Extra Algebra Pratice

1a) $\frac{2}{3}(t+9) - \frac{5}{6}(t+7) = \frac{3}{4}(t+3) + \frac{1}{3}t$

$\frac{2t+18}{3} - \frac{5t+35}{6} = \frac{3t+9}{4} + \frac{t}{3}$

$(8t+72) - (10t+70) = (9t+27) + (4t)$

$8t + 72 - 10t - 70 = 9t + 27 + 4t$

$-15t = 25$

$t = -1\frac{2}{3}$

b) $\frac{1}{x+2} + \frac{2}{x-2} = \frac{3}{x+3}$ $\quad x \neq -2, 2, -3$

$\frac{1}{x+2} + \frac{2}{x-2} - \frac{3}{x+3} = 0$

$\frac{1(x-2)(x+3) + 2(x+2)(x+3) - 3(x+2)(x-2)}{(x+2)(x-2)(x+3)} = 0$

$1(x-2)(x+3) + 2(x+2)(x+3) - 3(x+2)(x-2) = 0$

$x^2 + 3x - 2x - 6 + 2x^2 + 6x + 4x + 12 - 3x + 12 = 0$

$11x + 18 = 0$

$11x = -18$

$x = -1\frac{7}{11}$

c) $\frac{m+3}{m-1} = 2 - \frac{m+1}{m-3}$ $\quad m \neq 1, 3$

$\frac{m+3}{m-1} + \frac{m+1}{m-3} = 2$

$\frac{(m+3)(m-3) + (m+1)(m-1)}{(m-1)(m-3)} = 2$

$\frac{2m^2 - 10}{(m-1)(m-3)} = 2$

$2m^2 - 10 = 2(m-1)(m-3)$

$2m^2 - 10 = 2m^2 - 6m - 2m + 6$

$-10 = -8m + 6$

$8m = 16$

$m = 2$

Lesson 3

Extra Algebra Practice

1d) $\frac{3x-2}{x-2} - 6 = \frac{x-5}{x-3}$

$\frac{3x-2}{x-2} - 6 - \frac{x-5}{x-3} = 0 \quad x \neq 2, 3$

$\frac{(3x-2)(x-3) - 6(x-2)(x-3) - (x-5)(x-2)}{(x-2)(x-3)} = 0$

$(3x-2)(x-3) - 6(x-2)(x-3) - (x-5)(x-2) = 0$

$3x^2 - 2x - 9x + 6 - 6x^2 + 18x + 12x - 36 - x^2 + 7x - 10 = 0$

$-4x^2 + 26x - 40 = 0$

$2x^2 - 13x + 20 = 0$

$(2x-5)(x-4) = 0$

$x_1 = 2\frac{1}{2}$

$x_2 = 4$

2a) $\frac{A}{x-1} + \frac{B}{x-2} = \frac{2x-5}{x^2-3x+2}$

$\frac{A(x-2) + B(x-1)}{(x-1)(x-2)} = \frac{2x-5}{(x-1)(x-2)}$

$A(x-2) + B(x-1) = 2x - 5$

$Ax - 2A + Bx - B = 2x - 5$

$A + B = 2$

$-2A - B = -5$

$-A = -3$

$A = 3$

$3 + B = 2$

$B = -1$

$\therefore A = 3, B = -1$

b) $\frac{A}{2x-1} - \frac{B}{4x+5} = \frac{14x+28}{8x^2+6x-5}$

$\frac{A(4x+5) - B(2x-1)}{(2x-1)(4x+5)} = \frac{14x+28}{(2x-1)(4x+5)}$

$A(4x+5) - B(2x-1) = 14x + 28$

$4Ax + 5A - 2Bx + B = 14x + 28$

$4A - 2B = 14$

$5A + B = 26$

$4A - 2B = 14$

$10A + 2B = 56$

$14A = 70$

$A = 5$

$4(5) - 2B = 14$

$20 - 2B = 14$

$-2B = -6$

$B = 3$

$\therefore A = 5, B = 3$

Extra Algebra Practice

3a) $\dfrac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}}$

$= \dfrac{1}{x^{-1}+y^{-1}}$

$= \dfrac{1}{\frac{1}{x}+\frac{1}{y}}$

$= \dfrac{1}{\frac{x+y}{xy}}$

$= \dfrac{xy}{x+y}$

b) $x^{-2}-3(x+1)x^{-3}$

$= \dfrac{1}{x^2} - \dfrac{3x+3}{x^3}$

$= \dfrac{x}{x^3} - \dfrac{3x+3}{x^3}$

$= \dfrac{x-3x+13}{x^3}$

$= -\dfrac{2x+3}{x^3}$

c) $x^{-1}+x^{-2}(1-x)$

$= \dfrac{1}{x} + \dfrac{1-x}{x^2}$

$= \dfrac{x}{x^2} + \dfrac{1-x}{x^2}$

$= \dfrac{x+1-x}{x^2}$

$= \dfrac{1}{x^2}$

d) $3x(x-3)^{-1} - 2x^2(x-3)^{-2}$

$\dfrac{3x}{x-3} - \dfrac{2x^2}{(x-3)^2}$

$= \dfrac{3(x-3)}{(x-3)^2} - \dfrac{2x^2}{(x-3)^2}$

$= \dfrac{3(x-3)-2x^2}{(x-3)^2}$

$= \dfrac{3x^2-9x-2x^2}{x^2-6x+9}$

$= \dfrac{x^2-9x}{x^2-6x+9}$

e) $(x+1)(x-1)^{-1} - 2(x+1)^2(x-1)^{-2}$

