Derivatives Test

1) $f(x) = \sqrt{2x-3}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sqrt{2(x+h)-3} - \sqrt{2x-3}}{h}\right)$$

$$= \frac{1}{\sqrt{2x-3}}$$

2a) $y = 3x^5 - 2x^3 + 1$

$y' = 3(5x^4) - 2(3x^2) + 0$

$= 15x^4 - 6x^2$

b) $f(x) = -\frac{5}{\sqrt{3x^2-x}}$

$$f'(x) = \frac{(-5)'(\sqrt{3x^2-x}) - (\sqrt{3x^2-x})'(-5)}{\sqrt{3x^2-x}^2}$$

$$= \frac{(0)(\sqrt{3x^2-x}) - \left(\frac{1}{2}(3x^2-x)^{-\frac{1}{2}}\right)(6x-1))(-5)}{3x^2-x}$$

$$= \frac{-(6x-1)\left(2(3x^2-x)^{\frac{1}{2}}\right)^{-1}}{3x^2-x}$$

$$= \frac{30x-5}{2\sqrt{3x^2-x}\,(3x^2-x)}$$

c) $k(w) = \sqrt{w}\cdot \tan 3w$

$k'(w) = (\sqrt{w})'(\tan 3w) + (\sqrt{w})(\tan 3w)'$

$= \left(\frac{1}{2\sqrt{w}}\right)(\tan 3w) + (\sqrt{w})(\sec 3w)^2(3)$

$= \frac{\tan 3w}{2\sqrt{w}} + 3\sqrt{w}\sec^2 3w$

d) $c(d) = \log_4(2d-1)$

$c'(d) = \frac{1}{(\ln 4)(2d-1)}\cdot 2$

$= \frac{2}{(\ln 4)(2d-1)}$

e) $F(x) = \operatorname{arccsc}(6e^{2x})$

$$f'(x) = -\frac{1}{|6e^{2x}| \cdot \sqrt{(6e^{2x})^2 - 1}} \cdot 6(2)e^{2x}$$

$$= -\frac{2}{\sqrt{36e^{4x} - 1}}$$

f) $$w(v) = \frac{(v^3 - 2v)^3}{(4v^2 - 3)^2}$$

$$w'(v) = \frac{((v^3-2v)^3)'(4v^2-3)^2 - ((4v^2-3)^2)'(v^3-2v)^3}{(4v^2-3)^4}$$

$$= \frac{3(v^3-2v)^2(3v^2-2)(4v^2-3)^2 - (v^3-2v)^3(2(4v^2-3))(4)(2v)}{(4v^2-3)^4}$$

$$= \frac{36v^{10} - 195v^8 + 366v^6 - 276v^4 + 72v^2 - 16v(v^3-2v)^3}{(4v^2-3)^3}$$

g) $y = 5^b \cos b$

$y' = \ln(5)(5^b)(\cos b) + 5^b(-\sin b)$

$= \ln(5)(5^b \cos b) - 5^b \sin b$

h) $y^3x^2 - 3y\ln x = 3x^4$

$0 = y^3x^2 - 3y\ln x - 3x^4$

$$f_x = \frac{2y^3x^2 - 3y - 12x^4}{x}$$

$$f_y = 3x^2y^2 - 3\ln(x)$$

$$\frac{dy}{dx} = -\frac{\frac{2y^3x^2 - 3y - 12x^4}{x}}{3x^2y^2 - 3\ln(x)}$$

$$= -\frac{2x^2y^3 - 3y - 12x^4}{x(3x^2y^2 - 3\ln(x))}$$

3) $f(x) = \cot(3x)$

$f'(x) = 3(-\csc(3x)^2)$

$= -3\csc(3x)^2$

$f''(x) = (-3\csc(9x^2))'$

$= (-3)(\csc(9x^2))'$

$= (-3)(-\cot(9x^2)\csc(9x^2)\cdot 9\cdot 2x)$

$= \frac{54x\cos(9x^2)}{\sin(9x^2)^2}$

4) $y = \ln(3x^2+4)$ at $x = 2$

$y' = \frac{1}{3x^2+4}\cdot 3\cdot 2x$

$y' = \frac{6x}{3x^2+4}$

$m = \frac{6(2)}{3(2^2)+4}$

$= \frac{3}{4}$

5) $y = 3x^2 - x - 2$

$y' = 6x - 1 - 0$

$= 6x - 1$

$y = 3(-1)^2 - (-1) - 2$

$= 2$

$\therefore$ the point on the parabola is $(-1, 2)$

$x - 7y = -3$

$-7y = -3 - x$

$y = \frac{-3-x}{-7}$

$y = \frac{1}{7}x + \frac{3}{7}$

↑ slope $= \frac{1}{7}$

perpendicular slope $= -7$

$-7 = 6x - 1$

$x = -1$