Derivatives Applications Ex. 3

1) $x + y = 54$

$x = 54 - y$

$p(x, y) = x \cdot y$

$p(y) = (54 - y) \cdot y$

$= -y^2 + 54y$

$p'(y) = -2y + 54$

$0 = -2y + 54$

$2y = 54$

$y = 27$

$x = 54 - y$

$x = 54 - 27$

$= 27$

$\therefore$ the two numbers are 27 and 27

2) Let y represent the sum of the squares

$y = x^2 + (30 - x)^2$

$y' = 2x + 2(30 - x)(-1)$

$= 2x - 60 + 2x$

$= 4x - 60$

$0 = 4x - 60$

$x = 15$

sub in extremes since $0 \le x \le 30$

$y = x^2 + (30 - x)^2$

$= 0^2 + (30 - 0)^2$

$= 900$

$y = x^2 + (30 - x)^2$

$= 30^2 + (30 - 30)^2$

$= 900$ ← maximum sum

$y = x^2 + (30 - x)^2$

$= 15^2 + (30 - 15)^2$

$= 450$

↑ minimum sum

$\therefore$ largest possible sum: $0^2 + 30^2 = 900$ OR $30^2 + 0^2 = 900$

smallest possible sum: $15^2 + 15^2 = 450$

3) let x represent the length of one of the sides

$A = \frac{1}{2} \cdot x \cdot \sqrt{15^2 - x^2}$

$A' = \frac{-2x^2 + 225}{-2(225 - x^2)^{\frac{1}{2}}}$

$0 = -2x^2 + 225$

$x = \frac{15}{\sqrt{2}}$

$A = \frac{1}{2} \cdot \frac{15}{\sqrt{2}} \cdot \sqrt{15^2 - (\frac{15}{\sqrt{2}})^2}$

$= 56.25$

$56.25 \times 2 \div \frac{15}{2}$

$= \frac{15}{\sqrt{2}}$

$\therefore$ the largest possible area is $56.25 cm^2$ with dimensions of $\frac{15}{\sqrt{2}} \times \frac{15}{\sqrt{2}}$

4) let x represent one dimension of the rectangle.

$f(x) = 3x + 2(\frac{184}{x})$

$= 3x + \frac{368}{x}$

$f'(x) = 3 - \frac{368}{x^2}$

$0 = 3 - \frac{368}{x^2}$

$x = \frac{4\sqrt{69}}{3}$ OR $-\frac{4\sqrt{69}}{3}$ Reject, we are only interested in magnitude.

$f(x) = 3x + 2(\frac{184}{x})$

$f(\frac{4\sqrt{69}}{3}) = 3(\frac{4\sqrt{69}}{3}) + 2(2\sqrt{69})$

$= 8\sqrt{69}$

$\therefore$ it will require $8\sqrt{69}$ m of fencing if the rectangle is $\frac{4\sqrt{69}}{3}$ m $\times$ $8\sqrt{69}$ m

5) $R = (10 - 0.25x)(10000 + 1000x)$
$R = -250x^2 + 7500x + 100000$
$R' = -500x + 7500$

$0 = -500x + 7500$
$500x = 7500$
$x = 15$

$R = (10 - 0.25(15))(10000 + 1000(15))$
$= 156250$

$10 + 15 \cdot 0.25 = 13.75$

$\therefore$ a price of \$13.75 will yield the highest revenue of \$156250

6) $A = \frac{1}{2}ab\sin\theta$
$A' = \frac{1}{2}ab\cos\theta$

$0 = \frac{1}{2}ab\cos\theta$
$\cos\theta = 0$
$\theta = \cos^{-1}(0)$
$= 90°$

$\therefore$ an angle of 90° between them will maximize the area,

7) $A = 2x(8 - x^2)$ ← 2x because it's width goes into negative side too
$= 16x - 2x^3$
$A' = 16 - 6x^2$

$0 = 16 - 6x^2$
$x = \frac{2\sqrt{6}}{3}$

8) $SA = 2\pi x^2 + (800 \div \pi x^2)$

$SA' = 4\pi x - 1600 \div \pi x^3$

$0 = 4\pi x - 1600 \div \pi x^3$

$x = \frac{2\sqrt{5\pi}}{\pi}$

$800 = \pi \left(\frac{2\sqrt{5\pi}}{\pi}\right)^2 h$

$h = 40$

∴ to minimize the amount of materials needed, it will have a radius of $\frac{2\sqrt{5\pi}}{\pi}$ cm and a height of 40 cm