Algebraic Equations Problem Set

Final Answers for questions #1-16

1)

2) If Wilson was a teacher, then that means Blair is a lawyer. Since Blair supposedly says he is a lawyer (which should be a lie) his statement is contradictory to our original assumption. Therefor, Wilson is the lawyer

3) 7, 17, 37, 47, 67, 97 = 6 total

4) C

5) (1,1,1), (-1,-1,1), (1,-1,-1), (-1,1,-1) = 4 total

6) 69

7) Ann, Dave, Bill, Carol

8) 440 yards/hr, 1 hour ← This is because the current is affecting the boat and the hat

9) 6:7

10) D ; 45231

11) 704

12)

13)

14) 9

15) C ; 200

16) 750

Full Solutions for questions #17-20

17) $x^3 - 8$

$= x^3 - 2^3$ Difference of cubes fomula: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= (x-2)(x^2 + 2x + 2^2)$

$= (x-2)(x^2 + 2x + 4)$

18) $x - y = 2$
$cx + y = 3$

$$x = \frac{5}{c+1} \qquad y = \frac{3-2c}{c+1}$$

$x > 0 \rightarrow c+1 > 0 \rightarrow c > -1$
$y > 0 \rightarrow 3-2c > 0 \rightarrow 3 > 2c \rightarrow \frac{3}{2} > c$

$-1 < c < \frac{3}{2}$

19) $x^4 + 324 = x^4 + (4)(3^4)$

$$= (x^2 + 2(3^2) - 2(3x)(x^2 + 2(3^2) + 2(3x))$$

$$= (x(x-6)+18)(x(x+6)+18)$$

$$\frac{(10(10-6)+18)(10(10+6)+18)(22(22-6)+18)(22(22+6)+18)(34(34-6)+18)(34(34+6)+18)(46(46+6)+18)(46(46-6)+18)(58(58+6)+18)(58(58-6)+18)}{(4(4-6)+18)(4(4+6)+18)(16(16-6)+18)(16(16+6)+18)(28(28-6)+18)(28(28+6)+18)(40(40-6)+18)(40(40+6)+18)(52(52-6)+18)(52(52+6)+18)}$$

$$= \frac{\cancel{(58)}\cancel{(178)}\cancel{(370)}\cancel{(634)}\cancel{(970)}\cancel{(1378)}\cancel{(1858)}\cancel{(2410)}\cancel{(3034)}(3730)}{(10)\cancel{(58)}\cancel{(178)}\cancel{(370)}\cancel{(634)}\cancel{(970)}\cancel{(1378)}\cancel{(1858)}\cancel{(2410)}\cancel{(3034)}}$$

$$= \frac{3730}{10}$$

$$= 373$$

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20) $y=(x^2-bc)(2x-b-c)^{-1}$

$$y=\frac{x^2-bc}{2x-b-c}$$

$$x^2-bc-y(2x-b-c)=0$$

$$x^2-2yx+y(b+c)-bc=0$$

To have real roots, the disciminant must be ≥ 0

$$(-2y)^2-4(1)(y(b+c)-bc)\geq 0$$

$$4y^2-4y(b+c)+4bc\geq 0$$

$$y^2-y(b+c)+bc\geq 0$$

$$(y-b)(y-c)\geq 0$$

$y-b\leq 0$ OR $y-b\geq 0$

$y-c\leq 0$ $\quad$ $y-c\geq 0$

$y\leq b$ $\quad$ $y\geq b$

$y\leq c$ $\quad$ $y\geq c$

$\boxed{b<c}$ ← from the question

$\downarrow$ $\quad$ $\downarrow$

$y\leq b$ $\quad$ $y\geq c$

$\therefore$ Range $=(-\infty,b]\cup[c,+\infty)$