Math Challenger 2

1) $\left(\dfrac{3x+4}{5x+1}\right)^2 + \dfrac{3x+4}{5x+1} = 12$

$\left(\dfrac{3x+4}{5x+1}\right)^2 + \dfrac{3x+4}{5x+1} - 12 = 0$

$\underbrace{\qquad\qquad\qquad\qquad\qquad}$

$a^2 + a - 12$
(can be seen
as a polynomial)

let $a = \dfrac{3x+4}{5x+1}$

$a^2 + a - 12 = 0$ $a = -4$ or 3
$(a+4)(a-3) = 0$

i. $\dfrac{3x+4}{5x+1} = -4$

$3x+4 = -4(5x+1)$

$3x+4 = -20x-4$

$23x = -8$

$x = -\dfrac{8}{23}$

ii. $\dfrac{3x+4}{5x+1} = 3$

$3x+4 = 3(5x+1)$

$3x+4 = 15x+3$

$12x = 1$

$x = \dfrac{1}{12}$

2) $(x-7)(x^2+6x+4) = -(x-7)$

$$x^2+6x+4 = -1$$

$$x^2+6x+5 = 0$$

$$(x-5)(x-1) = 0$$

$x = 5$ or 1 or 7

3) If there are 10 children, you have 9 siblings (excluding you).

5 more brothers than sisters.

$7-2=5$

7 brothers

2 sisters

Since each of your brothers have 7 brothers too, that means you identify as male as well.

$\therefore$ there are 8 brothers and 2 sisters

4) $\dfrac{n(n+1)}{2} = 1+2+3+...+n$

$$20000\left(\dfrac{20001 \cdot 20002}{2}\right) = 20000 \cdot 20001 \cdot 20002 \div 2$$

$$20001\left(\dfrac{20000 \cdot 20001}{2}\right) = 20000 \cdot 20001 \cdot 20001 \div 2$$

$\therefore A > B$

5) $4^{2x-3} = 4^{0.5}$

$$2x-3 = 0.5$$

$$2x = 3.5$$

$$x = 1.75 = 1\tfrac{3}{4}$$

6) $(x^2 - 10x + 12^2)^2 = 144$

$x^2 - 10x + 12^2 = \pm 12$

i. $x^2 - 10x + 12^2 = -12$

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$ $x = 4$ or 6 or 10

ii. $x^2 - 10x + 12^2 = 12$

$x^2 - 10 = 0$

$(x)(x-10) = 0$

7) $4^{27} + 4^{27} + 4^{27} + 4^{27} = 4^{(y-1)}$

$4(4^{27}) = 4^{(y-1)}$

Multiplication
Exponant Rule $\hookrightarrow$ $4^1 \cdot 4^{27} = 4^{(y-1)}$

$a^b \cdot a^c = a^{b+c}$ $4^{28} = 4^{(y-1)}$

$y - 1 = 28$

$y = 29$

8) $(1)(1)(1) = 1$, $1^1 = 1$ ✓

$(-1)(-1)(-1) = -1$, $(-1)^1_3 = -1$ ✓

$(3)(3)(3) = 27$, $3^3 = 27$ ✓

$(-3)(-3)(-3) = -27$, $(-3)^{-3} = -\frac{1}{27}$ ✗