Spirit of Math schools

# Absolute Value, Exercise 2

Name: _____

Date: _____

*[39 marks]*

Solve and graph the solution sets. Show your work and graphs on lined paper, and write your solution sets on this page.

1) *[3 marks]* $2 < |y-7| < 9$

2) *[3 marks]* $-5 \le |f-5| < -2$

3) *[3 marks]* $8\frac{1}{5} > |3r+7| \ge 1\frac{2}{3}$

4) *[4 marks]* $-\frac{2}{3} < |4d-3| < \frac{2}{3}$

5) *[4 marks]* $\frac{3x-5}{6} - 2x = \frac{|x-7|}{5}$

6) *[4 marks]* $|h+3| + |h-3| < 6$

7) *[4 marks]* $|2(|x-3|)-7| < 5$

8) *[4 marks]* $\left|\frac{5-2x}{x+2}\right| \le 1$

9) *[5 marks]* $2|4-3x| - 3|2x+1| < 7$

10) *[5 marks]* $|3(|x-5|)-8| > 5$

1) $9 < y < 16$
$-2 < y < 5$

2) No solution

3) $-5\frac{1}{15} \le r < -2\frac{8}{9}$
$-1\frac{7}{9} \le r < \frac{2}{5}$

4) $\frac{7}{12} < d < \frac{11}{12}$

5) $x = -1\frac{28}{39}$

6) No solution
$-3 < x < 2$

7) $4 < x < 9$

8) $x > 1$

9) $-\frac{1}{6} < x$
$\frac{2}{3} < x < 6$

10) $x > 9\frac{1}{3}$

# AVE 2

1) $2 < |y-7| < 9$

**Positive Case**

$2 < y-7 < 9$

$9 < y < 16$

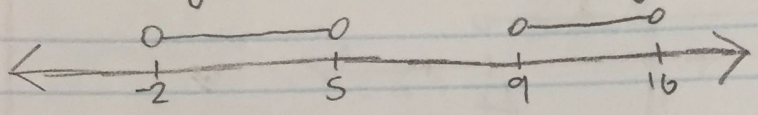
**Negative Case**

$2 < -(y-7) < 9$

$-2 > y-7 > -9$

$5 > y > -2$

$\therefore 9 < y < 16$ OR $-2 < y < 5$




2) $-5 \leq |f-5| < -2$

Since the solution of the absolute value will be positive, it will not be less than -2 which is a negative.

$\therefore$ there is no solution

3) $8\frac{1}{5} > |3r+7| \geq 1\frac{2}{3}$

$1\frac{2}{3} \leq |3r+7| < 8\frac{1}{5}$

**Positive Case**

$1\frac{2}{3} \leq 3r+7 < 8\frac{1}{5}$

$-5\frac{1}{3} \leq 3r < 1\frac{1}{5}$

$-1\frac{7}{9} \leq r < \frac{2}{5}$

**Negative Case**

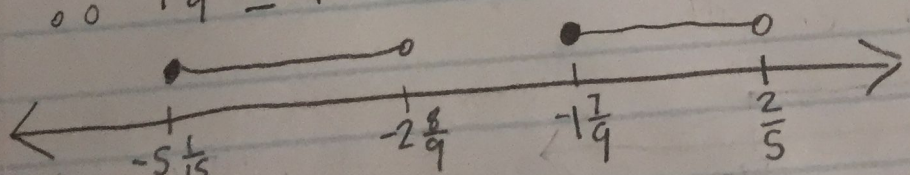
$1\frac{2}{3} \leq -(3r+7) < 8\frac{1}{5}$

$1\frac{2}{3} \leq -3r-7 < 8\frac{1}{5}$

$8\frac{2}{3} \leq -3r < 15\frac{1}{5}$

$-2\frac{8}{9} > r \geq -5\frac{1}{15}$

$\therefore -1\frac{7}{9} \leq r < \frac{2}{5}$ OR $-5\frac{1}{15} \leq r < -2\frac{8}{9}$



