Trig Exercise 11

1) $\frac{1+\cos\theta}{1+\sec\theta} = \cos\theta$

$LS = \frac{1+\cos\theta}{1+\sec\theta}$

$= \frac{1+\cos\theta}{1+\frac{1}{\cos\theta}}$

$= \frac{\cos\theta(1+\cos\theta)}{\cos\theta(1+\frac{1}{\cos\theta})}$

$= \frac{\cos\theta\cancel{(1+\cos\theta)}}{\cancel{\cos\theta+1}}$

$= \cos\theta$

$RS = \cos\theta$

$\because LS = RS$

$\therefore \frac{1+\cos\theta}{1+\sec\theta} = \cos\theta$

2) $\sin^2\theta\sec^2\theta = \sec^2\theta - 1$

$LS = \sin^2\theta\sec^2\theta$

$= (\sin\theta\sec\theta)^2$

$= (\sin\theta\frac{1}{\cos\theta})^2$

$= \left(\frac{\sin\theta}{\cos\theta}\right)^2$

$= (\tan\theta)^2$

$= \tan^2\theta$

$RS = \sec^2\theta - 1$

$= \tan^2\theta$

$\because LS = RS$

$\therefore \sin^2\theta\sec^2\theta$

3) $\csc^2\theta - \cot^2\theta = 1$

$LS = \csc^2\theta - \cot^2\theta$ pyth identity $\cot^2\theta + 1 = \csc^2\theta$

$= 1$

$RS = 1$

$\because LS = RS$

$\therefore \csc^2\theta - \cot^2\theta = 1$

4) $\sec^2\theta\cot^2\theta = 1+\cot^2\theta$

$LS = \sec^2\theta\cot^2\theta$
$= \frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}$
$= \frac{1}{\sin^2\theta}$
$= \csc^2\theta$

$RS = 1+\cot^2\theta$
$= \csc^2\theta$

$\because LS = RS$
$\therefore \sec^2\theta\cot^2\theta = 1+\cot^2\theta$

5) $(\cos\theta+1)^2 + \sin^2\theta = 2(1+\cos\theta)$

$LS = (\cos\theta+1)^2 + \sin^2\theta$
$= \cos^2\theta + 2\cos\theta + 1 + \sin^2\theta$ $\cos^2\theta+\sin^2\theta=1$
$= 2\cos\theta + 1 + 1$
$= 2\cos\theta + 2$

$RS = 2(1+\cos\theta)$
$= 2 + 2\cos\theta$
$= 2\cos\theta + 2$

$\because LS = RS$
$\therefore (\cos\theta+1)^2 + \sin^2\theta$

6) $\cos\theta\sec\theta = \sin^2\theta + \cos^2\theta$

$LS = \cos\theta\sec\theta$
$= \cos\theta \cdot \frac{1}{\cos\theta}$
$= \frac{\cos\theta}{\cos\theta}$
$= 1$

$RS = \sin^2\theta + \cos^2\theta$
$= 1$

$\because LS = RS$
$\therefore \cos\theta\sec\theta = \sin^2\theta + \cos^2\theta$

7) $\sin\beta - \csc\beta = -\cos\beta\cot\beta$

$LS = \sin\beta - \csc\beta$ $\qquad\qquad RS = -\cos\beta\cot\beta$

$= \sin\beta - \frac{1}{\sin\beta}$

$= \frac{\sin\beta}{1} - \frac{1}{\sin\beta}$

$= \frac{\sin^2\beta - 1}{\sin\beta}$

$= \frac{-\cos^2\beta}{\sin\beta}$

$= -\cos\beta \cdot \frac{\cos\beta}{\sin\beta}$

$= -\cos\beta\cot\beta$

$\because RS = LS$

$\therefore \sin\beta - \csc\beta = -\cos\beta\cot\beta$

8) $\tan^2\alpha\csc^2\alpha - \sin^2\alpha\sec^2\alpha = 1$

$LS = \tan^2\alpha\csc^2\alpha - \sin^2\alpha\sec^2\alpha$ $\qquad\qquad RS = 1$

$= \frac{\sin^2\alpha}{\cos^2\alpha} \cdot \frac{1}{\sin^2\alpha} - \frac{\sin^2\alpha}{1} \cdot \frac{1}{\cos^2\alpha}$

