Functions Problem Set

1) $f(x) = \dfrac{11-5x}{6x+1}$

$f(-1) = \dfrac{11-5(-1)}{6(-1)+1}$

$= -3.2$

2) $g(0) = f(-2)$

$f(-2) = 4 - (-2)^2$

$= 4 - 4$

$= 0$

$g(0) = 0$

3) $f(0) = 3 - 0$

$= 3$

$f(1) = 1^2 + 1$

$= 2$

$f(2) = 2^2 + 1$

$= 5$

$3 - 2 + 5 = 6$

4) $g(2-1) = \dfrac{2 - g(2)}{8}$

$g(1) = \dfrac{2 - g(2)}{8}$

$3 = \dfrac{2 - g(2)}{8}$

$24 = 2 - g(2)$

$g(2) = -22$

5) Let $x = 1$: $2 = m + b$ } solve system
   Let $x = 2$: $-1 = 2m + b$ } of linear eqns

$\rightarrow (1-1)g(1) + 2 = 2 = m + b$

$\rightarrow (2-2)f(2) - 1 = -1 = 2m + b$

$\begin{aligned} m + b &= 2 \\ 2m + b &= -1 \\ \hline -m &= 3 \\ m &= -3 \end{aligned}$

$\begin{cases} -3 + b = 2 \\ b = 5 \end{cases}$

6) Let $a = b = 5$

$F(2a-b) = F(a) \cdot F(b)$

$F(2(5)-5) = F(5) \cdot F(5)$

$f(5) = F(5)^2 \quad \neq 0$

Since the function cannot equal zero, the only other option is for $F(5)$ to equal 1

$\therefore f(5) = 1$

7)

Functions Problem Set

8) Let $x = 1$ ← sum of the coefficients is the same as the value for when $x = 1$

$$f(g(x)) = \frac{5 \cdot 1^4}{9} + \frac{17 \cdot 1^2}{3} + 21$$
$$= \frac{5}{9} + \frac{17}{3} + 21$$
$$= \frac{245}{9}$$

find x which in this ← case is $g(x)$

$$5x^2 - 3x + 7 = \frac{245}{9}$$
$$5x^2 - 3x - \frac{182}{9} = 0$$
$$45x^2 - 27x - 182 = 0$$
$$x = \frac{27 \pm \sqrt{27^2 - 4(45)(-182)}}{2(45)}$$

$$x_1 = \frac{7}{3}, \quad x_2 = -\frac{26}{15}$$
$$= 2\frac{1}{3}, \quad \quad = -1\frac{11}{15}$$

∴ the sum of the coefficients of $g(x)$ is either $2\frac{1}{3}$ or $-1\frac{11}{15}$

Functions Problem Set

9) $x \; f(2-x) - 2f(x) = x^2 + 1$ $f(x) = ?$

$(2-x) f(2 - (2-x)) - 2f(2-x) = (2-x)^2 + 1$
$(2-x) f(x) \qquad\qquad -2f(2-x) = x^2 - 4x + 5$

$x \, f(2-x) - 2f(x) = x^2 + 1$
$-2f(2-x) + (2-x)f(x) = x^2 - 4x + 5$
$\overline{2x \, f(2-x) - 4f(x) = 2x^2 + 2}$
$-2x f(2-x) + x(2-x)f(x) = x^3 - 4x^2 + 5x$
$\overline{(-x^2 + 2x - 4) \, f(x)} \qquad = x^3 - 2x^2 + 5x + 2$

$$f(x) = \frac{x^3 - 2x^2 + 5x + 2}{-x^2 + 2x - 4}$$

10a) Let $m = x-1$
Let $n = 1$

$f(mn) = f(m)f(n) - f(m+n) + 1001$

① $f((x-1)\cdot 1) = f(x-1) f(1) - f((x-1)+1) + 1001$

$f(x-1) = f(x-1) \cdot 2 - f(x-1+1) + 1001$

$f(x-1) = 2 \cdot f(x-1) - f(x) + 1001$

$f(x) = 2 \cdot f(x-1) + 1001 - f(x)$

② $f(x) = f(x-1) + 1001$

∴ equation ① is equal to equation ②

∴ $f(x) = f(x-1) + 1001$

b) $f(9999)$

$= f(1) + (9999-1) \cdot 1001$

$= 2 + 9998(1001)$

$= 10008000$