4) $-\frac{2}{3} < |4d - 3| < \frac{2}{3}$

**Positive Case**

$-\frac{2}{3} < 4d - 3 < \frac{2}{3}$

$2\frac{1}{3} < 4d < 3\frac{2}{3}$

$\frac{7}{12} < d < \frac{11}{12}$

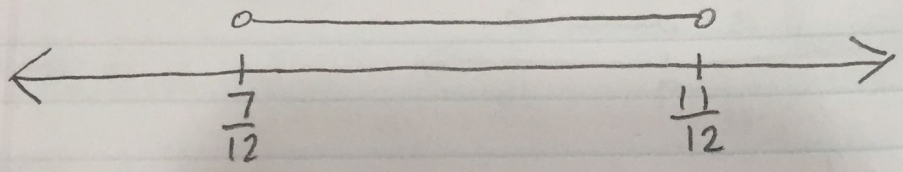
**Negative Case**

$-\frac{2}{3} < -4d + 3 < \frac{2}{3}$

$-3\frac{2}{3} < -4d < -2\frac{1}{3}$

$\frac{11}{12} > d > \frac{7}{12}$

$\therefore \frac{7}{12} < d < \frac{11}{12}$




5) $\frac{3x-5}{6} - 2x = \frac{|x-7|}{5}$

**Positive Case**

$\frac{3x-5}{6} - 2x = \frac{x-7}{5}$

$90x - 25 - 60x = 6x - 42$

$30x - 25 = 6x - 42$

$24x = -17$

$x = \frac{17}{24}$

**Negative Case**

$\frac{3x-5}{6} - 2x = \frac{-(x-7)}{5}$

$90x - 25 - 60x = -6x + 42$

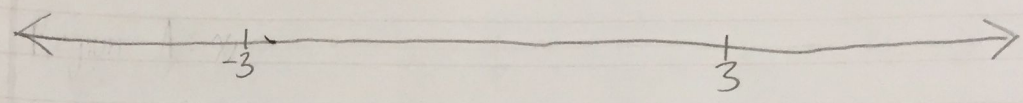
$30x - 25 = -6x + 42$

$36x = 67$

$x = 1\frac{31}{36}$

# AVE 2

6) $|h+3| + |h-3| < 6$




Region A: $h < -3$
Region B: $-3 \leq h < 3$
Region C: $3 \leq h$

Region A

$-(h+3) - (h-3) < 6$
$-h - 3 - h + 3 < 6$
$-2h < 6$
$h > -3$

$\therefore -3 < h < -3 \longrightarrow$ No Solution

Region B

$(h+3) - (h-3) < 6$
$h + 3 - h + 3 < 6$
$6 < 6$

$\therefore$ No Solution

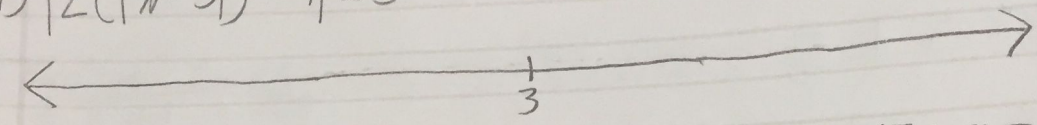
Region C

$h + 3 + h - 3 < 6$
$2h < 6$
$h < 3$

$\therefore 3 \leq h < 3 \longrightarrow$ No Solution

$\therefore$ NO SOLUTION

7) $|2(|x-3|)-7|<5$




Region A: $x<3$

$|-2(x-3)-7|<5$

$|-2x-1|<5$

Region C: $x<-\frac{1}{2}$

$-(-2x-1)<5$

$2x+1<5$

$2x<4$

$x<2$

$\therefore x<2$

Region F: $x\geq 6\frac{1}{2}$

$2x-13<5$

$2x<18$

$x<9$

$\therefore 6\frac{1}{2}\leq x<9$

Region D: $x\geq -\frac{1}{2}$

$-2x-1<5$

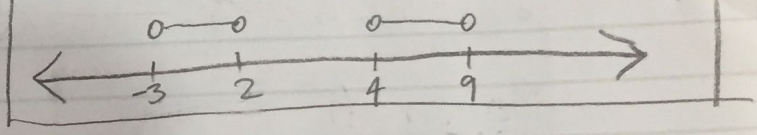
$-2x<6$

$x>-3$

$\therefore -3<x$

$0 \quad -3<x<2$

$0 \quad 0 \quad 4<x<9$




Region B: $x\geq 3$