$= \frac{1}{\cos^2\alpha} - \frac{\sin^2\alpha}{\cos^2\alpha}$

$= \sec^2\alpha - \tan^2\alpha$

$= 1$

$\because LS = RS$

$\therefore \tan^2\alpha\csc^2\alpha - \sin^2\alpha\sec^2\alpha = 1$

9) $\sec\theta\sqrt{1-\cos^2\theta} = \tan\theta$

$LS = \sec\theta\sqrt{1-\cos^2\theta}$ $\qquad$ $RS = \tan\theta$

$= \sec\theta\sqrt{\sin^2\theta}$

$= \sec\theta\sin\theta$

$= \frac{1}{\cos\theta}\cdot\sin\theta$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$\because LS = RS$

$\therefore \sec\theta\sqrt{1-\cos^2\theta} = \tan\theta$

10 $\csc\phi\cos\phi\sqrt{\sec^2\phi-1} = 1$

$LS = \csc\phi\cos\phi\sqrt{\sec^2\phi-1}$ $\qquad$ $RS = 1$

$= \frac{1}{\sin\phi}\cdot\cos\phi\sqrt{\tan^2\phi}$

$= \frac{\cos\phi}{\sin\phi}\cdot\tan\phi$

$= \cot\phi\tan\phi$

$= 1$

$\because LS = RS$

$\therefore \csc\phi\cos\phi\sqrt{\sec^2\phi-1} = 1$

11)

10) $\csc\varphi \cos\varphi \sqrt{\sec^2\varphi - 1} = 1$

$LS = \csc\varphi \cos\varphi \sqrt{\sec^2\varphi - 1}$
$= \frac{1}{\sin\varphi} \cos\varphi \sqrt{\tan^2\varphi}$
$= \frac{\cos\varphi}{\sin\varphi} \tan\varphi$
$= \cot\varphi \tan\varphi$
$= 1$

$RS = 1$

$\because LS = RS$
$\therefore \csc\varphi \cos\varphi \sqrt{\sec^2\varphi - 1} = 1$

11) $(\cos\beta + \sin\beta)^2 = 2\cot\beta \sin^2\beta + 1$

$LS = (\cos\beta + \sin\beta)^2$
$= \cos^2\beta + 2\cos\beta\sin\beta + \sin^2\beta$
$= 2\cos\beta \sin\beta + 1$

$RS = 2\cot\beta \sin^2\beta + 1$
$= 2\frac{\cos\beta}{\sin\beta} \cdot \sin^2\beta + 1$
$= 2\cos\beta\sin\beta + 1$

$\because LS = RS$
$\therefore (\cos\beta + \sin\beta)^2 = 2\cot\beta \sin^2\beta + 1$

12) $(\sec^2\theta - 1)(\csc^2\theta - 1) = 1$

$LS = (\sec^2\theta - 1)(\csc^2\theta - 1)$
$= (\tan^2\theta)(\cot^2\theta)$
$= \frac{\tan^2\theta}{1} \cdot \frac{1}{\tan^2\theta}$
$= \frac{\tan^2\theta}{\tan^2\theta}$
$= 1$

$RS = 1$

$\because LS = RS$
$\therefore (\sec^2\theta - 1)(\csc^2\theta - 1)$

13) $(\tan\varphi+1)^2 = \sec^2\varphi(\cos\varphi\sin\varphi)^2$

$LS = (\tan\varphi+1)^2$
$= \tan^2\varphi + 2\tan\varphi + 1$

$RS = \sec^2\varphi(\cos\varphi\sin\varphi)^2$
$= \sec^2\varphi\cos^2\varphi\sin^2\varphi$
$= (\tan^2\varphi+1)(\cos^2\varphi\sin^2\varphi)$
$= \sin^4 x + \cos^2\varphi\sin^2\varphi$

Not an identity?

14) $\frac{1}{\cot^2\theta} + 1 = \sec^2\theta$

$LS = \frac{1}{\cot^2\theta} + 1$
$= \tan^2\theta + 1$
$= \sec^2\theta$

$RS = \sec^2\theta$

$\because LS = RS$
$\therefore \frac{1}{\cot^2\theta} + 1 = \sec^2\theta$

15) $\cot^4\beta + \cot^2\beta = \csc^4\beta - \csc^2\beta$

$LS = \cot^4\beta + \cot^2\beta$
$= \cot^2\beta(\cot^2\beta+1)$
$= \cot^2\beta(\csc^2\beta)$
$= \cot^2\beta\csc^2\beta$

$RS = \csc^4\beta - \csc^2\beta$
$= \csc^2\beta(\csc^2\beta-1)$
$= \csc^2\beta(\cot^2\beta)$
$= \cot^2\beta\csc^2\beta$

$\because LS = RS$
$\therefore \cot^4\beta + \cot^2\beta = \csc^4\beta - \csc^2\beta$

