

Logarithms Exercise 2

1a) $3 \log_5 5$
 $= \log_5 5^3$
 $= \log_5 125$
 $= 3$

b) $\frac{\log_5 5 - \log_5 1}{\log_5 5 - \log_5 1}$
 $= \frac{\log_5 5 - \log_5 1}{\log_5 5 - \log_5 1}$
 $= \frac{1 - 0}{1 - 0}$
 $= \frac{1}{1}$
 $= 1$

c) $\log_2 1 - \log_2 1024$
 $= \log_2 1 - \log_2 2^{10}$
 $= \log_2 1 - \log_2 2^{10}$
 $= 0 - 10$
 $= -10$

d) $\log_6 4 + \log_6 9$
 $= \log_6 (4 \cdot 9)$
 $= \log_6 36$
 $= \log_6 6^2$
 $= 2$

e) $\log_5 1575 - \log_5 7$
 $= \log_5 (1575 \div 7)$
 $= \log_5 225$
 $= \log_5 15^2$
 $= 2$

f) $\log_6 25 + \log_6 16$
 $= \log_6 (25 \cdot 16)$
 $= \log_6 400$
 $= \log_6 10^4$
 $= 4$

g) $\log_6 14 + \log_6 15 - \log_6 35$
 $= \log_6 (14 \cdot 15) - \log_6 35$
 $= \log_6 210 - \log_6 35$
 $= \log_6 (210 \div 35)$
 $= \log_6 6$
 $= 1$

h) $\frac{\log_3 8 + \log_3 343}{\log_3 1083 - \log_3 3}$
 $= \frac{\log_3 (8 \cdot 343)}{\log_3 (1083 \div 3)}$
 $= \frac{\log_3 2744}{\log_3 361}$
 $= \frac{\log_3 14^3}{\log_3 19^2}$
 $= \frac{3}{2}$
 $= 1\frac{1}{2}$

i) $\log_2 7 + \log_2 27 - \log_2 125$
 $= \log_2 7 + \log_2 27 - \log_2 125$
 $= \log_2 (7 \cdot 27) - \log_2 125$
 $= \log_2 189 - \log_2 125$
 $= \log_2 \frac{189}{125}$
 $= \log_2 1.512$
 $= \log_2 2^{0.585}$
 $= 0.585$

j) $(\log_2 7)(\log_2 3)$
 $= \log_2 7 \times \log_2 3$
 $= 1$

k) $(2 \log_2 7)(\log_2 8 + \log_2 4)$
 $= (2 \log_2 7)(\log_2 (8 \cdot 4))$
 $= (2 \log_2 7)(\log_2 32)$
 $= \log_2 49 \times \log_2 32$
 $= \log_2 49 \times \log_2 2^5$
 $= \log_2 49 \times 5$
 $= 5 \log_2 49$
 $= 5 \log_2 7^2$
 $= 10 \log_2 7$
 $= 10$

l) $(\log_3 5)(\log_5 7)(\log_7 9)$
 $= \log_3 5 \times \log_5 7 \times \log_7 9$
 $= \log_3 9$
 $= \log_3 3^2$
 $= 2$

Logarithms Ex 2

2a) $\frac{10^{\log m^2} - 10^{2 \log m}}{m^{\log 100}} = (1-m)(1+m)$

LS = $\frac{10^{\log m^2} - 10^{2 \log m}}{m^{\log 100}}$
 $= \frac{10^{\log m^2} - 10^{\log m^2}}{m^{\log 100}}$
 $= \frac{m^2 - m^4}{m^2}$
 $= 1 - m^2$

RS = $(1-m)(1+m)$
 $= 1 + m - m - m^2$
 $= 1 - m^2$

$\therefore LS = RS$
 $\therefore \frac{10^{\log m^2} - 10^{2 \log m}}{m^{\log 100}} = (1-m)(1+m)$

b) $\log_3 \left(1 \frac{4}{11}\right) - \log_3 \left(\frac{2}{21}\right) - \log_3 \left(\frac{35}{66}\right) = 3$
 $LS = \log_3 \left(1 \frac{4}{11}\right) - \log_3 \left(\frac{2}{21}\right) - \log_3 \left(\frac{35}{66}\right)$
 $= \log_3 \left(1 \frac{4}{11} \div \frac{2}{21} \div \frac{35}{66}\right)$
 $= \log_3 27$
 $= \log_3 3^3$
 $= 3$

