \theta - \cot^2 \theta \\ &= 1 \end{aligned}$$

pyth identity
 $\cot^2 \theta + 1 = \csc^2 \theta$

$$RS = 1$$

$$\begin{aligned} \therefore LS &= RS \\ \therefore \csc^2 \theta - \cot^2 \theta &= 1 \end{aligned}$$

$$4) \sec^2 \theta \cot^2 \theta = 1 + \cot^2 \theta$$

$$\begin{aligned} LS &= \sec^2 \theta \cot^2 \theta \\ &= \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{1}{\sin^2 \theta} \\ &= \csc^2 \theta \end{aligned}$$

$$\begin{aligned} RS &= 1 + \cot^2 \theta \\ &= \csc^2 \theta \end{aligned}$$

$$\therefore LS = RS$$

$$\therefore \sec^2 \theta \cot^2 \theta = 1 + \cot^2 \theta$$

$$5) (\cos \theta + 1)^2 + \sin^2 \theta = 2(1 + \cos \theta)$$

$$\begin{aligned} LS &= (\cos \theta + 1)^2 + \sin^2 \theta \\ &= \cos^2 \theta + 2\cos \theta + 1 + \sin^2 \theta \\ &= 2\cos \theta + 1 + 1 \\ &= 2\cos \theta + 2 \end{aligned}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\begin{aligned} RS &= 2(1 + \cos \theta) \\ &= 2 + 2\cos \theta \\ &= 2\cos \theta + 2 \end{aligned}$$

$$\therefore LS = RS$$

$$\therefore (\cos \theta + 1)^2 + \sin^2 \theta$$

$$6) \cos \theta \sec \theta = \sin^2 \theta + \cos^2 \theta$$

$$\begin{aligned} LS &= \cos \theta \sec \theta \\ &= \cos \theta \cdot \frac{1}{\cos \theta} \\ &= \frac{\cos \theta}{\cos \theta} \\ &= 1 \end{aligned}$$

$$\begin{aligned} RS &= \sin^2 \theta + \cos^2 \theta \\ &= 1 \end{aligned}$$

$$\therefore LS = RS$$

$$\therefore \cos \theta \sec \theta = \sin^2 \theta + \cos^2 \theta$$

$$7) \sin \beta - \csc \beta = -\cos \beta \cot \beta$$

$$\begin{aligned} LS &= \sin \beta - \csc \beta \\ &= \sin \beta - \frac{1}{\sin \beta} \\ &= \frac{\sin^2 \beta - 1}{\sin \beta} \\ &= \frac{-\cos^2 \beta}{\sin \beta} \\ &= -\cos \beta \cdot \frac{\cos \beta}{\sin \beta} \\ &= -\cos \beta \cot \beta \end{aligned}$$

$$RS = -\cos \beta \cot \beta$$

$$\therefore RS = LS$$

$$\therefore \sin \beta - \csc \beta = -\cos \beta \cot \beta$$

$$8) \tan^2 \alpha \csc^2 \alpha - \sin^2 \alpha \sec^2 \alpha = 1$$

$$\begin{aligned} LS &= \tan^2 \alpha \csc^2 \alpha - \sin^2 \alpha \sec^2 \alpha \\ &= \frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \frac{1}{\sin^2 \alpha} - \frac{\sin^2 \alpha}{1} \cdot \frac{1}{\cos^2 \alpha} \\ &= \frac{1}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \sec^2 \alpha - \tan^2 \alpha \\ &= 1 \end{aligned}$$

$$RS = 1$$

$$\therefore LS = RS$$

$$\therefore \tan^2 \alpha \csc^2 \alpha - \sin^2 \alpha \sec^2 \alpha = 1$$

$$9) \sec \theta \sqrt{1 - \cos^2 \theta} = \tan \theta$$

$$\begin{aligned} LS &= \sec \theta \sqrt{1 - \cos^2 \theta} \\ &= \sec \theta \sqrt{\sin^2 \theta} \\ &= \sec \theta \sin \theta \\ &= \frac{1}{\cos \theta} \cdot \sin \theta \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \end{aligned}$$

$$RS = \tan \theta$$

$$\therefore LS = RS$$

$$\therefore \sec \theta \sqrt{1 - \cos^2 \theta} = \tan \theta$$

$$10) \csc \theta \cos \theta \sqrt{\sec^2 \theta - 1} = 1$$

$$\begin{aligned} LS &= \csc \theta \cos \theta \sqrt{\sec^2 \theta - 1} \\ &= \frac{1}{\sin \theta} \cdot \cos \theta \sqrt{\tan^2 \theta} \\ &= \frac{\cos \theta}{\sin \theta} \cdot \tan \theta \\ &= \cot \theta \tan \theta \\ &= 1 \end{aligned}$$

$$RS = 1$$

$$\therefore LS = RS$$

$$\therefore \csc \theta \cos \theta \sqrt{\sec^2 \theta - 1} = 1$$

11)