16) $(\cot\theta - 1)^2 + \frac{2}{\tan\theta} = \csc^2\theta$

$LS = (\cot\theta - 1)^2 + \frac{2}{\tan\theta}$

$RS = \csc^2\theta$

$= \left(\frac{\cos\theta}{\sin\theta} - 1\right)^2 + 2\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \left(\frac{\cos\theta - \sin\theta}{\sin\theta}\right)^2 + \frac{2\cos\theta}{\sin\theta}$

$= \frac{(\cos\theta - \sin\theta)^2}{\sin^2\theta} + \frac{2\cos\theta}{\sin\theta}$

$= \frac{\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin\theta}$

$= \frac{1}{\sin^2\theta} - \frac{2\cos\theta}{\sin\theta} + \frac{2\cos\theta}{\sin\theta}$

$= \frac{1}{\sin^2\theta}$

$= \csc^2\theta$

$\because LS = RS$

$\therefore (\cot\theta - 1)^2 + \frac{2}{\tan\theta} = \csc^2\theta$

17) $\sin\beta\sqrt{1+\tan^2\beta} = \sqrt{\sec^2\beta - 1}$

$LS = \sin\beta\sqrt{1+\tan^2\beta}$

$= \sin\beta\sqrt{1 + \frac{\sin^2\beta}{\cos^2\beta}}$

$= \sin\beta\sqrt{\frac{\cos^2\beta + \sin^2\beta}{\cos^2\beta}}$

$= \sin\beta\sqrt{\frac{1}{\cos^2\beta}}$

$= \sin\beta\left(\frac{1}{\cos\beta}\right)$

$= \tan\beta$

$RS = \sqrt{\sec^2\beta - 1}$

$= \sqrt{\frac{1}{\cos^2\beta} - 1}$

$= \sqrt{\frac{1-\cos^2\beta}{\cos^2\beta}}$

$= \sqrt{\frac{\sin^2\beta}{\cos^2\beta}}$

$= \frac{\sin\beta}{\cos\beta}$

$= \tan\beta$

$\because LS = RS$

$\therefore \sin\beta\sqrt{1+\tan^2\beta} = \sqrt{\sec^2\beta - 1}$

18) $\sec^2\beta - 1 = \sin\beta\cot\beta\sec\beta\tan^2\beta$

$LS = \sec^2\beta - 1$
$= \tan^2\beta$

$RS = \sin\beta\cot\beta\sec\beta\tan^2\beta$
$= \sin\beta\sec\beta\tan\beta$
$= \frac{\sin\beta}{1}\cdot\frac{1}{\cos\beta}\cdot\tan\beta$
$= \frac{\sin\beta}{\cos\beta}\cdot\tan\beta$
$= \tan^2\beta$

$\because LS = RS$
$\therefore \sec^2\beta - 1 = \sin\beta\cot\beta\sec\beta\tan^2\beta$

19) $(1+\sin\theta)^2 + (1+\cos\theta)^2 - 2(\cos\theta + \sin\theta) = 3$

$LS = (1+\sin\theta)^2 + (1+\cos\theta)^2 - 2(\cos\theta + \sin\theta)$
$= (1 + 2\sin\theta + \sin^2\theta) + (1+\cos\theta)^2 - 2(\cos\theta + \sin\theta)$
$= 1 + 2\sin\theta + \sin^2\theta + (1+\cos\theta)^2 - 2\cos\theta - 2\sin\theta$
$= 1 + 2\sin\theta + \sin^2\theta + (1 + 2\cos\theta + \cos^2\theta) - 2\cos\theta - 2\sin\theta$
$= 2 + \sin^2\theta + \cos^2\theta$
$= 2 + 1$
$= 3$

$RS = 3$

$\because LS = RS$
$\therefore (1+\sin\theta)^2 + (1+\cos\theta)^2 - 2(\cos\theta + \sin\theta) = 3$

Logarithms

20) $\cot^2\alpha + \csc^2\beta = \cot^2\beta + \csc^2\alpha$

$LS = \cot^2\alpha + \csc^2\beta$

$= \dfrac{\cos^2\alpha}{\sin^2\alpha} + \dfrac{1}{\sin^2\beta}$

$= \dfrac{\cos^2\alpha\sin^2\beta + \sin^2\alpha}{\sin^2\alpha\sin^2\beta}$

$=$

$RS = \cot^2\beta + \csc^2\alpha$

$= \dfrac{\cos^2\beta}{\sin^2\beta} + \dfrac{1}{\sin^2\alpha}$

$= \dfrac{\cos^2\beta\sin^2\alpha + \sin^2\beta}{\sin^2\beta\sin^2\alpha}$

$=$

Not an identity ?