$|2(x-3)-7|<5$

$|2x-13|<5$

Region E: $x<6\frac{1}{2}$

$-(2x-13) \quad <5$

$-2x+13 \quad <5$

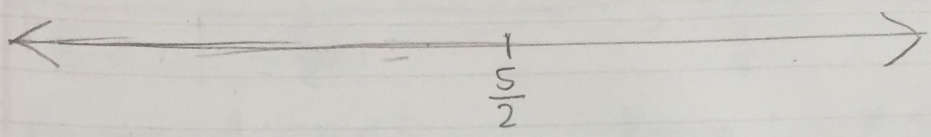
$-2x<-8$

$x>4$

$\therefore 4<x<6\frac{1}{2}$

# AVE 2

8) $\left|\frac{5-2x}{x+2}\right| \leq 1$




Region A: $x < \frac{5}{2}$

$-\left(\frac{5-2x}{x+2}\right) \leq 1$

$-\frac{5-2x}{x+2} \leq 1$

$-5-2x \leq x+2$

$-3x \leq 7$

$x \geq \frac{7}{3}$

Region B: $x \geq \frac{5}{2}$

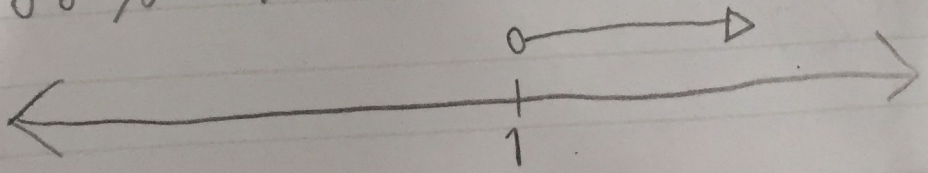
$\frac{5-2x}{x+2} \leq 1$

$5-2x \leq x+2$

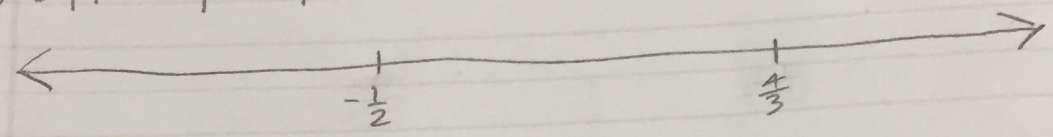
$-3x \leq -3$

$x > 1$

$\therefore x > 1$



9) $2|4-3x| - 3|2x+1| < 7$




Region A: $x < -\frac{1}{2}$

$2(4-3x) + 3(2x+1) < 7$

$8 - 6x + 6x + 3 < 7$

$8 + 3 < 7$

$11 < 7$

$\therefore$ No Solution

Region B: $-\frac{1}{2} \leq x < \frac{4}{3}$

$2(4-3x) - 3(2x+1) < 7$

$8 - 6x - 6x - 3 < 7$

$-12x + 5 < 7$

$-12x < 2$

$x > -\frac{1}{6}$

$\therefore -\frac{1}{6} < x < \frac{4}{3}$

Region C: $x \geq \frac{4}{3}$

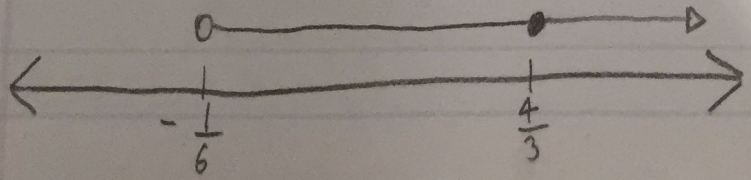
$-2(4-3x) - 3(2x+1) < 7$

$-8 + 6x - 6x - 3 < 7$

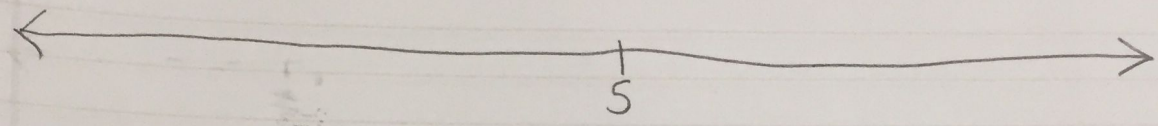
$-11 < 7$

$\therefore x \geq \frac{4}{3}$

$-\frac{1}{6} < x$



10) $|3(|x-5|)-8|>5$




Region A: $x<5$
$|3(x-5)-8|>5$
$|-3x-3|>5$

Region B: $x\geq 5$
$|3(x-5)-8|>5$
$|3x-23|>5$

Region C: $x<-1$
$-(-3x-3)>5$
$3x+3>5$
$3x>2$
$x>\frac{2}{3}$
$\therefore \frac{2}{3}<x<5$

Region D: $x\geq -1$
$-3x-3>5$
$-3x>8$
$x<-2\frac{2}{3}$
$\therefore$ NO Solution

Region E: $x<7\frac{2}{3}$
$-(3x-23)>5$
$-3x>-18$
$x<6$
$\therefore 5\leq x<6$

Region F: $x\geq 7\frac{2}{3}$
$3x-23>5$
$3x>28$
$x>9\frac{1}{3}$
$\therefore 9\frac{1}{3}<x$

$\frac{2}{3}<x<6$

$x>9\frac{1}{3}$