

Algebra Practice and Review

1a) $\frac{6}{a+9} - \frac{2}{a-7} = 1 \quad a \neq -9, 7$

$$\frac{6}{a+9} - \frac{2}{a-7} - 1 = 0$$

$$\frac{6(a-7) - 2(a+9) - (a+9)(a-7)}{(a+9)(a-7)} = 0$$

$$\frac{6a-42-2a-18-(a^2+2a-63)}{a^2+2a-63} = 0$$

$$\frac{6a-42-2a-18-a^2-2a+63}{a^2+2a-63} = 0$$

$$\frac{-a^2+2a+3}{a^2+2a-63} = 0$$

$$-a^2+2a+3 = 0 \quad a_1 = -1$$

$$(a+1)(a-3) = 0 \quad a_2 = 3$$

b) $\frac{6x+1}{4x} - \frac{2x-1}{2x} = 2 - \frac{4x+5}{3x+4} \quad x \neq 0, -\frac{4}{3}$

$$\frac{6x+1}{4x} - \frac{4x+2}{4x} = 2 - \frac{4x+5}{3x+4}$$

$$\frac{2x+3}{4x} + \frac{4x+5}{3x+4} - 2 = 0$$

$$\frac{(2x+3)(3x+4) + (4x+5)(4x) - 2(4x)(3x+4)}{(4x)(3x+4)} = 0$$

$$\frac{18x^2+3x+24x+4+12x^2+6x-16x+8-24x^2-32x+16x^2+20x}{12x^2+16x} = 0$$

$$\frac{-2x^2+5x+12}{12x^2+16x} = 0$$

$$-2x^2+5x+12 = 0 \quad x_1 = -\frac{3}{2}$$

$$-(2x+3)(x-4) = 0 \quad x_2 = 4$$

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$$2) \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} = \frac{2x^2 - 6x + 6}{(x-1)(x-2)(x-3)} \quad \left. \vphantom{\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}} \right\} \text{LCD} = (x-1)(x-2)(x-3)$$

$$A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) = 2x^2 - 6x + 6$$

$$(A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C) = 2x^2 - 6x + 6$$

$$A+B+C=2$$

$$-5A-4B-3C=-6$$

$$6A+3B+2C=6$$

$$C = 2 - A - B$$

$$-5A-4B-3(2-A-B)=-6$$

$$6A+3B+2(2-A-B)=6$$

$$-5A-4B-6+3A+3B=-6$$

$$6A+3B+4-2A-2B=6$$

$$-2A-B=0$$

$$4A+B=2$$

$$2A=2$$

$$A=1$$

$$-2(1)-B=0$$

$$-2-B=0$$

$$-B=2$$

$$B=-2$$

$$(1)+(-2)+C=2$$

$$-1+C=2$$

$$C=3$$

$$\therefore A=1, B=-2, C=3$$

$$3a) \frac{x}{(3-x)^4} + \frac{1}{(3-x)^3}$$

$$b) \rightarrow \frac{x + 3 - x}{(3-x)^4}$$

$$= \frac{3}{(3-x)^4}$$

$$4a) \frac{10x^{-2} + 7x^{-1} + 1}{1 - 4x^{-2}}$$

$$= \frac{(5x^{-1} + 1)(2x^{-1} + 1)}{(1 - 2x^{-1})(2x^{-1} + 1)}$$

$$= \frac{5x^{-1} + 1}{1 - 2x^{-1}} \cdot \frac{x}{x}$$

$$= \frac{5 + x}{x - 2}$$

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$$x \neq 0, -2, 2$$

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$$5a) \frac{10}{3\sqrt{6}-7} \times \frac{3\sqrt{6}+7}{3\sqrt{6}+7}$$
$$= \frac{30\sqrt{6}+70}{5}$$

