Logarithms Exercise 2

1a) $3\log_7 5$

$= \log_7 5^3$

$7^{\log_7 125}$

$= 125$

b) $\dfrac{10^{4\log 5} - \log 10^{27}}{\log_5 5 - \log_8 1}$

$= \dfrac{10^{\log 625} - \log 10^{27}}{\log_5 5 - \log_8 1}$

$= \dfrac{625 - 27}{1 - 0}$

$= \dfrac{598}{1}$

$= 598$

c) $\dfrac{\log 1 - \log_2 1024}{5\log_6 36}$

$= \dfrac{\log 10^0 - \log_2 2^{10}}{\log_6 6^{10}}$

$= \dfrac{0 - 10}{10}$

$= -1$

d) $\log_6 4 + \log_6 9$

$= \log_6 (4 \cdot 9)$

$= \log_6 36$

$= \log_6 6^2$

$= 2$

e) $\log_{15} 1575 - \log_{15} 7$

$= \log_{15} (1575 \div 7)$

$= \log_{15} 225$

$= \log_{15} 15^2$

$= 2$

f) $\log 625 + \log 16$

$= \log (625 \cdot 16)$

$= \log 10000$

$= \log 10^4$

$= 4$

g) $\log_6 14 + \log_6 15 - \log_6 35$

$= \log_6 (14 \cdot 15) - \log_6 35$

$= \log_6 210 - \log_6 35$

$= \log_6 (210 \div 35)$

$= \log_6 6$

$= 1$

h) $\dfrac{\log_{14} 8 + \log_{14} 343}{\log_{19} 1083 - \log_{19} 3}$

$= \dfrac{\log_{14} (8 \cdot 343)}{\log_{19} (1083 \div 3)}$

$= \dfrac{\log_{14} 2744}{\log_{19} 361}$

$= \dfrac{\log_{14} 14^3}{\log_{19} 19^2}$

$= \dfrac{3}{2}$

$= 1\frac{1}{2}$

i) $\log_{49} 7 + \log_{243} 27 - \log_{25} 125$

$= \frac{1}{2}\log_7 7 + \frac{3}{5}\log_{27} 27 - \frac{3}{2}\log_{125} 125$

$= \frac{1}{2}(1) + \frac{3}{5}(1) - \frac{3}{2}(1)$

$= \frac{1}{2} + \frac{3}{5} - \frac{3}{2}$

$= -\frac{2}{5}$

j) $(\log_3 7)(\log_7 3)$

$= \frac{\cancel{\log 7}}{\log 3} \times \frac{\cancel{\log 3}}{\cancel{\log 7}}$

$= 1$

k) $(2\log_2 7)(\log_7 8 + \log_7 4)$

$= (\log_2 49)(\log_7 (8 \cdot 4))$

$= (\log_2 49)(\log_7 32)$

$= \frac{\log 49}{\log 2} \times \frac{\log 32}{\log 7}$

$= \frac{\log 32}{\log 2} \times \frac{\log 49}{\log 7}$

$= \log_2 32 \cdot \log_7 49$

$= \log_2 2^5 \cdot \log_7 7^2$

$= 5 \cdot 2$

$= 10$

l) $(\log_3 5)(\log_5 7)(\log_7 9)$

$= \frac{\cancel{\log 5}}{\log 3} \times \frac{\cancel{\log 7}}{\cancel{\log 5}} \times \frac{\log 9}{\cancel{\log 7}}$

$= \frac{\log 9}{\log 3}$

$= \log_3 9$

$= \log_3 3^2$

$= 2$

Logarithms Ex 2

2a) $\dfrac{10^{\log m^2} - 10^{2\log m^2}}{m^{\log 100}} = (1-m)(1+m)$

$LS = \dfrac{10^{\log m^2} - 10^{2\log m^2}}{m^{\log 100}}$

$= \dfrac{10^{\log m^2} - 10^{\log m^4}}{m^{\log 10^2}}$

$= \dfrac{m^2 - m^4}{m^2}$

$= 1 - m^2$

$RS = (1-m)(1+m)$

$= 1 + m - m - m^2$

$= 1 - m^2$

$\because LS = RS$

$\therefore \dfrac{10^{\log m^2} - 10^{2\log m^2}}{m^{\log 100}} = (1-m)(1+m)$

b) $\log_3\left(1\frac{4}{11}\right) - \log_3\left(\frac{2}{21}\right) - \log_3\left(\frac{35}{66}\right) = 3$

$LS = \log_3\left(1\frac{4}{11}\right) - \log_3\left(\frac{2}{21}\right) - \log_3\left(\frac{35}{66}\right)$

$= \log_3\left(1\frac{4}{11} \div \frac{2}{21} \div \frac{35}{66}\right)$

$= \log_3 27$

$= \log_3 3^3$

$= 3$

$RS = 3$

$\because LS = RS$

$\therefore \log_3\left(1\frac{4}{11}\right) - \log_3\left(\frac{2}{21}\right) - \log_3\left(\frac{35}{66}\right) = 3$

c) $\log_2 9 = \frac{1}{\log_9 2}$

$LS = \log_2 9$ $\qquad$ $RS = \frac{1}{\log_9 2}$

$= \frac{\log 9}{\log 2}$ $\qquad$ $= \frac{\log 2}{\log 9}$

$\because LS \neq RS$

$\therefore \log_2 9 \neq \frac{1}{\log_9 2}$

d) $\log_{2^{500}}(2^{1000}) = 500$

$LS = \log_{2^{500}}(2^{1000})$ $\qquad$ $RS = 500$

$= \frac{1000}{500} \log_2 2$

$= 2 \log_2 2$

$= 2(1)$

$= 2$

$\because LS \neq RS$

$\therefore \log_{2^{500}}(2^{1000}) \neq 500$

e) $m^{\log_b n} = n^{\log_b m}$

$LS = m^{\log_b n}$ $\qquad$ $RS = m^{\frac{\log_m n}{\log_m b}}$

$=$ $\qquad$ $= (m^{\log_m n})^{\frac{1}{\log_m b}}$

$= n^{\frac{1}{\log_m b}}$

$= n^{\frac{\log m}{\log b}}$

$\because LS = RS$

$\therefore m^{\log_b n} = n^{\log_b m}$

Logarithms Ex 2

3) $63 = 3^2 \cdot 7$

$\log 3 + \log 3 + \log 7$
$= \log(3 \cdot 3 \cdot 7)$
$= \log 63$

$m + m + n$
$= 2m + n$

4) $\dfrac{\log(a^4 b)}{\log(a b^3)}$

$= \dfrac{\log a + \log a + \log a + \log a + \log b}{\log a + \log b + \log b + \log b}$

$= \dfrac{2+2+2+2+3}{2+3+3+3}$

$= \dfrac{11}{11}$

$= 1$

5a) $\log_x 729 = 3$
$x^3 = 729$
$x = \sqrt[3]{729}$
$x = 9$

b) $\log_x 729 = 2$
$x^2 = 729$
$x = \sqrt{729}$
$x = 27$

c) $4^x = 13^x$

As x increases, the two values get farther apart so the only possible value of x will be 0.

$4^0 = 13^0$

$1 = 1$

d)