~~$$10) \csc \phi \cos \phi \sqrt{\sec^2 \phi - 1} = 1$$~~

~~$$\begin{aligned} LS &= \csc \phi \cos \phi \sqrt{\sec^2 \phi - 1} \\ &= \frac{1}{\sin \phi} \cos \phi \sqrt{\tan^2 \phi} \\ &= \frac{\cos \phi}{\sin \phi} \tan \phi \\ &= \cot \phi \tan \phi \\ &= 1 \end{aligned}$$~~

$$RS = 1$$

~~$$\therefore LS = RS$$~~

~~$$\therefore \csc \phi \cos \phi \sqrt{\sec^2 \phi - 1} = 1$$~~

$$11) (\cos \beta + \sin \beta)^2 = 2 \cot \beta \sin^2 \beta + 1$$

$$\begin{aligned} LS &= (\cos \beta + \sin \beta)^2 \\ &= \cos^2 \beta + 2 \cos \beta \sin \beta + \sin^2 \beta \\ &= 2 \cos \beta \sin \beta + 1 \end{aligned}$$

$$\begin{aligned} RS &= 2 \cot \beta \sin^2 \beta + 1 \\ &= 2 \frac{\cos \beta}{\sin \beta} \cdot \sin^2 \beta + 1 \\ &= 2 \cos \beta \sin \beta + 1 \end{aligned}$$

$$\therefore LS = RS$$

$$\therefore (\cos \beta + \sin \beta)^2 = 2 \cot \beta \sin^2 \beta + 1$$

$$12) (\sec^2 \theta - 1)(\csc^2 \theta - 1) = 1$$

$$\begin{aligned} LS &= (\sec^2 \theta - 1)(\csc^2 \theta - 1) \\ &= (\tan^2 \theta)(\cot^2 \theta) \\ &= \frac{\tan^2 \theta}{1} \cdot \frac{1}{\tan^2 \theta} \\ &= \frac{\tan^2 \theta}{\tan^2 \theta} \\ &= 1 \end{aligned}$$

$$RS = 1$$

$$\therefore LS = RS$$

$$\therefore (\sec^2 \theta - 1)(\csc^2 \theta - 1)$$

$$13) (\tan \phi + 1)^2 = \sec^2 \phi (\cos \phi \sin \phi)^2$$

$$\begin{aligned} LS &= (\tan \phi + 1)^2 \\ &= \tan^2 \phi + 2\tan \phi + 1 \end{aligned}$$

Not an identity?

$$\begin{aligned} RS &= \sec^2 \phi (\cos \phi \sin \phi)^2 \\ &= \sec^2 \phi \cos^2 \phi \sin^2 \phi \\ &= (\tan^2 \phi + 1)(\cos^2 \phi \sin^2 \phi) \\ &= \sin^4 \phi + \cos^2 \phi \sin^2 \phi \end{aligned}$$

$$14) \frac{1}{\cot^2 \theta} + 1 = \sec^2 \theta$$

$$\begin{aligned} LS &= \frac{1}{\cot^2 \theta} + 1 \\ &= \tan^2 \theta + 1 \\ &= \sec^2 \theta \end{aligned}$$

$$RS = \sec^2 \theta$$

$$\therefore LS = RS$$

$$\therefore \frac{1}{\cot^2 \theta} + 1 = \sec^2 \theta$$

$$15) \cot^4 \beta + \cot^2 \beta = \csc^4 \beta - \csc^2 \beta$$

$$\begin{aligned} LS &= \cot^4 \beta + \cot^2 \beta \\ &= \cot^2 \beta (\cot^2 \beta + 1) \\ &= \cot^2 \beta (\csc^2 \beta) \\ &= \cot^2 \beta \csc^2 \beta \end{aligned}$$

$$\begin{aligned} RS &= \csc^4 \beta - \csc^2 \beta \\ &= \csc^2 \beta (\csc^2 \beta - 1) \\ &= \csc^2 \beta (\cot^2 \beta) \\ &= \cot^2 \beta \csc^2 \beta \end{aligned}$$

$$\therefore LS = RS$$

$$\therefore \cot^4 \beta + \cot^2 \beta = \csc^4 \beta - \csc^2 \beta$$

$$16) (\cot \theta - 1)^2 + \frac{2}{\tan \theta} = \csc^2 \theta$$

$$L.S. = (\cot \theta - 1)^2 + \frac{2}{\tan \theta}$$

$$= \left(\frac{\cos \theta}{\sin \theta} - 1 \right)^2 + 2 \left(\frac{\cos \theta}{\sin \theta} \right)$$

$$= \left(\frac{\cos \theta - \sin \theta}{\sin \theta} \right)^2 + \frac{2 \cos \theta}{\sin \theta}$$

$$= \frac{(\cos \theta - \sin \theta)^2}{\sin^2 \theta} + \frac{2 \cos \theta}{\sin \theta}$$