$$b) \frac{1}{\sqrt{w+5} - \sqrt{w+6}} \times \frac{\sqrt{w+5} + \sqrt{w+6}}{\sqrt{w+5} + \sqrt{w+6}}$$
$$= \frac{\sqrt{w+5} + \sqrt{w+6}}{-1}$$
$$= -\sqrt{w+5} - \sqrt{w+6}$$

$$6a) \frac{\sqrt{y}+3}{y-9}$$

$$= \frac{\sqrt{y}+3}{(\sqrt{y}-3)(\sqrt{y}+3)}$$

$$= \frac{1}{\sqrt{y}-3}$$

$$6b) \frac{\sqrt{3(x+h)-2} - \sqrt{3x-2}}{h}$$

$$= \frac{h \sqrt{3(x+h)-2} + h \sqrt{3x-2}}{3h}$$

$$\times \frac{\sqrt{3(x+h)-2} + \sqrt{3x-2}}{\sqrt{3(x+h)-2} + \sqrt{3x-2}}$$

$$7a) -\frac{5m^2}{6} (m^2+1)^{-\frac{5}{3}} + \frac{3}{2} (m^2+1)^{-\frac{2}{3}}$$

$$= -\frac{5m^2}{6} \left(\frac{1}{(m^2+1)^{\frac{5}{3}}} \right) + \frac{3}{2} \left(\frac{1}{(m^2+1)^{\frac{2}{3}}} \right)$$

$$= -\frac{5m^2}{6(m^2+1)^{\frac{5}{3}}} + \frac{3}{2(m^2+1)^{\frac{2}{3}}}$$

$$= -\frac{5m^2}{6(m^2+1) \sqrt[3]{(m^2+1)^2}} + \frac{3}{2 \sqrt[3]{(m^2+1)^2}}$$

$$= \frac{-5m^2 + 9m^2 + 9}{6 \sqrt[3]{(m^2+1)^2} (m^2+1)}$$

$$= \frac{4m^2 + 9}{6 \sqrt[3]{(m^2+1)^2} (m^2+1)}$$

$$7b) \frac{x^{-3} - y^{-3}}{xy^{-1} + x^{-1}y + 1} \cdot \frac{x^3y^3}{x^3y^3} \quad \leftarrow \begin{matrix} x \neq 0 \\ y \neq 0 \end{matrix}$$

$$= \frac{y^3 - x^3}{x^4y^2 + x^2y^4 + x^3y^3}$$

$$= \frac{(y - x)(y^2 + xy + x^2)}{x^2y^2(x^2 + y^2 + xy)} \quad \leftarrow \text{Difference of cubes}$$

$$= \frac{y - x}{x^2y^2}$$

7c) Difference of cubes formula:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(\sqrt[3]{x})^3 - (\sqrt[3]{a})^3 = (\sqrt[3]{x} - \sqrt[3]{a})(\sqrt[3]{x^2} + \sqrt[3]{ax} + \sqrt[3]{b^2})$$

$$\frac{x - a}{\sqrt[3]{x} - \sqrt[3]{a}} \times \frac{\sqrt[3]{x^2} + \sqrt[3]{ax} + \sqrt[3]{b^2}}{\sqrt[3]{x^2} + \sqrt[3]{ax} + \sqrt[3]{b^2}}$$

$$= \frac{(x - a)(\sqrt[3]{x^2} + \sqrt[3]{ax} + \sqrt[3]{b^2})}{(\cancel{x - a})}$$

$$= \sqrt[3]{x^2} + \sqrt[3]{ax} + \sqrt[3]{b^2}$$

$$7d) \frac{a^{-2} - b^{-2}}{a^{-1} - b^{-1}}$$

$$a^{-1} - b^{-1}$$

$$= \frac{(a^{-1} - b^{-1})(a^{-1} + b^{-1})}{a^{-1} - b^{-1}}$$

$$= a^{-1} + b^{-1}$$

$$= \frac{1}{a} + \frac{1}{b}$$

$$= \frac{b + a}{ab}$$