Spirit of Math schools

Factoring Polynomials: Exercise 3

Name: _____ Date: _____

[*45 marks total*]

Difference in Squares

A) [*38 marks*] Factor *fully* each of the following. In questions 11 to 16 also write the final calculated answer. Show work on lined paper for questions 18 to 20, and for part B. There is 1 mark per question except where indicated.

1) $a^2-49=$ $(a+7)(a-7)$

2) $m^2-729=$ $(m+27)(m-27)$

3) $x^2-16y^2=$ $(x+4y)(x-4y)$

4) $9b^2-25=$ $(3b+5)(3b-5)$

5) $-81+p^2=$ $(p+9)(p-9)$

6) $961-16d^2=$ $(4d+31)(-4d+31)$

7) $529f^2-1849g^2=$ $(23f+43g)$ $(23f-43g)(23f+43g)$

8) [*2 marks*] h^4-1
$=(h^2)^2-(1)^2$
$=(h^2+1)(h+1)(h-1)$

9) [*2 marks*] m^4-n^4
$=(m^2)^2-(n^2)^2$
$=(m^2+n^2)(m+n)(m-n)$

10) [*3 marks*] $a^8-b^8=$ $(a^4)^2-(b^4)^2$
$=(a^4+b^4)(a^2+b^2)(a+b)(a-b)$

11) [*2 marks*] $19^2-18^2=$ $(19-18)(19+18)$
$=37$

12) [*2 marks*] $19^2-17^2=$ $(19+17)(19-17)$
$=72$

13) [*2 marks*] $19^2-16^2=$ $(19+16)(19-16)$
$=105$

14) [*3 marks*] $19^4-18^4=$ $(19^2+18^2)(19^2-18^2)$
$=(685)(37)$
$=25345$

15) [*2 marks*] $1493^2-1492^2=$ $(1493+1492)(1493-1492)$
$=2985$

16) [*2 marks*] 1493^2-493^2
$=(1493+493)(1493-493)$
$=1986000$

17) [*2 marks*] $(x-2)^2-(x-3)^2$
$=((x-2)+(x-3))((x-2)-(x-3))$
$=2x-5$

7, 14, 19, 20 DO NOT EXPAND

18) [*3 marks*] $(a^2-6)^2-(4a+6)^2=$ $a(a+2)(a-6)(a+4)$

19) [*2 marks* $(2m-3)^2-(8m^2-4)^2=$ $-(2m-1)(4m+1)(2m-7+8m^2)$

+ 1 bonus]

20) [*4 marks*] $(17p^2-26)^2-(8p^2-10)^2=$ $(5p+6)(5p-6)(3p+4)(3p-4)$

B) [*7 marks*] In certain right triangles $\sqrt{h^2-a^2}=24$, where h represents the length of the hypotenuse and a is the length of one of the legs. Find all possible ordered pairs (h,a), where $h,a\in\mathbb{N}$. Be prepared to explain your reasoning.

b) $\sqrt{h^2 - a^2} = 24$

$h^2 - a^2 = 24^2$

$(h-a)(h+a) = 2^6 \cdot 3^2$

$h - a = 2^x \cdot 3^y$

$h + a = 2^{6-x} \cdot 32 - z$

$2h = (2^x \cdot 3^y) + (2^{6-x} \cdot 32 - z)$

$2a = 2^{6-x} \cdot 32 - z - 2^x \cdot 3^z$

$h = (2^{x-1} \cdot 3^z) + (2^{6-x-1} \cdot 3^{z-z})$

$a = 16(3^{z-z}) - 3^z$

$1 \leq x \leq 5$

1) $h = 25, a = 7$
2) $h = 74, a = 70$
3) $h = 40, a = 32$
4) $h = 26, a = 11$
5) $h = 288, a = 2$

18) $(a^2 - 6 - 4a - 6) \times (a^2 - 6 + 4a + 6)$

$(a^2 - 4a - 12) a \times (a + 4)$

$(a^2 + 2a - 6a - 12) a (a+4)$

$(a \times (a+2) - 6(a+2)) a \times (a+4)$

19) $(2m - 3 - 8m^2 + 4) \times (2m - 3 + 8m^2 - 4)$