$$= \frac{\cos^2 \theta - 2 \cos \theta \sin \theta + \sin^2 \theta}{\sin^2 \theta} + \frac{2 \cos \theta}{\sin \theta}$$

$$= \frac{1}{\sin^2 \theta} - \frac{2 \cos \theta}{\sin \theta} + \frac{2 \cos \theta}{\sin \theta}$$

$$= \frac{1}{\sin^2 \theta}$$

$$= \csc^2 \theta$$

$$R.S. = \csc^2 \theta$$

$$\therefore L.S. = R.S.$$

$$\therefore (\cot \theta - 1)^2 + \frac{2}{\tan \theta} = \csc^2 \theta$$

$$17) \sin \beta \sqrt{1 + \tan^2 \beta} = \sqrt{\sec^2 \beta - 1}$$

$$L.S. = \sin \beta \sqrt{1 + \tan^2 \beta}$$

$$= \sin \beta \sqrt{1 + \frac{\sin^2 \beta}{\cos^2 \beta}}$$

$$= \sin \beta \sqrt{\frac{\cos^2 \beta + \sin^2 \beta}{\cos^2 \beta}}$$

$$= \sin \beta \sqrt{\frac{1}{\cos^2 \beta}}$$

$$= \sin \beta \left(\frac{1}{\cos \beta} \right)$$

$$= \tan \beta$$

$$R.S. = \sqrt{\sec^2 \beta - 1}$$

$$= \sqrt{\frac{1}{\cos^2 \beta} - 1}$$

$$= \sqrt{\frac{1 - \cos^2 \beta}{\cos^2 \beta}}$$

$$= \sqrt{\frac{\sin^2 \beta}{\cos^2 \beta}}$$

$$= \frac{\sin \beta}{\cos \beta}$$

$$= \tan \beta$$

$$\therefore L.S. = R.S.$$

$$\therefore \sin \beta \sqrt{1 + \tan^2 \beta} = \sqrt{\sec^2 \beta - 1}$$

$$18) \sec^2 \beta - 1 = \sin \beta \cot \beta \sec \beta \tan^2 \beta$$

$$LS = \sec^2 \beta - 1$$

$$= \tan^2 \beta$$

$$RS = \sin \beta \cot \beta \sec \beta \tan^2 \beta$$

$$= \sin \beta \sec \beta \tan \beta$$

$$= \frac{\sin \beta}{1} \cdot \frac{1}{\cos \beta} \cdot \tan \beta$$

$$= \frac{\sin \beta}{\cos \beta} \cdot \tan \beta$$

$$= \tan^2 \beta$$

$$\therefore LS = RS$$

$$\therefore \sec^2 \beta - 1 = \sin \beta \cot \beta \sec \beta \tan^2 \beta$$

$$19) (1 + \sin \theta)^2 + (1 + \cos \theta)^2 - 2(\cos \theta + \sin \theta) = 3$$

$$LS = (1 + \sin \theta)^2 + (1 + \cos \theta)^2 - 2(\cos \theta + \sin \theta)$$

$$= (1 + 2\sin \theta + \sin^2 \theta) + (1 + \cos \theta)^2 - 2(\cos \theta + \sin \theta)$$

$$= 1 + 2\sin \theta + \sin^2 \theta + (1 + \cos \theta)^2 - 2\cos \theta - 2\sin \theta$$

$$= 1 + 2\sin \theta + \sin^2 \theta + (1 + 2\cos \theta + \cos^2 \theta) - 2\cos \theta - 2\sin \theta$$

$$= 2 + \sin^2 \theta + \cos^2 \theta$$

$$= 2 + 1$$

$$= 3$$

$$RS = 3$$

$$\therefore LS = RS$$

$$\therefore (1 + \sin \theta)^2 + (1 + \cos \theta)^2 - 2(\cos \theta + \sin \theta) = 3$$

$$20) \cot^2 \alpha + \csc^2 \beta = \cot^2 \beta + \csc^2 \alpha$$

$$\begin{aligned} \text{LS} &= \cot^2 \alpha + \csc^2 \beta \\ &= \frac{\cos^2 \alpha}{\sin^2 \alpha} + \frac{1}{\sin^2 \beta} \\ &= \frac{\cos^2 \alpha \sin^2 \beta + \sin^2 \alpha}{\sin^2 \alpha \sin^2 \beta} \\ &= \end{aligned}$$

$$\begin{aligned} \text{RS} &= \cot^2 \beta + \csc^2 \alpha \\ &= \frac{\cos^2 \beta}{\sin^2 \beta} + \frac{1}{\sin^2 \alpha} \\ &= \frac{\cos^2 \beta \sin^2 \alpha + \sin^2 \beta}{\sin^2 \beta \sin^2 \alpha} \\ &= \end{aligned}$$

Not an identity ?

1)