

Logarithms Exercise 2

$$\begin{aligned} 1a) \quad & 3 \log_7 5 \\ &= \log_7 5^3 \\ &= 7^{\log_7 125} \\ &= 125 \end{aligned}$$

$$\begin{aligned} b) \quad & \frac{10^{4 \log_5 5} - \log 10^{27}}{\log_5 5 - \log_8 1} \\ &= \frac{10^{\log_5 625} - \log 10^{27}}{\log_5 5 - \log_8 1} \\ &= \frac{625 - 27}{1 - 0} \\ &= \frac{598}{1} \\ &= 598 \end{aligned}$$

$$\begin{aligned} c) \quad & \frac{\log 1 - \log_2 1024}{5 \log_6 36} \\ &= \frac{\log 10^0 - \log_2 2^{10}}{\log_6 6^{10}} \\ &= \frac{0 - 10}{10} \\ &= -1 \end{aligned}$$

$$\begin{aligned} d) \quad & \log_6 4 + \log_6 9 \\ &= \log_6 (4 \cdot 9) \\ &= \log_6 36 \\ &= \log_6 6^2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} e) \quad & \log_{15} 1575 - \log_{15} 7 \\ &= \log_{15} (1575 \div 7) \\ &= \log_{15} 225 \\ &= \log_{15} 15^2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} f) \quad & \log 625 + \log 16 \\ &= \log (625 \cdot 16) \\ &= \log 10000 \\ &= \log 10^4 \\ &= 4 \end{aligned}$$

$$\begin{aligned} g) \quad & \log_6 14 + \log_6 15 - \log_6 35 \\ &= \log_6 (14 \cdot 15) - \log_6 35 \\ &= \log_6 210 - \log_6 35 \\ &= \log_6 (210 \div 35) \\ &= \log_6 6 \\ &= 1 \end{aligned}$$

$$\begin{aligned} h) \quad & \frac{\log_{14} 8 + \log_{14} 343}{\log_{19} 1083 - \log_{19} 3} \\ &= \frac{\log_{14} (8 \cdot 343)}{\log_{19} (1083 \div 3)} \\ &= \frac{\log_{14} 2744}{\log_{19} 361} \\ &= \frac{\log_{14} 14^3}{\log_{19} 19^2} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$

$$\begin{aligned}
 \text{i) } & \log_{49} 7 + \log_{243} 27 - \log_{25} 125 \\
 &= \frac{1}{2} \log_7 7 + \frac{3}{5} \log_{27} 27 - \frac{3}{2} \log_{125} 125 \\
 &= \frac{1}{2}(1) + \frac{3}{5}(1) - \frac{3}{2}(1) \\
 &= \frac{1}{2} + \frac{3}{5} - \frac{3}{2} \\
 &= -\frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{j) } & (\log_3 7)(\log_7 3) \\
 &= \frac{\log 7}{\log 3} \times \frac{\log 3}{\log 7} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{k) } & (2 \log_2 7)(\log_7 8 + \log_7 4) \\
 &= (\log_2 49)(\log_7 (8 \cdot 4)) \\
 &= (\log_2 49)(\log_7 32) \\
 &= \frac{\log 49}{\log 2} \times \frac{\log 32}{\log 7} \\
 &= \frac{\log 32}{\log 2} \times \frac{\log 49}{\log 7} \\
 &= \log_2 32 \cdot \log_7 49 \\
 &= \log_2 2^5 \cdot \log_7 7^2 \\
 &= 5 \cdot 2 \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{l) } & (\log_3 5)(\log_5 7)(\log_7 9) \\
 &= \frac{\log 5}{\log 3} \times \frac{\log 7}{\log 5} \times \frac{\log 9}{\log 7} \\
 &= \frac{\log 9}{\log 3} \\
 &= \log_3 9 \\
 &= \log_3 3^2 \\
 &= 2
 \end{aligned}$$