$= \dfrac{x+1}{x-1} - \dfrac{2(x+1)^2}{(x-1)^2}$

$= \dfrac{(x+1)(x-1)}{(x-1)^2} - \dfrac{2(x+1)^2}{(x-1)^2}$

$= \dfrac{(x+1)(x-1)-2(x+1)^2}{(x-1)^2}$

$= \dfrac{-x^2-3-4x}{x^2-2x-1}$

$= \dfrac{-x^2-4x-3}{x^2-2x-1}$

f) $\dfrac{3x^{-1}+2y^{-1}}{9x^{-2}+12x^{-1}y^{-1}+4y^{-2}}$

$= \dfrac{3x^{-1}+2y}{(3x^{-1}+2y)^2}$

$= \dfrac{1}{3x^{-1}+2y^{-1}}$

$= \dfrac{1}{\frac{3}{x}+\frac{2}{y}}$

$\rightarrow = \dfrac{1}{\frac{3y+2x}{xy}}$

$= \dfrac{xy}{3y+2x}$

g) $\frac{3-2x}{4\sqrt{x}} - 2\sqrt{x}$

$= \frac{3-2x}{4\sqrt{x}} - \frac{8x}{4\sqrt{x}}$

$= \frac{3-2x-8x}{4\sqrt{x}}$

$= \frac{3-10x}{4\sqrt{x}}$

$= \frac{\sqrt{x}(3-10x)}{4x}$

h) $\sqrt{5-3x} + \frac{x}{\sqrt{5-3x}}$

$= \frac{5-3x+x}{\sqrt{5-3x}}$

$= \frac{5-2x}{\sqrt{5-3x}}$

$= \frac{(5-2x)\sqrt{5-3x}}{5-3x}$

i) $\frac{3x^2}{\sqrt{x^2-2}} + \sqrt{x^2-2}$

$= \frac{3x^2+x^2-2}{\sqrt{x^2-2}}$

$= \frac{4x^2-2}{\sqrt{x^2-2}}$

$= \frac{(4x^2-2)2\sqrt{x^2-2}}{\sqrt{x^2-2}}$

j) $\frac{\sqrt{2+x} + \frac{3x}{4\sqrt{2+x}}}{2+x}$

$= \frac{\frac{4(2+x)+3x}{4\sqrt{2+x}}}{2+x}$

$= \frac{\frac{8+7x}{4\sqrt{2+x}}}{2+x}$

$= \frac{8+7x}{4\sqrt{2+x}(2+x)}$

$= \frac{(8+7x)\sqrt{2+x}}{4(2+x)^2}$

k) $\frac{(x+2)^{\frac{1}{2}} - (2x-1)(x+2)^{-\frac{1}{2}}}{x-3}$

$= \frac{(x+2)^{\frac{1}{2}} - (2x+1) \times \frac{1}{(x+2)^{\frac{1}{2}}}}{x-3}$

$= \frac{(x+2)^{\frac{1}{2}} + \frac{-2x+1}{(x+2)^2}}{x-3}$

$= \frac{-x+3}{\frac{(x+2)^{\frac{3}{2}}}{x-3}}$

$= -\frac{1}{(x+2)^{\frac{1}{2}}}$

$= -\frac{1}{\sqrt{x+2}}$

$= -\frac{\sqrt{x+2}}{x+2}$

Extra Algebra Practice

4a) $\frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$= \frac{\sqrt{6}+\sqrt{5}}{6-5}$

$= \frac{\sqrt{6}+\sqrt{5}}{1}$

$= \sqrt{6}+\sqrt{5}$

b) $\frac{1}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{\sqrt{3}+2}{3-4}$

$= \frac{\sqrt{3}+2}{-1}$

$= -\sqrt{3}-2$

c) $\frac{1}{q+\sqrt{q-2}} \times \frac{q-\sqrt{q-2}}{q-\sqrt{q-2}}$

$= \frac{q-\sqrt{q-2}}{q^2-(q-2)}$

$= \frac{q-\sqrt{q-2}}{q^2-q+2}$

d) $\frac{1}{2\sqrt{d}-\sqrt{f}} \times \frac{2\sqrt{d}+\sqrt{f}}{2\sqrt{d}+\sqrt{f}}$

$= \frac{2\sqrt{d}+\sqrt{f}}{4d-f}$

5a) $\frac{\sqrt{t-5}-1}{t-6} \times \frac{\sqrt{t-5}+1}{\sqrt{t-5}+1}$

$= \frac{t-5-1}{(t-6)(\sqrt{t-5}+1)}$

$= \frac{t-6}{(t-6)(\sqrt{t-5}+1)}$

$= \frac{1}{\sqrt{t-5}+1}$

b) $\frac{\sqrt{x+1}-1}{x} \times \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$

$= \frac{x+1-1}{x(\sqrt{x+1}+1)}$

$= \frac{x}{x(\sqrt{x+1}+1)}$

$= \frac{1}{\sqrt{x+1}+1}$

c) $\frac{\sqrt{3g+3}-3}{g-2} \times \frac{\sqrt{3g+3}+3}{\sqrt{3g+3}+3}$

$= \frac{3g-6}{g-2(\sqrt{3g+3}+3)}$

$= \frac{3(g-2)}{(g-2)(\sqrt{3g+3}+3)}$

$= \frac{3}{\sqrt{3g+3}+3}$

d) $\frac{\sqrt{4+h}-2}{h} \times \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \frac{4+h-4}{h(\sqrt{4+h}+2)}$

$= \frac{h}{h(\sqrt{4+h}+2)}$

$= \frac{1}{\sqrt{4+h}+2}$