# Limits Exercise 3

8) $\lim_{x \to \frac{1}{3}^-} \left(\frac{1}{1-3x}\right) = +\infty$ $\lim_{x \to \frac{1}{3}^+} \left(\frac{1}{1-3x}\right) = -\infty$

$\lim_{x \to \frac{1}{3}} \left(\frac{1}{1-3x}\right) = \text{DNE}$

9) $\lim_{x \to 0^-} \left|\frac{1}{x}\right| = +\infty$ $\lim_{x \to 0^+} \left|\frac{1}{x}\right| = +\infty$

$\lim_{x \to 0} \left|\frac{1}{x}\right| = +\infty$

10) $\lim_{x \to 0^-} \left(-\frac{2007}{x}\right) = +\infty$ $\lim_{x \to 0^+} \left(-\frac{2007}{x}\right) = -\infty$

$\lim_{x \to 0} \left(-\frac{2007}{x}\right) = \text{DNE}$

23) $\lim_{x \to -\infty} \left(\frac{x^4 - 3x^2 + 2}{3 - 2x^4}\right)$

$= \lim_{x \to -\infty} \frac{x^4\left(1 - \frac{3}{x^2} + \frac{2}{x^4}\right)}{x^4\left(\frac{3}{x^4} - 2\right)}$

$\lim_{x \to -\infty} \left(\frac{1 - \frac{3}{x^2} + \frac{2}{x^4}}{\frac{3}{x^4} - 2}\right)$

Any number divided by $\infty$ is zero

$= \frac{1 - 0 + 0}{0 - 2}$

$= -\frac{1}{2}$

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24) $\lim_{x \to +\infty} \left( \frac{(x+1)^3 - x^3}{x^2} \right)$

$= \lim_{x \to +\infty} \left( \frac{x^3 + 3x^2 + 3x + 1 - x^3}{x^2} \right)$

$= \lim_{x \to +\infty} \left( 3 + \frac{3}{x} + \frac{1}{x^2} \right)$

$= 3 + 0 + 0$

$= 3$

# Limits Exercise 3

1) $\lim_{x \to -\infty} (5x)$

$= -\infty$

2) $\lim_{x \to \infty} (\sqrt{x+3})$

$= \sqrt{\infty}$

$= \infty$

3) $\lim_{x \to -\infty} (-x^3 + x - 1)$ ← odd degree, negative leading coefficient

$= \infty$

4) $\lim_{x \to -\infty} (\sqrt{5-2x})$

$= \lim_{x \to -\infty} (5-2x)$

$= \lim_{x \to -\infty} (-2x+5)$

$= \infty$

5) $\lim_{x \to -3^-} \left(\frac{1}{x+3}\right) = -\infty$

$\lim_{x \to -3^+} \left(\frac{1}{x+3}\right) = \infty$

$\lim_{x \to -3} \left(\frac{1}{x+3}\right) = \text{DNE}$

6) $\lim_{x \to 1^-} \left(\frac{x^2}{|x-1|}\right) = \infty$

$\lim_{x \to 1^+} \left(\frac{x^2}{|x-1|}\right) = \infty$

$\lim_{x \to 1} \left(\frac{x^2}{|x-1|}\right) = \infty$

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7) $\lim_{x\to 0^+}\left(-\frac{3}{x}\right)$

$= -\infty$

11) $\lim_{x\to\infty}\left(\frac{3x-1}{2x+5}\right)$

$= \lim_{x\to\infty}\left(\frac{x\left(3-\frac{1}{x}\right)}{x\left(2+\frac{5}{x}\right)}\right)$

$= \lim_{x\to\infty}\left(\frac{3-\frac{1}{x}}{2+\frac{5}{x}}\right)$

$= \frac{3-0}{2+0}$

$= \frac{3}{2}$

12) $\lim_{x\to-\infty}\left(\frac{3x}{2x^4-x+3}\right)$

$= \lim_{x\to-\infty}\left(\frac{x^4\left(\frac{3}{x^3}\right)}{x^4\left(2-\frac{1}{x^3}+\frac{3}{x^4}\right)}\right)$

$= \lim_{x\to-\infty}\left(\frac{\frac{3}{x^3}}{2-\frac{1}{x^3}+\frac{3}{x^4}}\right)$

$= \frac{0}{2-0+0}$

$= 0$

13) $\lim_{x\to\infty}\left(\frac{4x^3+3x}{-2x^3+x^2+1}\right)$

$= \lim_{x\to\infty}\left(\frac{x^3\left(4+\frac{3}{x^2}\right)}{x^3\left(-2+\frac{1}{x}+\frac{1}{x^3}\right)}\right)$

$= \lim_{x\to\infty}\left(\frac{4+\frac{3}{x^2}}{-2+\frac{1}{x}+\frac{1}{x^3}}\right)$

$= \frac{4+0}{-2+0+0} = -2$

14) $\lim_{x\to\infty}\left(\frac{x^2-2}{5x^4+2x+1}\right)$

$= \lim_{x\to\infty}\left(\frac{x^4\left(\frac{1}{x^2}-\frac{2}{x^4}\right)}{x^4\left(5+\frac{2}{x^3}+\frac{1}{x^4}\right)}\right)$

$= \lim_{x\to\infty}\left(\frac{\frac{1}{x^2}-\frac{2}{x^4}}{5-\frac{2}{x^3}+\frac{1}{x^4}}\right)$

$= \frac{0-0}{5-0+0}$

$= 0$

15) $\lim_{x\to\infty}\left(\frac{-3x^4+1}{x^3+2x+1}\right)$

$= \lim_{x\to\infty}\left(\frac{x\left(-3x+\frac{1}{x^3}\right)}{x^3\left(1+\frac{2}{x^2}+\frac{1}{x^3}\right)}\right)$

$= \lim_{x\to\infty}\left(\frac{-3x+\frac{1}{x^3}}{1+\frac{2}{x^2}+\frac{1}{x^3}}\right)$

$= \frac{-\infty+0}{1+0+0}$

$= -\infty$

16) $\lim_{x\to-\infty}\left(\frac{x^3+5x}{-x^4+1}\right)$

$= \lim_{x\to-\infty}\left(\frac{x^4\left(\frac{1}{x}+\frac{5}{x^3}\right)}{x^4\left(-1+\frac{1}{x^4}\right)}\right)$

$= \lim_{x\to\infty}\left(\frac{\frac{1}{x}+\frac{5}{x^3}}{-1+\frac{1}{x^4}}\right)$

$= \frac{0+0}{-1+0}$

$= 0$

# Limits Exercise 3

17) $\lim_{x\to-\infty}\left(\frac{2x^6}{7x^3+1}\right)$

$=\lim_{x\to\infty}\left(\frac{x^3(2x^3)}{x^3\left(7+\frac{1}{x^3}\right)}\right)$

$=\lim_{x\to\infty}\left(\frac{2x^3}{7+\frac{1}{x^3}}\right)$

$=\frac{-\infty}{7+0}$

$=-\infty$

18) $\lim_{x\to-\infty}\left(x+\frac{1}{x}\right)$

$=-\infty+0$

$=-\infty$

19) $\lim_{x\to\infty}\left(\frac{1}{2^{|x-1|}-1}\right)$

$=\frac{1}{\infty-1}$

$=0$

20) $\lim_{x\to\infty}\left(\frac{1+3^{-x}}{x^3+1}\right)$

$=\frac{1+0}{\infty+1}$

$=0$

22) $\lim_{x\to-\infty}\left(xe^{x^2}\right)$

$=(-\infty)(\infty)$

$=-\infty$