Logarithms Ex 2

$$2a) \frac{10^{\log m^2} - 10^{2 \log m^2}}{m^{\log 100}} = (1-m)(1+m)$$

$$\begin{aligned} \text{LS} &= \frac{10^{\log m^2} - 10^{2 \log m^2}}{m^{\log 100}} \\ &= \frac{10^{\log m^2} - 10^{\log m^4}}{m^{\log 10^2}} \\ &= \frac{m^2 - m^4}{m^2} \\ &= 1 - m^2 \end{aligned}$$

$$\begin{aligned} \text{RS} &= (1-m)(1+m) \\ &= 1 + m - m - m^2 \\ &= 1 - m^2 \end{aligned}$$

$$\therefore \text{LS} = \text{RS}$$

$$\therefore \frac{10^{\log m^2} - 10^{2 \log m^2}}{m^{\log 100}} = (1-m)(1+m)$$

$$b) \log_3 \left(1 \frac{4}{11} \right) - \log_3 \left(\frac{2}{21} \right) - \log_3 \left(\frac{35}{66} \right) = 3$$

$$\begin{aligned} \text{LS} &= \log_3 \left(1 \frac{4}{11} \right) - \log_3 \left(\frac{2}{21} \right) - \log_3 \left(\frac{35}{66} \right) & \text{RS} &= 3 \\ &= \log_3 \left(1 \frac{4}{11} \div \frac{2}{21} \div \frac{35}{66} \right) \\ &= \log_3 27 \\ &= \log_3 3^3 \\ &= 3 \end{aligned}$$

$$\therefore \text{LS} = \text{RS}$$

$$\therefore \log_3 \left(1 \frac{4}{11} \right) - \log_3 \left(\frac{2}{21} \right) - \log_3 \left(\frac{35}{66} \right) = 3$$

$$c) \log_2 9 = \frac{1}{\log_9 2}$$

$$\begin{aligned} LS &= \log_2 9 \\ &= \frac{\log 9}{\log 2} \end{aligned}$$

$$\begin{aligned} RS &= \frac{1}{\log_9 2} \\ &= \frac{\log 2}{\log 9} \end{aligned}$$

$$\therefore LS \neq RS$$

$$\therefore \log_2 9 \neq \frac{1}{\log_9 2}$$

$$d) \log_{2^{500}} (2^{1000}) = 500$$

$$\begin{aligned} LS &= \log_{2^{500}} (2^{1000}) = 500 \quad RS = 500 \\ &= \frac{1000}{500} \log_2 2 \\ &= 2 \log_2 2 \\ &= 2(1) \\ &= 2 \end{aligned}$$

$$\therefore LS \neq RS$$

$$\therefore \log_{2^{500}} (2^{1000}) \neq 500$$

$$e) m^{\log_b n} = n^{\log_b m}$$

$$\begin{aligned} LS &= m^{\log_b n} \\ &= \end{aligned}$$

$$\begin{aligned} RS &= m^{\frac{\log n}{\log m b}} \\ &= (m^{\log_m n})^{\frac{1}{\log_m b}} \\ &= n^{\frac{1}{\log_m b}} \\ &= n^{\frac{\log m}{\log b}} \end{aligned}$$

$$\therefore LS = RS$$

$$\therefore m^{\log_b n} = n^{\log_b m}$$

Logarithms Ex 2

3) $63 = 3^2 \cdot 7$

$$\begin{aligned} & \log 3 + \log 3 + \log 7 \\ &= \log (3 \cdot 3 \cdot 7) \\ &= \log 63 \end{aligned}$$

$$\begin{aligned} & m + m + n \\ &= 2m + n \end{aligned}$$

4) $\frac{\log(a+b)}{\log(ab^3)}$

$$= \frac{\log a + \log a + \log a + \log a + \log b}{\log a + \log b + \log b + \log b}$$

$$= \frac{2+2+2+2+3}{2+3+3+3}$$

$$= \frac{11}{11}$$

$$= 1$$

5a) $\log_x 729 = 3$

$$x^3 = 729$$

$$x = \sqrt[3]{729}$$

$$x = 9$$

b) $\log_x 729 = 2$

$$x^2 = 729$$

$$x = \sqrt{729}$$

$$x = 27$$

$$c) 4^x = 13^x$$

As x increases, the two values get farther apart so the only possible value of x will be 0.

$$4^0 = 13^0$$

$$1 = 1$$

d)