RS = 3

$\therefore LS = RS$
 $\therefore \log_3 \left(1 \frac{4}{11}\right) - \log_3 \left(\frac{2}{21}\right) - \log_3 \left(\frac{35}{66}\right) = 3$

c) $\log_2 9 = \frac{1}{\log_2 3}$

LS = $\log_2 9$
 $= \frac{\log_2 9}{\log_2 2}$

RS = $\frac{1}{\log_2 3}$

$\frac{1}{3}$

$\therefore LS \neq RS$
 $\therefore \log_2 9 \neq \frac{1}{\log_2 3}$

d) $\log_{2^{1000}} (2^{1000}) = 500$

LS = $\log_{2^{1000}} (2^{1000})$
 $= \frac{1000}{1000} \log_2 2$
 $= 1 \log_2 2$
 $= 1(1)$
 $= 1$

RS = 500

$\therefore LS \neq RS$
 $\therefore \log_{2^{1000}} (2^{1000}) \neq 500$

e) $m^{\log n} = n^{\log m}$

LS = $m^{\log n}$
 $= m^{\log n}$

RS = $m^{\frac{\log n}{\log m}}$
 $= (m^{\frac{1}{\log m}})^{\log n}$
 $= n^{\log m}$

$\therefore LS = RS$
 $\therefore m^{\log n} = n^{\log m}$

Logarithms Ex 2

3) $63 = 3^2 \cdot 7$

$\log 3 + \log 3 + \log 7$
 $= \log (3 \cdot 3 \cdot 7)$
 $= \log 63$

$m + m + n$
 $= 2m + n$

4) $\frac{\log(a \cdot b)}{\log(ab)}$
 $= \frac{\log a + \log b}{\log a + \log b}$
 $= \frac{2 + 2 + 2 + 3}{2 + 3 + 3 + 3}$
 $= \frac{11}{11}$
 $= 1$

5a) $\log_x 729 = 3$
 $x^3 = 729$
 $x = \sqrt[3]{729}$
 $x = 9$

b) $\log_x 729 = 2$
 $x^2 = 729$
 $x = \sqrt{729}$
 $x = 27$

c) $4^x = 13^x$

As x increases, the two values get farther apart so the only possible value of x will be 0.

$4^0 = 13^0$
 $1 = 1$

d) $7^x = 11^{x+2}$

$\frac{7^x}{11^x} = 121$

$\log_7 121 = x$

$x \approx -10.61$

Round to 3 decimal places

e) $6^x = 5^{2x+1}$

$x \log 6 = (2x+1)(\log 5)$

$\log 5 = x(\log 6 - 2 \log 5)$

$\log 5 = x$

$x \approx 1.126$

52

63

Logarithms Exercise 2

$$\begin{aligned} 1a) & 3 \log_7 5 \\ &= \log_7 5^3 \\ &= \log_7 125 \\ &= 125 \end{aligned}$$

$$\begin{aligned} b) & \frac{10^{4 \log 5} - \log 10^{27}}{\log_5 5 - \log_5 1} \\ &= \frac{10^{\log 625} - \log 10^{27}}{\log_5 5 - \log_5 1} \\ &= \frac{625 - 27}{1 - 0} \\ &= \frac{598}{1} \\ &= 598 \end{aligned}$$

$$\begin{aligned} c) & \frac{\log 1 - \log_3 1024}{5 \log_6 36} \\ &= \frac{\log 10^0 - \log_3 2^{10}}{\log_6 6^{\log_6 36}} \\ &= \frac{0 - 10}{10} \\ &= -1 \end{aligned}$$

$$\begin{aligned} d) & \log_6 4 + \log_6 9 \\ &= \log_6 (4 \cdot 9) \\ &= \log_6 36 \\ &= \log_6 6^2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} e) & \log_{15} 1575 - \log_{15} 7 \\ &= \log_{15} (1575 \div 7) \\ &= \log_{15} 225 \\ &= \log_{15} 15^2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} f) & \log 625 + \log 16 \\ &= \log (625 \cdot 16) \\ &= \log 10000 \\ &= \log 10^4 \\ &= 4 \end{aligned}$$

$$\begin{aligned} g) & \log_6 14 + \log_6 15 - \log_6 35 \\ &= \log_6 (14 \cdot 15) - \log_6 35 \\ &= \log_6 210 - \log_6 35 \\ &= \log_6 (210 \div 35) \\ &= \log_6 6 \\ &= 1 \end{aligned}$$

$$\begin{aligned} h) & \frac{\log_{11} 8 + \log_{11} 343}{\log_{11} 1083 - \log_{11} 3} \\ &= \frac{\log_{11} (8 \cdot 343)}{\log_{11} (1083 \div 3)} \\ &= \frac{\log_{11} 2744}{\log_{11} 361} \\ &= \frac{\log_{11} 14^3}{\log_{11} 19^2} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$

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