

Algebraic Equations Problem Set

Final Answers for questions #1-16

- 1)
- 2) If Wilson was a teacher, then that means Blair is a lawyer. Since Blair supposedly says he is a lawyer (which should be a lie) his statement is contradictory to our original assumption. Therefore, Wilson is the lawyer
- 3) 7, 17, 37, 47, 67, 97 = 6 total
- 4) C
- 5) (1, 1, 1), (-1, -1, 1), (1, -1, -1), (-1, 1, -1) = 4 total
- 6) 69
- 7) Ann, Dave, Bill, Carol
- 8) 440 yards/hr, 1 hour ← This is because the current is affecting the ^{boat} and the ^{hat}
- 9) 6:7
- 10) D ; 4 5 2 3 1
- 11) 704
- 12)
- 13)
- 14) 9
- 15) C ; 200
- 16) 750

Full Solutions for questions #17-20

17) $x^3 - 8$
 $= x^3 - 2^3$ Difference of cubes formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 $= (x - 2)(x^2 + 2x + 2^2)$
 $= (x - 2)(x^2 + 2x + 4)$

$$18) \begin{cases} x - y = 2 \\ cx + y = 3 \end{cases}$$

$$x = \frac{5}{c+1} \quad y = \frac{3-2c}{c+1}$$

$$x > 0 \rightarrow c+1 > 0 \rightarrow c > -1$$

$$y > 0 \rightarrow 3-2c > 0 \rightarrow 3 > 2c \rightarrow \frac{3}{2} > c$$

$$-1 < c < \frac{3}{2}$$

$$19) \begin{aligned} x^4 + 324 &= x^4 + (4)(3^4) \\ &= (x^2 + 2(3^2) - 2(3x))(x^2 + 2(3^2) + 2(3x)) \\ &= (x(x-6) + 18)(x(x+6) + 18) \end{aligned}$$

$$\begin{aligned} & (10(10-6)+18)(10(10+6)+18)(22(22-6)+18)(22(22+6)+18)(34(34-6)+18) \\ & (34(34+6)+18)(46(46-6)+18)(46(46+6)+18)(58(58-6)+18)(58(58+6)+18) \\ & (4(4-6)+18)(4(4+6)+18)(16(16-6)+18)(16(16+6)+18)(28(28-6)+18) \\ & (28(28+6)+18)(40(40-6)+18)(40(40+6)+18)(52(52-6)+18)(52(52+6)+18) \\ & = \frac{(58)(178)(370)(634)(970)(1378)(1858)(2410)(3034)(3730)}{(10)(58)(178)(370)(634)(970)(1378)(1858)(2410)(3034)} \\ & = \frac{3730}{10} \\ & = 373 \end{aligned}$$

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20) $y = (x^2 - bc)(2x - b - c)^{-1}$

$$y = \frac{x^2 - bc}{2x - b - c}$$

To have real roots, the discriminant must be ≥ 0

$$x^2 - bc - y(2x - b - c) = 0$$

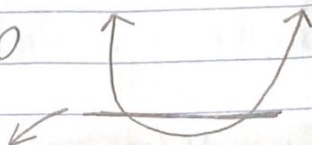
$$x^2 - 2yx + y(b+c) - bc = 0$$

$$(-2y)^2 - 4(1)(y(b+c) - bc) \geq 0$$

$$4y^2 - 4y(b+c) + 4bc \geq 0$$

$$y^2 - y(b+c) + bc \geq 0$$

$$(y-b)(y-c) \geq 0$$



$$\begin{aligned} y-b &\leq 0 \\ y-c &\leq 0 \end{aligned}$$

$$\text{OR} \begin{aligned} y-b &\geq 0 \\ y-c &\geq 0 \end{aligned}$$

$$\begin{aligned} y &\leq b \\ y &\leq c \\ \downarrow \\ y &\leq b \end{aligned}$$

$$\begin{aligned} y &\geq b \\ y &\geq c \\ \downarrow \\ y &\geq c \end{aligned}$$

$$\boxed{b < c} \leftarrow \text{from the question}$$

$$\therefore \text{Range} = (-\infty, b] \cup [c, +\infty)$$